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# Lane Change Strategy for Autonomous Vehicle

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# Lane Change Strategy for Autonomous Vehicle

Ву

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## Abstract

Recently, people's demand for smart vehicles continues to improve. As the core of smart driving, driverless vehicle becomes the most concerned technology. Lane change, the most common behavior in driverless situation, greatly affect the road efficiency. Fast and safe lane change operations have very practical significance in reducing traffic accidents. This paper uses driverless vehicle as research object, and the pathing planning and pathing tracking for lane change situation are studied. An efficient path planning method and trajectory tracking controller are designed and simulated. The main content contains the three following aspects:

- (1) A set of comprehensive lane change strategy is designed for different working conditions. Then path planning for lane change is researched based on mass point model and an efficient path planning method based on polynomial is proposed and optimized.
- (2) Kinematic model and 3 DOFs dynamic model of driverless vehicle based on magic tire model are established using SIMULINK. Several simulation and test are done to verify the rationality of the model.
- (3) The trajectory -tracking control system based on PID controller is designed. Then run simulation based on the model established and according to the results, the trajectory -tracking control system can track the lane-changing path accurately and analysis is made.

#### Key word: Driverless vehicle, Lane change, Path planning, Trajectory tracking control

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# Chapter 1 Introduction

#### 1.1 Research background and significance

Vehicle is a machine that has changed the world and a machine with 100- year history. As a kind of travel tool, vehicle brings great convenience to people and help them save a lot of time. But while it is promoting the process of human civilization, the negative impacts that cars bring us are growing. The increase of vehicle ownership brings a large amount of traffic accidents, resulting in serious casualties and major economic loses. As an essential part of vehicle usage, driving is full of danger and tedious. Drivers need to pay attention to the surrounding environment and other vehicles all the time and then make corresponding decision. Usually the decision depends on the drivers' driving experience and how concentrated the drivers are. In fact, the drivers' judgement error is the main reason for the vast majority of accidents, and about half of these accidents are due to the drivers' reaction delay.

With the rapid development of advanced technologies such as artificial intelligence and pattern recognition, vehicle industry is undergoing dramatic changes. Car is no longer a mere mechanical structure. It corporates many advanced scientific research to improve the existing functions and increases the safety of the car. Some examples for this are assistance driving and ABS system. Through these technologies, vehicles are getting more and more intelligent. Driverless vehicles have become the technology of greatest concern as the core of smart vehicles. Because some features like automatic road identification, route design and vehicle body state adjustment free drivers from tedious driving operations and make driving safer and easier. According to HIS, a consultancy for automotive industry, the growth of autonomous vehicles is catching up with the electric vehicles, and the average family will be able to enjoy the safe and convenient driving experience by autonomous driving around 2025. Autonomous driving integrated the use of many advanced technologies<sup>[1]</sup>, such as active control, artificial intelligence and visual analysis, so it is a measure of state research and industrial manufacturing.

Obstacle avoidance is an important feature of autonomous driving and lane change is a frequent operation for autonomous vehicle. In the face of obstacles, the autonomous

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vehicle can obtain the obstacle and vehicle position information and select the best obstacle avoidance method and routes, then control the speed and steering flexibly to achieve a smoothie and safe driving. Therefore, in the process of obstacle avoidance, it is necessary to realize the lateral and longitudinal control at the same time. In addition, the smoothness, safety and speed of the obstacle avoidance process also need to be considered. It is reported that there are about 1.2 million deaths caused by man-made traffic accidents each year, most of them are caused by drivers' negligence and fatigue. Obstacle avoidance system can significantly reduce the drivers' sense of tension and then assist or replace the driver to take obstacle avoidance measures, thus minimize the casualties caused by the accident.

In the context of booming electric vehicle and autonomous driving technology, research on the obstacle avoidance strategy, especially for the lane change situation, has theoretical and practical significance for the future development of obstacle avoidance control system of self-driving vehicle.

#### 1.2 Driverless vehicle overview

The superior performance of driverless technology over traditional vehicle in terms of safety and accuracy has been fully demonstrated and its future development is also generally accepted. First, driverless vehicle can reduce the number of man-made accidents on a large scale, reduce the number of casualties and reduce the huge amount of medical compensation and handling costs. Statistics show that unmanned technology can reduce the number of traffic accidents by about 90%, and it can also significantly reduce the car's travel time and energy consumption, while reducing the number of vehicles. Besides, autonomous vehicles also have the function of traffic congestion avoidance, speed improvement and optimal path planning, which can greatly enhance the efficiency of the road.

Since the 1970s, some developed countries, such as the United States, Britain, Japan and Germany, have started their research on driverless vehicle. From IT companies to car manufactures, governments and organizations, it's a growing consensus that autonomous vehicles will be the future of the entire automotive industry. A lot of money

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and energy have been invested in related research and breakthroughs have been made both in terms of feasibility and practicality.

The United States has achieved a high-level research on the key technologies of unmanned vehicles. The most famous one is driverless car developed by Google, as shown in Figure 1.1. Google Driverless Car is a fully autonomous driving car developed by Google X Labs and it can be started, traveled and stopped without driver. This project is led by Sebastian Trong, the inventor of Google Street View and the director of the artificial intelligence lab of Stanford. These vehicles use cameras, radar sensors and laser range finders to "see" other traffic conditions and use detailed maps to navigate the road ahead. Just set the destination, the vehicle will automatically plan the optimal route. When the vehicle is traveling according to the planned route, it will upload the route and traffic to the central data processing center, which will further improve the map and make the route planning more accurate. Allegedly, recorded a total mileage data of Google driverless cars has reached 700,000 miles.



Fig 1.1 Google Driverless Car

As soon as the British government announced the new rules that UAVs would be legally available, the British company RDM Group could not wait to release the prototype Lutz Pathfinder, the country's first unmanned vehicle, as shown in Figure 1.2. Lutz Pathfinder is designed to help passengers, shoppers and the elderly travel short distances. It can only carry two passengers with top speed of 15mph and battery duration of 8 hours. The Mobile Robotics Group research team at Oxford University designed a full range of image capture systems for unmanned vehicles, including 22 sensors, laser imaging



components, panoramic cameras and radar positioning systems.

