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Issues in Global Supply Chain: Uncertainty and Technology

Xiao Tan
Washington University in St. Louis, xtan@wustl.edu

WASHINGTON UNIVERSITY IN ST. LOUIS

Olin Business School
Supply Chain, Operations, and Technology

Dissertation Examination Committee:
Panos Kouvelis, Chair
Fuqiang Zhang
Lingxiu Dong
Sammi Yu Tang
Kaitlin Daniels

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by

Xiao Tan

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Dedicated to my family.

ABSTRACT OF THE DISSERTATION

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Professor Panos Kouvelis, Chair

The reason I decide on the title of my dissertation is that we are living at a tremendously special moment in history. The trade war between U.S. and China, the real war between Russia and Ukraine, and COVID-19 all indicate the turbulence and uncertainty we face. Artificial intelligence and the potential room temperature superconductor are the technology part. I believe high-quality research work should be based on history and reflect the timely reality. I am passionate about combining the "major event" with my relatively small "specialization field," which is supply chain management.

Talking about big events may be too broad and hollow. Hence I start with a small and specific question in my first Chapter: what is the impact of uncertain trade policy on a multinational company's manufacturing strategy in a global supply chain setting? In **Chapter 1 "Flexibility Value of Reshoring Capacity under Import Cost Uncertainty and Domestic Competition"**, companies that operate global supply chains are facing increasing uncertainty in the cost of imported goods—both finished products as well as raw materials. This has prompted companies to rethink the need for a diversified global supply chain, particularly by adding an onshore/nearshore production location to the current offshore location. This chapter adopts a game-theoretic model to analyze a global firm's reshoring capacity, output quantity, and production decisions in the presence of domestic market competition. We account for uncertainties around market demand and import costs

at both the raw-material (RM) and finished-goods (FG) level. We show that an increase in the average RM cost will reduce reshoring capacity investment, but the impact of the FG cost is ambiguous. We identify two opposite effects in the increase of FG cost: (1) an overflow demand effect—present when demand is high and exceeds an overflow threshold, and unsatisfied production must overflow to the offshore location. This effect encourages more reshoring capacity investment when the FG cost increases; and (2) an output quantity effect that leads to a reduction of the reshoring investment due to a higher expected unit cost of production. The direction of change in the reshoring investment depends on the dominant effect among the above two. We also find that when the cost disadvantage of onshore relative to offshore sourcing is large (small), the presence of domestic competition can lead to more (less) reshoring. Although higher import costs hurt the global firm's profit, the domestic competitor can sometimes benefit. Our research shows that reshoring some manufacturing back to the home country can provide operational flexibility and increase a global firm's competitiveness in an uncertain environment. The reshoring investment decision depends critically on which type (RM or FG) of imports is most affected by industrial and trade policies, and the intensity of competition in the domestic market.

An ongoing follow-up work in the second Chapter asks the question: What if the uncertainties are rooted in the supply chain structure itself? How should a global firm determine his sourcing strategy facing demand-side and supply-side uncertainty? **Chapter 2 "Sourcing Strategy under Demand Uncertainty and Supply Disruption"** considers a global firm trying to serve country U's domestic market through offshoring production in a foreign country C. Due to the COVID-19 pandemic, international trade tensions, wars, typhoons, or other unexpected disruptions, container ships get stuck outside congested ports. It leads to the uncertain time between placing an order and actually receiving the order, i.e., supply disruption. In addition, demand uncertainty usually comes along with and is even more prevalent than supply disruption. To hedge against such risks, the global firm can install domestic inventory, or source and produce locally. The former strategy requires establishment costs to set inventory. A pretty natural question we would like to ask is: What is the global firm's best sourcing strategy? Specifically, under what condition should the global firm keep offshoring? We find that Only inventory cost matters when there is no uncertainty at all. With the presence of demand uncertainty, the firm is less willing to offshore when he has flexibility. He could be more or less willing to offshore without flexibility. Another question is how supply disruption affects the global firm's sourcing strategy. The answer is that supply disruption increases the offshore inventory for low inventory prices, and decreases the inventory for high prices. In addition, we define "Strategic capacity" as the expectation of the output quantity that is locally sourced and produced. Strategic capacity partially accounts for the inventory difference due to the global firm's flexibility.

The third chapter studies the influence of technology innovation on a traditional agricultural supply chain. Specifically, how farmers' purchasing behavior and the firm's pricing strategy respond with respect to emerging agricultural drones. **Chapter 3 "Selling Agri-Tech Products: Firm Strategy, Farmer Incentives, and Government Subsidy"** notices with the development of technology, there are many emerging agri-technology products that can help with improving output. However, new products may be expensive or hard to use. We study the impact of agri-tech product adoption, like agricultural drones, on the traditional agriculture supply chain. Farmers' purchasing strategies, the firm's pricing decisions, and government subsidy schemes are considered. Since it requires a high capability of farmers to use the product properly, apart from selling agri-tech products, the firm may also sell professional services to help farmers. We find the best pricing strategy for the firm is to achieve either complete bundle selling or no bundle selling at all. And firm can free-ride farmers' high capability and gain more profits. In addition, four subsidy schemes are considered. Per-unit purchasing and per-unit selling subsidies are equivalent and are both dominated by the service subsidy if the government's budget is adequate. Output subsidy can be the best scheme if the government cares more for farmers.

Chapter 1

Flexibility Value of Reshoring Capacity under Import Cost Uncertainty and Domestic Competition

1.1 Introduction

Companies that operate global supply chains, especially those that source from and/or manufacture at offshore locations, can face significant uncertainties in the total landed cost of imported goods. The total landed cost includes not only the sourcing or production cost at the origin, but also logistics costs associated with shipping the goods to the domestic market, and customs duties and tariffs that are collected when goods cross national borders. In recent years, the trade frictions between the U.S. and other regions, the pandemic crisis, and the transformation to socially and environmentally responsible operations have all impacted the total landed cost of imported goods.

Since 2012, the U.S. has imposed high tariffs on solar cells and panels [11]. Starting in 2018, the U.S. imposed additional tariffs on raw materials like steel and aluminum and on a wide range of parts, components, and finished goods imported from China [49]. These steep tariffs intended to make imported materials and goods more expensive than equivalent goods produced domestically and help move American companies overseas back to the U.S. The Reshoring Initiative reported that tariffs had indeed surged as one of the most significant positive factors that drive reshoring [47]. Many of the previous administration's China tariffs are still maintained by the current administration [48].

In addition to tariffs, recent industrial policies in the U.S.—such as the CHIPS Act and the Inflation Reduction Act (IRA)—are also providing support for domestic manufacturing through subsidization. These policies can have implications for affected products. For example, for electric vehicle (EV) manufacturers with battery minerals sourced from the U.S., battery components manufactured, and final assembly done in the U.S., the IRA provides a $7,500 tax credit for consumers who purchase these EVs in the U.S. For car manufacturers that source and produce elsewhere and their products do not qualify for the tax credit, it is equivalent to their "total landed cost" in consumers' perception having increased by the tax credit amount [43]. The IRA also offers tax credits for solar developers that assemble solar panels in the U.S. [46].

Global shipping costs have gone through roller-coaster years since the pandemic. Cost per container increased by ninefold during the pandemic. Although trans-Pacific shipping rates had plummeted 75% from the peak post-pandemic, the shipping cost uncertainty persists. In April 2023, the average spot market price from Asia to the U.S. West Coast jumped 34%, which added planning uncertainty for the importers who had to deal with both uncertainty in economic conditions and shipping rates [6].

In a fragile environment with hard-to-forecast demand and supply shocks, and unstable geopolitics, multinational manufacturers need to have diversified global supply chains—being global and local at the same time. The local production capability, either onshore or nearshore, allows firms to mitigate various risks, such as country-specific tariffs and cost increases that affect the supply to the domestic market. For example, Emerson and Henkel have increased their share of production in Mexico and Eastern Europe to supply North America and the European market, with their Chinese facilities covering the remaining balance [15]. Mattel has shifted some of its production out of Asia to Mexico [22]. Global solar developers like Enel started building a massive solar panel factory in the U.S. to take advantage of the IRA tax credit [21]. Cisco increased its nearshoring investments in Mexican manufacturing facilities in 2016 [3]. Having a reasonably distributed supply chain gives the company flexibility to move production around in response to realized market and demand conditions. Consequently, the impact of the tariff policy on Cisco was minimal [59]. This flexibility value of reshoring or nearshoring is often undervalued in the presence of uncertain import costs and hard-to-predict demand conditions.

Despite these benefits of reshoring, we see companies still resisting drastic changes. There are several challenges to reshoring or nearshoring. One challenge is the significant cost of building new facilities and the time in establishing a new supply ecosystem, as in the case of Mattel, which has been in Mexico for years but still relies on Asian suppliers for certain parts [50]. High costs for local labor, supply, and energy are another challenge for reshoring. Another concern is the uncertain business environment and market competition. The growing reshoring trend makes companies more likely to face competitors that have already made such investments at home. Furthermore, possible actions on future trade and environmental policies and impacts of global supply shocks remain ambiguous.

The above industry observations demonstrate the importance of deciding not just whether to reshore, but the level of reshoring capacity investment in the face of import cost uncertainty and competition in the domestic market. In our analysis, we take the perspective of a global firm that already has facilities in a low-cost country but faces uncertainties in the total landed cost of serving the domestic market from these facilities (in our model, "domestic market" is interpreted as the market that tariffs, trade, and other industrial policies increase the cost of material and product imported into it; "reshoring" is interpreted as a firm increasing its manufacturing investment in the "domestic market"). We aim to provide answers to the following research questions:

- How do uncertain import costs affect the global firm's reshoring investment and profitability in serving the domestic market? How is the domestic competitor affected?

- What is the differential impact of raw-material (RM) versus finished-goods (FG) cost on the reshoring investment?
- Does competition in the domestic market hinder or encourage the reshoring investment?

We develop a three-stage model reflecting the global firm's operational decisions in different phases: a long-term decision on the domestic capacity to install, a medium-term decision on the desired output target based on the observed market demand and competition, and finally, a short-term sourcing and production decision based on the realized RM and FG import costs. The domestic capacity, if installed, provides sourcing and production flexibility for the firm. Given the observed import cost realization, the firm may meet the output target by sourcing and producing in the existing offshore location, producing in the domestic facility using local input, or producing in the domestic facility using imported input. To examine how domestic competition affects the reshoring decision, we consider a competition model where the global firm engages in an output quantity competition with a domestic manufacturer and compare the results with a single-firm model.

We show that the optimal reshoring decision follows a threshold policy defined by a capacity cost threshold above which reshoring becomes infeasible. The optimal reshoring capacity investment depends critically on two defined costs: the expected offshoring cost (when sourcing and producing in the low-cost country) and the expected global optimal cost (when the sourcing and production decisions optimize the use of the global supply chain).

We analyze the impact of import costs on reshoring investment and profitability. We find that an increased RM import cost does not affect the offshoring cost but leads to an increased expected global optimal cost (when reshoring capacity uses imported raw materials). Therefore, an increased RM import cost dampens the incentives for reshoring, leading to a lower reshoring threshold and capacity level. FG import cost, however, increases both the expected offshoring cost and the global optimal cost. These increases have opposite effects. On the one hand, an increased global optimal cost means a higher average unit production cost for serving the market, which leads to a lower output quantity and less need for reshoring capacity. We refer to this effect as the *output quantity effect*. However, when demand is high and exceeds an "overflow threshold", the global firm has to overflow excess production offshore. An increased offshoring cost makes offshore production more expensive and thus encourages more reshoring investment. This effect is referred to as the *overflow demand effect*. Whether a higher FG import cost leads to more or less reshoring capacity investment depends on the dominant effect.

The effect of competition on the reshoring decision has not undergone adequate investigation in previous literature. We show that when the input cost in the home country is relatively low, reshoring will lead to head-to-head competition with an efficient domestic competitor. In this case, as common wisdom suggests, the existence of domestic competition dampens the global firm's reshoring incentive. However, this intuition requires fine-tuning when the input cost in the home country is relatively high. We find that domestic competition may induce more reshoring capacity investment. The global firm may leverage its added sourcing and production flexibility due to the installment of a domestic facility. For moderate capacity costs, the reshoring capacity not only protects the global firm from increasing import costs but also keeps domestic competitors out of the market over a large range of demand realization. Consequently, the global firm has a lower expected cost and increased incentives to set a higher capacity level than the monopoly case. Our work argues convincingly that, for certain instances, domestic competition may encourage reshoring investment.

We also find that the increasing import costs indirectly affect the domestic manufacturer. As the import cost increases, although the global firm's profit always decreases, the domestic manufacturer may earn a higher (lower) profit when the global firm reduces (increases) its investment in reshoring capacity due to the dominance of the previously described output quantity (overflow demand) effect. A lower (higher) reshoring investment weakens (strengthens) the global firm's competitive position. In certain cases, a highly competitive global firm with added reshoring capacity may significantly increase its output, eventually leading to a higher industry output to the domestic market.

The rest of this paper is organized as follows. We review the relevant literature in section 1.2. The model formulation is presented in section 1.3. In section 1.4, we analyze the global firm's optimal solution to the three-stage game in a single-firm model and derive the equilibrium in the competition model. Equipped with the optimal solutions, we investigate the effect of import costs on reshoring capacity, output quantity, and profits in section 1.5. In section 1.6, we examine how competition affects the incentives for reshoring. Finally, we conclude in section 1.7.

1.2 Related Literature

Our paper is closely related to the global supply chain configuration literature. An important aspect of companies' global supply chain strategy is where to locate production/warehouse facilities and whether to source products from a foreign country or a domestic location in an uncertain environment. For example, [42] considers a two-stage production system in which the upstream stage produces raw materials (RM), and the downstream stage performs the final assembly of finished goods (FG). While RM can be produced in domestic and foreign markets and transshipped between markets, FG production only localizes to every market. The studied decision includes the locations of the RM facilities and the capacity levels for the RM and FG facilities. The modeled uncertainty is demand uncertainty in every market, with prices ex-ante set. They show that when transshipment costs are low, centralized configurations (with RM produced in only one market) are preferred. Market-focused configurations (with RM produced in each market) are preferred for high transshipment costs. Along with a similar modeling approach, [20] endogenizes prices in a responsive pricing Newsvendor network under both demand and exchange rate uncertainties. They only deal with RM capacity investment, while FG facilities are assumed to have ample capacities. One could interpret their results as follows: an increase in the "inbound transshipment cost" to the domestic market favors market-focused network configuration and the reshoring of material sourcing activities. Our paper accounts for uncertainties in both the demand and costs of RM and FG. We model the created "real option" for a global firm's existing facilities in an offshore location through capacity investment in the domestic market. The flexibility of such an added option in the presence of domestic market competition offers newly discovered and somewhat counter to prevailing insights on how RM and FG import costs and competition affect reshoring investment.

Several studies in the global supply chain literature do capture supply uncertainties in the form of random yield [34], production cost uncertainty [52], or exchange rate uncertainty [32, 20]. These studies consider cost uncertainty at either the RM or FG level. The explicit modeling of cost uncertainty at both the RM and FG levels differentiates our work from those. Trade policies, for instance, as considered in our paper, affect the cost of both imported RM and FG, and the import cost increases may be different for RM and FG. For companies that have production facilities in different countries and must decide where to source RM and where to conduct FG production, it is important to consider the relevant costs at both RM and FG levels.

Although our modeling of import cost uncertainty is not limited to tariffs, the results from our paper can help us understand the effect of recent tariff policies. Therefore, our research also contributes to the economics and Operations Management (OM) literature on tax and tariff policies. The economics literature that studies the latest tariff policies mainly focuses on the impact of tariffs on macroeconomic indicators such as prices, quantities of imports and exports, and welfare [2, 23, 27]. The recent paper by [12] empirically examines the impact of trade policy uncertainty on the supply chain networks of American firms. Operational details such as capacity investment and sourcing are not explicitly modeled. Complementary to the economics literature are OM studies that assess international tax policy implications at the firm level, including global firms' supply chain strategies under local content tariff rules [39], supply chain structure, and procurement strategy under China's export-oriented tax rules [30, 62], and capacity, sourcing, and pricing strategies when tax rates are different across operating countries [53, 61, 29].

Two recent papers in this literature compare reshoring and offshoring under tariffs or customs duties. Assuming deterministic demand and tariffs, [63] analyzes Cournot competition between a domestic firm and a multinational firm that can produce onshore or offshore. Their results support the government policy that imposing high tariffs will induce reshoring. In a monopolistic model with demand uncertainty, [14] compares offshore and reshore of finished goods production with offshore sourced components. They find that reshoring becomes less appealing when customs duties for components increase or those for finished goods decrease. These studies assume unlimited reshoring capacity to produce the desired output. We explicitly model the reshoring capacity decision, and the flexibility of allocating production between offshore and onshore. Our model captures the "real option" value of a global firm's partial reshoring capacity investment in the presence of industrial and trade policy uncertainties.

