Vladimir Bolotnikov

The College of William and Mary

Beurling-Lax type theorems in weighted Bergman-Fock spaces

Abstract

Since the shift operator $M_z : f(z) \to zf(z)$ is an isometry on the $\mathcal{Y}$-valued Hardy space $H^2_{\mathcal{Y}}$ of the open unit disk, any M_z-invariant closed subspace $\mathcal{M} \subset H^2_{\mathcal{Y}}$ is generated by the wandering subspace $\mathcal{E} = \mathcal{M} \ominus z\mathcal{M} = P_{\mathcal{M}} z\mathcal{M}^{\perp}$. Furthermore, $z^k\mathcal{E} \perp z^\ell\mathcal{E}$ for $k \neq \ell$, and any wandering subspace has the form $\mathcal{E} = \Theta\mathcal{U}$ for some $\mathcal{L}(\mathcal{U}, \mathcal{Y})$-valued inner function Θ and an appropriate coefficient space $\mathcal{U}$, which in turn leads to the representations

$$\mathcal{M} = \bigoplus_{k\geq 0}(z^k\mathcal{M} \ominus z^{k+1}\mathcal{M}) = \bigoplus_{k\geq 0} z^k\mathcal{E} = \bigoplus_{k\geq 0} z^k(P_{\mathcal{M}} z\mathcal{M}^{\perp}) = \bigoplus_{k\geq 0} z^k\Theta\mathcal{U} = \Theta H^2_{\mathcal{U}}$$

for an M_z-invariant subspace $\mathcal{M} \subset H^2_{\mathcal{Y}}$. These equivalent representations display the Beurling-Lax theorem and admit extensions to the noncommutative Fock space setting of formal power series in several non-commuting variables. We will discuss their possible extensions in the context of weighted Bergman-Fock spaces where they are produce several non-equivalent representations for closed subspaces invariant under multiplication by coordinate functions.

Talk time: 07/19/2016 3:30PM— 07/19/2016 3:50PM
Talk location: Crow 206

Special Session: Multivariable operator theory. Organized by H. Woerdeman.