Fig 1.2 Lutz Pathfinder

France INRIA company spent ten years developing autonomous vehicle "Cycab", which looks like a future golf carts, as shown in Figure 1.3. Ordinary GPS system accuracy can only reach a few meters, while "Cyber" is equipped with a special GPS system called "real-time motion GPS", its accuracy is up to 1 cm. This driverless car is equipped with laser sensors that act as "eyes", which helps the car Avoid obstacle along the way. It is also equipped with twin cameras to follow signposts. People can even drive their cars through their cell phones and every unmanned vehicle can communicate with each other over the internet. This means that information sharing can be done between such driverless cars, so that multiple driverless cars can make up a team and run in small intervals. This car can also get real traffic information through the traffic network to prevent traffic jams and it will automatically issue a warning to remind past pedestrians' attention.



Fig 1.3 Cycab

In China, there are also several companies that are worth our attention. The most well-known one is Baidu, which has been developing driverless technology since 2013, shown in Fig 1.4. In December 2015, it announced an establishment of an autonomous driving business unit, and also announced a development plan of "three-year business, five-year volume production". In the round-trip test in April 2016, Baidu's self-driving vehicles successfully achieved the following complex driving actions such as deceleration, lane change, overtaking, up-and-down ramping, and turning around. It also completed the switching between different road scenarios like entering express way and driving out of express way. The maximum speed was up to 100km/h in the test.



Fig 1.4 Baidu autonomous vehicle

Unmanned driving technology is the ultimate manifestation of intelligence. With the continuous advancement of high technologies such as cloud computing, artificial intelligence, modern sensing, information fusion and automatic control, the future development speed of driverless car will accelerate.

#### 1.3 Key technologies of autonomous vehicle

#### 1.3.1 Environmental perception technology

Environmental perception technology is equivalent to the eyes and ears of unmanned vehicles. Driverless vehicles identify surrounding environmental information support for their decision making through this technology. Environmental perception includes the perception of the position and attitude of driverless cars and the perception of the surrounding environments. A single sensor can measure only one aspect of the measured object, which is unable to meet demands. Therefore, multiple sensors must be used to measure one or more features of a measured object at the same time. After the measured data is processed by computer, a useful signal is extracted.

The position and attitude information of the driverless vehicle mainly includes the speed, acceleration, inclination, position, etc. This type of information is easy to measure and is mainly measured with sensors such as drive motors, electronic compasses, tilt sensors and gyroscopes.

Perception of the surrounding environments is realized mainly by active ranging sensors such as radars, supplemented by passive ranging sensors. Because the combination of laser, radar, ultrasonic and other active ranging sensors can meet the needs of the task under complex and harsh conditions. Besides, the most important thing is good real-time performance.

#### 1.3.2 Navigation and positioning technology

The driverless car's navigation module is used to determine the location, which is the support for the mission and path planning of the unmanned vehicle. Navigation can be divided into autonomous navigation and network navigation.

Autonomous navigation technology can complete the navigation task only with the help of positioning assistance. Autonomous navigation technology stores geolocation data locally and all calculations are done at the terminal. This can be done in any situation, but the limited computing resources of autonomous navigation devices result in poor computing power and sometimes fail to provide accurate and real-time navigation services. Existing autonomous navigation can be divided into three categories: relative positioning, absolute positioning and combination positioning. Relative positioning mainly relies on the odometer, gyroscope and other internal sensors. It measures the displacement of unmanned vehicles relative to the initial position to determine the current location. Absolute positioning mainly uses navigation beacons, active or passive identification, map matching or global positioning system positioning. The combination positioning is the combination of the previous two, which can make up for the lack of

single method.

Network navigation can carry out information exchange anytime and anywhere through wireless network and traffic information center. Mobile devices are connected to a direct server via a mobile communication network. The server performs map storage and complex calculations and other function and users can download map data from the server. Advantage of network navigation is that there are no storage capacity constraints, strong computing power and it's able to store any elaborate map, and map is always upto-date.

#### 1.3.3 Path planning technology

Path planning is a bridge for information perception and intelligent control of driverless vehicles and it is also the basis for autonomous driving. The mission of path planning is to find a path from the initial state to the target state according to certain evaluation criteria in an environment with obstacles. Path planning can be divided into global path planning and local path planning. In the case of a known map, the global plan uses known local information such as obstacles and road boundaries to determine the feasible optimal route, which combines the optimization and feedback mechanisms well. Local planning is based on the driving area of global planning under the guidance of the sensor. It determines the driving trajectory of the road ahead that vehicle needs. Global planning applies to circumstances with known environment. Local planning applies to situations where environment is unknown.

#### 1.3.4 Control technology

The main function of the control module is to make the estimation based on the information obtained by the sensing system, and then make decisions on the next action and then control the vehicle. Control system constantly adjust the steering and wheel angle and speed based on the deviation between the current position and desired path until reaching the destination.

#### **1.4 Research content**

This paper uses autonomous vehicle as research object. The research studies path planning and trajectory tracking control in the obstacle avoidance conditions, especially lane changing obstacle avoidance process. The main research includes three aspects:

(1) A set of comprehensive obstacle avoidance strategy is designed for different working conditions. Safe distance between vehicle is studied to avoid accidents. Then path planning for lane change is researched based on mass point model and an efficient path planning method based on polynomial is proposed.

(2) Kinematic model and 3 DOFs dynamic model of driverless vehicle based on magic tire model are established using SIMULINK and MATLAB. Several simulations and tests are done to verify the rationality of the model.

(3) The trajectory -tracking control system based on PID controller is designed. Then run simulation based on the model established and according to the results, the trajectory -tracking control system can track the lane-changing path accurately and analysis is made.