Moreover, the above studies that explicitly model tariffs assume that tariff rates are deterministic. Our paper models the uncertainty of import costs, not only because of tariffs but also shipping costs, tax credits, and other factors affecting them. A recent global field study [15, 16] among leading manufacturers indicates that trade and industrial policies, and the uncertainty around them, have gained more weight in influencing companies' global supply chain strategies. As part of their think piece on the impact of tariffs on the global supply chain, [19] use simple models to highlight—but not analyze in detail—the implication of tariff policy uncertainty, product portfolio flexibility, and competition for firms' global supply chain design decisions. However, the literature lacks a comprehensive model that captures critical operational details on reshoring capacity investment in the presence of tariff (or more generally, import cost) uncertainty and domestic competition. Our paper addresses this gap.

1.3 Model

Consider a global firm that currently serves the domestic market in country U through offshore production in a low-cost country C. Producing one unit of FG requires one unit of input. We denote the baseline FG cost as w_C per unit, which includes the current sourcing and production cost in country C. Without loss of generality, we normalize the current unit production cost in country C to 0. Due to the newly imposed tariffs on imported finished products, increasing shipping costs, the opportunity cost of tax credits, and sustainable operations-related costs in country C, the total landed cost of imported FG will become $w_C(1+t_F)$, where $t_F > 0$ represents the additional cost expressed as a percentage of the merchandise's value (e.g., ad valorem tariff rates). Assume the marginal *pdf*, *cdf*, and mean of t_F as $f_1(\cdot)$, $F_1(\cdot)$, and μ_F, respectively.

To hedge against the increasing FG cost, the global firm has the option of installing domestic capacity and sourcing and/or producing locally. Denote the unit capacity installment cost as C_U (which includes the amortized variable cost for installing capacity) and the capacity to be installed as K_U. When RM is sourced from country C, as for FG, the imported RM may incur additional costs. Denote the total landed cost of RM as $w_C(1+t_R)$, where $t_R > 0$ represents the additional RM cost expressed as a percentage of the RM value. Assume the marginal *pdf*, *cdf*, and mean of t_R as $f_2(\cdot)$, $F_2(\cdot)$, and μ_R, respectively. RM can also be sourced locally from country U, with a cost w_U. We assume $w_U > w_C$ to reflect the reality that country C used to be a low-cost country. When FG is produced locally in country U, the firm incurs a higher unit production cost than in country C (which is normalized to 0). The production cost difference is denoted as δ ($\delta > 0$). To summarize, with production facilities in both countries, the global firm has the following three sourcing and production options ([14] considers similar reshoring structures):

- Offshore sourcing and production: that is, produce FG in country C and import to country U; the total landed cost of FG is $w_C(1+t_F)$.
- Offshore sourcing and onshore production: that is, source RM from country C and produce FG in country U; the cost of FG is $w_C(1+t_R)+\delta$.
- Onshore sourcing and production: that is, source and produce locally in country U; the cost of FG is $w_U+\delta$ (for convenience, we define $w'_U := w_U+\delta$).

In the domestic market, the global firm faces competition from a domestic manufacturer, referred to as firm L. Firm L sources and operates only in country U. Like the global firm, firm L incurs the total unit cost w'_U. We consider a Cournot competition between the two firms. Assuming that the products of the two firms are perfect substitutes, we define the market price by a linear inverse demand function $p = \epsilon - (Q + Q_L)$, where ϵ denotes the random market size, and Q and Q_L denote the output quantity of the global firm and firm L, respectively. Assume that ϵ is distributed according to *pdf* $g(\cdot)$ and *cdf* $G(\cdot)$. To focus on the global firm's reshoring capacity decision, we assume that both firms do not face any capacity constraints in their current facility—that is, the global firm's offshore facility and firm L's domestic facility. The assumption implies that historically the firms have successfully supplied their markets, and these capacity investments are sunk.

With the option of investing in the domestic capacity, the global firm faces a series of decisions. First, there is the capacity investment decision in K_U units of domestic capacity. Capacity investments are long-term irreversible nature decisions made ex-ante to demand and trade uncertainties [18]. Second, the firm makes medium-term planning decisions in setting output quantity Q (and the price) for the market based on demand forecast and market competition. Such planning allows the firm to put in place needed auxiliary assets (e.g., supply contracts, optional subcontractor capacity, shipping and port contracts, etc.) Finally, in a pure ex-post mode after realization of all uncertainties, the firm optimizes the efficiency of its delivery to the market, deciding how to deploy its global facility network and other planned assets in a way that fully accounts for supply constraints, available trade incentives, and the application of specific tariff policies to its products. Since firm L has invested capacity in place, it only faces medium-term planning and short-term execution decisions. We assume that firm L makes the output quantity decision at the same time as the global firm (allowing the local firm to postpone output decisions to a later time would give the global firm Stackelberg leadership advantage in the output game [31]; to isolate this effect from the flexibility value we focus on in this research, we assume simultaneous decisions for a fair treatment). Given the absence of any future uncertainty realization for firm L, its short-term execution is straightforward.

We assume that once both firms set their output target, they do not adjust it in the short term. As often seen in the news, firm performance, e.g., stock price, is closely tied to whether or not the set production target is achieved. Being able to maintain the set production target can give investors confidence in the company and send the stock price soaring, while companies' shares often plunge when missing their targets [54, 40]. However, the global firm has the flexibility to reallocate the total production quantity in its global production network based on the realization of short-term economic parameters. Cisco, for example, was able to cut back manufacturing in China by utilizing a production network in 13 countries when a 25% tariff was levied on imports from China. By doing so, Cisco greatly reduced its trade war exposure and had a performance in line with the near-term forecast [59].

Based on the nature of the three decisions, we formulate the following three-stage game, as illustrated in Figure 1.1:

- Stage 1: The global firm decides on the domestic capacity size K_U to maximize its expected profit: $\Pi = \max_{K_U \geq 0} \; R(K_U) - C_U K_U$, where $R(K_U) = E_\epsilon[\pi(\epsilon, K_U)]$ denotes the global firm's long-term average profit over all possible market conditions ϵ and $\pi(\epsilon, K_U)$ is the realized profit for a given ϵ (defined below).

- Stage 2: After demand realization, the global firm and firm L set the output quantity Q and Q_L simultaneously. For a given ϵ, firm L solves the profit maximization problem: $\max_{Q_L \geq 0}[(\epsilon - (Q + Q_L)) - w'_U]Q_L$. The global firm maximizes its medium-term expected operating profit: $\pi(\epsilon, K_U) = \max_{Q \geq 0} \; (\epsilon - (Q + Q_L))Q - EC(Q, K_U)$, where $EC(Q, K_U) = E_{t_F, t_R}[TC(Q, K_U, t_F, t_R)]$ denotes the expected cost of producing the output quantity Q for the market.

- Stage 3: After import cost realizations, the global firm decides on the production quantities q_C, q_{UC}, and q_{UU}, where q_C denotes the sourcing and production quantity in country C. q_{UC} denotes the quantity produced in country U using imported raw materials from country C. q_{UU} denotes the quantity produced in country U using locally sourced raw materials. The global firm solves the following cost minimization problem:

$$
\begin{aligned}
TC(Q, K_U, t_F, t_R) = \min_{q_C, q_{UC}, q_{UU} \geq 0} & \; w_C(1 + t_F)q_C + [w_C(1 + t_R) + \delta]q_{UC} + w'_U q_{UU} \\
\text{s.t.} \quad & q_C + q_{UC} + q_{UU} = Q \\
& q_{UC} + q_{UU} \leq K_U.
\end{aligned}
\tag{1.1}
$$

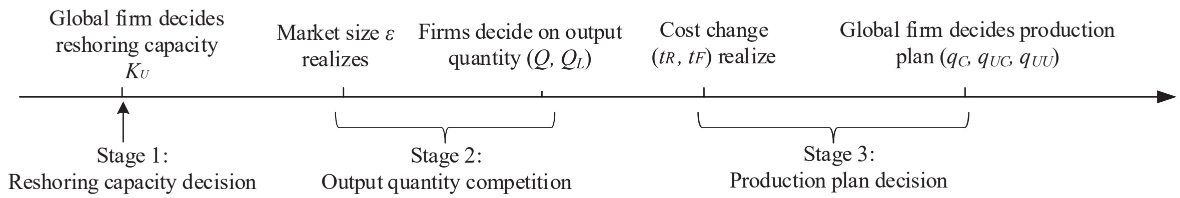


Figure 1.1: Sequence of Events

In the solar panel industry, we have seen the implementation of the described sequential strategic decisions. Global firms, like Meyer Burger, in 2021, in response to the U.S. government's announcement in support of clean energy, but prior to any realization of economic condition and specific trade policies, pursued investment in the U.S. solar panel capacity [45]. Then, accounting for existing local competitor (e.g., American solar panel manufacturer First Solar) capacity and the strong demand for solar installation due to the tax credit incentives, global firms are setting medium-term (2023–2025) output targets and prices for these products [21]. Finally, short-term production allocation decisions for global firms have to be postponed to the latest possible time due to the execution specifics of trade policies (e.g., how tax credits will be allocated among participants in the solar supply chain, whether suspended tariffs will be reinstated, etc.).

Assumption 1.3.1 *t_F and t_R are subject to an upper bound $\bar{t}$.*

Assumption 1.3.2 *$w_C < w'_U < w_C(1+\bar{t})$.*

These assumptions are imposed to avoid uninteresting cases in the analysis and are reasonable. For example, textiles imported to the U.S. in 2020 faced a maximum tariff of 40% [60].

Assumption 2 states that before the import cost increases, offshore sourcing and production are cheaper than doing those onshore. However, when the import costs are too high, moving those activities to country U may save on costs (otherwise, the global firm has no incentive to reshore, and the problem is trivial). Throughout this paper, we use the superscripts m and d to denote expressions pertaining to the single-firm (monopoly) and competition (duopoly) model, respectively.

1.4 Analysis

1.4.1 The Single-Firm Model

Before analyzing the competition model, we first consider a benchmark case where the global firm is a monopolist in the domestic market. The three-stage game is modified, with only the global firm making the output quantity decision in stage 2. We analyze the three-stage game using the standard backward approach. Starting from stage 3, we see that problem (1.1) is a linear program. The optimal sourcing and production decisions depend on the comparison among the three costs, $w_C(1+t_F)$, $w_C(1+t_R)+\delta$, and w'_U.

Lemma 1.4.1 *For given $K_U, Q, \epsilon, t_R, t_F$, the optimal sourcing and production decisions are:*

Regions	$(q_C^*, q_{UC}^*, q_{UU}^*)$
Ω_1: $t_R \in (0, \bar{t})$ and $t_F \in \left(0, \min\{t_R + \frac{\delta}{w_C}, \frac{w_U' - w_C}{w_C}\}\right)$	$(Q, 0, 0)$
Ω_2: $t_R \in (0, \frac{w_U - w_C}{w_C})$ and $t_F \in \left(t_R + \frac{\delta}{w_C}, \bar{t}\right)$	$((Q - K_U)^+, \min\{Q, K_U\}, 0)$
Ω_3: $t_R \in (\frac{w_U - w_C}{w_C}, \bar{t})$ and $t_F \in \left(\frac{w_U' - w_C}{w_C}, \bar{t}\right)$	$((Q - K_U)^+, 0, \min\{Q, K_U\})$

Table 1.1: Optimal Sourcing and Production Decisions in Stage 3

Figure 1.2(i) illustrates the result in Lemma 1.4.1. In the region Ω_1, t_F is relatively small; offshore sourcing and production provide the lowest cost. Similarly, in the region Ω_2, t_R is relatively small while t_F is relatively large; it is optimal to produce in country U using imported RM from country C. If the output quantity Q is larger than the available capacity K_U, the remaining quantity will be satisfied by the production in country C. Finally, in the region Ω_3, when imported costs for both RM and FG turn out to be high, domestic sourcing and production become the preferred choice. Again, production in country C will satisfy any quantity beyond the domestic capacity. This result reflects the global firm's *sourcing and production flexibility* when operating a global production network with both foreign and domestic production capabilities. Such a network enables the global firm to optimally utilize its production facility in different regions in response to import cost changes, like what Cisco did in response to the rising tariff.

Taking into account the optimal sourcing and production decision for all possible import cost realizations, the global firm then determines the output target in stage 2 based on the observed market size ϵ. Using Lemma 1.4.1, we can express the expected cost of producing

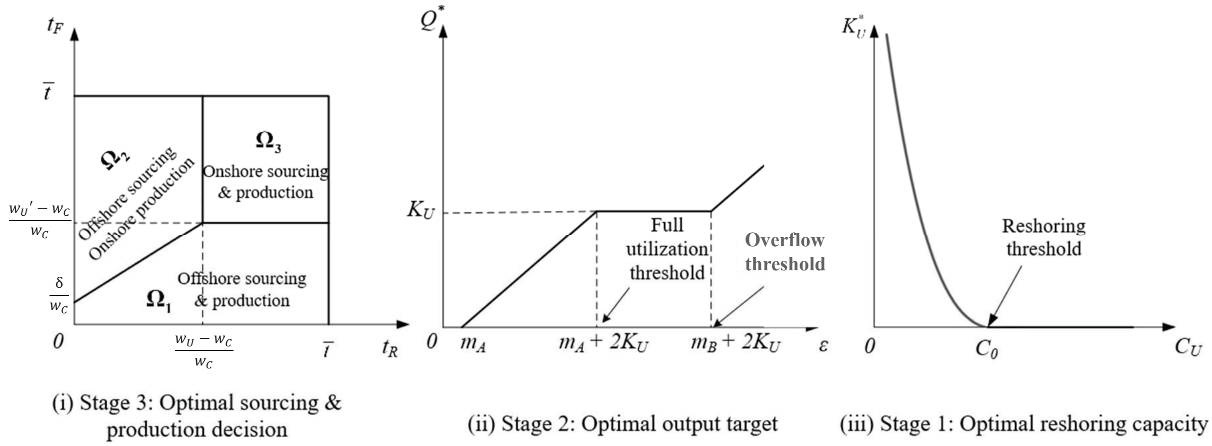


Figure 1.2: Optimal Solution in the Single-Firm Model

an output quantity Q as follows:

$$EC(Q, K_U) = E_{t_F,t_R}[TC(Q, K_U, t_F, t_R)] = \begin{cases} m_A \cdot Q, & \text{if } 0 \le Q \le K_U \\ m_A \cdot K_U + m_B \cdot (Q - K_U), & \text{if } Q > K_U, \end{cases} \tag{1.2}$$

where $m_A = E_{t_F,t_R}[\min\{w_C(1+t_F), w_C(1+t_R)+\delta, w_U'\}]$ represents the expected unit output cost when optimally utilizing the global production network, referred to as the (expected) *global optimal cost* in short hereafter, and $m_B = E_{t_F}[w_C(1+t_F)]$ represents the expected unit output cost when always producing in the offshoring country C, referred to as the (expected) *offshoring cost* (for brevity, when not confusing, we drop "expected" when referring to these costs hereafter). Clearly, we have $m_A < m_B$. Lemma 1.4.2 states the solution to the stage 2 problem, $\max_{Q\geq 0} (\epsilon - Q)Q - EC(Q, K_U)$.

Lemma 1.4.2 *The objective function in the stage 2 problem is continuous and concave in Q. The optimal output target Q^m can be characterized as the following:*

$$Q^m = \begin{cases} 0, & \text{if } \epsilon \in [0, m_A), \\ \frac{\epsilon - m_A}{2}, & \text{if } \epsilon \in [m_A, m_A + 2K_U), \\ K_U, & \text{if } \epsilon \in [m_A + 2K_U, m_B + 2K_U), \\ \frac{\epsilon - m_B}{2}, & \text{if } \epsilon \in [m_B + 2K_U, \infty). \end{cases}$$

Figure 1.2(ii) shows that the optimal output quantity is piecewise linear in ϵ. Production is not profitable when the market size is too small (i.e., $\epsilon < m_A$). When $m_A \le \epsilon < m_A + 2K_U$, the global firm optimally utilizes its production facilities in the two countries, and the domestic capacity K_U can fully satisfy the required production needs. As the market size increases (i.e., $\epsilon > m_B + 2K_U$), the domestic capacity will fall short when the global firm favors production in country U. In this case, any remaining output quantity that cannot be satisfied by K_U requires fulfilling by the capacity in country C. We define the two threshold values $m_A + 2K_U$ and $m_B + 2K_U$ as the *full-utilization threshold* and *overflow threshold*, respectively. Notice that when the market size is between $m_A + 2K_U$ and $m_B + 2K_U$, the optimal output target remains at a constant level K_U. The global firm does not raise its output target beyond K_U when the market size increases, because a higher output target would require utilizing the production facility in country C. Since the offshoring cost exceeds the global optimal cost, meeting excess domestic demand through offshoring does not prove profitable for these demand realizations. Only when the market size reaches a sufficiently large size (i.e., $\epsilon > m_B + 2K_U$) does overflowing production to country C become profitable.

