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JME 4110: Data Center Cooling

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At the current time there is a over reliance on digital data, algorithms and hardware. It became apparent that in order to decrease energy usage on reliable digital infrastructure that submerging the data centers in cooling dielectric fluid was the logical next step. The report presents the benefits of that next step to a large audience scientifically, technically as well as in report format.

JME 4110 Mechanical Engineering Design Project

Data Center Cooling

Brennan Fogarty
Christopher Schmidt
Aryal Suruchi
TABLE OF CONTENTS

1 Introduction 1
   1.1 Value proposition / project suggestion 1
   1.2 List of team members 1

2 Background Information Study 2
   2.1 22
   2.2 Background summary 2

3 Concept Design and Specification 3
   3.1 User Needs and Metrics 3
      3.1.1 Record of the user needs interview 3
      3.1.2 List of identified metrics 3
      3.1.3 Table/list of quantified needs equations 4
   3.2 Concept drawings 5
   3.3 A concept selection process. 7
      3.3.1 Concept scoring (not screening) 7
      3.3.2 Preliminary analysis of each concept’s physical feasibility 7
      3.3.3 Final summary statement 8
   3.4 Proposed performance measures for the design 8

4 Embodiment and fabrication plan 9
   4.1 Embodiment/Assembly drawing 9
      4.1.1 Initial Rough Sketch 9
      4.1.2 Initial CAD Drawings 10
   4.2 Parts List 12
   4.3 Draft detail drawings for each manufactured part 14
   4.4 Description of the design rationale 16

5 Engineering analysis 17
   5.1 Engineering analysis proposal 17
      5.1.1 Signed engineering analysis contract 17
   5.2 Engineering analysis results 17
5.2.1 Motivation
5.2.2 Summary statement of analysis done
5.2.3 Methodology
5.2.4 Results
5.2.5 Significance

6 Risk Assessment
6.1 Risk Identification
6.2 Risk Analysis
6.3 Risk Prioritization

7 Codes and Standards
7.1 Identification
7.2 Justification
7.3 Design Constraints
7.3.1 Functional
7.3.2 Safety
7.3.3 Quality
7.3.4 Manufacturing
7.3.5 Timing
7.3.6 Economic
7.3.7 Ergonomic
7.3.8 Ecological
7.3.9 Aesthetic
7.3.10 Life cycle
7.3.11 Legal
7.4 Significance

8 Working prototype
8.1 Prototype Photos
8.2 Working Prototype Video
8.3 Prototype components

9 Design documentation
9.1 Final Drawings and Documentation
9.1.1 Engineering Drawings
<table>
<thead>
<tr>
<th>Section Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1.2 Sourcing instructions</td>
<td>30</td>
</tr>
<tr>
<td>9.2 Final Presentation</td>
<td>31</td>
</tr>
<tr>
<td>Appendix A – Complete Engineering Drawings and Parts List</td>
<td>32</td>
</tr>
<tr>
<td>Appendix B – Calculations</td>
<td>37</td>
</tr>
<tr>
<td>Appendix C – Complete BOM for Data Center</td>
<td>53</td>
</tr>
<tr>
<td>Annotated Bibliography</td>
<td>54</td>
</tr>
</tbody>
</table>
1 INTRODUCTION

1.1 VALUE PROPOSITION / PROJECT SUGGESTION
A data center submerged into dielectric fluid so that heat can be transported more efficiently than air to air cooling is currently used. This method is theorized to make the data center operate on a third of the power needed to cool a comparable air to air cooled data center.

1.2 LIST OF TEAM MEMBERS
Brennan and Chris ended up on the same team because of similar work styles, in that they like to turn in good work but also have lots of other commitments. For Brennan, it’s a blossoming family and for Chris, it’s also a family with some civil service. Chris and Brennan also have a few years of history working with one another towards their degrees in the UMSL/WashU Joint Program. Suruchi joined the group when Chris and Brennan indicated they were open to adding a third member. She jumped at the opportunity to join a group that needed another person and has worked alongside her two other members in her past courses.

Chris and Brennan discussed the available projects prior to the first class period. They settled on a top three choices and project 5 - Data Center Cooling was the group’s second choice. By the time Suruchi was selected to pick a project on behalf of the group, the first choice was gone. Chris and Brennan confirmed that project 5 was something Suruchi would be willing to work on and she affirmed that it was. The group then selected project 5 – Data center Cooling.
2 BACKGROUND INFORMATION STUDY

2.1 DESIGN BRIEF
The group will design a way to efficiently move heat within a framework of using dielectric fluid as a heat transfer medium. The heat will be removed from a data center or a simulated data center. The group will evaluate the best possible design with the understanding that cooling data centers generate a large amount of heat. The design will also be constructed in such a manner that it can be scaled up to a larger size. Dielectric fluid will be used due to its efficiency compared to that of an air cooled system or a compressed refrigerant system. Account for safety factors to both the equipment and the environment while designing a system that addresses current design faults in data center cooling. Continue to be mindful of faults that may be encountered due to the proposed design as the group will be forced to design around not only engineering constraints but also resource constraints.

2.2 BACKGROUND SUMMARY
Our initial design intent centers on immersion cooling. In immersion cooling, the electronic components are immersed in a dielectric fluid that is readily accessible. The heat from the electronic components is transferred to the fluid. This has a great advantage from an efficiency standpoint when compared to traditional air cooling. It also has the potential to reduce the volume of space required for the data center.
3 CONCEPT DESIGN AND SPECIFICATION

3.1 USER NEEDS AND METRICS

3.1.1 Record of the user needs interview

What is the life expectancy of the system in order to determine quality of materials (Are you mining or using GPUs or servers)?
The system is expected to run daily for at least 5 years based on the warranty.

Is there any opposition to using Deionized water and Glycol mixture as the Dielectric fluid (this is to ensure value added performance and electronic reliability)?
There is opposition to using deionized water due to how corrosive it is to the electronics. The deionized water will damage the equipment so that it degrades sooner.

What is the T-initial and T-final (35 degrees or higher)?
To overclock the system provided it can be cooled, the initial temperature will be circa 120F, and the cooling needs to cool to at least 90 degrees Fahrenheit.

How many servers will you need to cool and what are their dimensions in the Conex (in order to accommodate total cooling load)?
The number of servers for this experiment will be 2 servers. The dimensions are listed as 28” by 16” by 10”. The total amount of cooling will be 3000 watts

Where will the Conex be located (both for construction and environmental considerations)?
The Conex will be stationary and located around accessible power lines.

Will conex be mobile (as it bears on HX placement)?
No. The Conex will be for all intents and purposes of this project stationary.

What will your maintenance schedule look like (weekly, biweekly, monthly or longer)?
The maintenance schedule will be bi-weekly to monthly maintenance on air filters. Need to ensure there are adequate safeties.

3.1.2 List of identified metrics

3.1.2.1 Reduced energy usage.

3.1.2.2 Submerged electronics into dielectric fluids.

3.1.2.3 Fits into a Conex
3.1.2.4 Easy to maintain

3.1.3 Table/list of quantified needs equations

<table>
<thead>
<tr>
<th>Power Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immersion Cooled</td>
</tr>
<tr>
<td>Dielectric Fluid</td>
</tr>
<tr>
<td>Noise Level (excluding exterior components)</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>Height</td>
</tr>
<tr>
<td>Standard Maintenance Time</td>
</tr>
<tr>
<td># of Major System Components</td>
</tr>
<tr>
<td>Cost</td>
</tr>
<tr>
<td>Assembly Time</td>
</tr>
</tbody>
</table>
3.2 CONCEPT DRAWINGS

Design 1: Fluid & Air absorption cooling

Need 1: Reduced energy use
Need 2: Submerged
Need 3: Dielectric Fluid
Need 4: Quiet running
Need 5: Fit in 20m3
Need 6: Easy maintenance

Design 2: Submerged 1 phase dielectric fluid
Design 3: Submerged 2 phase dielectric fluid

Design 4: Air to Air cooling tight envelope
3.3 CONCEPT SELECTION PROCESS.

3.3.1 Concept scoring (not screening)

<table>
<thead>
<tr>
<th>#</th>
<th>Need</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reduce/Minimize Energy Usage</td>
</tr>
<tr>
<td>2</td>
<td>Uses Immersion Cooling</td>
</tr>
<tr>
<td>3</td>
<td>Uses Dielectric Fluid</td>
</tr>
<tr>
<td>4</td>
<td>Runs Quietly</td>
</tr>
<tr>
<td>5</td>
<td>Fits in Connex</td>
</tr>
<tr>
<td>6</td>
<td>Ease of Maintenance</td>
</tr>
<tr>
<td>7</td>
<td>Limit the parts</td>
</tr>
<tr>
<td>8</td>
<td>Cost of entire system components</td>
</tr>
<tr>
<td>9</td>
<td>Time to assemble</td>
</tr>
</tbody>
</table>

3.3.2 Preliminary analysis of each concept's physical feasibility

During a meeting on 7/8/21, we discussed all five of our designs with the project sponsor, Mr. Molitor, and professor Giesmann. It was determined that the air to air heat exchanger and compressor would be unnecessary components and a radiator and fan combination would suffice. This decision was made following the concept design stage of this assignment but prior to the preliminary physical feasibility review.

As such, our initial designs will be discussed without these components.

i. There were no foreseen issues with this design.

ii. Obtaining standard dielectric fluid was seen as a potential obstacle but is believed possible.

iii. Obtaining two phase dielectric fluid was determined to be too difficult. This design was ruled out by the feasibility review due to timeframe.

iv. There were no foreseen issues with this design.
3.3.3 Final summary statement

The design with the most needs met, is design 2 and that can be seen under design 2 towards the center of the figure. Towards the lower portion of the figure you can see that design 2 got the most points on a scale of 0-1 with 0.7. Design 3 is the runner up and design 1 comes in 3rd. The second design was chosen to be modified without the air to air heat exchanger and compressor but replace them with a radiator and fan off of the dielectric fluid closed system. This design was discussed during the group’s 7/8/21 meeting as the best design to move forward with. The design passed the physical feasibility review and uses immersion cooling which was something both the group and the project sponsor wanted to implement.