#### Chapter 2 Lane Change Strategy Design and Path Planning

Obstacle avoidance, especially lane change, is a common driving behavior in driverless situations. The autonomous vehicle can select the optimal obstacle avoidance method based on the perceived obstacles and the location of the vehicle and control the speed and steering flexibly to achieve a safe and stable driving. Successful lane change is related to whether the lane change strategy is scientific. At present, path planning has been the hotspot of research, and many research results have been obtained. Some theories have been verified on real vehicles. However, in actual driving process, the existing optimization process of the lane change planning is complicated and not conductive to real-time calculation. Therefore, it is more practical to design an effective and flexibly calculated lane change trajectory planning method.

This chapter first introduces the characteristics of various lane changing process, and designs a comprehensive lane changing control strategy for different lane changing conditions encountered during driving. Then according to the specific lane change conditions, the path planning is studied.

#### 2.1 Lane change strategy design

According to the different lane changing environment, the lane change behavior can be divided into two situations, which are mandatory lane change and discretionary lane change. Mandatory is the normal driving process in which the vehicle has to change lane. The key point of such behavior is that there is a latest lane change point that the unmanned vehicle must complete lane change before travelling to that point, which usually occurs at intersections, lane merging, diversion and obstacles, as shown in Fig 2.1. The discretionary lane change is the lane changing behavior of unmanned vehicles, the purpose of which is to reach the driving target and surpass the front vehicle. This behavior is mainly to complete the purpose of driving to reduce driving time, which usually occurs in the situation where the vehicle behind travels faster than the front vehicle and the driving conditions in the adjacent lane are more superior.



Fig 2.1 Mandatory lane change

When vehicles make lane change decisions, there may be three kinds of situations: stop, follow and lane change. Stop is implemented by braking system to gradually decelerate the vehicle to a stationary state. Stop is a safe and effective method when there is a mandatory lane change for unmanned vehicles, but the actual situation cannot meet the necessary conditions for lane change. Following the front car means the car behind follow the front car to continue driving in the original lane. When autonomous vehicles are under discretionary lane change situation, but the reality cannot meet the necessary conditions for lane change, follow the car is a viable method. While driving continues, control system will maintain the original state and wait for the opportunity to change lanes. If there is enough lane change space on the target lane and the distance to the front car then the autonomous vehicle will be able to change lanes.

In this paper, a lane changing strategy is designed improve traffic efficiency and ensure safety. The flow chart of decision-making is shown in Figure 2.2. When a driverless vehicle encounters a mandatory lane change, the vehicle will change lanes if the target lane has sufficient lane change space, otherwise, the vehicle will only be able to stop and wait for the next lane change opportunity. When driverless vehicle change lane in order to avoid collisions with the front car or other driving purpose, which is discretionary lane change, if the target lane has enough lane space and the distance to the front vehicle is greater than the safe distance, then vehicle will change lane; otherwise, the vehicle needs to continue to follow the original lane until the new lane changing opportunities occur.



#### Fig 2.2 Decision making strategy

Autonomous vehicles make the path planning based on the decision of lane change strategy. Stop and following front car mainly based on the judgement of lane change conditions and lane space. When lane change is viable, a reasonable path will be planned.

#### 2.2 Lane change path planning

Lane change is a very common behavior. Smooth transition to an adjacent lane is a basic requirement of autonomous driving. Especially when vehicles encounter obstacles that block the original route, the importance of lane change movements is quite prominent. It is common sense that the lane changing movements are related to the continuity of autonomous driving. Choosing the appropriate lane change path is important for the smoothness of the transition.

A variety of reasonable lane change trajectories have been studied, refined and validated. Sine lane change trajectory design is simple and easy to adjust, but the initial lateral acceleration is not zero, which means a lateral load will be applied on passengers immediately. This kind of impact damages the travel experiences a lot. Although the arc trajectory minimizes the time consumption of the lane change process, but there is step change in the lateral acceleration, which will increase the difficulty of lateral control.

Trapezoidal acceleration lane change trajectory can continuously change, but it is necessary to set planning parameters, so the flexibility is poor. At present, the polynomial trajectory<sup>[2]</sup> used in the research of lane change does not have the defects above. The displacement, velocity and acceleration of the planned polynomial function are smooth. The path planning is carried out according to the actual driving conditions, and lane change is stable and easy to control. This section will design the path based on polynomial and analyze feasible methods for generating the path under obstacle constraints conditions.

#### 2.2.1 Path planning method based on polynomial function

Lane change diagram is shown in Figure 2.3, assuming the initial state of the vehicle at  $t_0$  is  $p_0 = [X_0, \dot{X}_0, \ddot{X}_0, Y_0, \dot{Y}_0, \ddot{Y}_0]$ . The six state variables indicate the longitudinal and lateral displacement, velocity and acceleration respectively. The status at end time  $t_f$  is then  $p_f = [X_f, \dot{X}_f, \ddot{X}_f, Y_f, \dot{Y}_f, \ddot{Y}_f]$ .



#### Fig 2.3 Lane change diagram

The initial state can be measured and estimated by various sensors, while the final state can be estimate based on the driving demand. From the vector, it is easy to know that the lateral and longitudinal states have 6 constraints respectively. Therefore, a quantic polynomial path with 6 parameters can be selected to define the longitudinal and lateral path planning and relation is given as follows<sup>[3]</sup>:

$$X(t) = \sum_{i=0}^{5} a_i \times t^i \tag{2-1}$$

$$Y(t) = \sum_{i=0}^{5} b_i \times t^i$$
<sup>(2-2)</sup>

Polynomial coefficient can be expressed in vectors:

$$\boldsymbol{a} = [a_0, a_1, a_2, a_3, a_4, a_5]^T, \quad \boldsymbol{b} = [b_0, b_1, b_2, b_3, b_4, b_5]^T$$
(2-3)

The vectors are calculated as follows:

$$\mathbf{a} = T^{-1} \cdot q_x, \quad \mathbf{b} = T^{-1} \cdot q_y \tag{2-4}$$

Where,

$$q_x = \left[ X_0, \dot{X}_0, \ddot{X}_0, X_f, \dot{X}_f, \ddot{X}_f \right]^T$$
(2-5)

$$q_{y} = \left[Y_{0}, \dot{Y}_{0}, \ddot{Y}_{0}, Y_{f}, \dot{Y}_{f}, \ddot{Y}_{f}\right]^{T}$$
(2-6)

$$\mathbf{T} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix}$$
(2-7)

If the time interval, the initial state and final state can be known, the polynomial coefficient can be calculated and then the ideal path can be obtained. Here's how to design a lane changing path using the above polynomial path planning approach.