Next, we analyze the global firm's capacity decision problem in stage 1: $\max_{K_U \geq 0} R(K_U) - C_U K_U$, where

$$R(K_U) = E_\epsilon[\pi(\epsilon, K_U)] = \int_{m_A}^{m_A+2K_U} \frac{(\epsilon - m_A)^2}{4} dG(\epsilon) + \int_{m_A+2K_U}^{m_B+2K_U} \left[-K_U^2 + (\epsilon - m_A)K_U\right] dG(\epsilon)$$
$$+ \int_{m_B+2K_U}^{\infty} \left[\frac{(\epsilon - m_B)^2}{4} + (m_B - m_A)K_U\right] dG(\epsilon). \tag{1.3}$$

The long-term average profit over all possible market conditions, $E_\epsilon[\pi(\epsilon, K_U)]$, follows from the optimal output target decision in Lemma 1.4.2. When the market size is smaller than the full-utilization threshold $m_A + 2K_U$, increasing K_U does not benefit the global firm since the capacity is never fully utilized. When ϵ is between the two thresholds, the firm's output target is constrained by the domestic capacity K_U. More domestic capacity implies more output to the market, thus leading to a higher profit. When ϵ is beyond the overflow threshold $m_B + 2K_U$, a unit increase in K_U will enable the firm to switch one unit of overflow production from country C back to country U. This can bring a cost saving of $(m_B - m_A)$. Following this, the optimal capacity decision is characterized below and illustrated in Figure 1.2(iii).

Proposition 1.4.1 *The stage 1 objective function is concave in K_U. There exists a threshold $C_0^m = (m_B - m_A) - \int_{m_A}^{m_B} G(\epsilon) d\epsilon$ such that the optimal capacity is $K_U^m = 0$ if $C_U \geq C_0^m$; otherwise,*

K_U^m *satisfies:*

$$FOC^m: \quad \int_{2K_U^m+m_A}^{2K_U^m+m_B} (\epsilon - 2K_U^m - m_A) dG(\epsilon) + \int_{2K_U^m+m_B}^{\infty} (m_B - m_A) dG(\epsilon) = C_U. \qquad (1.4)$$

We define the unit capacity cost threshold, C_0^m, as the *reshoring threshold*. It defines the feasible region for reshoring to happen. The higher (lower) C_0^m is, the more (less) likely the firm will reshore. The global firm only finds it profitable to install capacity in the home country when the capacity cost is not too high. The optimal capacity size increases as the capacity cost becomes cheaper, i.e., K_U^m decreases in C_U.

1.4.2 The Competition Model

First, notice that in stage 3, the global firm's problem is identical to the single-firm model, with the optimal solution given by Lemma 1.4.1 and the expected production cost $EC(Q, K_U)$ given by (1.2). Next, we derive the equilibrium of stage 2 Cournot competition. Firm L solves the profit maximization problem: $\max_{Q_L \geq 0} (\epsilon - (Q + Q_L)) Q_L - w_U' Q_L$. The best response function is $Q_L(Q) = \frac{\epsilon - Q - w_U'}{2}$. The global firm solves the profit maximization problem: $\max_{Q \geq 0} (\epsilon - (Q + Q_L)) Q - EC(Q, K_U)$. The best response function can be derived as the following:

$$Q(Q_L) = \begin{cases} \frac{\epsilon - m_A - Q_L}{2}, & \text{if } \frac{\epsilon - m_A - Q_L}{2} < K_U, \\ K_U, & \text{if } \frac{\epsilon - m_B - Q_L}{2} \leq K_U \leq \frac{\epsilon - m_A - Q_L}{2}, \\ \frac{\epsilon - m_B - Q_L}{2}, & \text{if } \frac{\epsilon - m_B - Q_L}{2} > K_U. \end{cases}$$

The equilibrium of the Cournot competition can be derived from the intersection of the two best response functions.

Proposition 1.4.2 *Figure 1.3 characterizes the equilibrium output quantities* (Q^d, Q_L^d) *in stage 2.* Q^d *and* Q_L^d *are represented by the solid black and blue lines, respectively; the quantities are shown in parentheses.*

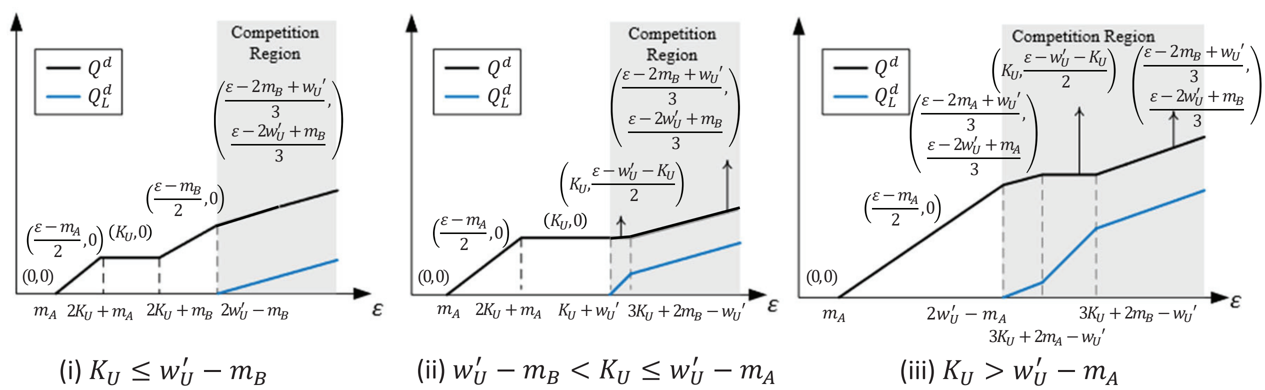


Figure 1.3: Output Quantity Equilibrium in Cournot Competition

The equilibrium result indicates that both firms will not find production profitable when the market size is too small. However, the two firms differ in their sourcing and production flexibility: The global firm can flexibly utilize the cheaper production site upon import cost realization, while the firm L has no such flexibility. Therefore, the global firm is more competitive with a lower average output cost than its domestic competitor. This is reflected in the equilibrium in that for a medium-sized market, only the global firm finds production profitable, and the firm L stays out of the market. In this case, production is not profitable for the firm L even if doing so in the domestic location is profitable for the global firm (i.e., onshore sourcing and production). Given the global firm's output target set in stage 2 (which is not adjusted in stage 3; see discussion in section 1.3), it can be verified that: when $Q_L^d = 0$, firm L will earn a negative profit if it produces a positive amount in stage 3. In this case, the global firm behaves like a monopolist. Its optimal output target is the same as Lemma 1.4.2.

The two firms actively engage in quantity competition only when the market size is large enough so that firm L enters (in the competition region shown in Figure 1.3). Once firm L joins the competition, the global firm must adjust its output target from the monopoly level. When and how it is adjusted depends critically on the capacity size K_U. When K_U is small (Figure 1.3 case (i)), in the presence of competition, the global firm will always exhaust its domestic capacity when producing in the home country and must overflow the remaining production to country C. The domestic competitor only competes for demands above the global firm's overflow threshold. For intermediate capacity size (Figure 1.3 case (ii)), the global firm's overflow threshold is above the smallest market size for the competitor to enter the competition. When competing with firm L, the global firm fully utilizes K_U when producing in country U and only overflows production to country C when the market size (hence the output target) is large. Finally, when the capacity size is large, the smallest market size that introduces competition falls below the full-utilization and overflow thresholds (Figure 1.3 case (iii)). The global firm faces competition even when its home country capacity K_U is not fully utilized.

Equipped with the stage 3 and stage 2 solutions, we next analyze the global firm's stage 1 capacity decision. We can verify that the global firm's stage 1 objective function is concave when $w_U' < 2m_B - m_A$ (or equivalently $w_U' - m_B < m_B - m_A$) holds. This condition implies that the cost saving of offshore sourcing and production compared to doing so onshore, $w_U' - m_B$, does not exceed the benefit of optimally utilizing the global production network, $m_B - m_A$. In other words, the home country cost w_U' is not too high. This assumption is reasonable given that we focus on situations where domestic industries remain reasonably competitive after the import cost increases so that reshoring is likely to be attractive to global firms. Meanwhile, we assume that the sourcing and production cost in the home country U is not too low, i.e., $w_U' > m_A$, to avoid the uninteresting case that domestic production is always preferred.

Proposition 1.4.3 *Assume that $m_A < w_U' < 2m_B - m_A$. Then the global firm's stage 1 objective function in the competition model, $R(K_U) - C_U K_U$, is concave in K_U, and the optimal capacity K_U^d satisfies the following characterizing equation (the thresholds C_0^d, C_1^d, C_2^d and first order conditions FOC_1^d, FOC_2^d are defined in Appendix):*

$w'_U \le m_B$	$w'_U > m_B$
$\begin{cases} FOC_1^d, & \text{if } C_U \in [0, C_2^d) \\ FOC_2^d, & \text{if } C_U \in [C_2^d, C_0^d) \\ 0, & \text{if } C_U \in [C_0^d, \infty) \end{cases}$	$\begin{cases} FOC_1^d, & \text{if } C_U \in [0, C_2^d) \\ FOC_2^d, & \text{if } C_U \in [C_2^d, C_1^d) \\ FOC^m, & \text{if } C_U \in [C_1^d, C_0^m) \\ 0, & \text{if } C_U \in [C_0^m, \infty) \end{cases}$

Table 1.2: Optimal Capacity Decision for Competition Model

The concavity of the objective function guarantees that the optimal reshoring capacity decision follows a structure similar to the single-firm model, except that the first-order condition now depends on which stage-2 equilibria are achieved. Specifically, we notice that when $w'_U > m_B$, the reshoring threshold C_0^m and the characterizing equation FOC^m when $C_U \ge C_1^d$ are identical to those of the single-firm model. In this case, the unit capacity cost is so high that the global firm only invests in a low level of domestic capacity. The resulting Cournot competition follows Proposition 1.4.2 case (i). Since competition only has an effect beyond the overflow threshold, the global firm's optimal capacity decision is the same as the single-firm model. Competition has no effect when making the trade-off between capacity utilization and overflow. A low capacity cost, i.e., $C_U < C_2^d$, means the installed capacity in country U is large. The resulting output quantity follows Proposition 1.4.2 case (iii). The corresponding characterizing equation is FOC_1^d. For an intermediate capacity cost, the resulting output quantity follows Proposition 1.4.2 case (ii). The corresponding characterizing equation is FOC_2^d.

1.5 Effect of Import Costs

This section answers the first two research questions: How do import costs affect the global firm's reshoring decision and profitability in serving the domestic market, as well as the output quantity and profitability of the domestic competitor? What is the differential impact of increased import cost at the raw-material versus finished-goods level on reshoring investment? For a clear understanding of these different costs, we assume in this section that the distributions for t_R and t_F are independent. As the distribution for one changes, the other is held fixed.

1.5.1 Impact on Reshoring Capacity

We first examine how the import cost magnitude affects the global firm's reshoring decision. We adopt the concept of deterministic (specifically, linear) transformation of a random variable [44] to model the change in the magnitude of uncertain import costs. We say t'_i ($i = R, F$) is an increasing deterministic transformation of t_i if t'_i is obtained by transforming every realization of t_i by an increasing amount (for simplicity, we assume the same positive amount hereafter). Therefore, the marginal *pdf* satisfies $f'_i(t_i + a) = f_i(t_i)$, $a > 0$. It is straightforward that t'_i has a larger mean than t_i. It also implies that t'_i first-order stochastically dominates t_i.

We first look at the RM import cost:

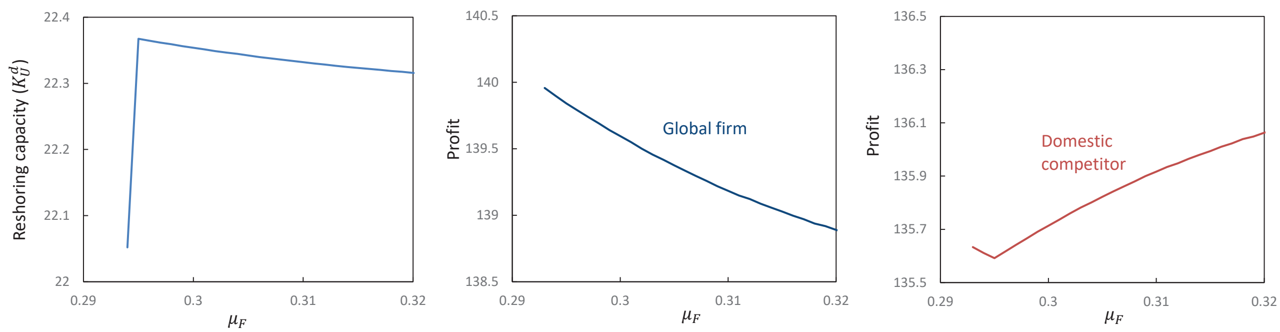


Figure 1.4: Impact of the FG Import Cost on Optimal Reshoring Capacity ($w_C = 10, w'_U = 13, C_U = 0.05, t_R \sim U[0.25, 0.35], t_F \sim U[\mu_F - 0.05, \mu_F + 0, 05], D \sim U[0, 80]$)

Proposition 1.5.1 *When t_R undergoes an increasing deterministic transformation, the global firm's reshoring threshold and optimal reshoring capacity both decrease, i.e., $\frac{\partial C_0^j}{\partial \mu_R} < 0$, $\frac{\partial K_U^j}{\partial \mu_R} < 0$, $(j = m, d)$.*

An increased RM import cost does not affect the offshoring cost as $m_B = E_{t_F}[w_C(1 + t_F)]$ is independent of t_R. However, a higher RM import cost does increase the global optimal cost, especially when utilizing the reshoring capacity using imported raw materials (when the optimal stage 3 decision is in Ω_2, i.e., offshore sourcing and onshore production). Therefore, as the RM import cost increases in magnitude, the global firm's incentive of reshoring reduces, reflected by a lower reshoring threshold and optimal reshoring capacity.

The FG import cost, however, affects both the expected offshoring and global optimal costs and can lead to different outcomes. Figure 1.4 illustrates an example in which an increased FG import cost can increase or decrease the optimal reshoring capacity.

Proposition 1.5.2 *When t_F undergoes an increasing deterministic transformation, there exist thresholds $\Theta^j_{K_U}$ (defined in Appendix A) such that $\frac{\partial K_U^j}{\partial \mu_F} > 0$ if $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} < \Theta^j_{K_U}$, and $\frac{\partial K_U^j}{\partial \mu_F} < 0$ otherwise, $(j = m, d)$.*

The impact of the FG import cost depends on how it affects the two costs m_A and m_B. Using the single-firm model as an example, we see that:

$$\frac{\partial K_U^m}{\partial \mu_F} = \frac{\frac{\partial m_B}{\partial \mu_F}\bar{G}(2K_U^m + m_B) - \frac{\partial m_A}{\partial \mu_F}\bar{G}(2K_U^m + m_A)}{2[G(m_B + 2K_U^m) - G(m_A + 2K_U^m)]}.$$

A higher average FG import cost increases both costs, i.e., $\frac{\partial m_A}{\partial \mu_F} \geq 0$, $\frac{\partial m_B}{\partial \mu_F} = w_C > 0$. However, the two expected costs m_A and m_B have opposite effects on the optimal reshoring capacity, as explained below.

First, an increased FG import cost makes offshore production more expensive and may make the global firm rely more on its domestic facility, a previously expensive option. The consequence is a higher unit cost of producing for the market, reflected by an increased m_A. This, in turn, will make the firm set a lower output quantity and reduce the overall capacity needs. Specifically, recall from Figure 1.2(ii) that when demand is above the full-utilization threshold, the output quantity is at least at the reshoring capacity level, i.e., $Q^* \geq K_U$. The total production cost is $m_A K_U + m_B(Q - K_U)$ based on (1.2). A higher m_A will thus increase the total production cost. As a result, the firm will have incentives to set a lower output quantity and, therefore has less need for reshoring capacity. We refer to this effect associated with m_A as the *output quantity effect*. Second, as the above total production cost shows, when demand is above the overflow threshold, the global firm must utilize not only its domestic capacity (when favorable) but also the offshore facility in country C. The overflowed quantity is produced at the higher offshoring cost m_B (recall that $m_B > m_A$). Therefore, a higher m_B would induce the global firm to invest in a larger domestic capacity, requiring it to overflow less production to country C. We refer to this effect associated with m_B as the *overflow demand effect.* The impact of FG import cost on reshoring investment depends on which of these two opposing effects dominates.