3.4 PROPOSED PERFORMANCE MEASURES FOR THE DESIGN

The main performance measure was initially chosen to be energy usage. However, as the design process progressed the main performance measure was updated to be the change in temperature and the ability of the tank to maintain a temperature within the GPU operating range of 75-85 °C.
4 EMBODIMENT AND FABRICATION PLAN

4.1 EMBODIMENT/ASSEMBLY DRAWING

4.1.1 Initial Rough Sketch
4.1.2 Initial CAD Drawings

Assembly 3-View & Isometric

Exploded View without Tank
4.2 PARTS LIST

*The initial BOM with sourcing can be found in the above drawings. The final parts list is below and located in Appendix C.

<table>
<thead>
<tr>
<th>Item</th>
<th>Name</th>
<th>Quantity</th>
<th>Piece Cost</th>
<th>Total Cost</th>
<th>Website/Location &amp; Hyperlink</th>
<th>Scrounged Description</th>
<th>Lead Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fan/Air Movers</td>
<td>2</td>
<td>$99.00</td>
<td>$198.00</td>
<td>Lowes</td>
<td>Over the counter (OTC)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Radiator</td>
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<td>$125.70</td>
<td>Advance Auto Parts</td>
<td>Over the counter (OTC)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3/4in</td>
<td>Copper Pipe</td>
<td>5ft Long</td>
<td>1</td>
<td>$13.96</td>
<td>$13.96</td>
<td>Lowes</td>
</tr>
<tr>
<td>4</td>
<td>Radiator Hose</td>
<td>Outlet</td>
<td>1</td>
<td>$9.95</td>
<td>RockAuto.com</td>
<td>5 days</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Radiator Hose</td>
<td>Inlet</td>
<td>1</td>
<td>$3.49</td>
<td>RockAuto.com</td>
<td>5 days</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Hose Clamps</td>
<td>4</td>
<td>$1.29</td>
<td>$5.16</td>
<td>Home Depot</td>
<td>Over the counter (OTC)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Tank Seal - Inlet</td>
<td>1</td>
<td>$-</td>
<td>$-</td>
<td>3D Printed</td>
<td>3D printed on Formlabs Form 3 SLA printer with Flexible 80A resin</td>
<td>~2 hours print time</td>
</tr>
<tr>
<td>8</td>
<td>Tank Seal - Outlet</td>
<td>1</td>
<td>$-</td>
<td>$-</td>
<td>3D Printed</td>
<td>3D printed on Formlabs Form 3 SLA printer with Flexible 80A resin</td>
<td>~2 hours print time</td>
</tr>
<tr>
<td>9</td>
<td>3/4&quot; GHT to 1-1/4&quot; Radiator Hose</td>
<td>1</td>
<td>$-</td>
<td>$-</td>
<td>3D Printed</td>
<td>3D printed on Formlabs Form 3 SLA printer with Tough 1500 resin</td>
<td>~5 hours print time</td>
</tr>
<tr>
<td>10</td>
<td>3/4&quot; GHT to 1&quot; Pipe</td>
<td>1</td>
<td>$-</td>
<td>$-</td>
<td>3D Printed</td>
<td>3D printed on Formlabs Form 3 SLA printer with Tough 1500 resin</td>
<td>~5 hours print time</td>
</tr>
<tr>
<td>11</td>
<td>1in</td>
<td>Copper Pipe</td>
<td>5ft Long</td>
<td>1</td>
<td>$29.45</td>
<td>$29.45</td>
<td>Lowes</td>
</tr>
<tr>
<td>12</td>
<td>Pump</td>
<td>1</td>
<td>$98.00</td>
<td>$98.00</td>
<td>Home Depot</td>
<td>Over the counter (OTC)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>3/4in</td>
<td>90° Elbow Copper Fitting</td>
<td>2</td>
<td>$10.98</td>
<td>$21.96</td>
<td>Lowes</td>
<td>Over the counter (OTC)</td>
</tr>
<tr>
<td>14</td>
<td>1in</td>
<td>90° Elbow Copper Fitting</td>
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<td>$17.98</td>
<td>$35.96</td>
<td>Lowes</td>
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<td>15</td>
<td>Heater</td>
<td>1500 Watt</td>
<td>2</td>
<td>$43.99</td>
<td>$87.98</td>
<td>Amazon</td>
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<tr>
<td></td>
<td>Description</td>
<td>Qty</td>
<td>Unit Cost</td>
<td>Total Cost</td>
<td>Source</td>
<td>Delivery Time</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>------------------------------</td>
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<td>------------</td>
<td>--------------</td>
<td>---------------</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Strut Channel</td>
<td>10 ft</td>
<td>1</td>
<td>$22.58</td>
<td>$22.58</td>
<td>Lowes</td>
<td>Over the counter (OTC)</td>
</tr>
<tr>
<td>17</td>
<td>GPUs</td>
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<td>$-</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Tank Support Plank</td>
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<td>$-</td>
<td>$-</td>
<td>Scrounged</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Strapping</td>
<td>4</td>
<td>$-</td>
<td>$-</td>
<td>Scrounged</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>PMMA Sheet</td>
<td>48&quot; x 96&quot; x 1/8&quot;</td>
<td>1</td>
<td>$153.94</td>
<td>$153.94</td>
<td>McMaster-Carr</td>
<td>1 day</td>
</tr>
<tr>
<td>21</td>
<td>PVC 90° Angle</td>
<td>1-1/4&quot; x 1-1/4&quot; x 1/8&quot;</td>
<td>6</td>
<td>$12.62</td>
<td>$75.72</td>
<td>McMaster-Carr</td>
<td>1 day</td>
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<tr>
<td>22</td>
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<td>50 Pack</td>
<td>2</td>
<td>$8.28</td>
<td>$16.56</td>
<td>McMaster-Carr</td>
<td>1 day</td>
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<tr>
<td>23</td>
<td>18-8 Stainless Steel Hex Nut</td>
<td>100 Pack</td>
<td>1</td>
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<td>McMaster-Carr</td>
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<td></td>
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<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>$1,020.88</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.3 DRAFT DETAIL DRAWINGS FOR EACH MANUFACTURED PART

3/4" GHT TO RADIATOR HOSE

3/4" GHT TO 1" PIPE

NOTE:
PIPE ADAPTERS WERE PRINTED ON FORMLABS FORM 2 & FORMLABS FORM 3 SLA 3D PRINTERS USING TOUGH 1500 RESIN.

SECTION C-C

SECTION D-D

DETAIL E
SCALE 2:1

INLET SEAL

OUTLET SEAL

NOTE:
SEALS WERE PRINTED ON FORMLABS FORM 3 SLA 3D PRINTERS USING FLEXIBLE BPA RESIN.
4.4 DESCRIPTION OF THE DESIGN RATIONALE

The major design changes from initial to final involved the removal of branching of inlet and outlet pipes. This was done to simplify calculations, eliminate some of the possibility of error in calculations, and reduce the cost of additional fittings. Additionally, the 3D printed nozzle/diffuser parts that direct the flow through the GPUs were removed. This was done as the fluid was changed to water and the GPUs were simulated by two 1500W heaters. A more accurate representation of the pump and radiator were also modeled to show the physical parts purchased and more accurately reflect the size of the connecting fittings.

Some pipe dimensional changes were made due to the availability of scrounged fittings and parts for an intermediate build. Final parts were purchased unless explicitly stated.
5 ENGINEERING ANALYSIS

5.1 ENGINEERING ANALYSIS PROPOSAL

ANALYSIS TASKS AGREEMENT

PROJECT: Data Center   NAMES: Brennan Fogarty   INSTRUCTOR: Prof. Jakiela

Christopher Schmid
Suruchi Aryan

The following engineering analysis tasks will be performed:

1. Analysis of heat transfer and flow rate
2. Pipe flow analysis

The work will be divided among the group members in the following way:

1. Christopher | CS
2. Christopher, Suruchi, and Brennan | CS, BF, SA

Instructor signature: ____________________________  Print instructor name: Craig Giesmann

Instructor signature: ____________________________  Print instructor name: Mark Jakiela

(Group members should initial near their name above.)
5.2 ENGINEERING ANALYSIS RESULTS

5.2.1 Motivation
We will be looking into the rate of heat transfer within the data center so that we can better calculate the amount of heat that is being moved from the fluid being heated by the working electronics to the surrounding environment. Along the same line of analysis and covered under heat transfer will be flow rate analysis. Both these analyses will carry the project forward as they deal exclusively with how heat will be transferred both within and through the system.

5.2.2 Summary statement of analysis done
The heat transfer rate for what the user needed was 3000 watts. So in order to find the heat transfer rates’ minimum the experiment included calculations to determine the minimum mass flow rate of both fluid and air. Lastly for engineering analysis of the pipe flow the group will be looking at how the fluid moves through the system via the pipe and how this can contribute to major and minor flow losses.

5.2.3 Methodology
The analysis of heat transfer and flow rate was done using thermodynamic formulas for q and mass flow rate. The analysis of pipe flow was done using fluid dynamic formulas such as Darcy-Weisback formula, Reynolds number formula and summing major and minor losses. Please refer to the calculations for reference on formulas and to review calculations.