To reduce the difficulty of the implementation of control, autonomous vehicles normally choose a constant speed when driving forward. Assume the car moves to position (0,0) at constant speed 10m/s at t<sub>0</sub>, and there is a static obstacle 50m in front of it. Then it is necessary to make the lane change. First, estimate the end position. The lateral end position is generally the width of the road, which is 3.5m. Longitudinal end position is then 50m. Since the constant speed can reduce the difficulty of the control of the autonomous vehicle, we try to make a constant speed planning in the longitudinal movement. Assume that the final longitudinal velocity is the same as the initial velocity. Both are 10m/s, then it takes 50/10=5s to accomplish the lane change process. Then we can get  $q_x = [0,10,0,50,10,0]$  and  $q_y = [0,0,0,3.5,0,0]$ . Using equation (2-4), we can get the longitudinal and lateral state as shown in Figure 2.4. It can be seen from the planning results that the path planned can reach the desired position accurately and the changes of speed and acceleration are relatively smooth, which is very beneficial to the control system for trajectory tracking.



Fig 2.4 Longitudinal and lateral state

As can be seen from Fig 2.4, the planned motion is longitudinal constant. This polynomial lane change, which is planned by selecting the appropriate lane change time and longitudinal displacement at the end, is called a longitudinal constant velocity polynomial lane change. This kind of lane change design can reduce the difficulty of longitudinal control, and the output of each lateral state quantity has regularity, which facilitates the rapid calculation.

#### 2.2.2 Constraints analysis and optimization

The research in this section mainly focuses on the situation with obstacles. The situation of discretionary lane change is relatively simple, so we will not discuss too much in this section. The purpose of the lane change is usually to avoid the obstacles ahead, but the lane change process may also lead to collision, therefore it is necessary to set reasonable constraints to limit the lane change process. Meanwhile, the dynamic characteristics of the vehicle must also be taken into account, which ensure the planned lane change trajectory is executable. Defining the appropriate conditions for obstacle avoidance is an important step in completing the lane change planning. The following sections define the external and internal constraints.



#### Fig 2.5 Constraint analysis

The collision that occurs during the lane changing mainly refers to the situation when the vehicle approaches the obstacle and fails to fully enter the adjacent lane, which results in contact and collision with the obstacle. In order to avoid collision, the first constraint can be defined as follows:

When the front end of car A reaches the end of the obstacle B, the lateral distance between the two should be greater than the width of the obstacle, otherwise collision occurs.

Because the general heading angle is relatively small, while the lateral displacement of the car is much smaller than the longitudinal displacement, it can be assumed that the angle between the car and longitudinal axis of the road is approximately zero. The diagram of critical collision is shown in Fig 2.5. After time t<sub>c</sub>, the longitudinal displacement of car A is L and the lateral displacement is  $y_c$ , and the front end of car A and end of obstacle are close to collinear state. Assume the width of the obstacle, which is another vehicle in most of the cases, is w. Then the first constraint can be represented by  $y_c > w$ . If the current speed is V, then the critical time t<sub>c</sub> can be decided by L/V.

In the case of unreasonable lane change route planning, it is possible that a collision may occur during the lane changing process. Based on the research on the polynomial function, the lateral displacement at same longitudinal displacement can be increased by reducing the longitudinal final displacement. In this way, collision will be avoided. On the basis of the planned routes in Fig 2.4, reduce the longitudinal displacement to 40m and the lane change time to 4s. Fig 2.6 shows the results of comparison of displacements before adjustment and after adjustment. Obviously, there is a significant increase in the lateral displacement at the same longitudinal displacement.



Fig 2.6 Before & after Comparison

But the adjustment also causes an increase in the maximum lateral acceleration. Too large lateral acceleration will result in more difficulty in control and bad riding comfort, so the second constraint is defined as follows:

The lateral acceleration of the vehicle should be controlled within a reasonable range. In this paper, the lateral acceleration  $a_y$  is between  $-2 \text{ m/s}^2$  and  $-2 \text{ m/s}^2$ .

The vehicle will have tendency to roll under the action of lateral acceleration, which not only affects the comfort of riding but also poses a great threat to driving safety. Take both comfort and safety into consideration, assume the maximum lateral acceleration is 2 m/s<sup>2</sup>. According to Fig 2.4, we find that the maximum lateral acceleration occurs at about a quarter of the time consumption of whole lane change process. According to the calculation, the error of this estimation is merely 2%. Thus, we can approximate that the lateral acceleration corresponding to the 1/4 lane change time is the maximum lateral acceleration. This method can improve the efficiency of calculation. When the maximum lateral acceleration exceeds the constraint limit, the lane change need to be adjusted. The maximum lateral acceleration can be reduced by increasing the longitudinal displacement in the end. Again, on the basis of planned route in Fig 2.4. increase the longitudinal displacement to 60m, Figure 2.7 shows the comparison result of before and after adjustment.

