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Authors: Terry Tidwell, Robert Glaubius, Christopher D. Gill, and William D. Smart

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Type of Report: Other

# Optimal Time Utility Based Scheduling Policy Design for Cyber-Physical Systems

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**Abstract**—Classical scheduling abstractions such as deadlines and priorities do not readily capture the complex timing semantics found in many real-time cyber-physical systems. Time utility functions provide a necessarily richer description of timing semantics, but designing utility-aware scheduling policies using them is an open research problem. In particular, optimal utility accrual scheduling design is needed for real-time cyber-physical domains.

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## I. INTRODUCTION

Computing systems increasingly interact with the physical world through sensing, actuation, or both. Timing constraints imposed by these interactions are of paramount concern for the safe and correct operation of these cyber-physical systems.

Because size, weight, power consumption or other physical restrictions may strongly influence their deployment and operation, such cyber-physical systems may be resource constrained and different jobs may contend for their shared common resources. When that happens, a scheduling policy must arbitrate access to shared resources while satisfying system timing and other constraints.

Real-time systems traditionally model timing constraints as deadlines. In *hard real-time systems* [1] any deadline miss may be considered equivalent to total system failure, or at least to the loss of any value from the corresponding job. Such a system is not feasibly schedulable unless all jobs meet all their deadlines. In contrast, *soft real-time systems* do not assume deadline misses are catastrophic. This implies some flexibility in how missed deadlines are handled. In some soft real-time systems, a deadline miss may mean that the job simply should be aborted. In others a late job should be completed with as little overrun as possible. However, none of these cases cover the full range of possible timing semantics that may be inherent to cyber-physical systems.

Time utility functions (TUFs) are a powerful abstraction for expressing more general timing constraints [2], [3], which

characterize the *utility* of completing particular jobs as a function of time and thus capture more complex semantics than simple deadlines. To satisfy temporal constraints in a system whose semantics are described by time utility functions, a scheduling policy must maximize system-wide utility accrual.

However, jobs in cyber-physical systems have other features that complicate the scheduling problem. Specifically, jobs in many cyber-physical systems may be non-preemptable and may have stochastic duration. For example, jobs may involve actuation, such that preemption of a job may require restoring the state of a physical apparatus such as a robot arm. In such cases the cost of preemption may be unreasonably high. Therefore scheduling algorithms for cyber-physical systems must be able to consider non-preemptable jobs. Interaction with the physical environment also may lead to unpredictable job behavior, most notably in terms of job durations. A scheduling policy should anticipate this variability not only by considering the worst case execution time (as is often done in deadline-based scheduling) but also the probabilistic distribution of job durations. This is especially important when the goal is to maximize utility accrual, which depends on the timing of job completion.

Scheduling problems with these concerns may arise in a variety of cyber-physical systems as well as in traditional real-time systems. In mobile robotics, jobs competing for use of a robotic arm must be scheduled for efficient alternation. In CPU scheduling, quality of service (QoS) for an application may be specified as a time utility function, and the jobs to be scheduled may have long critical sections where preemption is not allowed. Finally, in critical real-time systems access to the bus by COTS peripherals may have to be scheduled in order to guarantee real-time performance [4].

In this paper we extend our previous work on Markov Decision Process (MDP) based scheduling policy design [5]–[7] to generate *utility-aware* scheduling policies appropriate for real-time cyber-physical systems with non-preemptable jobs that run with stochastic duration. This extension enables us to derive *optimal utility-aware* scheduling policies for this important class of systems. It also improves on current state of the art utility accrual schedulers, which rely on heuristics and do not take advantage of predictable future job releases such as jobs released under a periodic task model.

This paper is organized as follows. In Section II we provide a brief survey of related work. In Section III we describe

our system and task model. In Section IV we formulate the scheduling problem as a Markov decision process and describe our solution approach. In Section V we examine the performance of scheduling policies for different classes of time utility functions in our system model. We offer conclusions in Section VI and propose directions for further investigation.

## II. RELATED WORK

Utility accrual scheduling primarily has been restricted to heuristics based on maximizing instantaneous potential utility density, which is the expected utility of running a job normalized by its expected duration [2].

The Generic Benefit Scheduler (GBS) [8] schedules tasks under resource contention using the potential utility density heuristic without assigning deadlines. If there is no resource contention, the proposed scheduling policy simply greedily schedules jobs according to the highest potential utility density.

Locke’s Best Effort Scheduling Algorithm (LBESA) [9] schedules jobs with stochastic durations and non-convex time utility functions using a variation of Earliest Deadline First (EDF) [10], where jobs with the lowest potential utility density are dropped from the schedule if the system becomes overloaded. This technique requires an assignment of job deadlines along their time utility curves; optimal selection of those deadlines is itself an open problem.

Other research on utility accrual scheduling has crucially relied on restricting the shapes of the time utility curves to a single class of functions. The Dependent Activity Scheduling Algorithm (DASA) [11] assumes time utility functions are non-increasing downward step functions. The Utility Accrual Packet Scheduling Algorithm (UPA) [12], which extends an algorithm presented by Chen and Muhlethaler [13], assumes time utility functions can be approximated using a strictly linearly decreasing function. Gravitational task models [14] assume that the shapes of the time utility functions are symmetric and unimodal. In addition it is assumed that utility is gained when the non-preemptable job is scheduled, which is equivalent to assuming deterministic job durations with utility gained on job completion.