5.2.4 Results

5.2.4.1 The flow rate depends entirely on radiator size/efficiency; our flow needs to be 2 CFM of air and 0.2 GPM fluid thru pipes. *Apply specific heat changes for dielectric fluids.*

5.2.4.2 Head pressure was 14.24 ft of head loss. *Apply viscosity changes to account for the dielectric fluid pump needed.*

5.2.4.3 Conclusion: The radiator determines the bulk of calculations and the engineer should design around the radiator's limitations and strengths.

5.2.5 Significance
The results have influenced the final prototype in the materials we need to use. For the heat transfer we will need a motor strong enough to move the fluid more than 2 gallons a minute. And the significance of the pipe will be crucial in finding a pump large enough to overcome the head pressure caused by the small, planned diameter of the pipe, causing the pipe diameter to increase to lower head pressure.
6 RISK ASSESSMENT

6.1 RISK IDENTIFICATION

6.1.1 Traveling

6.1.2 Equipment Safety

6.1.3 Environmental

6.1.4 Horse Play

6.1.5 Lifting and Rigging

6.1.6 Weather

6.1.7 Electrical Safety

6.2 RISK ANALYSIS

6.3 RISK PRIORITIZATION:

As group members, what was discussed was the possible risks that could occur during our project. Also, as future engineers, we thought that safety should be the main risk and should be taken seriously. For that, we were cautious while handling chemicals and using electrical equipment. Also, following the codes and standards needed for the prototype.
7  CODES AND STANDARDS

7.1 IDENTIFICATION
It is a regionally adoptable standard for the safe installation of electrical wiring and equipment in the United States. It is part of the National Fire Code series. As we deal with wirings that is the reason we chose this code. It requires users to apply a permanent label to all service equipment rated 1,200 amps or higher.

7.2 JUSTIFICATION
This is significant to the group as we use these standards and codes because we are dealing with electrical equipment and also it is very necessary to conserve energy and follow precautions with using any form of energy. Though we are just building the prototype all the constraints like the power of the load and building space are hard to apply on our project. However, we tried to limit the maximum power in our prototype to be 3kw which is under 100kw. Moreover, this helps us build a safe device.

7.3 DESIGN CONSTRAINTS:
Location of the prototype was a major constraint when planning for this project. The system the way the user described required a lot of electricity that would need specially trained re-wiring. In addition, we in the group were spread across MO and IL and we were all performing this experiment during another covid outbreak which limited any in person contact. This influenced who had and made modifications to the design. Electrical safety was another main constraint we had to face for this project, as we were dealing with high energy electrical equipment but needed to use power tools to construct our prototype. The final constraint to be discussed in this review is that of resources. We had to buy components of major end items and fabricate them ourselves. An example would be the tank that was made for the prototype or the copper pipes that we used because they belonged to the program where other pipes may have been better to used due to adiabatic responses.

7.3.1 Functional
For the functional design the constraint that we had to work within was to cool through a process that is normally considered a pre-cooling process in HVAC. The best way to cool electronics is using a heat pump but it’s not efficient. Resources led to a redesign to only cool using a radiator and a fan.

7.3.2 Safety
For the design constraints as it pertains to safety the constraint that most concerned the group was the use of multiple 220VAC wires for each data center. This constraint made us change our design to simulate the electronics using heaters. Also since we could not get dielectric fluid in time we needed to use water and no electricity in the fluid.

7.3.3 Quality
The constraint when it came to quality is the same constraint that most engineers have in that there are never quite enough resources to do everything you design. Whether it be material or time the quality suffered from lack of resources and time.
7.3.4  **Manufacturing**  
The manufacturing design constraint that the group fell under was that of a design that could be scaled up to a larger size and number. That made the design more modular so that it could be moved, enlarged or modified for future use.

7.3.5  **Timing**  
The time constraint was the worst one during this experiment due to the fact that we all had other projects that needed our work along with the work for the design. This put a real strain on the project more than most constraints.

7.3.6  **Economic**  
The economic constraint was such that we knew any money we spent on this project would likely be used and not reimbursed. This has led to less quality on the prototype.

7.3.7  **Ergonomic**  
The ergonomic constraint was that the project would be a tank that was filled with liquid. Making the project a heavy carry when it was time to work on it or move it. This led to the design being plastic and steel reinforced with the option of adding handles to future versions.

7.3.8  **Ecological**  
The ecological constraints were that we used items that should not be thrown away in nature. Whether it was waste from the design or the design's product life cycle the group wanted to make the project as recyclable as possible.

7.3.9  **Aesthetic**  
The design constraints that the group suffered from was letting people see the design work in real time. In that sense the group member Brennan insisted that we build a tank from scratch out of acrylic that allowed the project to be seen in action.

7.3.10  **Life Cycle**  
Pertaining to the life cycle constraint the most important thing was the dielectric fluid. In that the lifecycle of the system relied entirely on fluid moving through the system to cool it. The fluid and the pump are the lifecycle that should be reviewed. If the fluid only lasts 3 months then that's the life cycle before return on investment has reached its conclusion as that is one of the most expensive costs to replace. The pump is cheap and replaceable and holes in the system can be patched.

7.3.11  **Legal**  
Legal constraints are what to do with the waste heat and to make it as safe as possible to prevent further engineering disasters that can result in displaced resources.
7.4 SIGNIFICANCE

7.4.1 Effects on the Group
In the future when the prototype begins to serve the heat transfer of the actual data center there will need to be proper labeling of the equipment and it will have to be installed and inspected by a licensed electrician. During that inspection the electrician will respond to all the legal regulations and to assist the prototype will have a shut-off breaker that is able to be locked out so as to perform maintenance on the system. It will also be located away from spaces that will need to be conditioned and the exhaust will go by the guidance laid out by ASHARE.

The design constraints have had the affect on delaying production of the prototype due to all the happiness factors that need to be met.
In this photograph, the tank is shown on the left side of the picture. Inside the tank are two 1500W heaters which simulate the GPUs, or data miners. The pipe exiting the tank on the left side is the hot fluid outlet. It goes around the backside of the tank and then enters the top left side of the radiator, cooling the fluid slightly along the pipe’s length. The fluid then flows through the radiator where the two fans blow cool air over the radiator fins. The fluid receives the majority of its cooling while flowing through the radiator. The heat is expelled into the environment surrounding the radiator. The cooled fluid exits the bottom right side of the radiator where it flows through an EPDM hose into the pump. Note the system behaves as a closed system. So, the pump simultaneously pulls the hot fluid through outlet pipe and radiator while pushing the cooled fluid from the radiator through the cooled inlet pipe back into the tank.
In this photograph, the system is shown from the hot water outlet side of the tank. The two heaters can be seen in the center of the tank. Also the temperature monitoring device is shown clamped to the tank (right side of the photo). The radiator is shown in the background along with the two fans assisting in heat transfer through forced convection.

8.2 WORKING PROTOTYPE VIDEO

The main performance measure of our system was the change in temperature and the ability to maintain a temperature below 90°C or 194°F. This performance measure was chosen as the maximum safe operating temperature of the data mining equipment considered for our project is 90°C. The two 1500W heaters shown are assumed to be equivalent to the two 1500W GPUs in terms of heat production.
Note: The video shows some leakage at some pipe unions. This is due to the inability to source fittings in the condensed time frame. As a substitute, 3D printed SLA fittings were created which did not seal completely. The minor leakage resulting from these components did not appear to significantly affect the heat transfer observed or inhibit the group from achieving success on the main performance measure.

8.3 PROTOTYPE COMPONENTS

PUMP JOINTS

In this photograph, the pump joints are shown. This joint was responsible for most of the leakage in the system. The joint is composed of a 1 ¼" EDPM hose coming from the radiator into a 3D printed SLA 1 ¼” to ¾” GHT adapter with a 3D printed rubber like gasket on the inside. The pump inlet side did not have any leakage at this joint. The pump outlet side, however, did have a moderate amount of leakage. It consisted of a 3D printed SLA ¾” GHT to 1” pipe adapter. On the 1” side the adapter had a slot for which the pipe was meant to slide into then be caulked to seal. Unfortunately this seal did not hold. However, the overall performance of the system was still able to be observed.
In this photograph, the tank is shown. The tank is responsible for holding the primary heat transfer medium, the fluid. In the prototype demonstration, the fluid used was water. As a result, the GPUs were simulated with the two 1500W heaters that can also be seen in the above photo.
Photograph number 3 showcases the two ¼ HP 925 CFM fans used to enable the heat transfer method of forced convection between the cool air and heated radiator fins. It should be noted that during the prototype testing the ambient air temperature of the room was also monitored and seen to have increased by nearly ten degrees. This is a good indication that a secondary application for data cooling could also include the heating of a room or rooms with the expelled heat from the system.
The final photograph shows a closer image of the radiator and the inlet and outlet pipes. The two EPDM hoses entering and leaving it can be seen to have hose locks where necessary. Additionally, a secondary port that could be used for draining or recycling cool water can be seen about halfway up the left side of the radiator. It too has a hose lock ensuring leakage will not occur.
9 DESIGN DOCUMENTATION

