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A New Extension of Wray-Agarwal Wall Distance Free Turbulence Model to Rough Wall Flows

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This paper provides a roughness correction to the latest version of Wall-Distance-Free Wray-Agarwal (WA) one equation turbulence model (WA2018). The results from WA 2018 rough wall model are compared to Spalart-Allmaras model and the previous version of WA roughness model (WA2017). The results from WA2018-Rough model for flow over a flat plate show substantial improvement from the previous version WA2017-Rough and a good agreement with a semi-empirical formula based on experimental results. For flow past a S809 airfoil with surface roughness, WA2018-Rough model performs quite well compared to SA-Rough model.

Nomenclature

AOA = angle of attack
C_f = skin friction coefficient
C_l = lift coefficient
k = turbulence kinetic energy
k_s = sand grain roughness height
l = length of the plate
Ma = Mach number
Re = Reynolds number
S = mean strain
u^+ = mean velocity normalized by the friction velocity
W = vorticity magnitude
y = Cartesian coordinate
κ = Karman constant
ν = kinematic viscosity
μ_t = turbulent eddy viscosity
ρ = density
ω = dissipation rate per unit turbulent kinetic energy

I. Introduction

Computational Fluid Dynamics (CFD) is widely utilized in aerospace , turbomachinery, automobiles and a multitude of industrial applications. The analysis of the effect of surface roughness due to manufacturing, erosion or cavitation is very important in the real-world applications since roughness can significantly affect the performance of industrial products. The accurate roughness modification to a turbulence model is especially important since they affect the computational simulation results of all industrial products influenced by fluid flow; these results are important in the design and optimization of products.

This paper extends the Wall-Distance-Free (WDF) one equation Wray-Agarwal (WA) model to rough wall flows. As shown by Han et al. [1], WA-WDF (WA2018) model has several advantages compared to WA2017 model [2]: (a) it is accurate and robust in nearly zero-strain rate flow field encountered in some applications and (b) the wall distance free nature of the WA model enhances its accuracy near curved surfaces [1]. Hence, to take advantage of WA2018 model, a new version WA model that includes the effect of surface roughness is developed in this paper. The validation and verification of WA2018-Rough includes two cases: (a) flow past a rough flat plate with various roughness heights and flow over rough S809 airfoil. It is shown that WA2018-Rough can accurately predict the flow past objects with surface roughness.

II. Wall Roughness Extension of Original Wray-Agarwal WA2017 Turbulence Model

A. The Original Model – WA2017

The original WA2017 turbulence model is also used in this study; it is the listed on the NASA Turbulence Modeling Resource (TMR) website [3]. The WA one-equation model solves for the variable $R = k/\omega$.

$$\frac{\partial R}{\partial t} + \frac{\partial u_j R}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\sigma_R R + \nu)\frac{\partial R}{\partial x_j}\right] + C_1 RS + f_1 C_{2kw}\frac{R}{S}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j} - (1 - f_1)C_{2k\varepsilon}R^2\left(\frac{\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}}{S^2}\right) \tag{1}$$

The turbulent eddy viscosity is given by:

$$\mu_t = \rho f_\mu R \tag{2}$$

where ρ is the density. S is the main strain given by:

$$S = \sqrt{2S_{ij}S_{ij}}\,,\ S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

To ensure there is no division by zero, S is bounded by:

$$S = \max(S, 10^{-16}s^{-1}) \tag{4}$$

The damping function f_μ is used to account wall for blocking:

$$f_\mu = \frac{\chi^3}{\chi^3 + C_\omega^3}\,, \quad \chi = \frac{R}{\nu} \tag{5}$$