No existing utility accrual scheduling approach anticipates future job arrivals. Therefore, existing techniques are sub-optimal for systems in which future arrivals can be accurately predicted, such as those encountered under a periodic task model [10].

MDPs have been used to model sequential decision problems including applications in cyber-physical domains such as helicopter control [15], [16] and mobile robotics [17], [18]. Our previous work [5]–[7] formulated MDPs to design scheduling policies in soft real-time environments with always-available jobs, but did so only for simple utilization-share-based semantics. In this work we extend such use of MDPs to design new classes of *utility-aware* scheduling policies for periodic tasks with stochastic duration.

Several other attempts have been made to address the difficulties that arise from non-preemptive and stochastic tasks in real-time systems. Statistical Rate Monotonic Scheduling

(SRMS) [19] extends the classical Rate Monotonic Scheduling (RMS) [10] algorithm to deal with periodic tasks with stochastic duration. Constant Bandwidth Servers (CBS) [20] allow resource reservation in real-time systems where tasks have stochastic duration. Manolache, et al. [21], estimate deadline miss rates for non-preemptive tasks with stochastic duration. These works use classical scheduling abstractions such as priority and deadlines, rather than time utility functions, and are thus not appropriate for systems with the more complex timing semantics considered by our approach.

## III. SYSTEM MODEL

We consider a system of non-preemptable jobs with stochastic durations and timing semantics formulated as time utility functions (TUFs). More formally, in our system model  $n$  periodic tasks  $(T_i)_{i=1}^n$  require mutually exclusive use of a single common resource. Each task  $T_i$  consists of an infinite sequence of non-preemptable jobs, where  $J_{i,j}$  denotes the  $j$ th job in  $T_i$ . A new job of  $T_i$  is added to the ready queue every  $p_i$  time units, where  $p_i$  is the period of task  $T_i$ . Each job of a task has stochastic duration distributed according to probability density function  $D_i$ , where  $D_i(t)$  is the probability of the job’s duration being exactly  $t$ . Each task has an associated time utility function  $U_i(t)$  that maps the time elapsed since the release time  $r_{i,j}$  of job  $J_{i,j}$  to a real number representing the utility of completing that job at time  $r_{i,j} + t$ .

At each invocation the scheduler can choose to run any available job  $J_{i,j}$  in the ready queue. Doing so results in the job occupying the scheduled resource stochastically for some number of quanta distributed according to  $D_i$ . The scheduler may instead choose to keep the resource idle for the next quantum. The optimal scheduling policy under this system model chooses scheduling actions (dispatching an available job on the scheduled resource or keeping the resource idle for a quantum) so as to maximize long term system utility accrual. Note that schedules produced under this model need not be work conserving (scheduling some job if any job is available) since a work conserving schedule may not be optimal given arbitrarily shaped time utility functions. For example, unimodal TUF  $U_i$  may peak too late relative to job  $J_{i,j}$  if the job were to be run at its earliest opportunity.

To make this MDP-based scheduling policy design approach feasible we place additional constraints on the task duration distribution,  $D_i$ , and the time utility function  $U_i$ . We assume that each task has a worst-case execution time  $w_i$  that serves as an upper bound on the support of  $D_i$ . We also assume that each task has a termination time  $tm_i$  after which  $U_i(t)$  becomes zero. At time  $r_{i,j} + tm_i$  job  $J_{i,j}$  is removed from the ready queue, if it hasn’t yet been chosen to be run. We discuss these requirements in greater detail in Section IV based on the illustrative example task set shown in Figure 1.

## IV. SOLUTION APPROACH

In previous work [5]–[7] we presented methods for formulating scheduling problems with non-preemptable jobs with stochastic duration as MDPs. Using existing techniques from