9.1 FINAL DRAWINGS AND DOCUMENTATION

9.1.1 Engineering Drawings

See Appendix A for the complete drawings.
### 9.1.2 Sourcing instructions

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9.2 FINAL PRESENTATION

File Locations of CAD drawings, videos and PPT: [https://gowustl-my.sharepoint.com/personal/bpfogarty_wustl_edu/_layouts/15/onedrive.aspx?id=%2Fpersonal%2Fbpfogarty%5Fwustl%5Fedu%2FDocuments%2FSenior%20Design%2FPublic%20Share&ct=1629167812873&or=OWA%2DNT&cid=f01f2cfa%2De685%2D25d6%2D69a3%2D9a1d72c55d6&originalPath=aHR0cHM6Ly9nb3d1c3RsLW15LnNoYXJlcG9pbnQuY29tLzpmOi9nL3BlcnNvbnJvc2FsdF91ZHUvRXFVjemJuVDEtYzI2b2pUVIVkM1F6SDhCaDVLbYkxoS2pISG1xVHYwUkRXRXg1Zz9ydGljZTI0dTlaNENkaDJVZw](https://gowustl-my.sharepoint.com/personal/bpfogarty_wustl_edu/_layouts/15/onedrive.aspx?id=%2Fpersonal%2Fbpfogarty%5Fwustl%5Fedu%2FDocuments%2FSenior%20Design%2FPublic%20Share&ct=1629167812873&or=OWA%2DNT&cid=f01f2cfa%2De685%2D25d6%2D69a3%2D9a1d72c55d6&originalPath=aHR0cHM6Ly9nb3d1c3RsLW15LnNoYXJlcG9pbnQuY29tLzpmOi9nL3BlcnNvbnJvc2FsdF91ZHUvRXFVjemJuVDEtYzI2b2pUVIVkM1F6SDhCaDVLbYkxoS2pISG1xVHYwUkRXRXg1Zz9ydGljZTI0dTlaNENkaDJVZw)

Link to online paper: [https://openscholarship.wustl.edu/cgi/preview.cgi?article=1170&context=mems411](https://openscholarship.wustl.edu/cgi/preview.cgi?article=1170&context=mems411)
APPENDIX A - COMPLETE ENGINEERING DRAWINGS
NOTE:
ALL INTERIOR TANK JOINTS ARE SEALED WITH SILICONE CAULK. PVC ANGLES ARE CAULKED IMMEDIATELY PRIOR TO ASSEMBLY. NUTS AND SEALS ARE CAULKED OVER AFTERTANK ASSEMBLY.
NOTE: THE TANK SUPPORT WAS MADE FROM SCRAGGED WOOD. THE STRAPPING WAS SCRAGGED AND FASTENED WITH MISCELLANEOUS FASTENERS (NOT SHOWN IN DRAWING).

3/4" COPPER PIPE

1" COPPER PIPE

PIPE CUT LIST

DATA COOLING CENTER
11  APPENDIX B – CALCULATIONS (FINAL DESIGN)

Assumptions and Constants:
The system is to be treated as a closed system. The system will be evaluated at the worst case scenario
temperatures for the following calculations.

Flow rate: \( Q = 5 \text{ gal/min} \times \frac{0.133681 \text{ ft}^3}{1 \text{ gal}} \times \frac{1 \text{ min}}{60 \text{ s}} = 0.01114 \text{ ft}^3/s \)

Pipe Inner Diameter Section 1: \( D_{1i} = 0.800 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.06667 \text{ ft} \)

Pipe Outer Diameter Section 1: \( D_{1o} = 0.880 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.07333 \text{ ft} \)

Pipe Inner Diameter Section 2: \( D_{2i} = 1.025 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.08542 \text{ ft} \)

Pipe Outer Diameter Section 2: \( D_{2o} = 1.125 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.09375 \text{ ft} \)

Radiator Hose Inside Diameter: \( D_{hose,i} = 1.1875 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.09896 \text{ ft} \)

Radiator Hose Outside Diameter: \( D_{hose,o} = 1.600 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.13333 \text{ ft} \)

Maximum Tank Temperature: \( T_{\text{max,t}} = 90^\circ C \times \frac{9}{5} + 32 = 194^\circ F \)

Maximum Ambient Temperature: \( T_{\text{max,A}} = 115^\circ F \)

Equivalent Roughness, new Copper pipe: \( \varepsilon \approx 4.92 \times 10^{-6} \text{ ft} \)

Total Pipe Length Section 1: \( l_1 = 48 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 4 \text{ ft} \)

Total Pipe Length Section 2: \( l_2 = 30 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 2.5 \text{ ft} \)

Cross-Sectional Area

Pipe Section 1: \( A_{p1} = \frac{\pi}{4} (0.06667)^2 = 3.491 \times 10^{-3} \text{ ft}^2 \)

Pipe Section 2: \( A_{p2} = \frac{\pi}{4} (0.08542)^2 = 5.731 \times 10^{-3} \text{ ft}^2 \)

Radiator Hose: \( A_{hose} = \frac{\pi}{4} (0.09896)^2 = 7.691 \times 10^{-3} \text{ ft}^2 \)

Flow Rate & Velocity Equations

\( Q_1 = Q_2 = Q_{hose} \)

\( V_1 A_1 = V_2 A_2 \)

\( V_1 A_{p1} = V_2 A_{p2} \rightarrow V_2 = \frac{V_1 A_{p1}}{A_{p2}} \)

\( V_1 A_1 = V_{hose} A_{hose} \rightarrow V_{hose} = \frac{V_1 A_{p1}}{A_{hose}} \)
Velocity:

\[ V_1 = \frac{Q}{A} = \frac{0.01114 \text{ ft}^3}{3.491 \times 10^{-3} \text{ ft}^2 \cdot \text{s}} = 3.191 \frac{\text{ft}}{\text{s}} \]

\[ V_2 = \frac{V_1 A_{p1}}{A_{p2}} = \frac{3.191 \frac{\text{ft}}{\text{s}} \cdot 3.491 \times 10^{-3} \text{ ft}^2}{5.731 \times 10^{-3} \text{ ft}^2} = 1.944 \frac{\text{ft}}{\text{s}} \]

\[ V_\text{hose} = \frac{V_1 A_{p1}}{A_{\text{Rose}}} = \frac{3.191 \frac{\text{ft}}{\text{s}} \cdot 3.491 \times 10^{-3} \text{ ft}^2}{7.691 \times 10^{-3} \text{ ft}^2} = 1.448 \frac{\text{ft}}{\text{s}} \]

**Section 1**

*Water Properties @ 14.7 psia & T_{\text{max,t}}:*

Density: \( \rho = 1.8732 \frac{\text{slugs}}{\text{ft}^3} \)

Dynamic Viscosity: \( \mu = 6.6015 \times 10^{-6} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2} \)

Specific Heat Ratio: \( k = 1.3983 \)

*Reynolds Number: \( Re = \frac{\rho V D_{\text{i}}}{\mu} \)

\[ Re_1 = \frac{\rho V_1 D_{\text{i}}}{\mu} = \frac{1.8732 \frac{\text{slugs}}{\text{ft}^3} \cdot 3.191 \frac{\text{ft}}{\text{s}} \cdot 0.06667 \text{ ft}}{6.6015 \times 10^{-6} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}} = 60,366.89 \]

Equivalent roughness:

\[ \frac{\varepsilon}{D_{\text{i}}} = \frac{4.92 \times 10^{-6}}{0.06667} = 7.380 \times 10^{-5} \]

Moody Diagram
Friction factor from Moody Diagram:

\[ f_1 = 0.021 \]

Head Loss:

Head loss is approximated using the Darcy-Weisbach equation. It consists of both major and minor losses. Major losses stem primarily from friction and minor losses from valves, bends, and tees.

The equation accounting for both frictional and minor losses is as follows:

\[
\Delta h_f = \left(f \frac{l}{D} + \sum K\right) \frac{V^2}{2g}
\]

Where \( f \) is the friction factor, \( l \) is the pipe length, \( D \) is the pipe diameter, \( V \) is the fluid velocity, \( g \) is the gravitational constant, and \( K \) is the loss coefficient. The loss coefficients are known values which are summed to find the total loss coefficient for the system or section being evaluated. The figure and table below show these losses.
![Figure 8.22](image) Entrance flow conditions and loss coefficient (Data from Refs. 28, 29).
(a) Reentrant, $K_L = 0.8$, (b) sharp-edged, $K_L = 0.5$, (c) slightly rounded, $K_L = 0.2$ (see Fig. 8.24),
(d) well-rounded, $K_L = 0.04$ (see Fig. 8.24).

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*See Fig. 8.32 for typical valve geometry.
Examining section one, there is one re-entrant pipe and two elbows.

$$\sum K_1 = K_{re-entrant} + 2 \cdot K_{elbow} = 0.8 + 2 \cdot 0.3 = 1.4$$

$$\Delta h_1 = \left( f_1 \frac{l_1}{D_{i1}} + \sum K_1 \right) \frac{V_1^2}{2g} = \left( 0.021 \frac{4}{0.06667} + 1.4 \right) \frac{3.191^2}{2 \cdot 32.174} = 0.421 \, ft$$

**Mass Flow Rate**

Mass flow rate, section 1: $$\dot{m}_1 = \rho H_2O A_1 V_1$$

$$\dot{m}_{1,b1} = \left( 1.8732 \frac{slugs}{ft^3} \right) \left( 3.491 \cdot 10^{-3} \, ft^2 \right) \left( 3.191 \frac{ft}{s} \right) = 20.867 \cdot 10^{-3} \frac{slug}{s}$$

$$\dot{m}_{1,b1} = 20.867 \cdot 10^{-3} \frac{slug}{s} \cdot \frac{32.174 \, lbm}{slug} = 0.671 \frac{lbm}{s}$$

**Heat Transfer**

Pipe Inner Diameter Section 1: $$D_{i1} = 0.800 \, in \cdot \frac{1 \, ft}{12 \, in} = 0.0667 \, ft$$

Pipe Outer Diameter Section 1: $$D_{o1} = 0.880 \, in \cdot \frac{1 \, ft}{12 \, in} = 0.07333 \, ft$$

$$Re_1 = \frac{\rho V_1 D_{i1}}{\mu} = \frac{1.8732 \frac{slugs}{ft^3} \cdot 3.191 \frac{ft}{s} \cdot 0.0667 \, ft}{6.6015 \times 10^{-6} \, lb \cdot s \cdot ft^{-2}} = 60,366.89$$

Convective Heat Transfer Coefficient, Air – Free Convection: $$h_A \approx 3 \frac{Btu}{h \cdot ft^2 \cdot ^\circ F}$$

