Positivity and representing measures in the truncated moment problem

Abstract

Let $K$ denote a nonempty closed subset of $\mathbb{R}^n$ and let $\beta \equiv \beta^{(m)} = \{\beta_i\}_{i \in \mathbb{Z}_+^n, |i| \leq m}$, $\beta_0 > 0$, denote a real $n$-dimensional multisequence of finite degree $m$. The Truncated $K$-Moment Problem (TKMP) concerns the existence of a positive Borel measure $\mu$, supported in $K$, such that

$$\beta_i = \int_{\mathbb{R}^n} x^i \, d\mu \quad (i \in \mathbb{Z}_+^n, |i| \leq m).$$

We describe a number of interrelated techniques for establishing the existence of such $K$-representing measures. We discuss $K$-representing measures arising from $K$-positivity or strict $K$-positivity of the Riesz functional $L_\beta$ associated with $\beta$; representing measures arising from extensions of moment matrices; Tchakaloff’s Theorem and its generalizations and applications to TKMP; representing measures arising from a nonempty core variety.

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