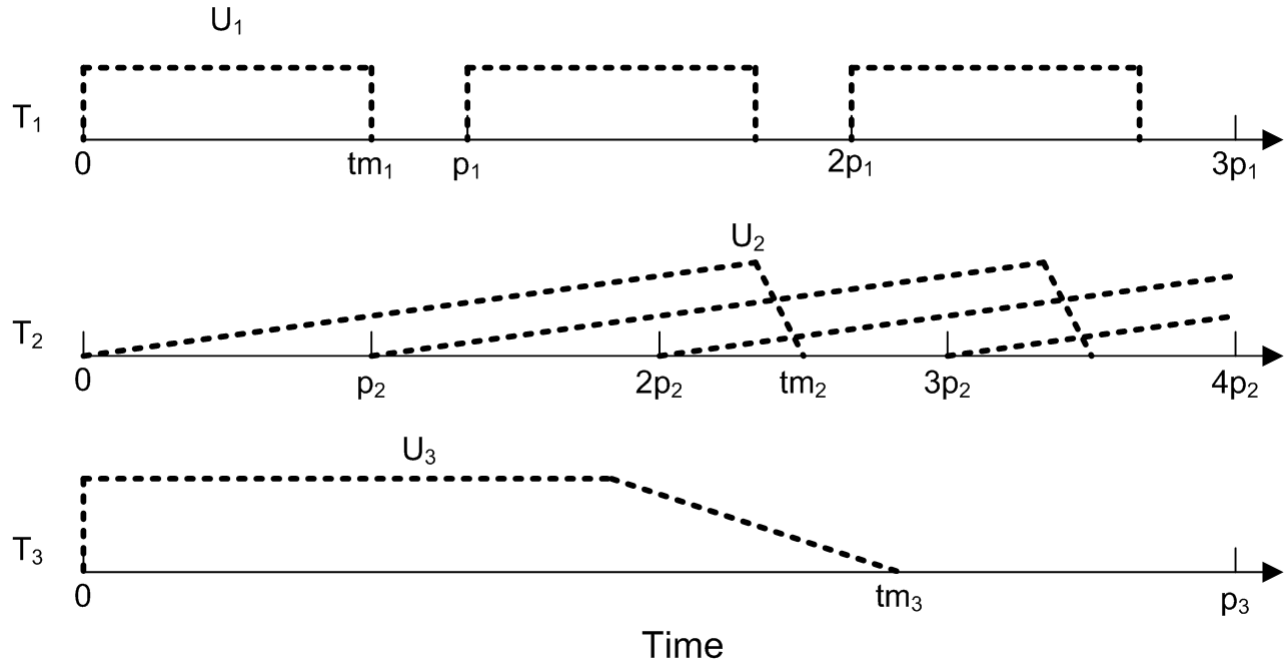


Fig. 1. Example 3 task system. Periods  $p_i$ , time utility curves  $U_i$ , and termination times  $tm_i$  are shown for each task.

the operations research literature [22] we were able to derive optimal policies for a particular class of scheduling problems. That work focused on scheduling always-available non-preemptable jobs with stochastic durations to adhere to a desired resource share. This was achieved by penalizing the system in proportion to its deviation from the desired share target.

In this work we extend those techniques to design *utility-aware* rather than *share-aware* scheduling policies. In addition, we extend our previous work from an always-available job model to a periodic task model.

#### A. Markov Decision Processes (MDPs)

An MDP is a four-tuple  $(\mathcal{X}, \mathcal{A}, P, R)$  consisting of a collection of states  $\mathcal{X}$  and actions  $\mathcal{A}$ , a transition system  $P$  that establishes the conditional probabilities  $P(y|x, a)$  of transitioning from state  $x$  to  $y$  on action  $a$ , and a reward function  $R$  that specifies the immediate utility of acting in each state. The reward function  $R$  is defined over the domain of state-action-state tuples such that  $R(x, a, y)$  is the immediate reward for taking action  $a$  in state  $x$  and ending up in state  $y$ .

A policy  $\pi$  for an MDP maps states in  $\mathcal{X}$  to actions in  $\mathcal{A}$ . At each discrete decision epoch  $k$  the agent observes the state of the MDP  $x_k$ , then selects an action  $a_k = \pi(x_k)$ . The MDP then transitions to state  $x_{k+1}$  with probability  $P(x_{k+1}|x_k, a_k)$  and the controller receives reward  $r_k = R(x_k, a_k, x_{k+1})$ . Given a discount factor  $\gamma \in [0, 1)$ , the *value* of a policy, written  $V^\pi$ , is the expected sum of long-term, discounted rewards obtained while following that policy:

$$V^\pi(x) = E \left\{ \sum_{k=0}^{\infty} \gamma^k r_k \mid x_0 = x, a_k = \pi(x_k) \right\}. \quad (1)$$

Overloading notation, we let

$$R(x, a) = \sum_{y \in \mathcal{X}} P(y|x, a) R(x, a, y)$$

denote the expected reward when executing  $a$  in  $x$ . Then we may equivalently define  $V^\pi$  as the solution to the linear system

$$V^\pi(x) = R(x, \pi(x)) + \gamma \sum_{y \in \mathcal{X}} P(y|x, \pi(x)) V^\pi(y)$$

for each state  $x$ . When  $|R(x, a)|$  is bounded for all actions in all states, the discount factor  $\gamma$  prevents  $V^\pi$  from diverging for any choice of policy, and can be interpreted as the prior probability that the agent survives from one decision epoch to the next [23].

Our objective is find some policy  $\pi^*$  that achieves maximal value  $V^*$  in every state.  $V^*$  satisfies the Bellman equation

$$V^*(x) = \max_{a \in \mathcal{A}} \left\{ R(x, a) + \gamma \sum_{y \in \mathcal{X}} P(y|x, a) V^*(y) \right\}$$

There are numerous algorithms for computing  $V^*$  for MDPs with finite state and action spaces [22]. Given  $V^*$ , an optimal policy can be obtained by choosing the action that results in the greatest expected successor state value.

