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Bessel sequences from iterated operator actions

Abstract

The subject of the talk is motivated by questions related to the new field of Dynamical Sampling which was recently initiated by A. Aldroubi et al. In short, the general Dynamical Sampling problem deals with sequences $(A^k x_i)_{i\in I,\,k=0,\dots,K_i}$, where A is a linear operator and the x_i are vectors. The question then is for which A, x_i, and K_i the sequence constitutes a frame for the underlying Hilbert space. Clearly, for this it is necessary that each of the subsequences $(A^k x_i)_{k=0,\dots,K_i}$ is a Bessel sequence. As this is only interesting when $K_i = \infty$, we consider sequences of the form $(A^k x)_{k\in\mathbb{N}}$. In order to exploit a comprehensive spectral theory, we restrict ourselves to normal operators A and completely characterize the normal operators A and vectors x for which $(A^k x)_{k\in\mathbb{N}}$ is a Bessel sequence. The characterization is formulated in terms of the measure $\mu_x := \|E(\cdot)x\|^2$, where E is the spectral measure of the operator A: *The sequence* $(A^k x)_{k\in\mathbb{N}}$ *is a Bessel sequence if and only if* (i) μ_x *is concentrated on the closed unit disc* $\overline{\mathbb{D}}$*,* (ii) *The restriction of* μ_x *to the unit circle is absolutely continuous with respect to arc length measure with* L^∞*-density function,* (iii) $\mu_x|\mathbb{D}$ *is a Carleson measure.* If time permits, we will apply the results to a certain sampling framework with the heat equation in the background.

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