A relatively small increase in m_A, specifically when compared with the possibility of having production overflow to country C (given the reshoring capacity is fully utilized), i.e., when $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} < \frac{\bar{G}(2K_U^m+m_B)}{\bar{G}(2K_U^m+m_A)} =: \Theta_{K_U}^m$, means the overflow demand effect of m_B dominates the output quantity effect of m_A. As a result, it is optimal to increase the reshoring capacity. This outcome is what the recent U.S. trade policy intends to achieve. The reshoring capacity creates several benefits. First, increased reshoring capacity allows the global firm to avoid costly offshore production, which is beneficial when the FG tariff significantly increases the offshoring cost. Second, increased reshoring capacity may enable the global firm to increase its output target, especially when the output is constrained by the reshoring capacity.

However, when the condition $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} > \frac{\bar{G}(2K_U^m+m_B)}{\bar{G}(2K_U^m+m_A)}$ holds, the cost change in m_A is relatively large, and the possibility of having production overflow to country C is relatively small. As the cost m_A has an associated output quantity effect, as explained above, that reduces the capacity needs, the optimal reshoring capacity decreases as the FG import cost increases in magnitude. Similar insights hold in the competition model, although the threshold values differ.

Propositions 1.5.1 and 1.5.2 can be applied to provide possible explanations for why the trade policy set by the U.S. government has brought jobs back for some industries and companies but not for others. There are several contributing factors. First, the type of imposed tariff and which part of the supply chain gets affected make a difference. Although the FG tariff tends to result in more reshoring, the RM tariff has the opposite effect. This explains the slowdown in reshoring at the U.S. bicycle manufacturer Kent due to increased tariffs on its raw materials and components [5]. Also, in 2020–2021, we observed a slowdown in building solar assembly capacity as there were fears of potential tariffs on solar components sourcing from Southeast Asia. Later, the two-year tariff moratorium on such components led to the manufacturing capacity push by global solar developers. Second, imposing a higher FG tariff can lead to the unintended outcome of less reshoring, particularly when the tariff impact on the global optimal cost is relatively large and it is less likely to have a strong demand scenario with overflow possibility. Finally, market demand can be another influencing factor of the tariff effect. High demand means the overflow possibility is also high. Consequently, a higher FG tariff will likely drive more reshoring. For popular electronic products, cars, and heavy agriculture and construction equipment, this appears to be the case. Companies such as HP, Apple, GM, Ingersoll Rand, and Caterpillar are pursuing, to different extents, reshoring or nearshoring investment for products in these categories. This is also the case for renewable energy-related products (e.g., solar panels, batteries for electric vehicles). Projected strong demand for sustainable solutions and high-demand overflow possibility has led to aggressive plans for reshored manufacturing investments.

1.5.2 Impact on Expected Output

In this section, we will examine how import costs affect firms' expected output target evaluated at stage 1. For the RM import cost, as explained before, it only affects m_A but not m_B, and a higher cost leads to a lower reshoring capacity. It can be shown that both the full-utilization threshold and overflow threshold decrease in μ_R, too. Consequently, the global firm's expected output is lower. The domestic competitor can take advantage of this and increase its output. The total expected output to the market is lower under an increased RM import cost.

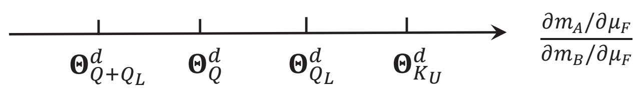
Proposition 1.5.3 *When t_R undergoes an increasing deterministic transformation, we have* $\frac{\partial E(Q^j)}{\partial \mu_R} < 0$ $(j = m, d)$, $\frac{\partial E(Q_L^d)}{\partial \mu_R} > 0$, *and* $\frac{\partial E(Q^d + Q_L^d)}{\partial \mu_R} < 0$.

Next, we look at the FG import cost. When the global firm operates as a monopolist, although a higher cost for imported FG can increase or decrease the reshoring capacity investment, it always leads to a lower expected output. Recall from Figure 1.2(ii) that the global firm only finds the business profitable when demand is higher than m_A. Since m_A increases in μ_F, a higher cost for imported FG reduces the profitability region. Although the global firm may invest more in the reshoring capacity to mitigate the negative impact of FG import cost increase, the firm finds the business less attractive and, thus, on average, outputs less to the market. This is not necessarily the case when competition exists, as stated in the following result:

Proposition 1.5.4 *When t_F undergoes an increasing deterministic transformation:*

(i) In the single-firm model, we have $\frac{\partial E(Q^m)}{\partial \mu_F} < 0$*;*

(ii) In the competition model, there exist thresholds Θ^d_Q, $\Theta^d_{Q_L}$, $\Theta^d_{Q+Q_L}$, *and* $\Theta^d_{K_U}$ *(defined in Appendix A) with* $\Theta^d_{Q+Q_L} < \Theta^d_Q < \Theta^d_{Q_L} < \Theta^d_{K_U}$ *such that the results in the following table hold:*



$\frac{\partial K^d_U}{\partial \mu_F}$	+	+	+	+	−
$\frac{\partial E(Q^d)}{\partial \mu_F}$	+	+	−	−	−
$\frac{\partial E(Q^d_L)}{\partial \mu_F}$	−	−	−	+	+
$\frac{\partial E(Q^d + Q^d_L)}{\partial \mu_F}$	+	−	−	−	−

Several observations can be made in the competition model. First, an immediate comparison to the single-firm model indicates that the global firm's expected output can increase in the FG import cost when facing competition. This occurs when the ratio $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F}$ is relatively small (specifically, lower than Θ^d_Q). The definitions of m_A and m_B suggest that $\frac{\partial m_A}{\partial \mu_F}$ tends to be small relative to $\frac{\partial m_B}{\partial \mu_F}$ when $w_C(1+t_R)+\delta$ or w'_U is smaller than $w_C(1+t_F)$ for most cost realizations. In other words, producing in the domestic facility (using the raw material from either country C or country U) tends to be more cost-effective. With the sourcing flexibility through the added reshoring capacity, the global firm has a competitive advantage over its domestic competitor. This allows the global firm to increase its expected output, supported by a significant increase in its reshoring capacity, while the local competitor has to reduce its output. Second, the global firm's increase in output may lead to the total industry outputting more to the market as a result. Finally, as the ratio $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F}$ becomes larger, the previously explained output quantity effect dominates. The global firm will lower its expected output, while the local competitor will take advantage and increase its expected output.

The above theoretical results on production quantities are largely in line with the empirical findings. A recent study conducted by the United States International Trade Commission [57] shows that the Section 232 tariffs on imported steel and aluminum had negatively impacted downstream industries that depend on these raw materials. Domestic production in the most affected industries decreased by 0.6% per year on average, with the largest annual decrease of 3.2% in 2018 in the cutlery and hand tool manufacturing industry. This finding supports our result in Proposition 1.5.3 that a higher cost for imported RM will reduce the expected industry output.

The study also analyzes the impact of Section 301 tariffs on imported goods from China. For the industries with the highest value of imports covered by the tariffs, the study finds that the value of U.S. production rose between 1.2% and 7.5% in 2021 as a result of the tariffs; meanwhile, U.S. imports from China declined by 13%. Using their estimated tariff impact, and the trade and production data, we calculate the impact of the tariffs on the combined output, i.e., the sum of domestic U.S. gross output and U.S. imports from China (the details are included in Appendix B). We find that Section 301 tariffs had negatively affected the total output in the studied industries. Although we do not observe any instance of increased total output—a possible outcome suggested by Proposition 1.5.4—we do find that industries with a larger portion of production done in the U.S. (vs. China) tend to have a much smaller decrease in total output (e.g., in the plastic products industry, the ratio of average domestic output to average imports from China is 11.65, and the tariffs decreased the total output by −0.23% on average; while in the audio and video equipment industry, the domestic-production-to-China-import ratio is 0.39, and the tariff effect is −17.7%). This finding partially supports our result that the global firm may increase its expected output when the ratio $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F}$ is relatively small (likely to occur when domestic production is cost-effective as discussed above). The increase in the global firm's output offsets (or can even dominate) the decrease in the domestic firm's output.

1.5.3 Impact on Profit

Proposition 1.5.5 *(i) When t_R undergoes an increasing deterministic transformation, the global firm's expected profit decreases in μ_R, and the local competitor's expected profit increases in μ_R.*

(ii) When t_F undergoes an increasing deterministic transformation, there exists a threshold Λ_L^d (defined in Appendix A) such that: the global firm's expected profit always decreases in μ_F; the local competitor's expected profit increases in μ_F if $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} > \Lambda_L^d$ and decreases in μ_F otherwise.

Recall from Propositions 1.5.1 and 1.5.3 that an increased RM import cost results in a lower reshoring capacity and expected output for the global firm. Since the reshoring capacity provides a lever for the global firm to respond to cost changes, the global firm's profit decreases as a result of lower reshoring capacity. This can, in turn, benefit the local competitor who is immune from the import cost changes and whose competitiveness strengthens as the global firm reduces its reshoring capacity investment and output.

Regarding the impact of FG import cost, Proposition 1.5.5 part (ii) states that an increased cost for imported FG always hurts the global firm. The previously cost-effective offshore production now becomes less attractive under the increased cost. Consequently, the global firm has to turn to the domestic location, which was relatively expensive before. Although the global firm may increase its investment in reshoring capacity and utilize it to respond to the cost increase, the net effect of increased FG import cost is a higher cost for the global firm, which hurts its profit. As in the case of the RM import cost, although the domestic competitor's cost is not directly affected by the cost of imported FG, the cost change can have an indirect effect through market competition. When the global firm increases (reduces) its reshoring capacity, the competition intensifies (dampens), which can hurt (benefit) the domestic competitor, as illustrated in Figure 1.4.

Since increases in the import cost for RM and FG both hurt the global firm's profit, it is natural to ask which cost has a more severe negative effect. While t_R and t_F have the same effect on the cost of input sourced from country C, they affect the cost of overflow production differently: an increase in the FG import cost always negatively affects the overflow cost when overflow production is necessary. The RM import cost, however, does not affect the overflow cost. This implies that t_F's negative impact on the global firm's profit is more significant than t_R's. Figure 1.5 illustrates one example where t_R and t_F are initially distributed uniformly on [0.25, 0.35]. The blue dotted line represents the global firm's profit when μ_R increases from 0.3 to 0.32, ceteris paribus. The orange dotted line represents the case when μ_F makes the same change instead. We observe that the cost increase in imported FG indeed results in more profit loss for the global firm compared to the RM cost increase.

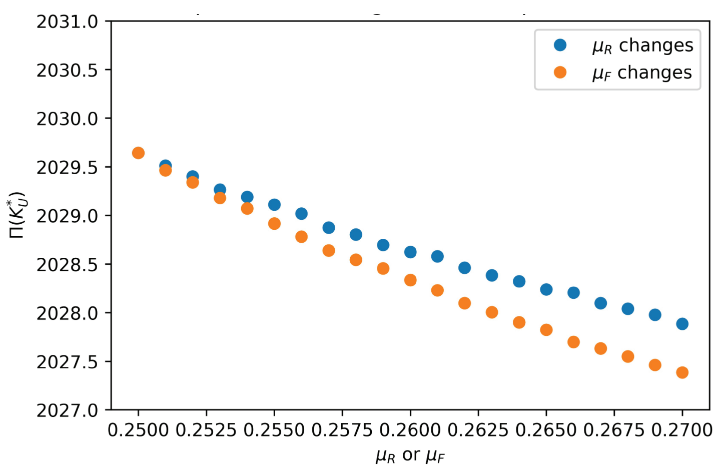


Figure 1.5: Impact of Import Cost Increase on the Global Firm's Optimal Profit ($w_C = 10, w_U' = 13, t_i \sim U[\mu_i - 0.05, \mu_i + 0.05], i \in \{R, F\}, C_U = 0.03$)

1.6 The Effect of Competition

As we have seen from our results so far, the impact of increased import costs on the global firm's sourcing actions and profitability depends on the market's competitive structure. One might expect that the presence of domestic competition would discourage the global firm's reshoring activity to avoid head-to-head competition. As shown later in this section, this is not necessarily the case. We will first present results and a counterexample for the general model before using a special case with uniform demand distribution to illuminate the competition effect more precisely.

Recall from Propositions 1.4.1 and 1.4.3 that the capacity cost thresholds C_0^m and C_0^d dictate when reshoring is feasible for the global firm. We first have:

Proposition 1.6.1 *The reshoring thresholds have the relationship: $C_0^d \leq C_0^m$, with equality holding when $w_U' > m_B$.*

Clearly, when $w_U' > m_B$, the reshoring threshold is the same regardless of competition (see Proposition 1.4.3). However, when $w_U' \leq m_B$, i.e., when the sourcing and production cost in the home country is relatively low compared to the offshoring cost, the reshoring threshold in the competition model, C_0^d, is smaller than that in the monopoly model, C_0^m. This implies that when $C_U \in (C_0^d, C_0^m)$, the global firm finds reshoring attractive when it does not face competition from firm L but will refrain from doing so under competition. This result confirms our intuition that competition can discourage reshoring activities, and this occurs when the domestic competitor, firm L, is relatively cost-competitive.

However, the next result provides a situation where the global firm invests in more capacity in the home country when facing a less competitive domestic rival.

Proposition 1.6.2 *When $w'_U > m_B$ and $C_U \in (C_2^d, C_1^d)$, we have $K_U^d > K_U^m$.*

This situation corresponds to Proposition 1.4.2 and Figure 1.3 case (ii). The two firms engage in Cournot competition when the market size ϵ is above $K_U + w'_U$. Comparing the characterizing equations FOC_2^d in the competition model with FOC^m in the single-firm model, we can identify the marginal revenue as the following:

$$
\begin{aligned}
FOC_2^d : & \int_{m_A+2K_U}^{K_U+w'_U} \underbrace{\left[\epsilon - m_A - 2K_U\right]}_{\text{marginal revenue under full utilization without competition}} dG(\epsilon) \\
& + \int_{K_U+w'_U}^{3K_U+2m_B-w'_U} \underbrace{\left[\frac{\epsilon - 2m_A + w'_U}{2} - K_U\right]}_{\text{marginal revenue under full utilization with competition}} dG(\epsilon) \\
& + \int_{3K_U+2m_B-w'_U}^{\infty} \underbrace{(m_B - m_A)}_{\text{marginal revenue under overflow}} dG(\epsilon) = C_U.
\end{aligned}
$$

The marginal revenue under overflow is the same in both models. A unit increase in K_U avoids overflowing a unit of production to country C. This leads to a cost-saving of $m_B - m_A$, regardless of the output quantity. However, the marginal revenue under full utilization depends on the market competition. When $\epsilon < K_U + w'_U$, firm L does not compete with the global firm. The global firm's marginal revenue is the same as the monopoly model.

When $\epsilon \in (K_U + w'_U, 3K_U + 2m_B - w'_U)$, competition between the two firms changes the total market output and, consequently, the global firm's marginal revenue. When $w'_U > m_B$, firm L's competitiveness is low with a relatively high cost, and the global firm obtains a higher marginal revenue than a monopoly firm. Therefore, the global firm's optimal capacity size K_U is higher in the competition model.

1.6.1 A Special Case: Uniform Demand Distribution

Next, we present a special case with uniform demand distribution to better understand the effect of competition. We assume the uniform distribution of ϵ on $[0, M]$. To avoid uninteresting cases, we assume that the upper bound of demand is sufficiently large, i.e., $M > 4\max\{E_{t_F}[w_C(1+t_F)], E_{t_R}[w_C(1+t_R)+\delta], w'_U\}$.