#### B. Utility-Aware Task Scheduling MDP

In our system model there are two salient features that determine the *scheduler state*: the system time, and the set of jobs available to run. We define our MDP over the set of scheduler states. The scheduler state is composed of a variable for tracking the elapsed time since the system was started,  $\tau$ ,

and indicator variables  $q_i$  for each task that track whether there are any jobs of  $T_i$  in the ready queue.

For the experiments presented in Section V we assume that  $tm_i \leq p_i$ . In this case  $q_i$  is defined over  $\{0, 1\}$  and is 1 if there is a job of task  $T_i$  in the ready queue and 0 otherwise. Because we require the termination time of the job to be less than or equal to the task period, we need only reason about one job per task at a time. For example, in Figure 1 tasks  $T_1$  and  $T_3$  have TUFs that satisfy this restriction, and so only a single job of each task may be in the ready queue at any time.

If we allow  $tm_i > p_i$  but assume that jobs of the task must be run in order,  $q_i$  is an integer which counts the number of jobs of  $T_i$  that are in the ready queue. The values that  $q_i$  can take are bounded by  $\lceil tm_i/p_i \rceil$ , the maximum number of jobs of  $T_i$  that can be in the ready queue. Note that this MDP is a strict generalization of the case where  $tm_i \leq p_i$ . For example, in Figure 1  $2p_2 < tm_2 < 3p_2$ . Consequently, at most three jobs of task  $T_2$  may be in the ready queue.

If we allow  $tm_i > p_i$  and also allow jobs of a task to be run out of order,  $q_i$  becomes an array in  $\{0, 1\}^{\lceil tm_i/p_i \rceil}$  that tracks which of the  $\lceil tm_i/p_i \rceil$  most recently released jobs of  $T_i$  are in the ready queue. Note that this again is a strict generalization of the previous two cases.

An action  $a_{i,j}$  in our MDP is the decision to dispatch job  $J_{i,j}$  and is only valid in a scheduler state if  $q_i$  indicates  $J_{i,j}$  is in the ready queue. If we assume that  $tm_i \leq p_i$  (or that jobs of  $T_i$  must be run in order), only a single job of  $T_i$  is ever eligible to be run. In this case we can simplify our set of actions to  $a_i$ , the action that runs that eligible job. In addition there is a special action  $a_{idle}$ , available in every state, which is the decision to advance time in the system by one quantum without scheduling any job.

With this mapping from scheduler states to MDP states in place, the transition function  $P(y|x, a)$  is the probability of reaching scheduler state  $y$  from  $x$  when choosing action  $a$ . We discuss the formulation of the reward function for our utility aware task scheduling MDP next, in Section IV-C.

### C. Reward Function

The reward function is defined similarly to the transition function. The scheduler accrues no reward if the resource is idled, i.e.,  $R(x, a_{idle}, y)$  is always zero. Otherwise,  $R(x, a_i, y)$  is the *utility density* of the job of  $T_i$  that was just run. The utility density of a job  $J_{i,j}$  completed at system time  $\tau$  is defined as  $U_i(\tau - r_{i,j})/(\tau - r_{i,j})$ . Utility density is used as the immediate reward as opposed to  $U_i(\tau - r_{i,j})$  in order to differentiate between jobs with different durations. It is then possible to define the expected potential utility density for running a job of  $T_i$  in terms of  $D_i$  and  $U_i$ , as a function of the age of the job  $t$  as shown in Equation 2.

$$\sum_{d=1}^{w_i} \frac{D_i(d)U_i(d+t)}{d} \quad (2)$$

An example calculation of the expected potential utility density as a function of job age is shown in Figure 2. Task



Fig. 2. Expected potential utility density.

$T_i$  has TUF  $U_i$  as shown in the upper graph.  $D_i$  is defined on the range  $[1, 2]$  with  $D_i(1) = 0.5$  and  $D_i(2) = 0.5$ . The bottom graph shows the result of applying Equation 2. This is defined to be the expected immediate reward for selecting the action that schedules a job of  $T_i$  with the given age.

If we assume  $tm_i < p_i$ , the age of the job of task  $T_i$  at time  $\tau$  is  $\tau \bmod p_i$ . Under this set of assumptions the expected immediate reward  $R(x, a_i)$  is:

$$R(x, a_i) = \sum_{d=1}^{w_i} \frac{D_i(d)U_i(d + (\tau \bmod p_i))}{d} \quad (3)$$

If we allow  $tm_i > p_i$  but assume that jobs must be run in order the age of the job of task  $T_i$  scheduled by  $a_i$  is  $p_i(q_i - 1) + (\tau \bmod p_i)$  and the immediate reward is:

$$R(x, a_i) = \sum_{d=1}^{w_i} \frac{D_i(d)U_i(d + p_i(q_i - 1) + (\tau \bmod p_i))}{d} \quad (4)$$

### D. Wrapped Utility-Aware Task Scheduling MDP

Because  $\tau$  is not bounded the resulting MDP has an infinite number of states. However, the hyperperiod  $H$  of the tasks, defined as the least common multiple of the task periods, allows us to wrap the state space of our MDP into a finite set of exemplar states as follows. The intuition is similar to that for hyperperiod analysis in classical real-time scheduling approaches. Given two states  $x$  and  $y$  with identical  $q_i$  for all tasks and times that map to the same point in the system hyperperiod,  $\tau_x \bmod H = \tau_y \bmod H$ , the two states will have the same relative distribution over successor states and rewards. This means that the optimal policy will be the same at both states. Thus it suffices to consider only the finite subset of states where  $\tau < H$ . Anytime the MDP would transition to a state where  $\tau \geq H$  it instead transitions to the otherwise identical state with system time  $\tau \bmod H$ .