The kinematic viscosity ν is defined as μ/ρ. The switching function f_1 is defined by:

$$f_1 = \min(\tanh(arg_1^4), 0.9)\,, \quad \arg_1 = \frac{1 + \frac{d\sqrt{RS}}{\nu}}{1 + \left[\frac{\max(d\sqrt{RS}, 1.5R)}{20\nu}\right]^2} \tag{6}$$

where d is the minimum distance to the nearest wall. The constants are defined as:

$$\begin{gathered} C_{1k\omega} = 0.0829, \quad C_{1k\varepsilon} = 0.1127 \\ C_1 = f_1(C_{1k\omega} - C_{1k\varepsilon}) + C_{1k\varepsilon} \\ \sigma_{kw} = 0.72, \quad \sigma_{k\varepsilon} = 1.0 \\ \sigma_R = f_1(\sigma_{kw} - \sigma_{k\varepsilon}) + \sigma_{k\varepsilon} \\ C_{2k\omega} = \frac{C_{1k\omega}}{\kappa^2} + \sigma_{kw}, \quad C_{2k\varepsilon} = \frac{C_{1k\varepsilon}}{\kappa^2} + \sigma_{k\varepsilon} \\ \kappa = 0.41, \quad C_\omega = 8.54 \end{gathered} \tag{7}$$

B. Modified Roughness Version of WA2017 Model

Nikuradse has shown that the idealized physical roughness can be represented by the equivalent sand grain approach with empirical correlations [4]. The basic idea to get the roughness effect is to increase the eddy viscosity as a function of the roughness height near the wall. The velocity will have a shift in boundary layer under fully rough condition. The velocity profile is given by:

$$u^{+} = \frac{1}{\kappa} \ln \frac{y}{k_s} + 8.5 \tag{8}$$

The WA2017-Rough model follows the approach of SA-Rough model. The wall distance d is replaced by d_{new} at all occurrences of the distance d in the original WA2017 model. d_{new} is given by:

$$d_{new} = d + 0.03 k_s \tag{9}$$

The viscous damping function, Eq. (5), must also be modified to get the accurate representation of viscous sublayer and buffer layer profiles. The modification is given by:

$$f_{\mu} = \frac{\chi^3}{\chi^3 + C_{\omega}^3}\ , \quad \chi = \frac{R}{\nu} + C_{r1} \frac{k_s}{d_{new}} \tag{10}$$

where$C_{r1} = 0.5$, and C_{ω} remains 8.54.

Since the modification of boundary condition does not give a large enough eddy viscosity near the wall, the coefficient $C_{2k\omega}$ of destruction term in $k - \omega$ is modified based on Wray and Agarwal's work [5]. It is given by:

$$(C_{2k\omega})_{new} = C_{2k\omega} \frac{d}{d_{new}} \tag{11}$$

Eq (16) is used to replace the $C_{2k\omega}$ coefficient in the original WA equation in Eq. (1).

III. The New Wall Roughness Extension to Wray-Agarwal Turbulence Model - WA2018

A. The Original Wall Distance Free WA Model – WA2018

Based on Han et al.'s paper, WA2018 is similar to WA2017 model except for several additional terms [1]. In WA2018 model, Eq. (1) is modified to Eq. (12) shown below:

$$\begin{aligned} \frac{\partial R}{\partial t} + \frac{\partial u_j R}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\sigma_R R + \nu)\frac{\partial R}{\partial x_j}\right] + C_1 RS + f_1 C_{2kw} \frac{R}{S} \frac{\partial R}{\partial x_j} \frac{\partial S}{\partial x_j} \\ -(1 - f_1) min \left[C_{2k\omega} R^2 \left(\frac{\frac{\partial S}{\partial x_j} \frac{\partial S}{\partial x_j}}{S^2} \right), C_m \frac{\partial R}{\partial x_j} \frac{\partial R}{\partial x_j} \right] \end{aligned} \tag{12}$$

In WA2018 model, Eq. (6) is replaced by Eq. (13) shown below:

$$f_1 = \tanh(arg_1^4),\ \ \mathrm{arg}_1 = \frac{\nu + R}{2} \frac{\eta^2}{C_{\mu} k \omega} \tag{13}$$

and,

$$k = \frac{\nu_T S}{\sqrt{C_{\mu}}}$$

$$\omega = \frac{S}{\sqrt{C_\mu}}$$

$$\eta = S \max(1, \left|\frac{W}{S}\right|)$$

$$W = \sqrt{2W_{ij}W_{ij}}, \quad W_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i})$$