The optimal capacity decision in the single-firm model can be solved as the following:

Corollary 1.6.1 *When $\epsilon \sim U[0, M]$, the global firm's optimal capacity is:*

$$K_U^m = \begin{cases} \frac{M - m_A - \sqrt{2MC_U}}{2}, & \text{if } C_U \le \frac{(m_B - m_A)^2}{2M}, \\ \frac{2M - m_B - m_A - \frac{2MC_U}{m_B - m_A}}{4}, & \text{if } \frac{(m_B - m_A)^2}{2M} < C_U \le \frac{(m_B - m_A)(2M - m_B - m_A)}{2M}, \\ 0, & \text{if } C_U > \frac{(m_B - m_A)(2M - m_B - m_A)}{2M}. \end{cases}$$

When the unit capacity cost is close to zero, the optimal capacity is bounded above at $\frac{M-m_A}{2}$ due to the upper bound of the market size. The optimal capacity decreases as C_U increases. When C_U is small, K_U^m convexly decreases in C_U. The reason is that a low capacity cost allows the global firm to build a large capacity. Consequently, overflow production to country C never occurs. In this case, since the domestic capacity can fully satisfy the market demand when producing onshore, a slight change in capacity cost can dramatically affect the capacity decision. When C_U increases, the capacity decreases, and it can no longer fully satisfy demand. The firm has to overflow excess demand to country C. In this case, the optimal capacity is linearly decreasing in the capacity cost.

The optimal capacity decision in the competition model follows directly from Proposition 1.4.3 and is illustrated by the orange lines in Figure 1.6. The expressions for the characterizing equations and threshold values are included in Appendix A.

Next, we focus on the comparison of the two models.

Proposition 1.6.3 *Assume that $\epsilon \sim U[0, M]$.*

(i) When $w'_U < \frac{m_A+m_B}{2}$, we have $K_U^d \leq K_U^m$, with equality holds when $C_U \geq C_0^m$.

(ii) When $m_B < w'_U < 2m_B - m_A$, we have: $K_U^d < K_U^m$ if $C_U < I_1$; $K_U^d > K_U^m$ if $I_1 < C_U < C_1^d$; and $K_U^d = K_U^m$ if $C_U \geq C_1^d$.

(iii) When $\frac{m_A+m_B}{2} < w'_U < m_B$, we have: $K_U^d < K_U^m$ if $C_U < I_1$ or $I_2 < C_U < C_0^m$; $K_U^d > K_U^m$ if $I_1 < C_U < I_2$; and $K_U^d = K_U^m = 0$ if $C_U \geq C_0^m$.

From Proposition 1.6.1, we see that when $w'_U < m_B$, the reshoring threshold is smaller under competition. Proposition 1.6.3 part (i) states a stronger result under uniform demand distribution. When w'_U is even smaller, i.e., $w'_U < \frac{m_A+m_B}{2}$, not only the reshoring threshold but the capacity installment is also lower under competition. The left panel in Figure 6 illustrates this result. A highly competitive domestic competitor dampens the global firm's incentive to reshore.

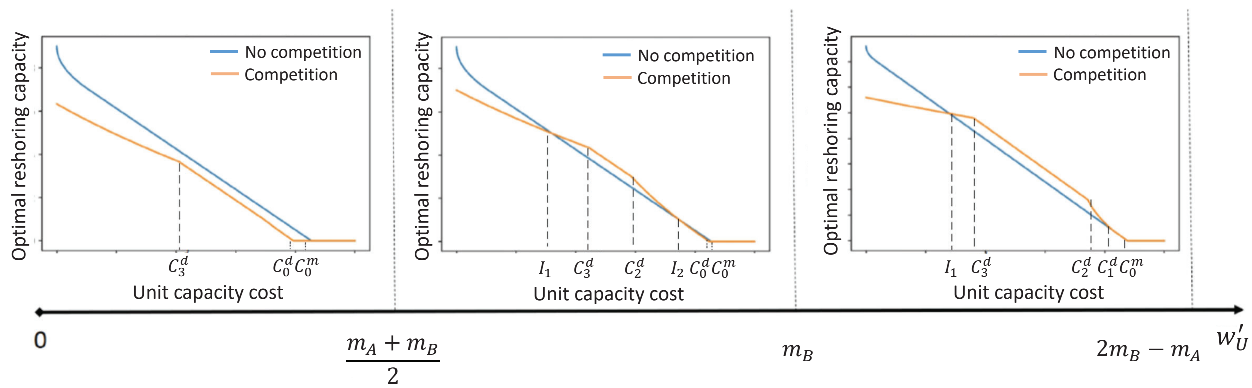


Figure 1.6: Comparison of Optimal Capacity Between the Single-Firm and Competition Models

Next, let us focus on the right panel of Figure 1.6. The threshold C_1^d follows from Proposition 1.4.3. When $C_U \geq C_1^d$, the capacity decision is the same in the two models. When $I_1 < C_U < C_1^d$, competition leads to an increased reshoring capacity. The expression for I_1 is given in the proof of Proposition 1.6.3, and it can be shown that $I_1 < C_2^d$. Therefore, this result generalizes Proposition 1.6.2, which only provides a sufficient condition (a subset of the entire set $I_1 < C_U < C_1^d$) for this outcome. Finally, when w'_U is between $\frac{m_A+m_B}{2}$ and m_B, there also exists a range of C_U where the capacity investment is higher under competition.

It is interesting to observe from Figure 1.6 that the unit capacity cost may affect the optimal reshoring capacity at different rates in the two models. When C_U is small, the global firm invests in a large reshoring capacity, which can fully satisfy its output without overflow when producing onshore. The optimal capacity convexly decreases in C_U in both models, as explained earlier. However, facing competition, the global firm's output quantity is smaller than the monopoly quantity. Therefore, under competition, not only is the reshoring capacity size smaller, but the change in the capacity cost affects capacity investment at a slower rate. We can observe this effect in all three panels of Figure 1.6. When C_U is moderately high, i.e., when $C_U > C_2^d$, under competition the global firm fully utilizes K_U when domestic production is preferred and will overflow production to country C when the output target is above K_U (corresponding to Figure 1.3 case (ii)). In this case, the change in the capacity cost has a stronger effect under competition. A unit decrease in C_U induces the global firm to increase the reshoring capacity by a larger amount when facing competition than when it is a monopoly. The higher capacity allows the global firm to compete with firm L more effectively without turning to the more costly alternative. We can observe this effect in the middle and right panels of Figure 1.6. Finally, when C_U is moderately low, i.e., when $C_U \in (C_3^d, C_2^d)$, the capacity cost affects the optimal capacity decision at the same rate, regardless of competition. The optimal K_U decreases linearly in C_U at the same rate in both models. It is worth noting that since K_U decreases in C_U at different rates in the two models, $K_U^d < K_U^m$ can occur in two disjoint regions ($C_U < I_1$ and $C_U \in (I_2, C_0^m)$ in Figure 1.6, middle panel).

The competitive environment affects the reshoring decision and the value of flexibility offered by such an option. Installing domestic capacity provides the global firm with a real option of sourcing and producing domestically in certain cost conditions. We can define the flexibility value (or real option value) of reshoring as the difference between the optimal profit with reshoring and without such an option. As the value of this real option depends on the size of the installed capacity, competition would increase (decrease) the flexibility value when it leads to a higher (lower) reshoring capacity. It can be shown that the comparison of flexibility value between the single-firm and competition model is similar to the reshoring capacity comparison shown in Figure 1.6.

1.7 Concluding Remarks

Recent trade protectionist policies in the U.S. and Europe, supply chain shocks due to the pandemic, industrial policies favoring the shift of certain supply chains, and sustainability issues have resulted in substantial cost increases for imported materials and finished products from certain countries, with Chinese exports the most frequent example. Moreover, the current geopolitical environment has intensified the uncertainty around cross-border trade and the regulatory risks of industrial and environmental policies. Global firms, in response to elevated risks and uncertain import costs, have considered restructuring their global supply chains, and one of those alternatives is to "reshore/nearshore" manufacturing activities in the home country or region. This is not an all-or-nothing proposition, and it typically results in rebalancing capacity allocations between offshore and onshore locations. As a result, this adds the "real option" of choosing between producing in the existing facilities with low cost or using reshored facilities to avoid tariffs, imposed penalties, and/or leverage subsidies and other incentives. The value of such an option is driven by uncertainties in the import cost and somewhat by future market demand.

We seek to rigorously argue how the "real option" of an added reshoring capacity investment allows for profitable and effective management of supply-side uncertainty. Our stylized model offers rich theoretical insights into factors affecting reshoring decisions. We often dismiss generic and unproven statements on how certain policies, for example, tariffs of any kind or rich subsidies for favored supply chain solutions, can result in increased reshoring investment, and how competition in the domestic market will dampen reshoring. We provide more nuanced answers, but also clearly identify when the counter to common intuition may occur and the factors that drive it.

To obtain our insights, we develop a three-stage model to analyze a global firm's long-term reshoring capacity decision in the face of all uncertainties, the medium-term output quantity equilibrium between the global firm and a domestic competitor upon demand realization, and the global firm's short-term sourcing and production decision after import cost realization. The main findings and managerial insights from our analysis can be summarized as follows.

First, we find that an increase in the import cost for raw materials versus finished goods can have different implications on companies' reshoring incentives. Ceteris paribus, a higher expected cost for imported finished goods can increase or decrease reshoring investment, while a higher cost for imported raw materials always results in less reshoring. Higher import costs for raw materials increase the production cost when producing domestically using imported raw materials ex-post, dampening the incentive for reshoring. The finished-goods cost has two opposite effects: an overflow demand effect—a higher cost for imported finished goods increases the cost of producing the overflow quantity at the offshore facility, which encourages a higher reshoring capacity; and an output quantity effect—a higher cost for imported finished goods increases the average cost of outputting to the market, which reduces the total output target and capacity needs. A higher cost for imported finished goods can lead to an increased or decreased reshoring capacity depending on the dominant effect. An implication of the trade policy is that imposing tariffs may not necessarily achieve the desired policy goal of bringing jobs back to the U.S. in certain industries. Careful consideration should be given to how to execute trade restrictions, especially what part of the supply chain or of the product tree to have tariffs applied.

Second, we find that when the cost disadvantage of domestic sourcing and production compared to offshoring is small, the presence of domestic competition will dampen companies' reshoring incentives. However, when the domestic location's cost disadvantage is big, domestic competition can induce more reshoring. In this case, the reshoring capacity and the flexibility it brings protect the global firm from cost increases and enable it to compete more effectively with the domestic competitor. Third, we show that the increase in import costs indirectly affects the domestic manufacturer through competition. Although exposure to higher import costs always hurts the global firm's profit, the domestic competitor may benefit when the global firm reduces its reshoring investment and cannot effectively mitigate the negative effect of cost increases.

Our research makes both theoretical and practical contributions. On the theoretical side, we contribute to the global supply chain literature by modeling input cost uncertainty at different supply chain stages and analyzing their impact on companies' operational decisions. We enrich the Newsvendor network literature by developing a three-stage modeling framework that incorporates market competition and two ex-post decision-making stages, one after demand realization and the other after cost realization. On the practical side, our research provides valuable insights to companies restructuring their global supply chains in reaction to the rapidly changing and more uncertain trade environment and supply chain conditions. We suggest that investing in domestic or reshore production can provide the value of exercising a valuable "real option" (shift production to the domestic or nearshore location in certain scenarios) and increase a global firm's competitiveness in an uncertain cost and demand environment. Our research also partially explains tariff policy outcomes observed in different industries.

Chapter 2

Sourcing Strategy under Demand Uncertainty and Supply Disruption

2.1 Introduction

Apart from the landed cost uncertainty caused by trade policy uncertainty, in my second Chapter, "Sourcing Strategy under Demand Uncertainty and Supply Disruption." We look at the structurally inherent uncertainties in a global supply chain, the demand side and the supply side.

The whole idea is inspired by the news that China's Covid-19 lockdown causes 1 in 5 container ships stuck outside congested ports in Shanghai. As we know, supply disruption includes disruption in production, sales, or distribution. This is an example of distribution disruption. According to Xinhua Net, Shanghai exported nearly 1 billion containers in 2021. The value is about $2.5 billion US dollars. You can see the severity of the disruption.

Consider a global firm trying to serve country U's domestic market through offshoring production in a foreign country C. Due to the COVID-19 pandemic, international trade tensions, wars, typhoons, or other unexpected disruptions, container ships get stuck outside congested ports [4]. It leads to the uncertain time between placing an order and actually receiving the order, i.e., supply disruption. In addition, demand uncertainty usually comes along with and is even more prevalent than supply disruption [37]. To hedge against such risks, the global firm can install domestic inventory, or source and produce locally. The former strategy requires establishment costs to set inventory. Through our paper, we only consider building finished-good (FG) inventory. Considering building raw material (RM) inventory does not change our insights a lot. The latter strategy may not always be available to the global firm. Since the global firm may have invested all capital into the foreign country. Moreover, there exists a local competitor waiting in the domestic country to grab the market. We call the global firm "he/him" and the local competitor "she/her" afterward. A pretty natural question we would like to ask is

Q: What is the global firm's best sourcing strategy?

There are three possible sourcing strategies. The first one is keeping offshore production and always using it. The second one is keeping offshore production while simultaneously starting local sourcing and production. The third one is reshoring, i.e., eliminating overseas options. To be more specific, we are especially interested in

Q1: Under what condition should the global firm keep offshoring?

The most intuitive answer is that it depends on the cost comparison. Suppose the offshoring costs less than local sourcing for every unit product, then definitely keeping offshoring is beneficial. It is true in some cases but not always true, especially when encountering uncertainties. Thus, our second question is

Q2: How do demand and supply disruption affect the global firm's sourcing strategy?

We study it from two perspectives: magnitude and variability. A larger demand variability brings more risks and calls for increased investment in flexibility. In a competitive setting, this, in turn, strengthens the global firm's competitiveness when facing domestic competition, and the domestic competitor suffers a setback. The investment in the offshoring inventory and sourcing flexibility it brings serve as a real option. With this flexibility, the global firm can benefit from variability. This insight is consistent with the findings in the operational flexibility literature [20]. However, an increasing demand magnitude can increase the global firm's willingness to set offshoring inventory and source overseas. When he lacks the local sourcing option, the global firm even chooses to offshore facing a cost disadvantage. The intuition is that the first movers' advantage attracts the global firm when the demand is often very large. The supply disruption is modeled in a different way, and we will see the details later. Last but not least,

Q3: Does deciding the sourcing strategy before the local competitor benefit or harm the global firm?

Common knowledge tells us early commitment gives the first mover an advantage. But we can see under uncertainties, there could exist an "early order's disadvantage". Specifically, the global firm chooses not to set offshoring inventory even if the inventory cost is slightly smaller than the local cost. Because he orders the inventory earlier than the local competitor. However, the inventory constraint prevents the global firm from occupying the extra market. It occurs when the demand has a relatively large magnitude.

Apart from studying the thresholds that enable setting up offshore inventory, we are also interested in comparing the inventory decisions when there is only demand uncertainty.

Q4: if the global firm has the flexibility of local sourcing and production, does he set a higher or lower inventory compared to the case when he has only an offshore option?

We find that the optimal inventory is larger when the global firm does not have sourcing flexibility. Although the proof is based on uniform demand distribution, the result can be generalized to general demand distribution. The intuition is that when the global firm has only the offshoring option, he has the incentive to set up a large inventory to satisfy the market. On the other hand, if he has the flexibility, then he may not want to invest too much in the inventory. Since local sourcing and production are a back-up.

Adding supply disruption incentivizes us to ask more questions about the inventory comparison.

Q5: If the global firm has the flexibility of local sourcing and production, does he set a higher or lower inventory when there is an additional supply disruption?

We find that the optimal inventory under disruption is smaller than the one without disruption if the inventory cost is high, which is straightforward. However, when the inventory cost is low, then the optimal inventory is larger when disruption happens. One reason is that the maximum possible inventory decision under disruption is larger than the one without supply disruption. On the other hand, the global firm only receives a proportion of inventory as he orders. When the cost of establishing inventory is small, the global firm anticipates the risk of a small inventory and thus intends to hedge more against supply disruption.

Call the inventory decisions under demand uncertainty only "safety stock". The difference between inventories driven by demand and supply disruption and the "safety stock" is called "strategic inventory". Then, we find that the strategic inventory is always larger for the offshore only case rather than the flexibility case. Because the flexible global firm has another "strategic capacity" from locally sourcing and production. With a numerical example, we find the strategic capacity is larger under two uncertainties when the inventory cost is moderate.