An example of a wrapped utility-aware task scheduling MDP is shown in Figure 3. In this example there are two tasks  $T_1$  and  $T_2$  with deterministic quantum durations ( $D_1(1) =$

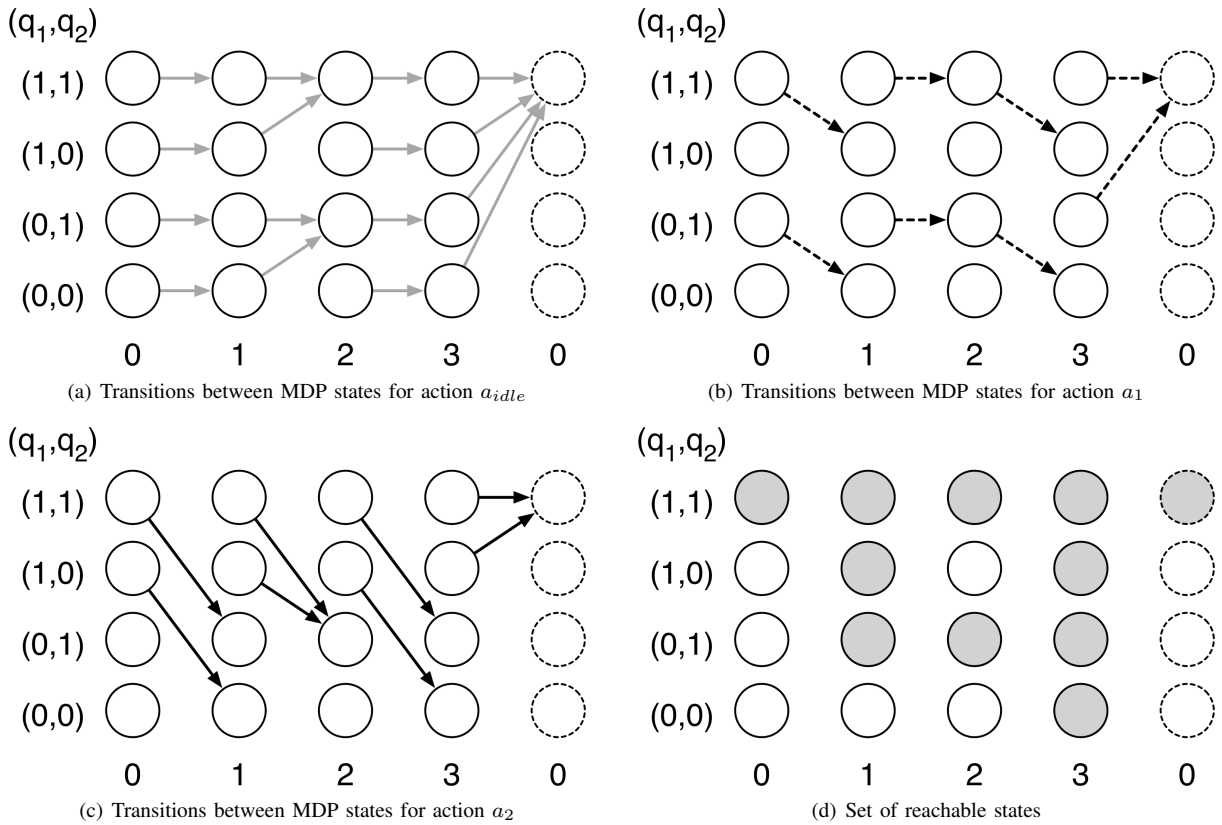


Fig. 3. Wrapped utility-aware task scheduling MDP for two tasks with periods  $p_1 = 4$  and  $p_2 = 2$ , termination times equal to periods, and deterministic single quantum duration.

$D_2(1) = 1$ ), and termination times equal to their period ( $tm_1 = p_1 = 4$ , and  $tm_2 = p_2 = 2$ ). The states of the MDP are projected so that states with the same  $\tau$  are arranged in columns, and states with the same tuple of indicator variables  $(q_1, q_2)$  are arranged in rows. Figures 3(a), 3(b), and 3(c) show the transitions between states for taking action  $a_{idle}$ ,  $a_1$  and  $a_2$  respectively. The states where  $\tau = 0$  are duplicated on the right side of each figure to illustrate our state wrapping over the hyperperiod, which for the example shown is 4. Figure 3(d) shows the set of reachable states from the state where  $q_1 = q_2 = 1$  and  $\tau = 0$ , which is the system's initial state.