We can clearly see that increasing the lane change time can reduce the maximum lateral acceleration, but this will also reduce the lateral displacement

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Fig 2.7 Before & after Comparison

On one hand, we hope to reduce the final longitudinal displacement to avoid obstacles and enhance safety, on the other hand, we also hope to increase the final longitudinal displacement to reduce the maximum lateral acceleration and improve the riding comfort. So, we need to take both of them into consideration, select the best longitudinal displacement at the end, so that the lane change process is safe and comfortable, and of course the constraints discussed previously must be satisfied. Now we have an optimization problem<sup>[4]</sup> here.

The design variable of this problem is the final longitudinal displacement. Assume the distance between the vehicle and obstacle is d. Then usually the range of the final longitudinal displacement is [0.7d, 1.3d], which is the design space. Our object is to enhance the safety as well as riding comfort. Since we have two different objectives, we add them linearly with different weight to transfer the problem from a multiple objective problem to a single objective problem. The constraints are introduced previously. Use the MATLAB genetic algorithm toolbox to optimize the problem under different conditions, and the results are as follows:

Current speed	Distance to the obstacle	Optimal final longitudinal displacement	
10 m/s	30 m	32.28 m	
20 m/s	40 m	no optimal found	
30 m/s	80 m	94.15 m	

**Table 2.1 Optimization results** 

It can be noticed that no optimal final displacement can be found under some situations, because the speed is too high for the distance to the obstacle, which means only a little time is given to the lane change process and this will lead to a very high lateral acceleration which probably exceed 2m/s<sup>2</sup>. For situation like this, the sensor need to detect obstacles earlier and the brake system should engage to decelerate the vehicle in the lane change process, and this will not be discussed in this paper.

#### 2.3 Chapter summary

The chapter discussed the design of a comprehensive and safe lane change strategy. First, characteristics of several different operations such as stop, following and lane change are analyzed. Then according to the specific condition and space, lane change strategy will be selected flexibly.

When studying the path planning, the longitudinal velocity is assumed to be constant. Use the polynomial to design the path and it works pretty well. We also figure out that both maximum lateral acceleration and lateral displacement are related to the final longitudinal displacement. The relation is studied, and optimization is made to enhance the safety and riding comfort during the lane change process.

#### Chapter 3 Vehicle Dynamic and Kinematic Modeling and Simulation

As an extremely complex non-linear model, a car can be regarded as a system composed of a car body and four wheels. In the last chapter of study, we take the car as a mass point and we designed the lane change path by studying its longitudinal and lateral motion. However, in fact, in addition to the longitudinal and lateral translation, yaw, roll and pitch are all very significant movement of vehicle. In order to study the path design more accurately, we need to establish the dynamics and kinematic model<sup>[5]</sup>.

In this chapter, a three-DOF vehicle dynamic model, a magic tire model and a kinematic model are built according to the characteristics of the automobile, then simulation framework is set up in SIMULINK and simulation results are analyzed.

#### 3.1 Vehicle dynamic model

Many scholars have made a great deal of research on vehicle dynamic modeling, and the most commonly used one is three-DOF vehicle dynamic model with longitudinal velocity, lateral velocity and yaw velocity. This is exactly the model used in this chapter, the model is simple, so it can reduce computational complexity. It can also reflect the research purpose of this paper. To use this model, we make the following idealized assumptions:

(1) Ignore the steering linkage mechanism, take the steering angle of the front wheel as input directly.

(2) Assume the autonomous vehicle runs on a flat road and neglect the vertical movement of the vehicle

(3) Assume the suspension system and vehicle are rigid, regardless of the effect of the suspension movement.

(4) Ignore the impact of air resistance and rolling resistance of the tires

The model includes longitudinal, lateral and yaw motion of the body, which is shown in Fig 3.1 and can be described by the following equation:

$$\mathbf{m}\dot{V}_{x} = \mathbf{m}V_{y}\dot{\varphi} + F_{x1} + F_{x2} + F_{x3} + F_{x4} \tag{3-1}$$

$$m\dot{V}_{y} = -mV_{x}\dot{\phi} + F_{y1} + F_{y2} + F_{y3} + F_{y4}$$
(3-2)

$$I_{z}\ddot{\varphi} = (F_{y1} + F_{y2})b - (F_{y3} + F_{y4})a + (-F_{x1} + F_{x2} - F_{x3} + F_{x4})c$$
(3-3)



Fig 3.1 3-DOF vehicle dynamic model

Where, the  $V_x$  and  $V_y$  are the longitudinal and lateral velocity in the vehicle coordinate system respectively.  $V_x$  and  $V_y$  are the change rate of the longitudinal and lateral velocity rather than the acceleration in longitudinal and lateral direction, since the yaw angle is always changing. The longitudinal and lateral acceleration can be calculated using  $F_{xi}$  and  $F_{yi}$  (longitudinal and lateral force on tire, i=1,2,3,4) respectively, and the accelerations calculated are the components of absolute acceleration in the vehicle coordinate system. Please think carefully about this.  $\dot{\phi}$  and  $\ddot{\phi}$  are yaw velocity and yaw acceleration respectively. m is the total mass of the vehicle and  $I_z$  is the moment of inertia. a and b are distance from the centroid to front axle and rear axle respectively. c is the distance from the tire to the longitudinal axis of the vehicle. Longitudinal and lateral force ( $F_{xi}$  and  $F_{yi}$ ) on the tire can be calculated using tire longitudinal and lateral force ( $F_{li}$  and  $F_{ti}$ ) as well as steering angel  $\delta$ :

$$F_{r} = F_{l} \cos \delta - F_{t} \sin \delta \tag{3-4}$$

$$F_{v} = F_{l} \sin \delta + F_{t} \cos \delta \tag{3-5}$$

The tire longitudinal and lateral forces ( $F_l$  and  $F_t$ ) can be obtained according to the tire model described below.

#### 3.1.1 Tire model

The road will exert friction on the tire when vehicle is moving, while tire is contacting with the road. The force will be transmitted to the car body through tire. The interaction between the wheel and the road surface generates tire longitudinal force and tire lateral force respectively. The longitudinal force can realize braking and driving functions of the vehicle, while tire lateral force provides the steering function for the vehicle. Therefore, tire model is the key to vehicle dynamics analysis.