Maximum Tank Temperature: $$T_{max,t} = 90^\circ C \cdot \frac{9}{5} + 32 = 194 \, ^\circ F$$

Prandtl number of tank water at $$T_{max,t}$$: $$Pr_{H2O} = 1.95$$

Thermal conductivity of water at $$T_{max,t}$$: $$k_{H2O, maxT} = 0.67589 \frac{W}{m \cdot K}$$

$$k_{H2O, maxT} = 0.67589 \frac{J}{s \cdot m \cdot K} \cdot \frac{1 \, Btu}{1055.06 \, J} \cdot \frac{3600 \, s}{1 \, h} \cdot \frac{1 \, m}{3.281 \, ft} \cdot \frac{1 \, K}{1.8 \, ^\circ F}$$

$$k_{H2O, maxT} = 0.390501 \frac{Btu}{h \cdot ft \cdot ^\circ F}$$

Nusselt Number: For cooling, $$Nu = 0.023 Re^{0.8} Pr^{0.3}$$

Section 1 (Pre-radiator): $$Nu_1 = 0.023 Re_1^{0.8} Pr_1^{0.3} = 0.023 \cdot 60,366.89^{0.8} \cdot 1.95^{0.3} = 187.66$$

Convective Heat Transfer Coefficient, Water – Forced Convection: $$h_{1,H2O} = \frac{k_{H2O}}{D_{pipe,i}} \cdot Nu_{4A}$$

$$h_{1,H2O} = 0.390501 \frac{Btu}{0.06667 \cdot 187.66} = 1.099 \cdot 10^3 \frac{Btu}{h \cdot ft^2 \cdot ^\circ F}$$
Thermal conductivity of copper: $k_{Cu} = 231.84 \frac{Btu}{h*ft*^\circ F}$

**Equivalent Resistance**

Maximum Tank Temperature: $T_{max,t} = 90^\circ C \times \frac{9}{5} + 32 = 194 ^\circ F$

Maximum Ambient Temperature: $T_{max,A} = 115 ^\circ F$

Heat Transfer, Section 4A: $Q = \frac{T_{max,t} - T_{max,A}}{R_{total}}$ where $R_{total}$ is the total resistance to heat transfer

Specific Heat of Water at $T_{max,tank}$: $c_{p,H2O,max} = 1.005 \frac{Btu}{lbm*^\circ F}$

\[ R_{tot} = R_{H2O,conv.} + R_{Cu,cond.} + R_{Air,conv.} \]

\[ R_{H2O,conv.} = \frac{1}{h_{H2O} * A_{surf}} = \frac{1}{h_{H2O} * \pi * D_{pipe,i} * l_{section}} \]

\[ R_{Cu,cond.} = \frac{\ln(D_{pipe,o} - D_{pipe,i})}{2\pi * k_{Cu} * l_{section}} \]

\[ R_{Air,conv.} = \frac{1}{h_{Air} * A_{surf}} = \frac{1}{h_{H2O} * \pi * D_{pipe,i} * l_{section}} \]

For section 1:

Length: $l_1 = 48 \text{ in} = 4 \text{ ft}$

\[ R_{tot,1} = \frac{1}{h_{H2O} * \pi * D_{pipe,i} * l_{section}} + \frac{\ln(D_{pipe,o} - D_{pipe,i})}{2\pi * k_{Cu} * l_{section}} + \frac{1}{h_{Air} * \pi * D_{pipe,i} * l_{section}} \]

\[ R_{tot,1} = \frac{1}{1.099 \times 10^3 * \pi * 0.06667 * 4} + \frac{\ln(0.08542 - 0.06667)}{2\pi * 231.84 * 4} + \frac{1}{3 * \pi * 0.06667 * 4} \]
\[ R_{tot,1} = 0.398 \text{ } ^\circ \text{F} \times h \text{ } \text{Btu} \]

\[ \dot{Q} = \frac{194 - 115}{0.398} = 198.36 \text{ } \text{Btu} \text{ } \text{h} \]

Surface Temperature of Pipe Section 1: \[ \dot{Q} = \frac{T_{surf,A} - T_{Air}}{R_{Air,conv.}} \]

\[ T_{surf,1} = T_{Air} + \dot{Q} \times R_{Air,conv.} = 115 \text{ } ^\circ \text{F} + 198.36 \times \frac{\text{Btu}}{\text{h}} \times \frac{1}{3 \pi + 0.06667 \times 4 \text{ } \text{Btu}} = 193.92 \text{ } ^\circ \text{F} \]

Outlet Temperature of Fluid through Pipe:

\[ \dot{Q} = \dot{m} c_{p,\text{H}_2\text{O}} (T_i - T_o) = h A_s \Delta T_{lm}, \text{ where } \Delta T_{lm} \text{ is a log-mean temperature defined by:} \]

\[ \Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln \frac{\Delta T_o}{\Delta T_i}} \]

Simplifying and solving for \( T_o \):

\[ T_o = T_s - (T_s - T_i) e^{\frac{-h A_s}{\dot{m} c_p}} \]

For section 1:

\[ T_{o,1} = 193.92 - (193.92 - 194) e^{\frac{-0.08}{1.448 \times 0.9896 \times 10^{-6}}} \]

\[ T_{o,1} = 193.92 - (-0.08) e^{-379} = 193.97 \text{ } ^\circ \text{F} = T_{i,rad} \]

**Radiator Hose (Section 1 to Radiator Transition):**

The small section of radiator hose between pipe section 1 and the radiator entrance produces negligible heat transfer. As such, it is ignored in the heat transfer calculations. Only head loss calculations are considered.

Velocity of fluid in hose: \( V_{hose} = 1.448 \text{ } \text{ft/} \text{s} \)

Length of section 1 to radiator transition hose: \( L_{hose,1-R} = .25 \text{ } \text{ft} \)

\[ Re_{hose,1-R} = \frac{\rho V_{hose} D_{hose,i}}{\mu} = \frac{1.8732 \text{ } \text{slugs} / \text{ft}^3 \times 1.448 \text{ } \text{ft/s}}{6.6015 \times 10^{-6} \text{ } \text{lb} \times \text{s} / \text{ft}^2} = 40,660.22 \]

Absolute Roughness, EPDM hose: \( \varepsilon_{EPDM} \approx 1.969 \times 10^{-1} \)

Equivalent Roughness:

\[ \frac{\varepsilon_{EPDM}}{D_{hose,i}} = \frac{1.969 \times 10^{-11}}{0.09896} = 1.989 \times 10^{-10} \]

From Moody Diagram, Friction Factor: \( f_{hose,1-R} = 0.022 \)
Examining the radiator hose connecting section one to the radiator, there are two unions. There is an additional union between the adapter and section 1. This is included in this calculation for simplicity.

\[
\sum K_{hose,1-R} = 3 \times K_{union} = 3 \times 0.08 = 0.24
\]

\[
\Delta h_{hose,1-R} = \left( f_{hose,1-R} \frac{l_{hose,1-R}}{D_{hose,1-R}} + \sum K_{hose,1-R} \right) \frac{V_{hose}^2}{2g} = \left( 0.022 \frac{0.25}{0.099896} + 0.24 \right) \frac{1.448^2}{2 \times 32.174}
\]

**Radiator:**

Number of tubes: \( N_{tubes} = 35 \)

Radiator Width/Tube Length: \( W_{rad} = l_{tube} = 26 \text{ in} = 2.1667 \text{ ft} \)

Radiator Height: \( H_{rad} = 14.25 \text{ in} = 1.1875 \text{ ft} \)

Radiator Depth: \( d_{rad} = 0.640 \text{ in} = 0.0533 \text{ ft} \)

Tube Interior Width: \( W_{tube,int} = 0.625 \text{ in} = 0.0521 \text{ ft} \)

Tube Interior Height: \( H_{tube,int} = 0.065 \text{ in} = 5.4167 \times 10^{-3} \text{ ft} \)

Cross-Sectional Area- Tube: \( A_{CS,tube} = H_{tube}W_{tube} = (0.0533) \times (5.4167 \times 10^{-3}) = 2.821 \times 10^{-4} \text{ ft}^2 \)

Perimeter of Tube: \( P_{tube} = 2H_{tube} + 2W_{tube} = 2(0.0533) + 2(5.4167 \times 10^{-3}) = 0.115 \text{ ft} \)

Hydraulic Diameter of Tube: \( D_{h,tube} = \frac{4A_{CS,tube}}{P_{tube}} = \frac{4 \times 2.821 \times 10^{-4}}{0.115} = 9.812 \times 10^{-3} \text{ ft} \)

Total Tube Area / Radiator: \( A_{rad,t} = N_{tubes}A_{CS,tube} = 35 \times (2.821 \times 10^{-4}) = 9.874 \times 10^{-3} \text{ ft}^2 \)

\[ Q_1 = Q_{rad} \rightarrow V_1A_1 = V_{rad}A_{rad} \]

Radiator Velocity: \( V_{rad} = \frac{V_1A_1}{A_{rad}} = \frac{3.191 \times 10^{-3}}{9.874 \times 10^{-3}} \text{ ft}^2 = 1.128 \text{ ft/s} \)

Equivalent Roughness, new Aluminum pipe: \( \varepsilon \approx 6 \times 10^{-5} \text{ in} \)

\[ Re = \frac{\rho V D}{\mu} \]

Density: \( \rho = 1.8732 \text{ slugs/ft}^3 \)

Dynamic Viscosity: \( \mu = 6.6015 \times 10^{-6} \text{ lbf \cdot s/ft}^2 \)

\[ Re_{rad} = 1.8732 \text{ slugs/ft}^3 \times 1.128 \text{ ft/s} \times 9.812 \times 10^{-3} \text{ ft} \times \frac{\text{lbf} \times \text{s}^2}{\text{slugs} \times \text{ft}} = 3.140.23 \]