Such state wrapping bounds the values that each of the state variables can take, and consequently bounds the size of the MDP. If we assume  $tm_i \leq p_i$ , an upper bound for the number of states in the scheduling MDP is

$$2^n H. \quad (5)$$

If instead we allow  $tm_i > p_i$  but assume jobs of a task must be run in order, an upper bound for the number of states in the scheduling MDP is

$$H \prod_{i=0}^n \lceil tm_i/p_i \rceil. \quad (6)$$

If we allow  $tm_i > p_i$  but let jobs of a task run in arbitrary

order, an upper bound for the number of states is

$$H 2^{\sum_{i=0}^n \lceil tm_i/p_i \rceil}. \quad (7)$$

In each case, the wrapped MDP has finitely many states. Consequently it can be solved using existing techniques from the operations research literature, and the resulting policy for this scheduling MDP is an optimal utility accrual scheduling policy for the modeled system. However, the size of the state space is sensitive not only to the number of tasks and their constraints but also to the size of the hyperperiod. As such, the solution approach presented in this paper is especially appropriate for systems with harmonic tasks since in this case  $H = \max(p_1, \dots, p_n)$ , which reduces the size of the state space that must be considered.

## V. EXPERIMENTAL EVALUATION

Although the solution outlined in Section IV can be applied to arbitrary time utility functions, for the purposes of evaluation we concentrate on three particular classes of representative time utility functions, of the form shown in Figure 4.

These three classes of time utility functions are both able to represent a wide range of timing constraints and suitable for evaluating the approach presented in this paper. The first curve, shown in Figure 4(a), is representative of time utility functions for hard real-time jobs, whose utility remains constant up until

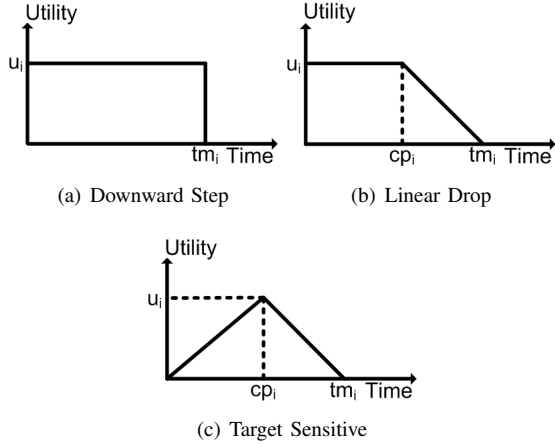


Fig. 4. Representative time utility functions.

a deadline after which the job provides no utility<sup>1</sup>, e.g., jobs in avionic or automotive control systems. In our experiments this family of curves is parameterized by two variables:  $tm_i$ , a termination time after which the expected utility of the tasks falls to zero, and  $u_i$  the upper bound on the utility curve, which for the experiments presented here was chosen uniformly at random from the range [2,32].

The second curve, shown in Figure 4(b), is representative of time utility functions for soft real-time jobs whose utility degrades with time but suffers no sharp drop, e.g., calculation of distant waypoints in on-board flight planning [24]. In our experiments this family of curves is parameterized by three variables:  $tm_i$ , a termination time after which the expected utility of the tasks falls to zero;  $u_i$  the upper bound on the utility curve (chosen uniformly at random from the range [2,32]); and  $cp_i$  the critical point at which the expected utility of the job starts to fall linearly toward zero (chosen uniformly at random from the range  $[0, tm_i]$ ).

The third curve, shown in Figure 4(c), is representative of time utility functions for target sensitive jobs - jobs sensitive to jitter but less sensitive to delays, e.g., in control systems where inter-job jitter can cause instabilities and failure [25], or in media-streaming applications in which buffering is not possible since inter-frame jitter can cause degradation of quality [26]. In our experiments this family of curves is parameterized by three variables:  $tm_i$ , a termination time after which the expected utility of the tasks falls to zero;  $u_i$  the upper bound on the utility curve (chosen uniformly at random from the range [2,32]); and  $cp_i$  the critical point before which the expected utility rises linearly toward  $u_i$  and after which the expected utility of the job starts to fall linearly toward zero (chosen uniformly at random from the range  $[0, tm_i]$ ).

Task sets were created by randomly generating duration distributions and periods. Periods were randomly chosen factors of 2400 in the range [100, 2400]. Choosing periods for tasks this way ensured that the hyperperiod of the task set was at

<sup>1</sup>The case where a deadline miss implies system failure can be modeled via a similar step-function TUF in which utility drops to  $-\infty$  instead of 0.

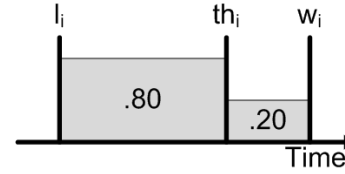


Fig. 5. Task duration distributions used in experiments.

most 2400.

