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Summation of Power Series by Continued Square Roots

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It is proposed that a power series may be summed (analytically continued outside its radius of convergence) by converting it to a continued square root of the form $a_0\sqrt{1+a_1 z\sqrt{1+a_2 z\sqrt{\cdots}}}$. The continued square root coefficients a_i for a given function f(z)) are determined by equating the Taylor coefficients of the continued square root with those of f(z). For most power scries, the continued square root has a region of convergence much smaller than that of the Taylor series, continued exponential, or continued fraction. However, when the power series is of the form

$$d_0 + d_1 z - \sum_{n=2}^{\infty} \frac{\Gamma(\frac{n-1}{2}) Cos(\frac{n\pi}{2}) \sum_{i=0}^{\infty} f_i^{g_i n}}{2\Gamma(\frac{1}{2})\Gamma(\frac{n+2}{2})} z^n$$

for $Re(d_n) \geq 0$, $Re(f_i) \geq 0$, and $g_i \in \mathbb{C}$, the Taylor series and continued fraction representations break down and the continued square root converges for all Re(z) > 0 on the complex plane.