The flow is turbulent as \( Re > 2300 \)
Friction factor: \[ f = \frac{1}{(1.58 \ln(Re_{rad}) - 3.28)^2} = \frac{1}{(1.58 \ln(3140.23) - 3.28)^2} = 0.0112 \]

From Moody Diagram, Radiator friction factor: \[ f \approx 0.044 \]

Head Loss radiator: The radiator tubes run in parallel. As such, it is only necessary to calculate the head loss of a single tube as the total head loss will be equivalent to the head loss of a single tube. Accounted for are a sharp edge entrance, the entrance, and the exit. The entrance and exit are treated as elbows.

\[
\sum K_{rad} = K_{re-entrant} + 2 \times K_{elbow} = 0.8 + 2 \times 0.3 = 1.4
\]

Head Loss radiator: \[
\Delta h_{rad} = \left(0.0112 \times \frac{2.1667}{9.874 \times 10^{-3}} + 1.4\right) \times \frac{(1.128)^2}{2 \times 32.174} = 76.279 \times 10^{-3} \text{ ft}
\]

**Radiator- Heat Transfer (ε-NTU Method):**

Coolant Side (Fluid)

Hydraulic Diameter Coolant: \[ D_{h,c} = D_{h,tube} = 9.812 \times 10^{-3} \text{ ft} \]

Reynolds Number: \[ Re_c = Re_{rad} = 3,140.23 \]

Fluid velocity: \[ V_c = V_{rad} = 1.128 \frac{ft}{s} \]

Because \( T_{max} \approx T_{rad} \) the properties at \( T_{rad} \) are assumed equal.

\[ \mu_c = 6.6015 \times 10^{-6} \frac{lb * s}{ft^2} \times 32.174 \frac{lbm * ft}{lb * s^2} = 2.124 \times 10^{-4} \frac{lbm}{ft * s} \]

\[ c_{p,H_2O,ti,rad} = 1.005 \frac{Btu}{lbm \cdot ^\circ F} \]

\[ k_{H_2O,ti,rad} = 0.390501 \frac{Btu}{h \cdot ft \cdot ^\circ F} \times \frac{1 \text{ h}}{3600 \text{ s}} = 1.085 \times 10^{-4} \frac{Btu}{s \cdot ft \cdot ^\circ F} \]

Prandtl Number: \[ P_c = \frac{c_{p,c} \mu_c}{k_c} = \frac{1.005 \frac{Btu}{lbm \cdot ^\circ F} \times 2.124 \times 10^{-4} \frac{lbm}{s \cdot ft^2 \cdot ^\circ F}}{1.085 \times 10^{-4} \frac{Btu}{s \cdot ft \cdot ^\circ F}} = 1.967 \]

**Nusselt Number:**

Laminar Flow: \[ Nu_c = \frac{h_c \cdot D_{h,c}}{k_c} = 1.86 (Re \cdot Pr)^{\frac{1}{3}} \left(\frac{L}{D}\right)^{\frac{1}{3}} \left(\frac{\mu}{\mu_s}\right)^{0.14} \]

Turbulent Flow: \[ Nu_c = \frac{h_c \cdot D_{h,c}}{k_c} = \frac{\left(\frac{f}{D}\right) (Re_c^{-1000}) Pr}{1 + 12.7(f/2)^{1/2} (Pr^{1/3} - 1)} \] where Friction Factor: \[ f = \frac{1}{(1.58 \ln(Re_c) - 3.28)^2} \]

Because \( Re > 2300 \), the flow in the tube is turbulent. The Nusselt calculation follows:

Friction factor: \[ f = \frac{1}{(1.58 \ln(Re_{rad}) - 3.28)^2} = \frac{1}{(1.58 \ln(3140.23) - 3.28)^2} = 0.0112 \]
\[
N_u = \left(\frac{0.0112}{2}\right) \times (3.140.23 - 1000) \times 1.967 \frac{1}{1 + 12.7 \left(\frac{0.0112}{2}\right)^\frac{1}{2} \left(1.967\frac{2}{3} - 1\right)} = 15.310
\]

Heat Transfer Coefficient: \(h_c = \frac{N_u \cdot K_c}{D_{h,c}} = \frac{15.310 \times 1.085 \times 10^{-5}}{9.812 \times 10^{-3}} = 0.0169 \frac{Btu}{s \cdot ft^2 \cdot ^\circ F}\)

**Air**

Hydraulic Diameter: \(D_{h,A} = \frac{4 \times d_{core} \cdot A_r}{A_A}\), where \(d_{core}\) is the core depth, \(A_r\) is the free flow area, and \(A_A\) is the total heat transfer area of the air

Number of tubes: \(N_{tubes} = 35\)

Core Depth: \(d_{core} = 0.640\) in = 0.0533 ft

Tube Exterior Width: \(W_{tube,ext} = 0.635\) in = 0.0529 ft

Tube Exterior Height: \(H_{tube,ext} = 0.075\) in = 0.00625 ft

Tube Length: \(L_{rad} = 26\) in. = 2.1667 ft

Free Flow Area: \(A_r \approx 434\) in\(^2\) = 3.014 ft\(^2\)

Heat Transfer Area of Air: \(A_A = N_{tubes} \times (2 \times W_{tube,ext} + 2 \times H_{tube,ext}) \times L_{rad}\)
\[
A_A = 35 \times (2 \times 0.635 + 2 \times 0.075) \times 26 = 1292.2\ in^2 = 8.97 ft^2
\]

\[
D_{h,A} = \frac{4 \times 0.0533 \times 3.014}{8.97} = 0.07 ft
\]

Density of Air at Max Ambient Temperature: \(\rho_A = 2.147 \times 10^{-3} \frac{slugs}{ft^3}\)

Dynamic Viscosity of Air at Max Ambient Temperature: \(\mu_A = 4.00 \times 10^{-7} \frac{lbm/s}{ft^2}\)

\[
\mu_A = 4.00 \times 10^{-7} \frac{lbm/s}{ft^2} \times 32.174 \frac{lbm*ft}{lbm*s^2} = 1.287 \times 10^{-5} \frac{lbm}{ft*s}
\]

Flow Rate: \(Q_A \approx 2 \times 925 \frac{ft^3}{min} = 1,850 \frac{ft^3}{min}\)

Velocity: \(V_A = \frac{Q_A}{A_r} = \frac{1850 \frac{ft^3}{min}}{3.014 \frac{ft^3}{ft\cdot min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 10.23 \frac{ft}{s}\)

Reynolds Number: \(Re_A = \frac{\mu_A \cdot V_A \cdot D_{h,A}}{\rho_A} = \frac{2.147 \times 10^{-3} \times 10.23 \times 0.07}{4.00 \times 10^{-7}} = 3,843.67\)

Specific Heat of Air at \(T_{Ambient,max}\): \(c_{p,air} = 0.2407 \frac{Btu}{lbm \cdot ^\circ F}\)

\[
k_{air} = 0.016030 \frac{Btu}{h \cdot ft \cdot ^\circ F} \times \frac{1 \text{ h}}{3600 \text{ s}} = 4.453 \times 10^{-6} \frac{Btu}{s \cdot ft \cdot ^\circ F}
\]

Prandtl Number: \(P_A = \frac{c_{p,air} \cdot \mu_A}{k_A} = \frac{0.2407 \times 1.287 \times 10^{-5}}{4.453 \times 10^{-6}} = 0.696\)
Colburn Factor: $J = \frac{0.174}{Re^{0.88}} = \frac{0.174}{3843.67^{0.88}} = 7.372 \times 10^{-3}$

Heat transfer Coefficient: $h_A = \frac{J * p_A * V_A * C_p a}{Pr_A^{1/3}}$

$$h_A = \frac{J * p_A * V_A * C_p a}{Pr_A^{1/3}}$$

$$h_A = \frac{7.372 \times 10^{-3} * 2.147 * 10^{-3}}{0.696^2} \frac{slugs}{ft^3} * \frac{10.23}{s} * \frac{Btu}{lbm * °F} * \frac{3600}{s} * \frac{32.174}{lbm}$$

$$h_A = 5.748 \frac{Btu}{h * ft^2 * °F}$$

Heat Rejection

Thermal conductivity of aluminum: $k_{Al} = k_f = 136 \frac{Btu}{h * ft * °F}$

Fin thickness: $t_f = 0.001 \text{ in} = 8.333 \times 10^{-5} \text{ ft}$

Fin height: $H_f = 0.310 \text{ in} = 0.03 \text{ ft}$

$$h_A = 5.748 \frac{Btu}{h * ft^2 * °F}$$

$$h_c = 0.0169 \frac{Btu}{s * ft^2 * °F} * \frac{3600}{s} = 60.84 \frac{Btu}{h * ft^2 * °F}$$

Fin Efficiency Factor: $F_f = \left[ \frac{2 * h_A}{k_f * t_f} \right]^{0.5} \left( \frac{H_f}{2} \right)$

$$F_f = \left[ \frac{2 * 5.748}{136} \frac{Btu}{h * ft^2 * °F} * 8.333 * 10^{-5} \text{ ft} \right]^{0.5} \left( \frac{0.03 \text{ ft}}{2} \right) = 0.466$$

Fin Efficiency: $n_f = \frac{\tanh F_f}{F_f} = 0.9334$

Fin Area: $A_f = 16.12 \text{ ft}^2$

$A_A = 8.97 \text{ ft}^2$

Effectiveness of Fins: $\varepsilon_f = 1 - (1 - n_f) \left( \frac{A_f}{A_A} \right) = 1 - (1 - 0.9334) \left( \frac{16.12}{8.97} \right) = 0.8803$