2.2 Benchmark Model

Starting from the benchmark without demand uncertainty or lead-time disruption, the global firm has the flexibility to choose between offshoring and onshoring. The term "offshoring" here means that the global firm only sources the product from the foreign country C. The term "onshoring" means that the global firm sources and produces locally. Denote the offshoring sourcing cost as w_C, the local sourcing cost as w_S, and the local production cost as w_P. The total local unit cost is $w_U = w_S + w_P$. Since there is no uncertainty, establishing inventory makes no sense. In the domestic market, the global firm faces competition from a domestic manufacturer, referred to as firm L. Firm L only sources and operates in country U. Like the global firm, firm L incurs the total unit cost w_U. We consider a Cournot competition between the two firms. The market price is governed by a linear inverse demand function $p = \epsilon - q - q_L$, where ϵ denotes the random market size, and q and q_L denote the output quantity of the global firm and firm L, respectively. Hence, it is a simple Cournot game. The global firm decides his output quantity q_C and q_U, while the local competitor decides her output quantity q_L to maximize their profits respectively:

$$\max_{q_C, q_U \geq 0} [\epsilon - q_C - q_U - q_L] \cdot (q_C + q_U) - w_C \cdot q_C - w_U \cdot q_U$$

$$\max_{q_L \geq 0} [\epsilon - q_C - q_U - q_L] \cdot q_L - w_U \cdot q_L$$

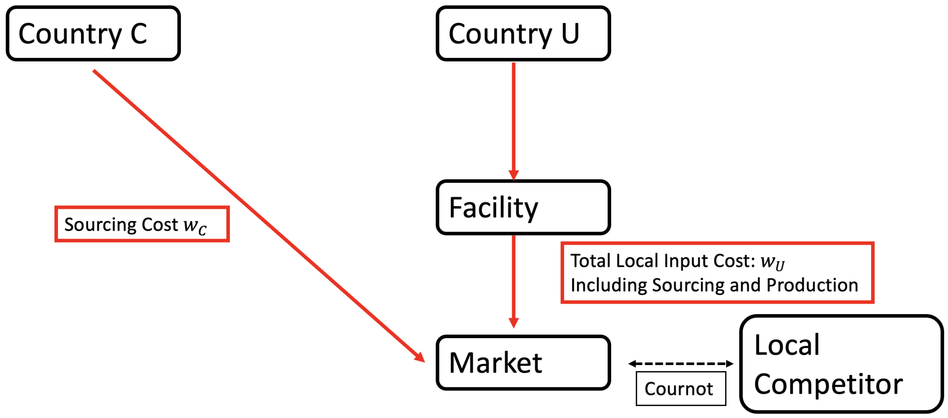


Figure 2.1: No Uncertainties, Global Firm Has the Sourcing Flexibility.

And the only reason behind these two choices is the cost. If the offshoring sourcing cost w_C is smaller than the local unit cost w_U, i.e., $w_C \leq w_U$, then the problem becomes

$$\max_{q_C \geq 0} [\epsilon - q_C - q_L] \cdot q_U - w_C \cdot q_C$$

$$\max_{q_L \geq 0} [\epsilon - q_C - q_L] \cdot q_L - w_U \cdot q_L$$

which gives us

$$\Rightarrow \begin{cases} q_C^* = \frac{\epsilon - 2w_C + w_U}{3} \\ q_L^* = \frac{\epsilon - 2w_U + w_C}{3} \end{cases}$$

This case is equivalent to the case where the global firm is a "purely global" one. It only offshores and sources from the foreign country C with cost w_C.

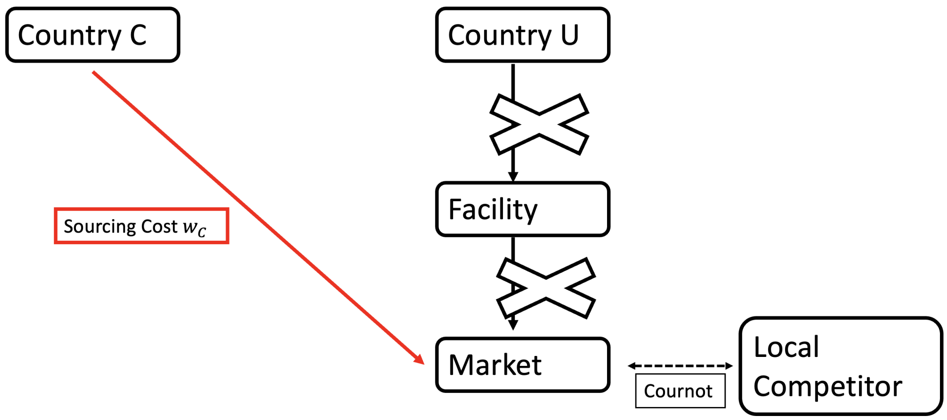


Figure 2.2: No Uncertainties, Global Firm Sources From the Foreign Country Only.

Otherwise, if $w_C > w_U$, then the problem becomes

$$\max_{q_U \geq 0} [\epsilon - q_U - q_L] \cdot q_U - w_U \cdot q_U$$

$$\max_{q_L \geq 0} [\epsilon - q_U - q_L] \cdot q_L - w_U \cdot q_L$$

Easily we have their optimal output quantities

$$\Rightarrow \begin{cases} q_U^* = \frac{\epsilon - w_U}{3} \\ q_L^* = \frac{\epsilon - w_U}{3} \end{cases}$$

In addition, the global firm's optimal profit is $\frac{(\epsilon - w_U)^2}{9}$. The global firm is more like a duplicate of the local competitor than a "global" firm. They share the same cost w_U, representing local sourcing and production costs.

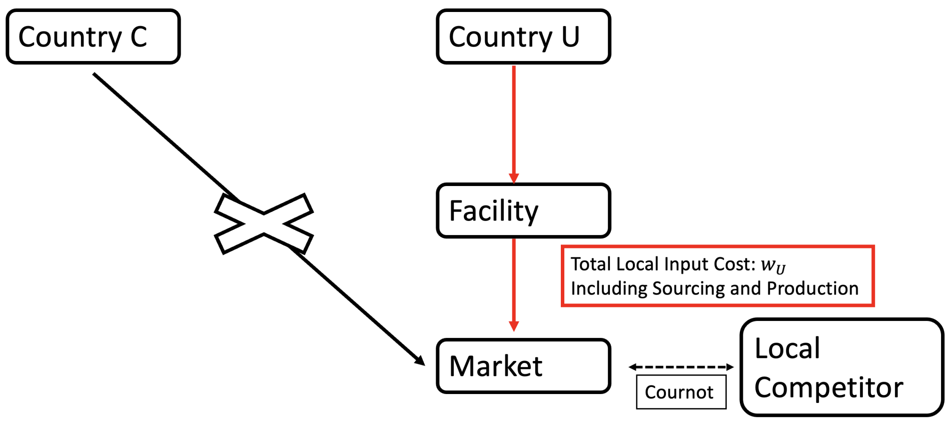


Figure 2.3: No Uncertainties, Global Firm Sources and Produces Locally Only.

In short, w_U is the threshold for the sourcing cost w_C that the offshoring occurs only and if only w_C is smaller than such threshold. We also call it the “break-even” point for the sourcing cost w_C.

2.3 Model with Demand Uncertainty

When there is demand uncertainty only, consider the case where the global firm only sources finished goods from country C and establishes FG inventory before demand realization. At the same time, the local competitor makes quantity decisions after demand realization. They engage in a Cournot competition. We use the subscript “regular” in this scenario. In the following case, we will add supply disruption and add the subscript “disruptive”.

Denote the costs of sourcing FG overseas as w_C. In other words, the global firm pays such costs to build inventories. We use “sourcing cost” for cases without uncertainties and "inventory cost" for cases with uncertainties. Explicitly model the transshipment costs across the two countries as w_{TF} for finished goods. Actually, we can include any cost only related to the quantity into w_{TF}. Under such settings, we can see that w_{HF}, and w_{TF} can be normalized to zero WLOG. Denote the cost of sourcing locally as w_S and the production cost as w_P. Without loss of generality, combine w_S and w_P as $w_U = w_S + w_P$ as before.

In the domestic market, the global firm faces competition from a domestic manufacturer, referred to as firm L. Firm L only sources and operates in country U. Like the global firm, firm L incurs the total unit cost w_U. We consider a Cournot competition between the two firms. The market price is governed by a linear inverse demand function $p = \epsilon - q - q_L$, where ϵ denotes the random market size, and q and q_L denote the output quantity of the global firm and firm L, respectively.

2.3.1 With Flexibility

Suppose the global firm has the flexibility to use local sourcing and production as a backup option to satisfy the market. Usually, the inventory cost is smaller than the local cost. After using all inventory, the global firm is able to satisfy extra demand using local sourcing and production.

Then the global firm decides its FG inventory I to maximize its expected profit in the first stage

$$\max_{I \geq 0} E_\epsilon[r_{regular}] - w_C \cdot I$$

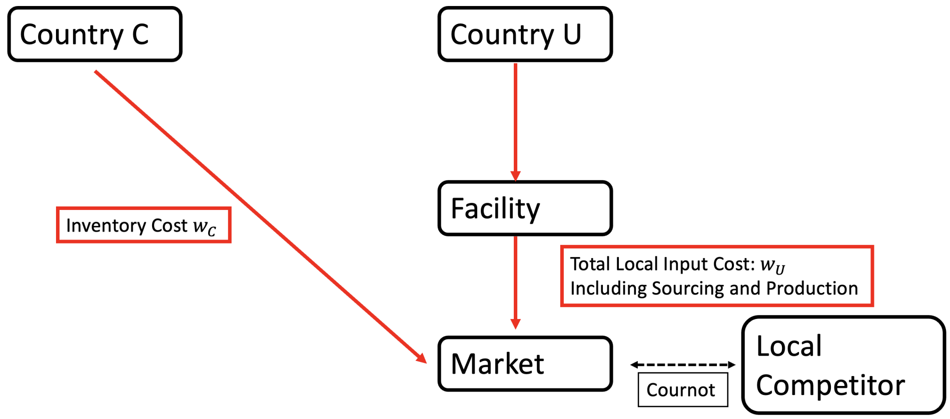


Figure 2.4: With Uncertainties, Global Firm Has the Sourcing Flexibility.

After demand realization, the global firm decides the output quantities q_C to maximize the ex-post profit in the second stage, i.e.

$$r_{regular} = \max_{q_C \geq 0} [\epsilon - q_C - q_U - q_L] \cdot (q_C + q_U) - 0 \cdot q_C - w_U \cdot q_U$$

$$\text{s.t.} \quad q_C \leq I$$

The cost associated with output q_C is 0 because we have already included the sourcing cost in the previous stage.

As for the local competitor,

$$\max_{q_L \geq 0} [\epsilon - q_C - q_U - q_L] \cdot q_L - w_U \cdot q_L$$

Using backward induction, it is straightforward to show

Lemma 2.3.1 *Given demand realization ϵ, two firms' output quantities are*

Demand Realization	q_C^*	q_U^*	q_L^*
$0 \le \epsilon < 2w_U$	$\frac{\epsilon}{2}$	0	0
$2w_U \le \epsilon < 3I - w_U$	$\frac{\epsilon + w_U}{3}$	0	$\frac{\epsilon - 2w_U}{3}$
$3I - w_U \le \epsilon < 3I + w_U$	I	0	$\frac{\epsilon - w_U - I}{2}$
$\epsilon \ge 3I + w_U$	I	$\frac{\epsilon - w_U}{3} - I$	$\frac{\epsilon - w_U}{3}$

if $I > w_U$*, otherwise,*

Demand Realization	q_C^*	q_U^*	q_L^*
$0 \le \epsilon < 2I$	$\frac{\epsilon}{2}$	0	0
$2I \le \epsilon < I + w_U$	I	0	0
$I + w_U \le \epsilon < 3I + w_U$	I	0	$\frac{\epsilon - w_U - I}{2}$
$\epsilon \ge 3I + w_U$	I	$\frac{\epsilon - w_U}{3} - I$	$\frac{\epsilon - w_U}{3}$

The reason why we have two different cases is that if the inventory is small ($I < w_U$), the global firm exhausts its inventory when the market size is still too small to enable the local competition to engage in the Cournot competition. Then the global firm has to let extra profit slip from his fingers due to the lack of inventory. In other words, the inventory constraint binds when the demand is small. On the other hand, if the inventory is large ($I > w_U$), they engage in the competition before the global firm uses all his inventory.

Then for the case $I > w_U$, we have

$$E_\epsilon[r_{regular}] = \int_0^{2I} \frac{\epsilon^2}{4} + \int_{2I}^{I+w_U} (\epsilon - I)I + \int_{I+w_U}^{3I+w_U} \frac{\epsilon + w_U - I}{2} \cdot I +$$
$$\int_{3I+w_U}^{\infty} \left[\frac{(\epsilon - w_U)^2}{9} + w_U I \right] dG(\epsilon) - w_C I$$

Take first order derivative with respect to I,

$$\frac{\partial E_\epsilon[r_{regular}]}{\partial I} = \int_{2I}^{I+w_U} (\epsilon - 2I) dG(\epsilon) + \int_{I+w_U}^{3I+w_U} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) + \int_{3I+w_U}^{\infty} w_U dG(\epsilon) - w_C$$

The second-order derivative

$$\frac{\partial^2 E_\epsilon[r_{regular}]}{\partial I^2} = \int_{2I}^{I+w_U} (-2) dG(\epsilon) + \int_{I+w_U}^{3I+w_U} (-1) dG(\epsilon) - \frac{I}{2} \cdot (g(I + w_U) - 3g(3I + w_U)) < 0$$

always negative when the demand pdf is a decreasing function. Thus, the optimal FG inventory in this case I^* solves

$$FOC_{F1}: \int_{2I}^{I+w_U} (\epsilon - 2I) dG(\epsilon) + \int_{I+w_U}^{3I+w_U} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) + \int_{3I+w_U}^{\infty} w_U dG(\epsilon) = w_C$$

The subscript “F” stands for “flexibility”. Without further ado,

Proposition 2.3.1 *The optimal inventory decision I^* satisfies the following characterizing equations:*

$$\begin{cases} 0, & \text{if } w_C \in [w_{F0}, \infty) \\ FOC_{F1}, & \text{if } w_C \in [w_{F1}, w_{F0}) \\ FOC_{F2}, & \text{if } w_C \in [0, w_{F1}) \end{cases}$$

where

$$FOC_{F1}: \int_{2I}^{I+w_U} (\epsilon - 2I) dG(\epsilon) + \int_{I+w_U}^{3I+w_U} \left[\frac{\epsilon + w_U}{2} - I\right] dG(\epsilon) + \int_{3I+w_U}^{\infty} w_U dG(\epsilon) = w_C$$
$$FOC_{F2}: \int_{3I-w_U}^{3I+w_U} \left[\frac{\epsilon + w_U}{2} - I\right] dG(\epsilon) + \int_{3I+w_U}^{\infty} w_U dG(\epsilon) = w_C$$

And

Proposition 2.3.2 *w_{F0} is the threshold for the sourcing cost w_C that the offshoring occurs only and if only w_C is smaller than such threshold. It always holds*

$$w_{F0} = \int_0^{w_U} \epsilon dG(\epsilon) + \int_{w_U}^{\infty} w_U dG(\epsilon) < w_U$$

i.e., even the global firm has a little cost advantage while offshoring. He prefers not to join the competition due to the negative effect of early commitment. Uncertainty makes the global firm give up his strategic advantage. Although the increase of demand magnitude could increase the threshold w_C^0, it never exceeds w_U.