Most of constants are remain the same as in WA2017, except for the constants shown below:

$$C_{1k\varepsilon} = 0.1284$$

$$C_\mu = 0.09$$

$$C_m = 8.0$$

B. Modified Roughness Version WA2018 Model

The current version of roughness modification to WA2018 is shown below:

$$k_{new} = \frac{\nu_T S}{\sqrt{C_\mu}} Cr_1 \tag{14}$$

where $Cr_1 = \frac{1}{1+\frac{Uks}{\nu}}$. Note that the term $\frac{Uks}{\nu}$ is a non-dimensional roughness height such that if ks →0, then $Cr_1 \rightarrow 1$, and roughness k keeps the original form as in the WA2018 model. Obviously, Cr_1 is adapted to roughness condition; if the roughness height is infinitesimal, this roughness extension will perform as if the surface is smooth.

The boundary condition $R_{wall} = 0$ is replaced by an equation:

$$R_{wall} = 18133k_s^3 - 58.4k_s^2 + 0.0999k_s + 0.0000354 \tag{15}$$

Note that Eq. (15) should be set at a fixed value on the boundary after substituting the value of k_s.

IV. Tests Cases and Results

The grids are generated using ANSYS ICEM. The maximum *y*+ is less than 1 to ensure that the near wall treatment for both WA and SA models is accurate. For the flat plate case, an alternative mesh from the NASA TMR website [3] was also employed to compare the results on several meshes. The simulations were conducted using the open source software OpenFOAM v3.1.0.

A. Flow past a 2D flat plate in Zero pressure gradient

This case is a 2D flat plate verification and validation test case from NASA Turbulence Modeling Resource (TMR) website [3]. Figure 1 shows the boundary conditions and Fig, 2 shows the mesh in the computational domain around the flat plate. In this case, a two-meter-long flat plate is employed. The Mach number Ma= 0.2 and Reynolds number at x=1m is $Re_L = 5 \times 10^6$. A velocity boundary condition of 66.3 m/s at inlet and pressure boundary condition at the outlet are used in this case.

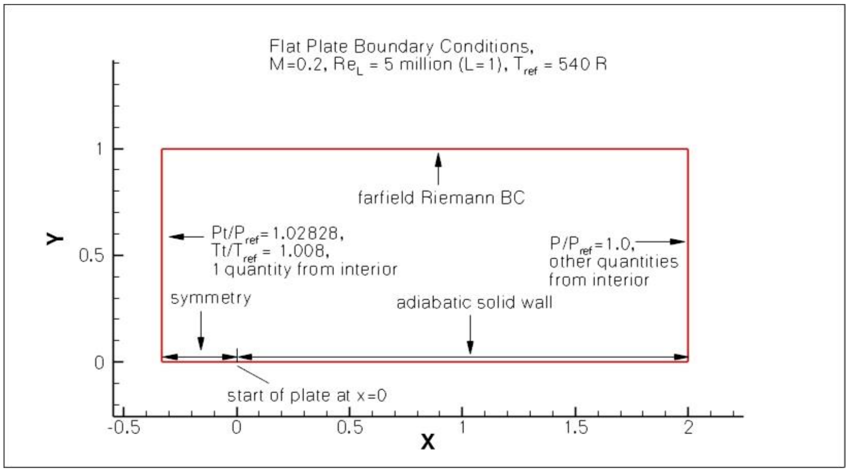

Fig. 1 Flat plate geometry and boundary conditions [3].

Since Spalart-Allmaras (SA) model is also one of the most widely used one equation turbulence model in aerodynamics, computations from WA –Rough model are also compared with SA-Rough model. The results from the two turbulence models are compared with a semi-empirical equation for the skin friction coefficient C_f on a rough flat plate. Based on Mills and Hang's work [6], the following equation is accurate within 1 percent of experimental values when $150 < x/k_s < 1.5 \times 10^7$:

$$C_f = \left(3.476 + 0.707 \ln \frac{x}{k_s}\right)^{-2.46} \tag{16}$$

Figure 3 shows the comparison of computed results obtained by WA2018 model, WA2017 model, SA model and Eq. (16). As the sand grain roughness height k_s increases, the error I results obtained from each model increases. When k_s is as small as 0.00025m, the flat plate has very small roughness, therefore the three turbulence models accurately predict the skin friction coefficient C_f. For $k_s = 0.0005$m, the SA model's predictions are more accurate compared to those from WA2017 and WA2018 models at the leading edge, especially in the range $0 \le x \le 0.4$m. When $x > 0.4$m, the two WA models show a better agreement with Eq. (16). For $k_s = 0.0010$m, WA2018 model shows the best agreement among the three models, while the SA model still has a better agreement in a very limited range near the leading edge ($x \le 0.4$m). At this high level of roughness, it is obvious that WA2017 model has the same result as WA2018 model in the range $x \le 0.4$m, and the similar result as SA model in the range $0.4 < x \le 2$, which makes the performance of WA2017 model the worst among the tree models. For $k_s = 0.0015$m, WA2018 model gives good result near the leading edge, and has the best agreement near the trailing edge. The overall results from WA2018 model are most accurate compared to the results from WA2017 and SA models.

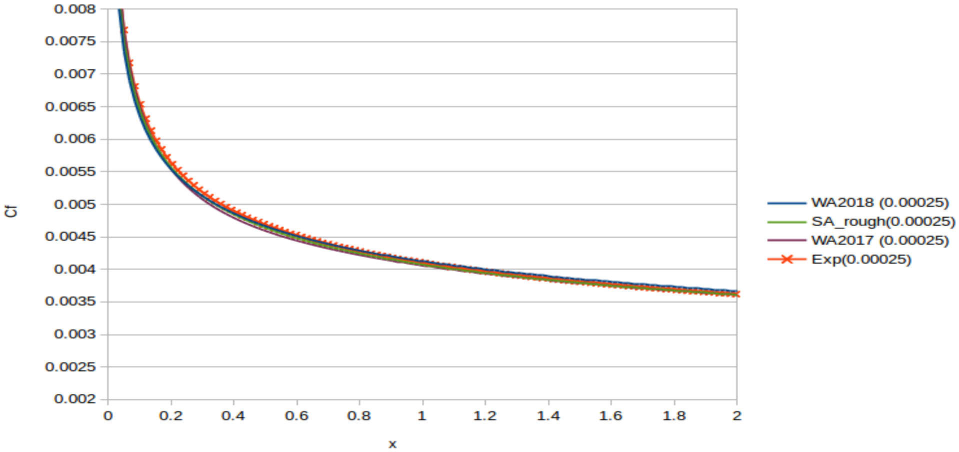

Fig. 2(a) Comparison of C_f for $k_s = 0.00025$.

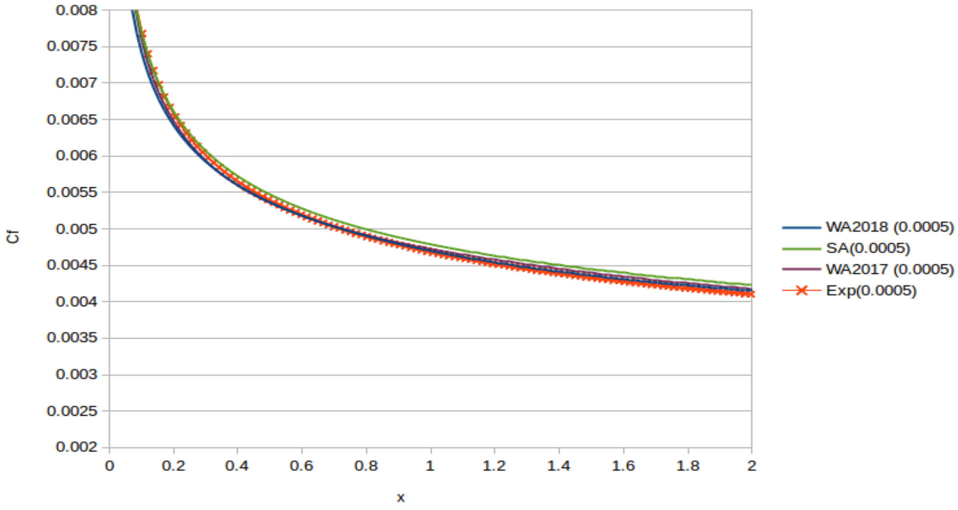

Fig. 2(b) Comparison of C_f for $k_s = 0.0005$.