Radiator Width/Tube Length: $W_{rad} = L_{tube} = 26 \text{ in} = 2.1667 \text{ ft}$

Radiator Height: $H_{rad} = 14.25 \text{ in} = 1.1875 \text{ ft}$

Radiator Depth: $d_{rad} = 0.640 \text{ in} = 0.0533 \text{ ft}$
Radiator Core Volume: \( V_c = 2.1667 \times 1.1875 \times 0.0533 = 0.137 \text{ ft}^3 \)

\( A_{\text{rad}, i} = A_c = 35 \times (2.821 \times 10^{-4}) = 9.874 \times 10^{-3} \text{ ft}^2 \)

Tube Thickness: \( t_{\text{Tube}} = 0.010 \text{ in.} = 8.333 \times 10^{-4} \text{ ft} \)

Overall Thermal Resistance: 
\[
R = \frac{1}{\varepsilon_f h_A} + \frac{1}{\frac{A_c A_p}{A_p A_v}} h_c + \frac{t_{\text{Tube}}}{k_{\text{Tube}}} \]
\[
= \frac{1}{0.8803 \times 5.748 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{°F}}} + \frac{1}{\frac{7.078 \text{ ft}^2 / 0.137 \text{ ft}^3}{8.97 \text{ ft}^2 / 0.137 \text{ ft}^3}} \times 60.84 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{°F}} + \frac{8.333 \times 10^{-4} \text{ ft}}{136 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{°F}}} = 0.2184 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{°F}} 
\]

Overall Heat transfer Coefficient: 
\[
U = \frac{1}{R} = \frac{1}{15.13 \frac{\text{h} \cdot \text{ft}^2 \cdot \text{°F}}{\text{Btu}}} = 4.58 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{°F}} 
\]

\( \dot{m}_A = \rho_A A_R V_A = 2.147 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3} \times 3.014 \text{ ft}^2 \times 10.23 \frac{\text{ft}}{s} = 6.620 \times 10^{-2} \frac{\text{slugs}}{s} \)

\( \dot{m}_A = 6.620 \times 10^{-2} \frac{\text{slugs}}{s} \times 32.174 \frac{\text{lb} \cdot \text{m}}{\text{slug}} = 2.130 \frac{\text{lb} \cdot \text{m}}{s} \)

\( \dot{m}_c = \rho_c A_{\text{cs. tube}} V_c = 1.8732 \frac{\text{slugs}}{\text{ft}^3} \times 2.821 \times 10^{-4} \text{ ft}^2 \times 1.128 \frac{\text{ft}}{s} = 5.961 \times 10^{-4} \frac{\text{slugs}}{s} \)

\( \dot{m}_c = 5.961 \times 10^{-4} \frac{\text{slugs}}{s} \times 32.174 \frac{\text{lb} \cdot \text{m}}{\text{slug}} = 1.918 \times 10^{-2} \frac{\text{lb} \cdot \text{m}}{s} \)

\( c_{p, \text{air}} = 0.2407 \frac{\text{Btu}}{\text{lb} \cdot \text{m} \cdot \text{°F}} \)

\( c_{p, \text{H}_2\text{O}, \text{ti}, \text{rad}} = c_{p, c} = 1.005 \frac{\text{Btu}}{\text{lb} \cdot \text{m} \cdot \text{°F}} \)

Stream heat capacity rate for air: 
\( C_A = \dot{m}_A c_{p, A} = 2.130 \frac{\text{lb} \cdot \text{m}}{s} \times 0.2407 \frac{\text{Btu}}{\text{lb} \cdot \text{m} \cdot \text{°F}} = 0.513 \frac{\text{Btu}}{s \cdot \text{°F}} \)

\( C_A = 0.513 \frac{\text{Btu}}{s \cdot \text{°F}} \times 3600 \frac{s}{1 \text{ h}} = 1845.69 \frac{\text{Btu}}{1 \text{ h} \cdot \text{°F}} \)

Stream heat capacity rate for coolant: 
\( C_c = \dot{m}_c c_{p, c} = 1.918 \times 10^{-2} \frac{\text{lb} \cdot \text{m}}{s} \times 1.005 \frac{\text{Btu}}{\text{lb} \cdot \text{m} \cdot \text{°F}} = 1.939 \frac{\text{Btu}}{s \cdot \text{°F}} \)

\( C_c = 1.913 \times 10^{-2} \frac{\text{Btu}}{s \cdot \text{°F}} \times 3600 \frac{s}{1 \text{ h}} = 69.39 \frac{\text{Btu}}{1 \text{ h} \cdot \text{°F}} \)

Stream heat capacity ratio: 
\( \frac{C_r}{C_r} = \frac{\min(C_A \text{ or } C_c)}{C_A} = \frac{C_c}{C_A} = \frac{69.39}{1845.69} = 3.759 \times 10^{-2} \)

\( C_r = 1.005 \frac{\text{Btu}}{\text{lb} \cdot \text{m} \cdot \text{°F}} \times 3600 \frac{s}{1 \text{ h}} = 897 \frac{\text{Btu}}{s \cdot \text{°F}} \)

Number of transfer units: 
\( N T U_{\text{max}} = \frac{U \cdot A_A}{\min(C_A \text{ or } C_c)} = \frac{4.577 \frac{\text{Btu}}{h \cdot \text{ft}^2 \cdot \text{°F}} \times 897 \frac{\text{ft}^2}{2}}{69.39 \frac{\text{Btu}}{h \cdot \text{°F}}} = 0.296 \)

Effectiveness of Heat Exchanger: 
\( \varepsilon_{HE} = 1 - e^{-C_r N T U_{\text{max}}^{-0.22}} = 1 - e^{-0.759 \times 10^{-2} \times 0.296^{-0.22}} = 0.255 \)
Radiator Inlet Temperature: $T_{i,rad} = 193.97 \, ^\circ F = T_c$

Maximum Ambient Temperature: $T_{max,A} = 115 \, ^\circ F = T_A$

Total Heat Transfer Rate: $Q_{tot} = \varepsilon_{HE} \cdot \min(C_A \text{ or } C_c) \cdot (T_c - T_A)$

$Q_{tot} = 0.255 \cdot 69.39 \, \frac{Btu}{h \cdot ^\circ F} \cdot (193.81^\circ F - 115^\circ F) = 1394.50 \, \frac{Btu}{h}$

Coolant Outlet Temperature: $T_{o,c} = T_{i,c} - \frac{Q_{tot}}{C_c} = 193.97^\circ F - \frac{1394.50 \, \frac{Btu}{h}}{69.39 \, \frac{Btu}{h \cdot ^\circ F}} = 173.87 \, ^\circ F$

Air Outlet Temperature: $T_{o,a} = T_{i,a} + \frac{Q_{tot}}{C_A} = 115^\circ F + \frac{1394.50 \, \frac{Btu}{h}}{1845.69 \, \frac{Btu}{h \cdot ^\circ F}} = 115.76 \, ^\circ F$

**Radiator Hose (Radiator to Section 2 Transition):**

The small section of radiator hose between pipe the radiator exit and section 2 produces negligible heat transfer. As such, it is ignored in the heat transfer calculations. Only head loss calculations are considered.

Temperature of fluid in hose: $T_{H2O,2-R} = 173.87 \, ^\circ F$

**Water Properties @ 14.7 psia & $T_{H2O,2}$:**

Density: $\rho = 1.887 \, \frac{slugs}{ft^3}$

Dynamic Viscosity: $\mu = 7.546 \cdot 10^{-6} \, \frac{lbf \cdot s}{ft^2}$

Velocity of fluid in hose: $V_{hose} = 1.448 \, \frac{ft}{s}$

Length of radiator to section 2 transition hose: $L_{hose,2-1} = 1 \, ft$

$Re_{hose,2-1} = \frac{\rho V_{hose}D_{hose,i}}{\mu} = \frac{1.887 \, \frac{slugs}{ft^3} \cdot 1.448 \, \frac{ft}{s} \cdot 0.09896 \, ft}{7.546 \times 10^{-6} \, \frac{lbf \cdot s}{ft^2}} = 35,883.01$  

Absolute Roughness, EPDM hose: $\varepsilon_{EPDM} \approx 1.969 \cdot 10^{-11}$

Equivalent Roughness: $\frac{\varepsilon_{EPDM}}{D_{hose,i}} = \frac{1.969 \cdot 10^{-11}}{0.09896} = 1.989 \cdot 10^{-10}$

From Moody Diagram, Friction Factor: $f_{hose,2-1} = 0.023$

Examining the radiator hose connecting the radiator to the pump, there are two unions and a curve that is the shape of two elbows. There is an additional union between the adapter and the pump. This is included in this calculation for simplicity.