Tire models can be divided into three kinds: the theoretical tire model, the experience tire model and the physical tire model. The most widely used of the these is Magic Formula proposed by Pacejka. It can express the tire characteristics under different driving conditions, the formula parameters are fitted according to the experimental data, and the tire longitudinal force and lateral force are solved by the slip ratio, slip angle and vertical load on tire. The following formula can be used to represent the relation between the force on tire and the related variables:

$$F_l = f_l(\alpha, s, F_z), \tag{3-6}$$

$$F_t = f_t(\alpha, s, F_z) \tag{3-7}$$

Solving process is shown as below:



#### Fig 3.2 Relation between input and output

During driving, acceleration will change the center of gravity and the weight distribution, so vertical load of each tire can be expressed as follows:

$$F_{z1} = \frac{mga}{2(a+b)} - \frac{mh_{CG}a_x}{2(a+b)} - \frac{mh_{CG}a_y}{4c}$$
(3-8)

$$F_{z2} = \frac{mga}{2(a+b)} - \frac{mh_{CG}a_x}{2(a+b)} + \frac{mh_{CG}a_y}{4c}$$
(3-9)

$$F_{z3} = \frac{mgb}{2(a+b)} + \frac{mh_{CG}a_x}{2(a+b)} - \frac{mh_{CG}a_y}{4c}$$
(3-10)

$$F_{z4} = \frac{mgb}{2(a+b)} + \frac{mh_{CG}a_x}{2(a+b)} + \frac{mh_{CG}a_y}{4c}$$
(3-11)

Where the *m* is the mass of the vehicle.  $h_{CG}$  is the centroid height.  $a_x$  and  $a_y$  are the longitudinal and lateral acceleration in vehicle coordinate system respectively. Notice the  $a_x$  and  $a_y$  are calculated using vehicle longitudinal and lateral force ( $F_{xi}$  and  $F_{yi}$ ) as well as Newton's second law, however they are not  $V_x$  and  $V_y$ . Please think carefully about this.

The slip angle of each tire can be calculated using the formulas below:

$$\alpha_1 = \tan^{-1} \frac{\left(V_y + b\dot{\varphi}\right)}{\left(V_x - c\dot{\varphi}\right)} - \delta \tag{3-12}$$

$$\alpha_{2} = \tan^{-1} \frac{(V_{y} + b\dot{\phi})}{(V_{x} + c\dot{\phi})} - \delta$$
(3-13)

$$\alpha_3 = \tan^{-1} \frac{V_y - a\dot{\varphi}}{V_x - c\dot{\varphi}} \tag{3-14}$$

$$\alpha_4 = \tan^{-1} \frac{V_y - a\dot{\phi}}{V_x + c\dot{\phi}} \tag{3-15}$$

The slip ratio of each tire can be expressed as follows:

$$\mathbf{s} = \frac{\omega \cdot r}{v_l} - 1 \tag{3-16}$$

Where r is the effective radius of the tire, and  $\omega$  is the angular velocity of the wheel.  $v_l$  is the tire longitudinal velocity (not vehicle longitudinal velocity  $V_x$ ), and they are calculated as follows:

$$v_{l1} = (V_x - c\dot{\phi})\cos\delta + (V_y + b\dot{\phi})\sin\delta$$
(3-17)

$$v_{l2} = (V_x + c\dot{\phi})\cos\delta + (V_y + b\dot{\phi})\sin\delta$$
(3-18)

$$v_{l3} = V_x - c\dot{\varphi} \tag{3-19}$$

$$v_{l4} = V_x + c\dot{\varphi} \tag{3-20}$$

According to the fitting parameters of the magic tire model, there is a linear region in the relationship between the tire longitudinal force and slip ratio. And linear region also exists in the relationship between the lateral force and slip angle. Therefore, the force on tire can be represented using following formulas:

$$F_l = C_l \cdot s \quad , \quad F_t = C_t \cdot \alpha \tag{3-21}$$

Where  $C_l$  is the longitudinal stiffness and  $C_t$  is the lateral stiffness. Research has shown

that when the lateral acceleration is lower than 0.4g, the linear tire model has relatively good fitting accuracy. This model can also reduce computational complexity.

According to the mathematical model introduced earlier, we can build a model with SIMULINK and packaged then into different sub-systems. Assume the vehicle is moving on a horizontal dry cement road, so the friction coefficient is constant. Neglect the steering linkage mechanism, take the steering angle as input directly. The vehicle dynamic model is shown as follows:



Fig 3.3 Vehicle dynamic SIMULINK model

Before we set up the input signal and run the simulation, considering that we may

want to visualize how the vehicle move under the control of the steering input, we need to build another sub-system, which is associated with vehicle kinematic modeling.

#### 3.2 Vehicle kinematic model

When we want to take vehicle as an independent individual to carry out research, the vehicle coordinate system is generally used, just like what we do when we build the dynamic model. When it is necessary to consider the relative movement of the vehicle and surrounding objects, a global inertia coordinate is required. The kinematic model of a vehicle which is turning on a flat road is shown in the Fig 3.4.





Where (X(t), Y(t)) is the coordinate of the midpoint of the rear axle in the global coordinate system.  $\varphi$  is the yaw angle and  $\delta$  is the steering angle.  $V_x$  and  $V_y$  are the longitudinal and lateral velocity in the vehicle coordinate system respectively, just as introduced dynamic model earlier. Here we should notice that the vehicle coordinate system is not a coordinate system fixed on the vehicle body. It is, in essence, a static coordinate system aligned with the vehicle longitudinal and lateral direction at every

moment. Otherwise, the  $V_x$  and  $V_y$  will be both 0. 1 is the wheelbase of the vehicle. According to the geometric relation, the vehicle kinematic model can be expressed using the following formula:

$$X(t) = V_x \cos \varphi - V_y \sin \varphi \tag{3-22}$$

$$Y(t) = V_v \cos \varphi + V_x \sin \varphi \tag{3-23}$$

Build a kinematic model subsystem in SIMULINK then connect it with the dynamic model we build previously, which is shown in Fig 3.5.