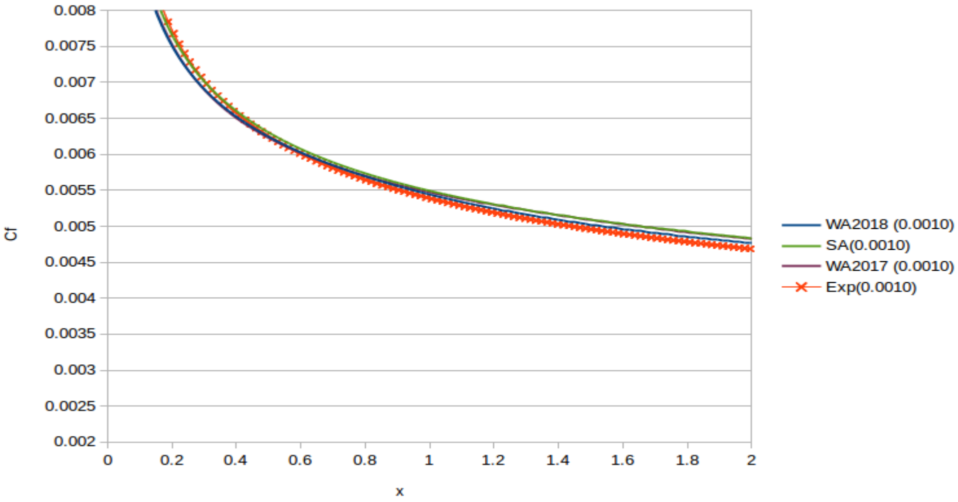

Fig. 2(c) Comparison of C_f for $k_s = 0.0010$.

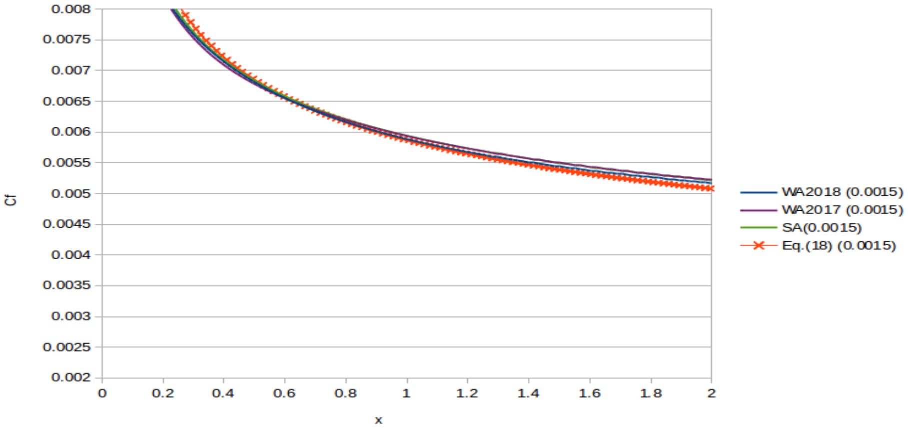

Fig. 2(d) Comparison C_f for $k_s = 0.0015$.

Fig. 2 Comparison of C_f for three turbulence models with roughness.

B. flow past a Rough S809 Airfoil

The second verification and validation case is that of flow over a rough S809 airfoil, which is commonly used on wind turbine blades. The working environment for a wind turbine may be harsh, and as a consequence the surface of the turbine blades may become rough due to erosion, sand grits and cavitation. The computation results are compared using the SA model, WA2018 model, WA2017 model and the experimental data collected by Ramsay of Ohio State University [7]. In this case, the chord length Reynolds number (x=c) is 1 million. Based on Ramsay's work, the standard #40 lapidary grit is chosen to give a relationship between the roughness height and chord length of *k*/c=0.0019.

Figure 3 shows the comparison of experimental and computational data for rough S809 airfoil. The results using the three turbulence models depict very similar behavior for pressure coefficient prediction, showing a very small drop in Cp at the leading edge which may be improved by using a finer mesh or a better defined geometry of S809.

