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SUMMATION OF POWER SERIES BY CONTINUED SQUARE ROOTS

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It is proposed that a power series may be summed (analytically continued outside its radius of convergence) by converting it to a continued square root of the form $a_0 \sqrt{1 + a_1 z} \sqrt{1 + a_2 z} \sqrt{\dots}$. The continued square root coefficients a_i for a given function $f(z)$ are determined by equating the Taylor coefficients of the continued square root with those of $f(z)$. For most power series, the continued square root has a region of convergence much smaller than that of the Taylor series, continued exponential, or continued fraction. However, when the power series is of the form

$$d_0 + d_1 z - \sum_{n=2}^{\infty} \frac{\Gamma(\frac{n-1}{2}) \text{Cos}(\frac{n\pi}{2}) \sum_{i=0}^{\infty} f_i g_i^n}{2\Gamma(\frac{1}{2})\Gamma(\frac{n+2}{2})} z^n$$

for $\text{Re}(d_n) \geq 0$, $\text{Re}(f_i) \geq 0$, and $g_i \in \mathbb{C}$, the Taylor series and continued fraction representations break down and the continued square root converges for all $\text{Re}(z) > 0$ on the complex plane.