To see more clearly how the inventory cost determines the optimal inventory, we study a special case where the demand follows a uniform distribution $\epsilon \sim U[0, M]$. We assume the maximum possible market size M is large enough ($M > 4w_U$) to avoid trivial cases. Hence,

Corollary 2.3.1 *When the demand follows an uniform distribution $\epsilon \sim U[0, M]$, the optimal inventory decision for the regular flexibility case is*

$$\begin{cases} 0, & \text{if } w_C \in [w_{F0}, \infty) \\ 2w_U - \sqrt{5w_U^2 - 2Mw_U + 2Mw_C}, & \text{if } w_C \in [w_{F1}, w_{F0}) \\ \frac{M(w_U - w_C)}{2w_U}, & \text{if } w_C \in [w_{F2}, w_{F1}) \\ \frac{2(M+w_U) - \sqrt{(M+w_U)^2 + 12Mw_C}}{3}, & \text{if } w_C \in [0, w_{F2}) \end{cases}$$

where

$$\begin{aligned} w_{F0} =& w_U - \frac{w_U^2}{2M} \\ w_{F1} =& w_U - \frac{2w_U^2}{M} \\ w_{F2} =& \frac{Mw_U + 2w_U^2}{3M} \end{aligned}$$

The above graph shows us how the optimal inventory changes with respect to the inventory cost. It firstly convexly decreases on w_C, then linearly decreases, then convexly decreases again. The very left segment ($0 \le w_C < w_{F2}$) actually degenerates to the "offshore only" case in the next chapter. This is why we have an additional break-even point w_{F2}. The intuition is that the uniformly distributed demand has an upper bound. So the optimal inventory can not increase to infinity, even the inventory cost is zero. The maximum inventory can not

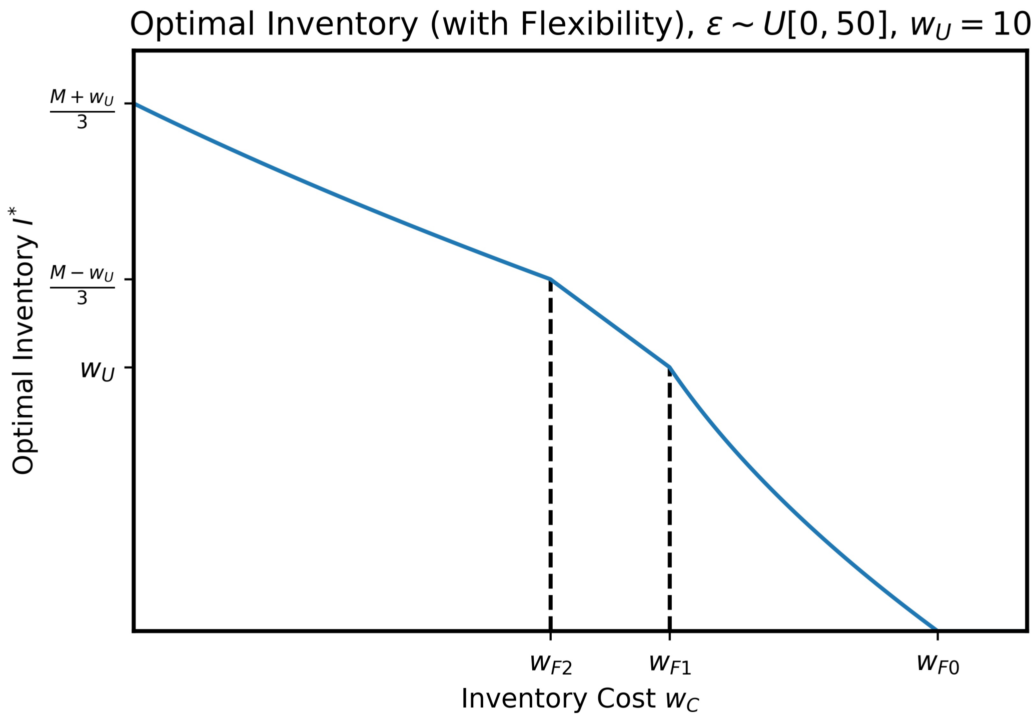


Figure 2.5: Optimal Inventory (with Flexibility)

exceed $\frac{M+w_U}{3}$. When the inventory is large enough to cover all possible demands, there is no need to use local options. Hence it degenerates into the "offshore only" case.

And $w_{F0} = w_U - \frac{w_U^2}{2M}$ increases as the maximum possible market size M increases, which means the global firm is more willing to set offshore inventory when the market has a larger potential.

2.3.2 Offshore Only

The graph below represents the case where the global firm has no access to local resources.

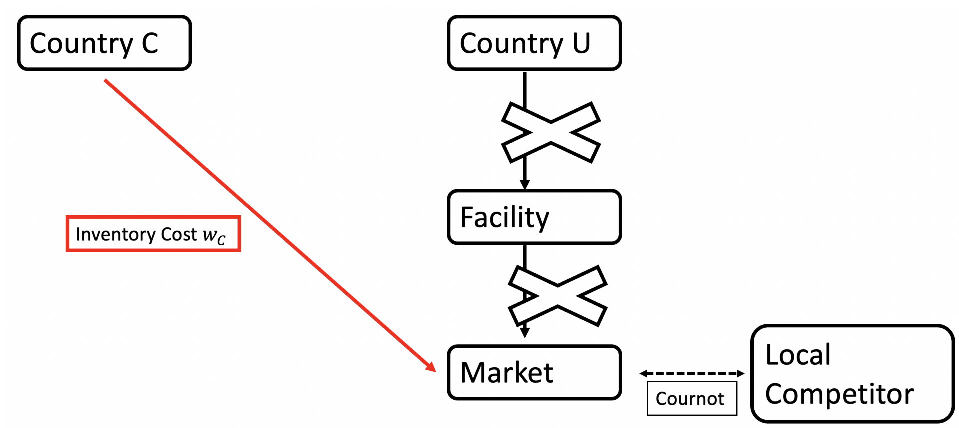


Figure 2.6: With Uncertainties, Global Firm Sources and Produces Overseas Only.

The stage 1 problem stays the same:

$$\max_{I \geq 0} E_\epsilon[r_{regular}] - w_C \cdot I$$

However, for the stage 2 problem, the global firm only one output decision q_C:

$$r_{regular} = \max_{q_C \geq 0} [\epsilon - q_C - q_L] \cdot q_C - 0 \cdot q_C$$
$$\text{s.t.} \quad q_C \leq I$$

As for the local competitor,

$$\max_{q_L \geq 0} [\epsilon - q_C - q_L] \cdot q_L - w_U \cdot q_L$$

Use the subscript “O” as for “Offshore Only”. Save all details in the Appendix, we conclude

Proposition 2.3.3 *The optimal inventory decision I^* satisfies the following characterizing equations:*

$$\begin{cases} 0, & \text{if } w_C \in [w_{O0}, \infty) \\ FOC_{O1}, & \text{if } w_C \in [w_{O1}, w_{O0}) \\ FOC_{O2}, & \text{if } w_C \in [0, w_{O1}) \end{cases}$$

where

$$FOC_{O1} : \int_{2I}^{I+w_U} (\epsilon - 2I) dG(\epsilon) + \int_{I+w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) = w_C$$

$$FOC_{O2} : \int_{3I-w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) = w_C$$

w_{O0} and w_{O1} have exactly the same meanings as the previous chapter, which represent the break-even points of $I = 0$ and $I = w_U$. And

Proposition 2.3.4

$$w_{O0} = \int_0^{w_U} \epsilon dG(\epsilon) + \int_{w_U}^{\infty} \frac{\epsilon + w_U}{2} dG(\epsilon)$$

is the threshold for the sourcing cost w_C that the offshoring occurs only and if only w_C is smaller than such threshold.

w_{O0} could be larger or smaller than w_U.

The intuition that $w_{O0} > w_U$ is straightforward. Since the global firm is the first mover, the benefit of occupying the whole market enables it to offshore, even facing a cost disadvantage. It more likely happens when the demand is "often" large. To be more specific,

Lemma 2.3.2 *w_{O0} increases with the demand magnitude.*

If the demand has a large probability of being quite small. Then $w_{O0} < w_U$ may occur. Under this condition, the global firm chooses not to set offshore inventory even if the cost equals the local cost. Because it is the case corresponding to the small inventory case ($I < w_U$). The global firm exhausts its inventory when the market size is still too small to enable the local competition to engage in the Cournot competition. Then the global firm has to let extra profit slip from his fingers due to the lack of inventory. He can't set a large inventory because of the inventory cost. Hence, the global firm directly gives up the offshoring option. Only when the advantage of cost is large enough does the global firm establish a positive inventory for overseas FG. We can also conclude that deciding the sourcing strategy before the competitor harms the global firm when the market size has a large probability of being quite small, which we call "early order's disadvantage". Compared with the benchmark, demand uncertainty is the archcriminal.

The aforementioned comparison leaves one question. The global firm is restricted to offshore only. When deciding the feasible range of a positive inventory, we compare the profit with zero. But local sourcing and production is an outside option. How about we compare the profit with the profit of the outside option, which is

$$E_\epsilon\left[\frac{(\epsilon - w_U)^2}{9}\right]$$

as shown in the benchmark? Direct comparison is not tractable because of a bunch of implicit FOCs.

Assume $\epsilon \sim U[0, M]$ where M is "sufficiently" large ($M > 4w_U$), then $dG(\epsilon) = \frac{1}{M}d\epsilon$. As they engage in a Cournot competition, the global firm's profit from the outside option is

$$\pi_0^* = E_\epsilon\left[\frac{(\epsilon - w_U)^2}{9}\right] = \int_{w_U}^{M} \frac{(\epsilon - w_U)^2}{9}\frac{1}{M}d\epsilon = \frac{(M - w_U)^3}{27M}$$

If $\epsilon \sim U[0, M]$, then simplifying FOC_1 gives us:

$$5I^2 - 4AI + B = 0$$

where

$$A = M + w_U$$

$$B = M^2 - w_U^2 + 2Mw_U - 4Mw_C$$

and $B = B(w_C)$ is decreasing in w_C. Solving FOC_1 gives us:

$$I^* = \frac{4A - \sqrt{4A^2 - 5B}}{5}$$

Therefore, the optimal profit

$$\pi_1^* = \frac{1}{150M}\left[15AB - 8A^3 + (4A^2 - 5B)^{3/2}\right]$$

To find the break-even point,

$$\pi_1^* > \pi_0^* \Leftrightarrow (4A^2 - 5B)^{3/2} > \frac{50}{9}(M - w_U)^3 + 8A^3 - 15AB$$

Denote $C = \frac{50}{9}(M - w_U)^3$, then

$$\pi_1^* > \pi_0^* \Leftrightarrow 125B^3 - 75A^2B^2 - 30ABC + 16A^3C + C^2 < 0$$

which is a cubic function of $B(w_C)$. The uniqueness of one real root can be proved. Apply the similar process to FOC_{O2}, we have

Lemma 2.3.3 *If $\epsilon \sim U[0, M]$, then $w_{O0} < w_U$ always holds.*

Notice the rationale of inequality $w_{O0} < w_U$ does not come from the structure of the uniformly distributed demand, but comes from the restriction that the potential market size could be large ($M > 4w_U$).

Lemma 2.3.4 *$w_{F0} < w_{O0}$ always holds.*

In addition, the threshold w_{F0} in the "with flexibility" case is smaller than the one in the "offshore only" case. Hence the flexibility does not eliminate the "early order's disadvantage".

Similar to the previous chapter, applying uniform distribution gives

Corollary 2.3.2 *When the demand follows a uniform distribution $\epsilon \sim U[0, M]$, the optimal inventory decision for the regular offshore only case is*

$$\begin{cases} 0, & \text{if } w_C \in [w_{O0}, \infty) \\ \frac{2(M+w_U)-\sqrt{-M^2-2Mw_U+9w_U^2+12Mw_C}}{5}, & \text{if } w_C \in [w_{O1}, w_{O0}) \\ \frac{2(M+w_U)-\sqrt{(M+w_U)^2+12Mw_C}}{3}, & \text{if } w_C \in [0, w_{O2}) \end{cases}$$

where

$$w_{O0} = \frac{M^2 - w_U^2 + 2Mw_U}{4M}$$
$$w_{O1} = \frac{M - 2w_U}{4}$$

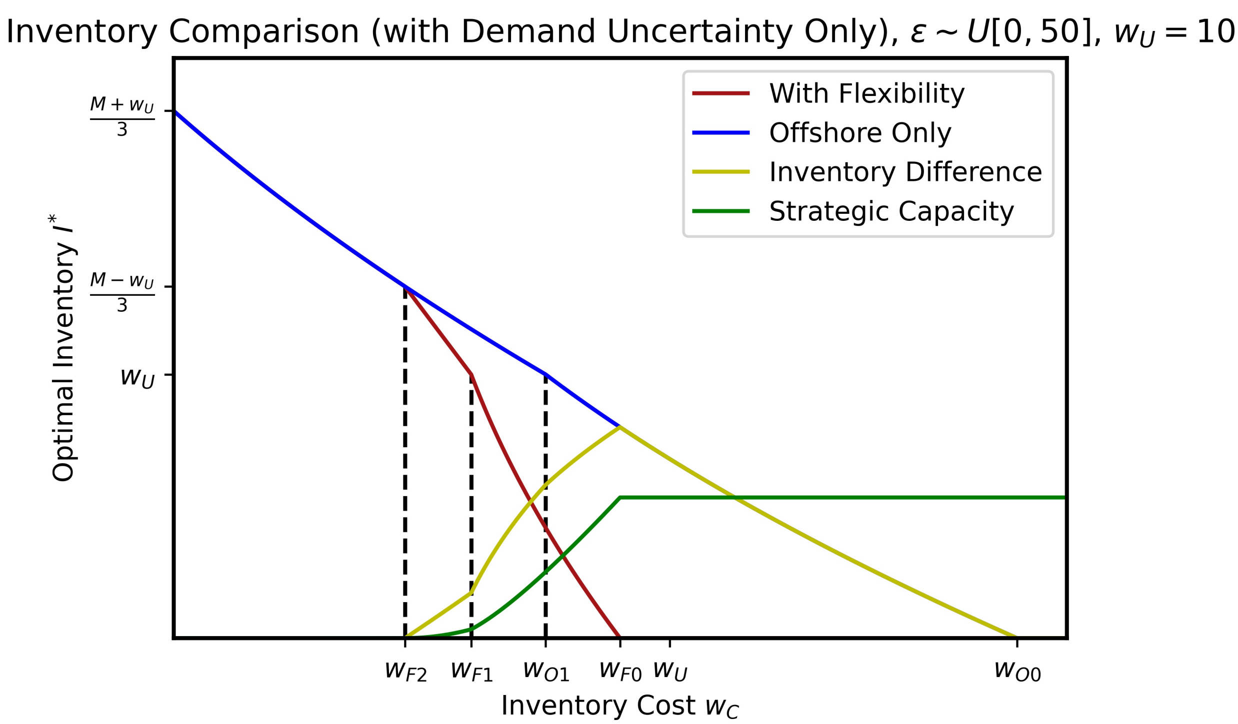


Figure 2.7: Inventory Comparison (with Demand Uncertainty Only)

We put two cases in one graph, which shows us how the optimal inventories with and without flexibility change the inventory cost.

Corollary 2.3.3 *If $\epsilon \sim U[0, M]$, then the optimal inventory is larger when the global firm does not have sourcing flexibility. When $w_C < w_{F2}$, these two inventories are the same.*

When the global firm has only the offshoring option, he has the incentive to set up a large inventory to satisfy the market. On the other hand, if he has the flexibility, then he may not want to invest too much in the inventory. Since the local sourcing and production is a backup. As we mentioned before, the intuition for the second result is that the uniformly distributed demand has an upper bound. So the optimal inventory can not increase to infinity, even if the inventory cost is zero. The maximum inventory can not exceed $\frac{M+w_U}{3}$. When the inventory is large enough to cover all possible demands, there is no need to use local options. Hence two cases degenerate into one "offshore only" case.

And $w_{O0} = \frac{M^2 - w_U^2 + 2Mw_U}{4M}$ increases as the maximum possible market size M increases, which means the global firm is more willing to set offshore inventory when the market has a larger potential.

Further, define "strategic capacity" as the expectation of the output quantity which is locally sourced and produced, i.e., $E[q_U^*]$. The way we call it reflects the fact that the flexible global firm can strategically use the local sourcing and production option as an additional capacity.

Obviously, strategic capacity only exists for the flexibility case,

$$E[q_U^*] = \int_{3I^*+w_U}^{M} \frac{\epsilon - w_U}{3} dG(\epsilon)$$

With uniform demand distribution $\epsilon \sim U[0, m]$, it becomes

$$\frac{(M - w_U - 3I^*)^2}{6M}$$

From the above graph, the strategic capacity is smaller than the inventory difference between the flexibility and offshore only cases in the interval $w_C \in [w_{F2}, w_{F0}]$. So the strategic capacity partially accounts for the inventory difference due to the global firm's flexibility.

2.4 Supply Disruption

To model the supply disruption of the FG inventory decision case, suppose there is a probability of p that the supply chain encounters a disruption. When the disruption happens, the firm receives a proportion α of what it orders. Therefore, the inventory decision problem under disruption is as follows.