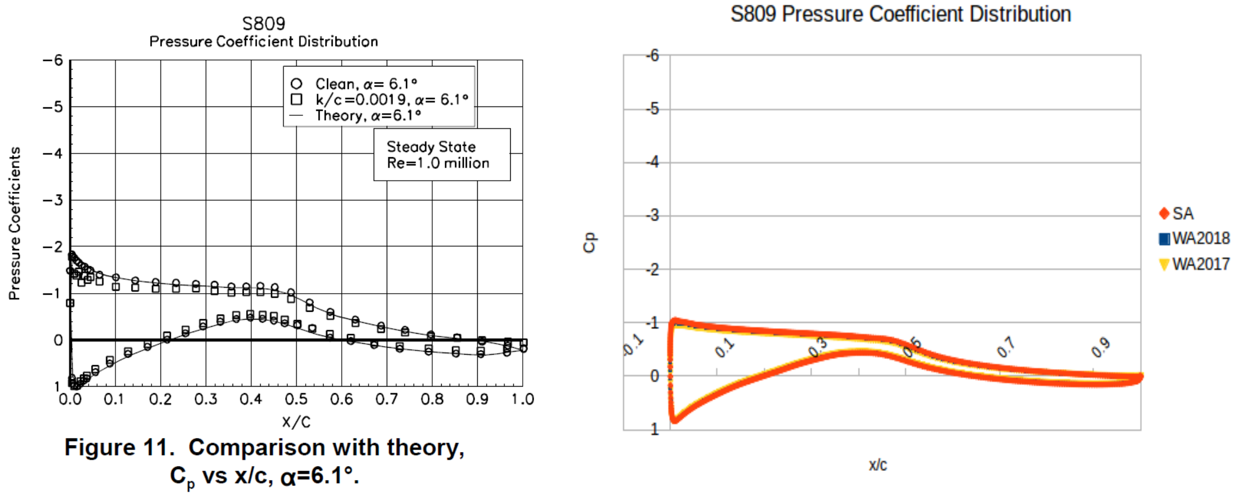

Figure 11. Comparison with theory, C_p vs x/c, α=6.1°.

Fig. 3 Comparison of computed and experimental Cp on rough S809 airfoil at α=6.1°.

Figure 4 shows the variation of computed lift coefficient with angle of attack for a smooth S809 airfoil and its comparison with experimental data. The results in Fig. 4 are quite reasonable since all of the three models show a quasi-linear relationship between the angles of attack (AOA) and lift coefficients when AOA is below 10°. White [8] has stated that an airfoil will have a stall when the AOA is about 10°, at which the flow separation occurs, and the theory fails to predict the lift coefficient. Figure 5 shows the variation of computed lift coefficient with angle of attack for a rough S809 airfoil and its comparison with experimental data. It can be seen that both WA2017 and WA2108 model with roughness fail to predict the data while the SA model performs reasonably well. According to Wray and Agarwal's work [5], the WA models require a laminar-turbulent transition model to predict the Cl for AOA > 8°. The overall performance of the three models for rough S809 case is not as good as for the smooth S809.

