

Raul Curto

University of Iowa, USA

A New Necessary Condition for the Hyponormality of Toeplitz Operators on the Bergman Space

Abstract

It is a well known result of C. Cowen that, for a symbol $\varphi \in L^\infty$, $\varphi = \bar{f} + g$ ($f, g \in H^2$), the Toeplitz operator T_φ acting on the Hardy space of the unit circle is hyponormal if and only if $f = c + T_{\bar{h}}g$, for some $c \in \mathbb{C}$, $h \in H^\infty$, $\|h\|_\infty \leq 1$. In this talk we will consider possible versions of this result in the Bergman space case.

Concretely, we consider Toeplitz operators on the Bergman space of the unit disk, with symbols of the form

$$\varphi \equiv \alpha z^n + \beta z^m + \gamma \bar{z}^p + \delta \bar{z}^q,$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ and $m, n, p, q \in \mathbb{Z}_+$, $m < n$ and $p < q$. By letting T_φ act on vectors of the form

$$z^k + cz^\ell + dz^r \quad (k < \ell < r),$$

we study the asymptotic behavior of a suitable matrix of inner products, as $k \rightarrow \infty$. As a consequence, we obtain a rather sharp inequality involving the above mentioned data:

$$|\alpha|^2 n^2 + |\beta|^2 m^2 - |\gamma|^2 p^2 - |\delta|^2 q^2 \geq 2 |\bar{\alpha}\beta mn - \bar{\gamma}\delta pq|.$$

This result is intended to be a precursor of basic necessary conditions for joint hyponormality of commuting tuples of Toeplitz operators acting on Bergman spaces in several complex variables.

The talk is based on joint work with Željko Čučković (University of Toledo, USA).

Talk time: 07/19/2016 3:00PM— 07/19/2016 3:20PM

Talk location: Crow 206

Special Session: Multivariable operator theory. Organized by H. Woerdeman.