Fig 3.5 Vehicle kinematic SIMULINK model

#### 3.3 Simulation and analysis

To verify the rationality of models built, we use different steering angle input to simulate and observe the rationality of the state variable and trajectories of the vehicle. In this section, we will run two simulations with different input steering angle signals, and then analyze the state variable output like yaw velocity, slip ratio, slip angle lateral acceleration and so on. Firstly, a ramp signal is used as input and then a sine wave signal. And in both simulation the vehicle velocity is 36km/h.

#### 3.3.1 Ramp input

The steering angle start with an initial value 0 which means no steering, and then it begins increasing at 1s, finally it reaches 3.438 degrees and then keeps constant. The input is shown as follow:



Vehicle state outputs are shown as follows:



As we can see, because the steering angle is 0 until 1s, the vehicle is moving in a straight line in fact. Correspondingly, the yaw velocity and lateral acceleration are both 0

until 1s. Once the vehicle begins turning, yaw velocity and lateral acceleration increases. Since the steering angle is positive, which means the vehicle turns left, the yaw velocity and lateral acceleration are both positive, which means the vehicle is rotates anticlockwise and the acceleration is to the left. When the steering angle reaches constant value, yaw velocity and lateral acceleration also reaches a constant, since it's a steady state then.



Since the vehicle moves in a straight line until 1s with constant speed, no acceleration causes the load redistribution. So, we can see same vertical load on tire 1 and 2, and same vertical load on tire 3 and 4. Once steering angle is positive, the vehicle begins turning left and there will be an acceleration to the left, which means the center of gravity of vehicle will move to the right. So, the load on the right two tires, 2 and 4, increases. The load on the left two tires, 1 and 3, decreases. Then all of them becomes steady state once



the steering angle reaches constant.

The motion of the four wheels are pure rolling until 1s, so the slip ratio is 0. When vehicle turns left, the right two wheels have a longer traveling curve than the left two wheels according to Ackerman steering geometry. We didn't build the model of differential in this paper, so the rotational speed of the left and right half shaft are same, which means the angular velocity of wheels cannot match the expected velocity for pure rolling motion. Wheels on the right has a higher velocity than expected which leads to the negatives slip ratio on tire 2 and 4. Wheels on the left has a smaller velocity than expected which leads to the positive slip ratio on tire 1 and 3. Then all of them becomes steady state once the steering angle reaches constant.



Slip angle of 4 tires are all 0 until 1s. When vehicle begins turning left, the slip angle becomes negative to produce positive tire lateral force(to the left) which provide the lateral acceleration to the left



The trajectory is shown by the kinematic model. There isn't steering angle until 1s so the vehicle moves straightly, then steering angle occurs and vehicle begins truing left. Finally, the steering angle reaches constant value and vehicle begins a circular motion just as expected.

#### 3.3.2 Sine input

The steering angle input is a sine wave with amplitude of 3.438 degrees and period of  $4\pi$ , which, in real world, is a driver turns left and right alternately. The input signal is shown as follow:



Vehicle state output shown as follows:



Both yaw velocity and lateral acceleration are sine wave with period of  $4\pi$  and their phase difference with steering angle input are 0. When steering angle is positive, which means vehicle turns left, both yaw velocity and lateral acceleration are positive, which means the vehicle is rotating anti-clockwise and accelerate to the left. Things are opposite when steering angle become negative.



All vertical load versus time of 4 tires are sine wave with period of  $4\pi$ . Vertical load on tire 2 and 4 have same phase as steering angle input, while tire 1 and 3 have phase difference of  $\pi$ . Because when vehicle turns left, centroid of gravity of vehicle moves to right. It moves to left when vehicle makes a right turn. So, the vertical load on tire 2 and 4 always have the same phase as steering angle and increases with the increase of steering angle, while load on tire 1 and 3 have an opposite phase and decrease with the increase of steering.



The slip ratio of each tire is sine wave with period of  $4\pi$ . Slip ratio of tire 1 and 3 have same phase as steering angle input, while tire 2 and 4 have phase difference of  $\pi$ . Because when steering angle is positive and vehicle turns left, tire 2 and 4 travel along a curve than tire 1 and 3, so expected velocity of tire 2 and 4 are higher, however tire 1 and 2 have same rotational speed, which leads to tire 1 and 3 rotate faster than expected and tire 2 and 4 rotate slower than expected. So, slip ratio is positive for tire 1 and 3 and it's negative for tire 2 and 4 when vehicle turns left. Things are opposite when steering angle

is negative. We can also see that the absolute value of slip ratio increases with increase of steering angle, this is because if steering angle increases, vehicle will make a sharper turn and the turning radius of vehicle will get smaller, and smaller turning radius will lead to a bigger difference between the expected velocity of tires on the right and tires on the left. Imagine the turning radius is infinitely long, the vehicle is then very much like moving straightly, and slip ratio is close to 0.



The slip angle of each tire is sine wave with period of  $4\pi$ . And they all have a difference of  $\pi$ . When vehicle turns left, negative slip angle will provide positive lateral force on tire, which will lead to positive lateral acceleration. Things are opposite when vehicle turns right.



Since the steering angle input is sine wave with 0 initial phase, so. in real world, the vehicle turns left and right alternately. A series curves are shown in the trajectory as expected. The vehicle moves at constant speed and steering is a sine wave, so the trajectory changes regularly.

#### 3.3.2 Conclusion

All the vehicle state variable output, including yaw velocity, lateral acceleration,

vertical load, slip ratio, slip angle and trajectory, are reasonable since their plot all perfectly follow the vehicle dynamic and kinematic theory and their magnitude are also reasonable.