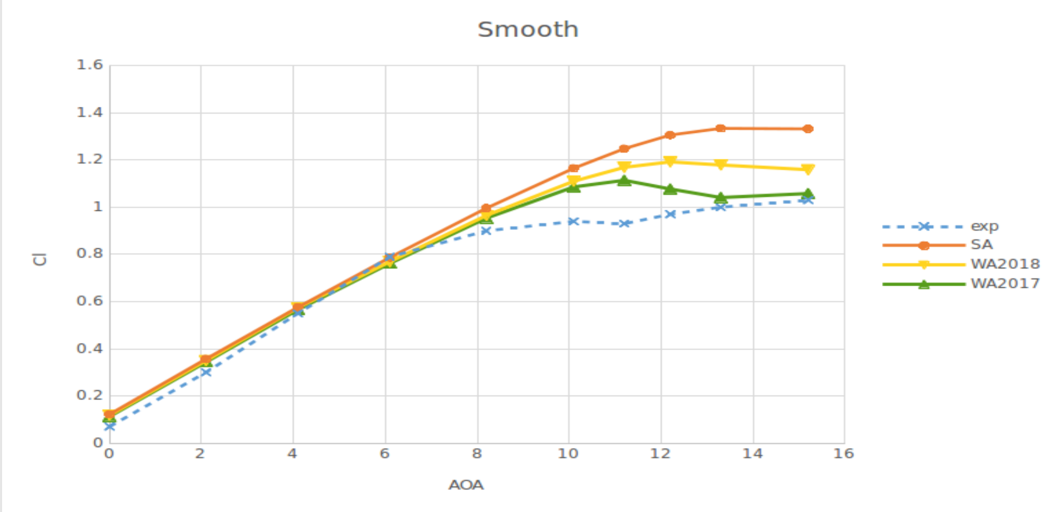

Fig. 4 Comparison of computations with three turbulence models and experimental data for smooth S809 airfoil.

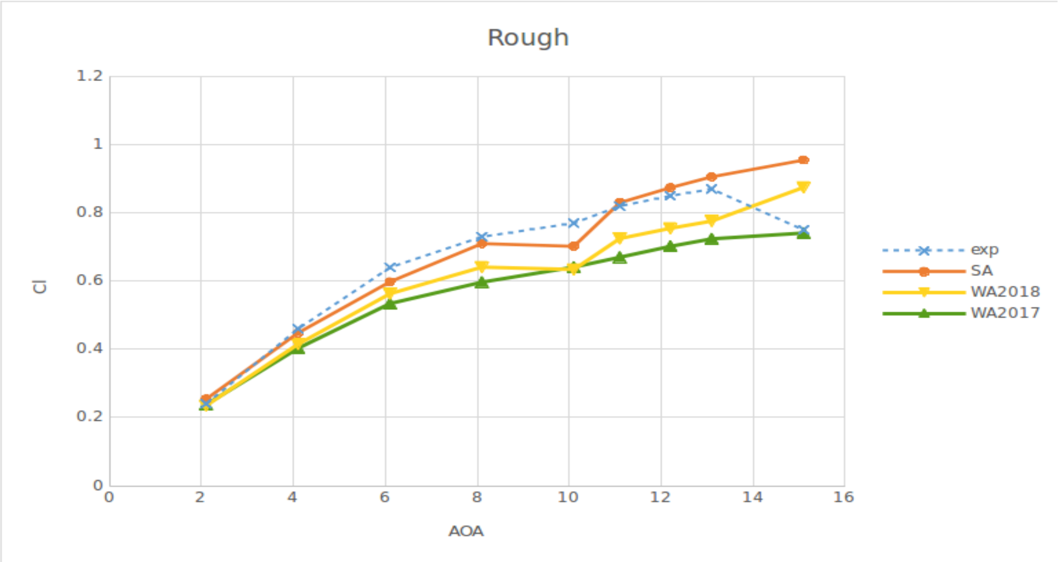

Fig. 5 Comparison of computations with three turbulence models and experimental data for rough S809 airfoil.

V. Conclusions

The roughness extensions for computing flows with surface roughness are developed for WA2017 and WA2018 models and are successfully applied for computing flow past a rough flat plate with varying roughness heights and flow past a S809 smooth and rough airfoil. The roughness extension for wall distance free version of Wray-Agarwal turbulence model (WA2018) gives better results compared to original WA2017 model. Overall, the new WA2018 performs even better than the SA model. The WA2018-Rough model can accurately predict the skin friction coefficient for any k_s in the range of $0.00025m \leq ks \leq 0.0015m$. For the S809 airfoil, SA-Rough model gives the best results compared to both WA2017 and WA2018 models. Since stall occurs when AOA is greater than 10°, WA model is unable to predict flow for AOA greater than 8°. A laminar to turbulent transitional flow model modification to WA model is needed to predict the flow separation on rough S809 airfoil. However, for small AOA below 8°, both WA2018 and SA models gives good results. The WA2018 model is an improvement over WA2017 model; it is demonstrated that WA2018-Rough model can be used to compute attached flows over objects with surface roughness quite accurately.

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