The duration distributions are balanced as shown in Figure 5, and are parameterized by three values: a lower bound  $l_i$  and upper bound  $u_i$  on the duration, and a threshold point  $th_i$  such that:

$$\sum_{t=l_i}^{th_i} D_i(t) = 0.80$$

$$\sum_{t=th_i+1}^{w_i} D_i(t) = 0.20$$

In addition, these points were chosen to meet the following constraints:

$$w_i/p_i \geq th_i/p_i \geq l_i/p_i \geq 0.05$$

$$th_i/p_i \geq 0.10$$

$$\sum_{i=1}^n l_i/p_i = 0.70$$

$$\sum_{i=1}^n th_i/p_i = 0.90$$

$$\sum_{i=1}^n w_i/p_i = 1.20$$

The values of the duration distribution are uniform in the ranges  $[l_i, th_i]$  and  $[th_i + 1, u_i]$ . Intuitively these constraints ensure that the task load on the system is normally between 0.70 and 0.90, with occasional transient overloads up to 1.20.

To evaluate the performance of our approach we consider a greedy policy  $\pi_g$  that chooses the action that yields the best immediate reward in expectation as defined in Equation 3.

$$\pi_g(x) = \operatorname{argmax}_{a \in \mathcal{A}} \{R(x, a)\}$$

As noted in Section IV-C taking the action with the highest expected immediate reward from scheduler state  $x$  corresponds to scheduling the job with the highest expected potential utility density. As discussed in [8] this is equivalent to the scheduling decision made by Generic Benefit Scheduler (GBS) in the special case where there is no resource contention between jobs. We use this policy as a baseline for comparing how much



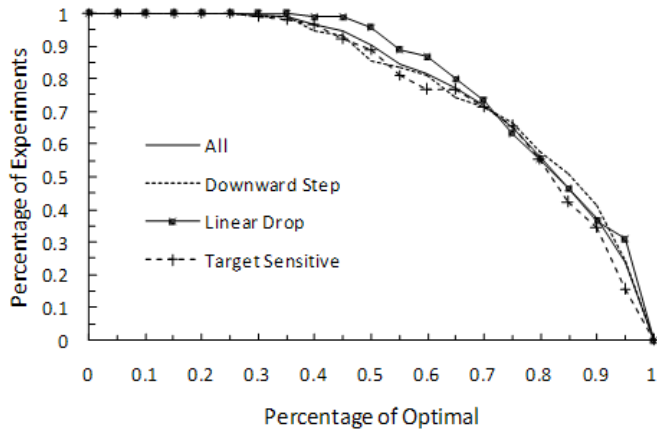


Fig. 6. Comparison of greedy policy performance to MDP-based optimal policies for 2 task scheduling problems for different classes of time utility functions.

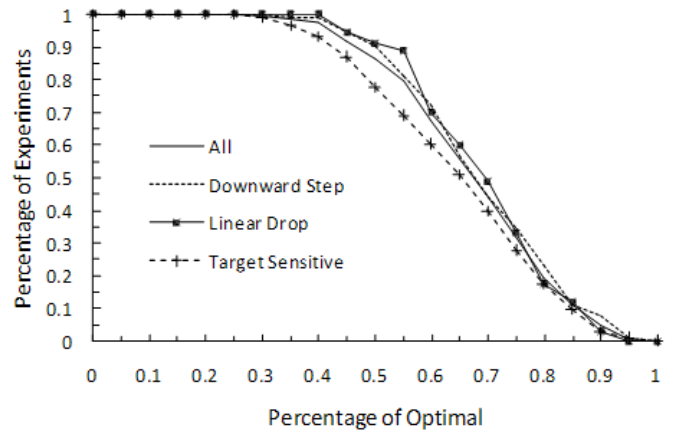


Fig. 8. Comparison of greedy policy performance to MDP-based optimal policies for 4 task scheduling problems for different classes of time utility functions.

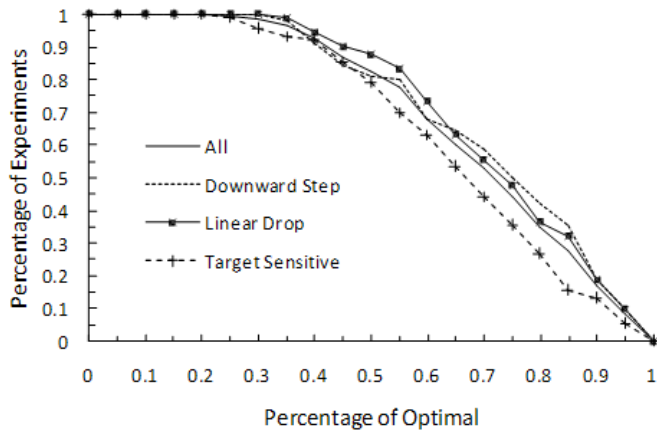


Fig. 7. Comparison of greedy policy performance to MDP-based optimal policies for 3 task scheduling problems for different classes of time utility functions.