$\sum K_{hose,2-1} = 3 \cdot K_{union} + 2 \cdot K_{elbow} = 3 \cdot 0.08 + 2 \cdot 0.3 = 0.84$
\[ \Delta h_{\text{hose,R-2}} = \left( f_{\text{hose,R-2}} \frac{l_{\text{hose,R-2}}}{D_{\text{hose,R-2}}} + \sum K_{\text{hose,R-2}} \right) \frac{V_{\text{hose}}^2}{2g} = \left( 0.023 \cdot \frac{1}{0.09896} + 0.84 \right) \frac{1.448^2}{2 \cdot 32.174} = 34.943 \times 10^{-3} \text{ ft} \]

**Section 2**

Convective Heat Transfer Coefficient, Air – Free Convection: \( h_A \approx 3 \frac{\text{Btu}}{h \text{ft}^2 \cdot \text{°F}} \)

\( T_{o,rad} = 173.87 \text{ °F} \)

Pipe Inner Diameter Section 2: \( D_{2i} = 1.025 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.08542 \text{ ft} \)

Pipe Outer Diameter Section 2: \( D_{2o} = 1.125 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 0.09375 \text{ ft} \)

Total Pipe Length Section 2: \( l_2 = 30 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 2.5 \text{ ft} \)

Area, Pipe Section 2: \( A_{p2} = \frac{\pi}{4} (0.08542)^2 = 5.731 \times 10^{-3} \text{ ft}^2 \)

\[ V_2 = \frac{V_1 A_{p1}}{A_{p2}} = \frac{3.191 \frac{\text{ft}}{s} \cdot 3.491 \times 10^{-3} \text{ ft}^2}{5.731 \times 10^{-3} \text{ ft}^2} = 1.944 \frac{\text{ft}}{s} \]

\[ R_2e = \frac{\rho V_2 D_{2i}}{\mu} = \frac{1.887 \frac{\text{slugs}}{\text{ft}^3} \cdot 1.944 \frac{\text{ft}}{s} \cdot 0.08542 \text{ ft}}{7.546 \times 10^{-6} \frac{\text{lb} \cdot \text{s}}{\text{ft} \cdot \text{ft}^2}} = 41,525.12 \]

Prandtl number of tank water at \( T_{o,rad} \): \( Pr_{H2O,o,rad} = 2.23 \)

Thermal conductivity of water at \( T_{o,rad} \): \( k_{H2O,o,rad} = 0.669 \frac{\text{W}}{\text{m} \cdot \text{K}} \)

\[ k_{H2O,o,rad} = 0.669 \frac{1 \text{ Btu}}{s \cdot m \cdot \text{K}} \cdot \frac{3600 \text{ s}}{1055.06 \text{ ft} \cdot \text{s}} \cdot \frac{1 \text{ m}}{3.281 \text{ ft}} \cdot \frac{1 \text{ K}}{18 \text{ °F}} \]

\[ k_{H2O,o,rad} = 0.38652 \frac{\text{Btu}}{h \cdot \text{ft} \cdot \text{°F}} \]

Nusselt Number: For cooling, \( Nu = 0.023 Re^{0.8} Pr^{0.3} \)

Section 2 (Post-radiator): \( Nu_2 = 0.023 Re^{0.8} Pr^{0.3} = 0.023 \times 41,525.12^{0.8} \times 2.23^{0.3} = 144.83 \)

Convective Heat Transfer Coefficient, Water – Forced Convection: \( h_{2,H2O} = \frac{k_{H2O}}{D_{\text{pipe,i}}} \cdot Nu_2 \)

\[ h_{2,H2O} = \frac{0.38652 \times 144.83}{0.08542} = 655.35 \frac{\text{Btu}}{h \cdot \text{ft}^2 \cdot \text{°F}} \]

Thermal conductivity of copper: \( k_{Cu} = 231.84 \frac{\text{Btu}}{h \cdot \text{ft}^2 \cdot \text{°F}} \)
Head Loss

\[ \frac{\varepsilon}{D_{2i}} = \frac{4.92 \times 10^{-6}}{0.08542} = 5.760 \times 10^{-5} \]

From Moody Diagram, Friction Factor: \( f_2 = 0.023 \)

Examining section two, there is one union and two elbows.

\[ \sum K_2 = K_{union} + 2 \times K_{elbow} = 0.08 + 2 \times 0.3 = 0.68 \]

\[ \Delta h_2 = \left( f_2 \frac{l_2}{D_{2i}} + \sum K_2 \right) \frac{V_2^2}{2g} = \left( 0.023 \times \frac{2.5}{0.08542} + 0.68 \right) \frac{1.944^2}{2 \times 32.174} = 0.0829 \text{ ft} \]

Equivalent Resistance

Maximum Ambient Temperature: \( T_{\text{max,A}} = 115 \degree F \)

Heat Transfer, Section 2: \( \dot{Q} = \frac{T_{\text{o,rad}} - T_{\text{max,A}}}{R_{\text{total}}} \) where \( R_{\text{total}} \) is the total resistance to heat transfer

Specific Heat of Water at \( T_{\text{o,rad}} \): \( c_{p,H_2O,\text{o,rad}} = 1.002 \text{ Btu lbm}^{-1} \text{°F}^{-1} \)

\( R_{\text{tot}} = R_{H_2O,\text{conv}} + R_{\text{Cu,cond}} + R_{\text{Air,conv}}. \)

\( R_{H_2O,\text{conv}} = \frac{1}{h_{H_2O} * A_{\text{surf}}} = \frac{1}{h_{H_2O} * \pi * D_{\text{pipe},i} * l_{\text{section}}} \)

\( R_{\text{Cu,cond}} = \frac{\ln(D_{\text{pipe,o}} - D_{\text{pipe,i}})}{2\pi * k_{\text{Cu}} * l_{\text{section}}} \)

\( R_{\text{Air,conv}} = \frac{1}{R_{\text{Air}} * A_{\text{surf}}} = \frac{1}{h_{H_2O} * \pi * D_{\text{pipe},i} * l_{\text{section}}} \)

\( R_{\text{tot}} = \frac{1}{h_{H_2O} * \pi * D_{\text{pipe},i} * l_{\text{section}}} + \frac{\ln(D_{\text{pipe,o}} - D_{\text{pipe,i}})}{2\pi * k_{\text{Cu}} * l_{\text{section}}} + \frac{1}{h_{\text{Air}} * \pi * D_{\text{pipe},i} * l_{\text{section}}} \)

\( R_{\text{tot,2}} = \frac{1}{655.35 * \pi * 0.08542 * 2.5} + \frac{\ln(0.09375 - 0.08542)}{2\pi * 231.84 * 2.5} + \frac{1}{3 * \pi * 0.08542 * 2.5} \)

\( R_{\text{tot,AB}} = 0.498 \text{ °F} \cdot \text{h} / \text{Btu} \)

\( \dot{Q} = \frac{173.87 - 115}{0.498} = 118.26 \text{ Btu} / \text{h} \)

Surface Temperature of Pipe Section 4: \( \dot{Q} = \frac{T_{\text{surf,4}} - T_{\text{Air}}}{R_{\text{Air,conv}}} \)

\( T_{\text{surf,4}} = T_{\text{Air}} + \dot{Q} * R_{\text{Air,conv}} = 115 \degree F + 118.26 \times \frac{\text{Btu}}{\text{h}} * \frac{1}{3 * \pi * 0.08542 * 2.5 * \text{Btu}} = 173.76 \degree F \)

Mass Flow Rate

Mass flow rate, section 2: \( \dot{m}_2 = \rho_{H_2O} A_2 V_2 \)
\[ \dot{m}_{1,b1} = \left( 1.887 \frac{slugs}{ft^3} \right) \left( 5.731 \times 10^{-3} ft^2 \right) \left( 1.944 \frac{ft}{s} \right) = 21.023 \times 10^{-3} \frac{slug}{s} \]

\[ \dot{m}_{1,b1} = 21.023 \times 10^{-3} \frac{slug}{s} \times 32.174 \frac{lbm}{slug} = 0.676 \frac{lbm}{s} \]

**Outlet Temperature of Fluid through Pipe:**

\[ \dot{Q} = \dot{m} c_{p, H2O} (T_i - T_o) = hA_s \Delta T_{lm}, \]  
where \( \Delta T_{lm} \) is a log-mean temperature defined by:

\[ \Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln \frac{\Delta T_o}{\Delta T_i}} \]

Simplifying and solving for \( T_o \):

\[ T_o = T_s - (T_s - T_i) e^{-\frac{hA_s}{mc_p}} \]

For section 2:

\[ T_{o,2} = 173.76 - (173.76 - 173.87) e^{- \left( \frac{655.35 \frac{Btu}{R \times ft^2 \times \circ F}}{0.676 \frac{lbm}{s}} \right) \left( 1 \frac{h}{3600 \text{ s}} \right) \left( 1.002 \frac{Btu}{lbm \circ F} \right)^2} \]

\[ T_{o,2} = 173.67 - (-0.11) e^{0.180} = 173.76 \circ F = 78.76 \circ C = T_{i, tank} \]

The tank inlet temperature of 78.76 \( \circ C \) is within the recommended maximum operating range of the GPUS of 75 \( \circ C \) – 85 \( \circ C \). This is under the conditions that the tank reached the maximum temperature before automatic shutoff of the GPUS of 90 \( \circ C \) (194 \( \circ F \)) and the maximum recorded temperature of the St. Louis, Missouri area in history, 115 \( \circ F \).

**Total Head Loss**

\[ \Delta h_{total} = \Delta h_1 + \Delta h_{ho \_1-R} + \Delta h_{rad} + \Delta h_{hos \_R-2} + \Delta h_2 \]

\[ \Delta h_{total} = 0.421 + 9.631 \times 10^{-3} + 76.279 \times 10^{-3} + 34.943 \times 10^{-3} + 0.0829 \]

\[ \Delta h_{total} = 0.625 \text{ ft} \]

**Design Requirement:** A pump is needed that can overcome 0.625 ft of head loss. The pump from the final design is rated for maximum of 48 ft of head.
## APPENDIX C – BILL OF MATERIALS

<table>
<thead>
<tr>
<th>Item</th>
<th>Name</th>
<th>Quantity</th>
<th>Price</th>
<th>Total</th>
<th>Website/Location &amp; Hyperlink</th>
<th>Screwned Description</th>
<th>Lead Time</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Familiar Movers</td>
<td>2</td>
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<td>$198.00</td>
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<td>$ -</td>
<td>3D Printed</td>
<td>3D printed on Formlabs Form 3 SLA printer with Flexible ASA resin</td>
<td>~2 hours print time</td>
</tr>
<tr>
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<td>$ -</td>
<td>3D Printed</td>
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<tr>
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<td>$ -</td>
<td>3D Printed</td>
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<td>$ -</td>
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