#### 3.4 Chapter summary

In this chapter, a 3-DOF vehicle dynamic model and a kinematic model are built to have a more real research on more details of vehicle. The model is built in SIMULINK and packaged into different subsystems. Assume the vehicle moves on a horizontal dry cement road with constant speed, then simulation is done using two different steering angle inputs. All the results are perfectly reasonable which means our model is rational and can be used to study the controller in the next chapter.

# **Chapter 4 Trajectory Tracking Control**

After getting the feasible lane changing route based on polynomial, it is necessary to choose a control method to track the lane change path. PID controller is widely accept in many practical applications. It's independent of the established system model and the control parameter can be obtained by trial and error method.

#### 4.1 Trajectory tracking PID controller

Today's closed-loop automation technology is based on the concept of feedback to reduce the uncertainty. The elements of feedback theory include three parts: measurement, comparison and implementation. The key to the measurement is the difference between the actual value of the control variable and the expected value, which is used to correct the system response and perform regulation control. In engineering practice, the most widely used regulator control law is proportional, integral and differential control, known as PID regulation

PID controller is common feedback loop components in industrial control applications. It consists of proportional unit P, integral unit I and differential unit D. PID control is based on proportional control. Integral control eliminates steady-state errors but may increase overshoot. Differential control accelerates the response of large inertial systems and weakens overshoot trends.

When design the lane change path in chapter 2, we make vehicle move at constant speed to reduce the difficulty of control. According to the path planning based on mass point model, the longitudinal velocity is constant, there is no need to implement control in longitudinal direction. The only thing need to be done is to control the lateral



Fig 4.1 Structure diagram of control system

displacement to make it close to the expected path. The structure diagram of control system is shown in Fig 4.1.

According to the diagram of control system, build the controller using SIMULINK and connect it with the vehicle model established earlier, which is shown in Fig 4.2. Control parameter is obtained by trial and error method.



Fig 4.2 SIMULINK model of controller

#### 4.2 Simulation and result analysis

Create a m document using MATLAB, which can integrate all the work in this paper. Run the document and first the users will be asked to give two inputs, current speed of the vehicle and distance to the obstacle and here we assume this information are already obtained by sensors. Then the optimal final longitudinal displacement is calculated using genetic algorithm. Finally, the document will call the SIMULINK model we have built earlier and transfer the current speed to the vehicle model and transfer the optimal path to the controller. The controller will keep the vehicle moving along the path by adjusting the steering angle.

We assume the current speed of the vehicle is 54km/h and there is an obstacle 50m in front of it. Run the m document then a window for user input shows up. The pop-up window is shown in Fig 4.3.

承 i.	—		$\times$			
current speed:						
how far the obstacle is:						
	确	定	取消			

Fig 4.3 Pop-up window

Set the current speed at 15 m/s (54km/h) and distance at 50m then click "OK". The result are as follows:



According to the optimization, the optimal final longitudinal displacement is 53.38m. The actual trajectory and expected trajectory overlap each other, which means the controller can help the vehicle track the path accurately. Let's look at more details.



Actual lateral displacement, velocity and acceleration all overlap the expected curve. Final lateral displacement is exactly 3.5m and lateral acceleration is less than 2m/s<sup>2</sup>, which satisfy the constraint.



The actual longitudinal displacement overlaps the expected curve, but both velocity and acceleration have deviation. This probably because we assume the vehicle is moving with constant longitudinal velocity, so we didn't complement any control in that direction. Since the magnitude of deviation is relatively small, we can still say the control is result is good.



Steering angle is output of the controller as well as the input of the vehicle dynamic and kinematic model. The controller measures the difference between the expected displacement and actual displacement then adjust the steering angle to make the vehicle

moves along the expected path. So steering angle is the result that we are most interested in. As we can see, the steering angle that the controller adopts is very close to a sine wave, which means the vehicle turns left first and then right, just as expected. We also notice that the steering angle curve is quite smooth, which means the control process and the change of steering angle is continuous without sudden change. The steering angle worked out by the controller is quite feasible.

#### 4.3 Chapter summary

In this chapter, a controller based on PID regulation is introduced and designed. Then the controller is built in SIMULINK and connected with the vehicle model. A m document which integrate all the work earlier is created and simulation is run. The result shows that the controller controls the vehicle accurately to make it move along the expected trajectory. Displacement, velocity and acceleration in both lateral and longitudinal direction are as expected. The change of steering angle is continuous and smooth. All the result indicate that the controller works pretty well, which means the path designed is feasible.

# **Chapter 5 Summary and outlook**

#### 5.1 Summary

In this paper our study focuses on obstacle avoidance, especially lane change situation, for autonomous vehicle. First, a comprehensive obstacle avoidance strategy is studied and lane change path is designed based on mass point model and optimized. Then, to study more real detail of vehicle, we build a vehicle dynamic model as well as a kinematic model. The dynamic model is 3-DOF model, and it includes subsystem like magic tire model. Two tests is done and the rationality of the model is verified. Finally, a controller based on PID regulation is designed, and according to the simulation result, the controller works pretty well, and all the result are very close to expectation which means our path designed is feasible.

#### 5.2 Outlook

This paper proposes some feasible path planning and control methods for lane change problem of autonomous vehicle. However, there are still many aspects which needs improvement, which are as follows:

(1) The paper only considers the situation with one static obstacle and assume the other lane is clear and open. More complicated situation is not studied, so more research is necessary to complement the strategy and path design.

(2) The lane change trajectory is designed assuming the longitudinal speed is constant when vehicle changes lane. But we do encounter some situations where the longitudinal velocity control is necessary. For example, when we deal with the optimization, there are some results without optimal because the distance to the obstacle is too small for vehicle speed in that situation. Then we do need to implement longitudinal control to slow down the vehicle to make the lane change doable.

(3) All the work in this paper is limited to theory analysis and simulation. Many features like steering linkage mechanism, wind resistance and tire resistance are ignored. So, it is necessary to further carry out testing and other work on real vehicles. Only then, will this project have real application and promotion significance.

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