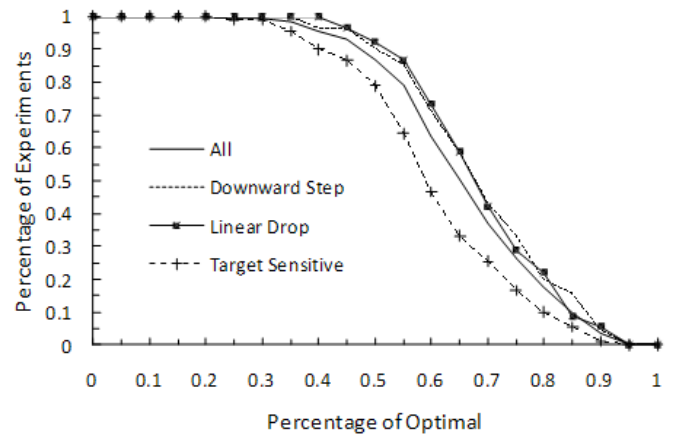


Fig. 9. Comparison of greedy policy performance to MDP-based optimal policies for 5 task scheduling problems for different classes of time utility functions.

improvement is made by solving for the optimal scheduling policy over using a heuristic approach.

The quality of  $\pi_g$ , and  $\pi^*$ , can be compared directly using our MDP model by evaluating the value of each policy at the initial state, denoted  $V^{\pi_g}(0)$  and  $V^*(0)$  respectively.

Figures 6, 7, 8 and 9 show results of comparing these values for a set of randomly generated task scheduling problems each with 2, 3, 4 or 5 tasks respectively. For each task set size, 270 problem instances were generated: 90 for each time utility function of the three classes shown in Figure 4. Each graph shows the percentage of random problem instances where  $V^{\pi_g}(0)$  was at least some percentage of  $V^*(0)$ . For example, in the 3 tasks problem instances shown in Figure 7, almost all the problem instances resulted in the  $\pi_g$  having  $V^{\pi_g}(0)$  at least 25% of  $V^*(0)$ . In only about 20% of problem instances was  $V^{\pi_g}(0)$  at least 90% of  $V^*(0)$ . There was very little variation for the respective quality of the exact and heuristic scheduling

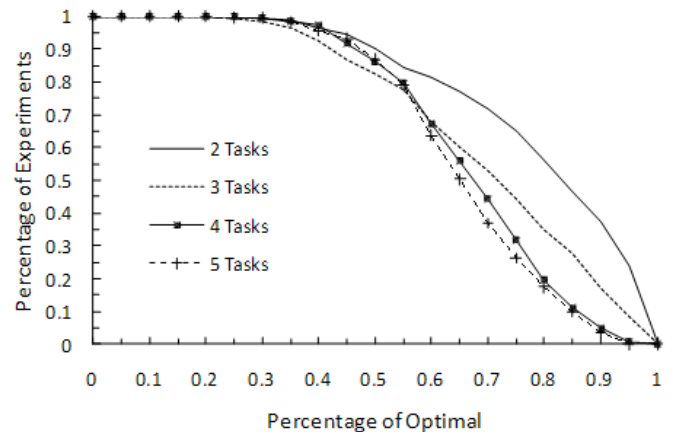


Fig. 10. Comparison of greedy policy performance to MDP-based optimal policies for scheduling problem instances with increasing number of tasks.

approaches for the different classes of time utility functions. However, target sensitive time utility functions resulted in better performance overall for the optimal scheduling policy compared to the heuristic scheduling policy. One reason for this is that the greedy scheduling policy is work conserving, and thus cannot let the scheduled resource become idle while there is an available job. The optimal scheduling policy, on the other hand, can delay the start of the job in order to maximize the expected potential utility density.

Figure 10 shows the effect of the number of tasks on the relative scheduling policy quality for the heuristic and optimal approaches. As Figure 10 illustrates, as the number of tasks increases the quality of the heuristic scheduling policy deteriorates relative to optimal.

These experiments show that existing heuristic approaches for utility accrual scheduling can perform poorly when tasks are non-preemptable and have stochastic duration. This is especially true as the number of tasks grows and if the jobs are target sensitive. In contrast, the policies produced by our approach maintain optimal solution quality in the face of these concerns.

## VI. CONCLUSIONS

In this paper we have introduced novel MDP based utility-aware scheduling policy design techniques for systems with periodic tasks and jobs that are non-preemptive and run with stochastic duration. This approach yields optimal utility accrual schedules appropriate for use in a variety of cyber-physical systems. These techniques are especially appropriate for systems with harmonic tasks, which reduces the size of the state space for the scheduling MDP.

Our utility-aware task scheduling MDP formulation also provides a framework for comparing the effectiveness of heuristic utility-aware scheduling policies. Our experiments demonstrate the superiority of MDP based optimal scheduling policies over state of the art heuristic utility accrual schedulers for several representative classes of time utility functions. Our approach is especially well suited for system configurations where the performance of heuristic policies is weakest: as the number of tasks increases and when time utility functions for the tasks are target sensitive. A more extensive characterization of when existing heuristics perform poorly is an area that merits further study.

Improving the scalability of the MDP based techniques presented here also merits further investigation. We plan to explore several approaches to that problem as future work. One approach is to develop parameterized policies that approximate the behavior of the optimal policies found by solving the utility-aware task scheduling MDP. A second approach is to look at simplified utility-aware task scheduling MDPs which approximate the structure of the original while being less sensitive to exponential growth in the number of states. A third approach is to consider compositional and hierarchical approaches, which use a polynomial number of exponentially smaller MDPs.

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