2.4.1 With Flexibility

Similar to the aforementioned global firm's flexibility to use local sourcing and production as a backup option to satisfy the market. Usually, the inventory cost is smaller than the local cost. After using all inventory, the global firm is able to satisfy extra demand using local sourcing and production.

$$\max_{I \geq 0} E_{\epsilon}[p \cdot r_{disruptive} + (1-p) \cdot r_{regular}] - w_C \cdot I$$

And the stage 2 problem becomes

$$\begin{aligned} r_{regular} = & \max_{q_C \geq 0} [\epsilon - q_C - q_U - q_L] \cdot (q_C + q_U) - 0 \cdot q_C - w_U \cdot q_U \\ \text{s.t.} \quad & q_C \leq I \end{aligned}$$

and

$$\begin{aligned} r_{disruptive} = & \max_{q_C \geq 0} [\epsilon - q_C - q_U - q_L] \cdot (q_C + q_U) - 0 \cdot q_C - w_U \cdot q_U \\ \text{s.t.} \quad & q_C \leq \alpha \cdot I \end{aligned}$$

Omit all details, we conclude

Lemma 2.4.1 *The optimal I^{**} satisfies the following characterizing equations:*

$$\begin{cases} 0, & \text{if } w_C \in [w_{FD0}, \infty) \\ FOC_{FD1}, & \text{if } w_C \in [w_{FD1}, w_{FD0}) \\ FOC_{FD2}, & \text{if } w_C \in [w_{FD2}, w_{FD1}) \\ FOC_{FD3}, & \text{if } w_C \in [0, w_{FD2}) \end{cases}$$

where

$$\begin{aligned} FOC_{FD1} :& p \cdot \alpha \cdot \left[\int_{2\alpha I}^{\alpha I + w_U} (\epsilon - 2\alpha I) dG(\epsilon) + \int_{\alpha I + w_U}^{3\alpha I + w_U} \left[\frac{\epsilon + w_U}{2} - \alpha I \right] dG(\epsilon) + \int_{3\alpha I + w_U}^{\infty} w_U dG(\epsilon) \right] + \\ & (1-p) \cdot \left[\int_{2I}^{I + w_U} (\epsilon - 2I) dG(\epsilon) + \int_{I + w_U}^{3I + w_U} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) + \int_{3I + w_U}^{\infty} w_U dG(\epsilon) \right] = w_C \\ FOC_{FD2} :& p \cdot \alpha \cdot \left[\int_{2\alpha I}^{\alpha I + w_U} (\epsilon - 2\alpha I) dG(\epsilon) + \int_{\alpha I + w_U}^{3\alpha I + w_U} \left[\frac{\epsilon + w_U}{2} - \alpha I \right] dG(\epsilon) + \int_{3\alpha I + w_U}^{\infty} w_U dG(\epsilon) \right] + \\ & (1-p) \cdot \left[\int_{3I - w_U}^{3I + w_U} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) + \int_{3I + w_U}^{\infty} w_U dG(\epsilon) \right] = w_C \\ FOC_{FD3} :& p \cdot \alpha \cdot \left[\int_{3\alpha I - w_U}^{3\alpha I + w_U} \left[\frac{\epsilon + w_U}{2} - \alpha I \right] dG(\epsilon) + \int_{3\alpha I + w_U}^{\infty} w_U dG(\epsilon) \right] + \\ & (1-p) \cdot \left[\int_{3I - w_U}^{3I + w_U} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) + \int_{3I + w_U}^{\infty} w_U dG(\epsilon) \right] = w_C \end{aligned}$$

The optimal inventory becomes smaller as the inventory cost w_C grows. Hence the threshold w_{FD0} is the break-even point for $I = 0$. Since supply could be disruptive, two more break-even points w_{FD1} and w_{FD2} corresponds to $I = w_U$ and $I = w_U/\alpha$ respectively. The subscript "FD" stands for "Flexibility case under supply disruption." And

Lemma 2.4.2

$$\begin{aligned} w_{FD0} &= [p \cdot \alpha + (1-p)] \cdot \left[\int_0^{w_U} \epsilon dG(\epsilon) + \int_{w_U}^{\infty} w_U dG(\epsilon) \right] \\ &= [p \cdot \alpha + (1-p)] \cdot w_{F0} \\ &< w_{F0} \end{aligned}$$

In short, the threshold for the disruption case is only a proportion of the threshold without disruption. The disruption prevents the global firm from setting offshoring inventory, which is intuitive. Then, applying the uniform demand distribution gives us.

Corollary 2.4.1 *When the demand follows an uniform distribution $\epsilon \sim U[0, M]$, the optimal inventory decision for the disruptive flexibility case is*

$$\begin{cases} 0 & \text{if } w_C \in [w_{FD0}, \infty) \\ \frac{2c_2 w_U - \sqrt{4c_2^2 w_U^2 - c_1 c_3 (2M w_U - w_U^2) + 2c_3 M w_C}}{c_3} & \text{if } w_C \in [w_{FD1}, w_{FD0}) \\ \frac{2c_2 w_U - \sqrt{4c_2^2 w_U^2 - p\alpha^3 [p\alpha(2M w_U - w_U^2) + (1-p) 2M w_U - 2M w_C]}}{p\alpha^3} & \text{if } w_C \in [w_{FD2}, w_{FD1}) \\ \frac{M(c_1 w_U - w_C)}{2c_2 w_U} & \text{if } w_C \in [w_{FD3}, w_{FD2}) \\ \frac{\mathcal{B} - \sqrt{4\mathcal{B}^2 - 3(1-p)[4p\alpha M w_U + (1-p)(M+w_U)^2 - 4M w_C]}}{3(1-p)} & \text{if } w_C \in [w_{FD4}, w_{FD3}) \\ \frac{2c_2 (M+w_U) - \sqrt{4c_2^2 (M+w_U)^2 - 3c_1 c_3 (M+w_U)^2 + 12 c_3 M w_C}}{3c_3} & \text{if } w_C \in [0, w_{FD4}) \end{cases}$$

where

$$
\begin{aligned}
c_1 =& p\alpha + (1-p) \\
c_2 =& p\alpha^2 + (1-p) \\
c_3 =& p\alpha^3 + (1-p) \\
\mathcal{B} =& 2p\alpha^2 w_U + (1-p)(M + w_U) \\
w_{FD0} =& c_1 \cdot \left[w_U - \frac{w_U^2}{2M} \right] \\
w_{FD1} =& \frac{p\alpha}{M} \cdot \left[\frac{(1-\alpha)^2 w_U^2}{2} - (\alpha+1) w_U^2 + M w_U \right] + \frac{1-p}{M} \cdot \left[M w_U - 2 w_U^2 \right] \\
w_{FD2} =& \frac{p\alpha}{M} \cdot \left[M w_U - 2 w_U^2 \right] + \frac{1-p}{M} \cdot \left[M w_U - \frac{2}{\alpha} w_U^2 \right] \\
w_{FD3} =& \frac{p\alpha}{M} \cdot \left[M w_U - \frac{2}{3}(M - w_U) M w_U \right] + \frac{1-p}{M} \cdot \frac{(M + w_U) w_U}{3} \\
w_{FD4} =& \frac{p\alpha}{M} \cdot \frac{(M + 2w_U) w_U}{3} + \frac{1-p}{4M} \cdot \left[M + w - \frac{M - w_U}{3\alpha} \right] \left[M + w - \frac{M - w_U}{\alpha} \right]
\end{aligned}
$$

The extra w_{FD3} is the break-even point for $I = \frac{M-w_U}{3}$, and w_{FD4} is for $I = \frac{M-w_U}{3\alpha}$. Although the threshold $w_{FD0} < w_{F0}$, comparing inventories for the flexible global firm with and without lead time uncertainty shows:

Proposition 2.4.1 *If $\epsilon \sim U[0, M]$, when $w_C < w_{F2}$, $I^{**} > I^*$, and when $w_C > w_{F2}$, $I^{**} < I^*$.*

In other words, the optimal inventory under disruption is smaller than the one without disruption if the inventory cost is high, which is straightforward. However, when the inventory cost is low, lower than the break-even point corresponding to $I^* = \frac{M-w_U}{3}$ without disruption, then the optimal inventory is larger when disruption happens. One reason is that the maximum possible inventory decision under disruption is larger than the one without supply disruption. Recall the very left segment of the blue line corresponds to the case when there is no need to use local options. The maximum possible inventory under disruptions is $1/\alpha$ times the one without disruption. On the other hand, the global firm only receives a proportion of inventory as he orders. When the cost of establishing inventory is small, the global firm anticipates the risk of a small inventory and thus intends to hedge more against the supply disruption.

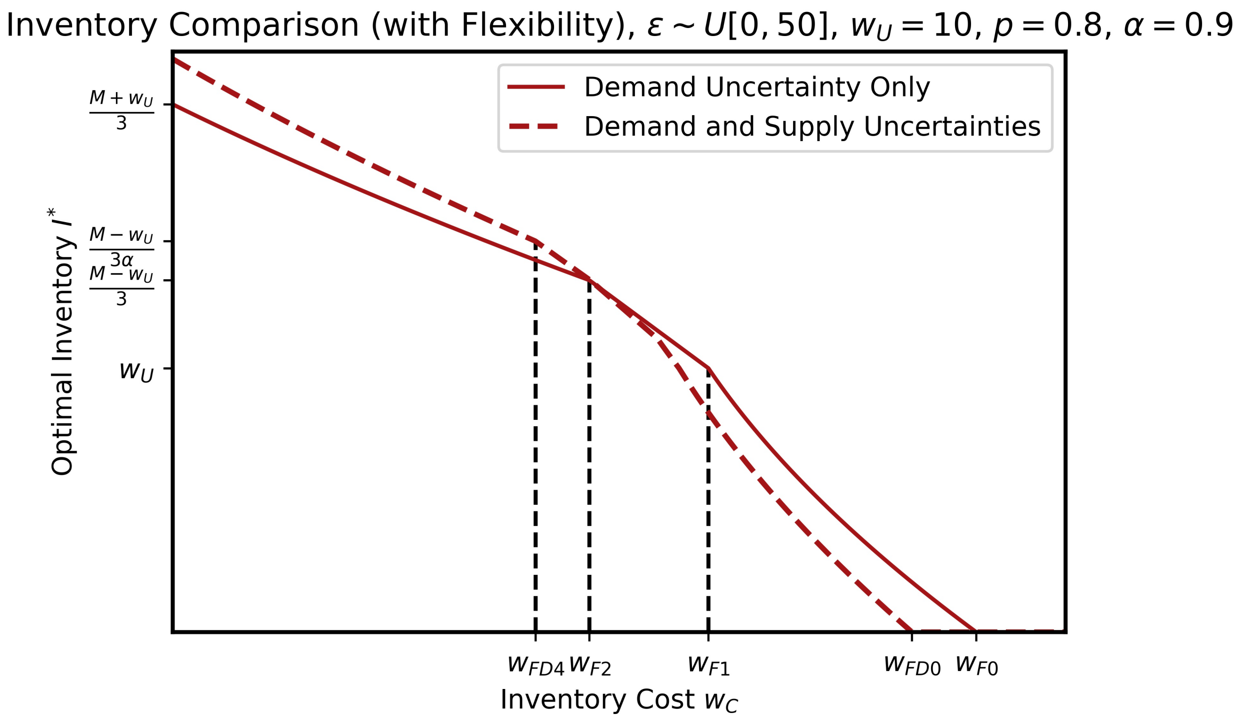


Figure 2.8: Inventory Comparison (with Flexibility)

2.4.2 Offshore Only

When the global firm lacks flexibility under supply disruption and demand uncertainty, the problem becomes

$$\max_{I\geq 0} E_{\epsilon}[p\cdot r_{disruptive}+(1-p)\cdot r_{regular}]-w_C\cdot I$$

And the stage 2 problem becomes

$$\begin{aligned} r_{regular} = &\max_{q_C\geq 0}\left[\epsilon-q_C-q_L\right]\cdot q_C-0\cdot q_C \\ &\text{s.t.}\quad q_C\leq I \end{aligned}$$

and

$$\begin{aligned} r_{disruptive} = &\max_{q_C\geq 0}\left[\epsilon-q_C-q_L\right]\cdot q_C-0\cdot q_C \\ &\text{s.t.}\quad q_C\leq \alpha\cdot I \end{aligned}$$

Without further ado,

Lemma 2.4.3 *The optimal I^{**} satisfies the following characterizing equations:*

$$\begin{cases} 0, & \text{if } w_C \in [w_{OD0}, \infty) \\ FOC_{OD1}, & \text{if } w_C \in [w_{OD1}, w_{OD0}) \\ FOC_{OD2}, & \text{if } w_C \in [w_{OD2}, w_{OD1}) \\ FOC_{OD3}, & \text{if } w_C \in [0, w_{OD2}) \end{cases}$$

where

$$\begin{aligned} FOC_{OD1}: & p \cdot \alpha \cdot \left[\int_{2\alpha I}^{\alpha I + w_U} (\epsilon - 2\alpha I) dG(\epsilon) + \int_{\alpha I + w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - \alpha I \right] dG(\epsilon) \right] + \\ & (1-p) \cdot \left[\int_{2I}^{I + w_U} (\epsilon - 2I) dG(\epsilon) + \int_{I + w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) \right] = w_C \\ FOC_{OD2}: & p \cdot \alpha \cdot \left[\int_{2\alpha I}^{\alpha I + w_U} (\epsilon - 2\alpha I) dG(\epsilon) + \int_{\alpha I + w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - \alpha I \right] dG(\epsilon) \right] + \\ & (1-p) \cdot \int_{3I - w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) = w_C \\ FOC_{OD3}: & p \cdot \alpha \cdot \int_{3\alpha I - w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - \alpha I \right] dG(\epsilon) + (1-p) \cdot \int_{3I - w_U}^{\infty} \left[\frac{\epsilon + w_U}{2} - I \right] dG(\epsilon) = w_C \end{aligned}$$

The optimal inventory becomes smaller as the inventory cost w_C grows. Hence the threshold w_{OD0} is the break-even point for $I = 0$. Since supply could be disruptive, two more break-even points w_{OD1} and w_{OD2} corresponds to $I = w_U$ and $I = w_U/\alpha$ respectively. The subscript "OD" stands for "Offshore only case under supply disruption." And similarly

Lemma 2.4.4

$$w_{OD0} = [p \cdot \alpha + (1-p)] \cdot w_{O0} < w_{O0}$$

Similar to the previous chapter, the threshold for the disruption case is only a proportion of the threshold without disruption. The disruption prevents the global firm from setting offshoring inventory, which is intuitive. However, since w_{O0} could be larger or smaller than w_U, so does $w_{OD0} = [p \cdot \alpha + (1-p)] \cdot w_{O0}$. Therefore, the "first mover's advantage" and "early order's disadvantage" are still there for the disruption case. Which one dominates depends on the magnitude of the uncertain demand. Apart from the demand uncertainty, the supply disruption also influences the global firm's willingness to set offshoring inventory. If the disruption is more liked to happen (p is large), or the disruption effect is severe (α is small), then the threshold w_{O0} is small. The global firm is less willing to establish more offshore inventory to hedge against the risk because the inventory establishment still brings risk.

Then, applying the uniform demand distribution gives us.

Corollary 2.4.2 *When the demand follows an uniform distribution $\epsilon \sim U[0, M]$, the optimal inventory decision for the disruptive offshore only case is*

$$\begin{cases} 0, & \text{if } w_C \in [w_{OD0}, \infty) \\ \frac{2c_2(M+w_U) - \sqrt{4c_2^2(M+w_U)^2 - 5c_1c_3B + 20c_3Mw_C}}{5c_3}, & \text{if } w_C \in [w_{OD1}, w_{OD0}) \\ \frac{2c_2(M+w_U) - \sqrt{4c_2^2(M+w_U)^2 - [5p\alpha^3 + 3(1-p)][p\alpha B + (1-p)(M+w_U)^2 - 4Mw_C]}}{5p\alpha^3 + 3(1-p)}, & \text{if } w_C \in [w_{OD2}, w_{OD1}) \\ \frac{2c_2(M+w_U) - \sqrt{4c_2^2(M+w_U)^2 - 3c_1c_3(M+w_U)^2 + 12c_3Mw_C}}{3c_3}, & \text{if } w_C \in [0, w_{OD2}) \end{cases}$$

where

$$
\begin{aligned}
B =& M^2 - w_U^2 + 2Mw_U \\
w_{OD0} =& c_1 \cdot \frac{B}{4M} \\
w_{OD1} =& \frac{p\alpha}{M} \cdot \left[\frac{(1-\alpha)^2 w_U^2}{2} + \frac{(M-(3-3\alpha)w_U)(M-(1+\alpha)w_U)}{4} \right] + \frac{1-p}{M} \cdot \frac{M(M-2w_U)}{4} \\
w_{OD2} =& \frac{p\alpha}{M} \cdot \frac{M(M-2w_U)}{4} + \frac{1-p}{4M} \cdot \left[M + (1-\frac{1}{\alpha})w_U \right] \left[M + (1-\frac{3}{\alpha})w_U \right]
\end{aligned}
$$

Compared with the case with only demand uncertainty,

Proposition 2.4.2 *If $\epsilon \sim U[0, M]$, there exists a threshold w_{OC} which is smaller than w_{O1}. When $w_C < w_{OC}$, $I^{**} > I^*$, and when $w_C > w_{OC}$, $I^{**} < I^*$.*

Although there is a maximum possible inventory for the offshore only case, the effect is different from the flexibility case. $\frac{M-w_U}{3}$ does not serve as a turning point for inventory decisions anymore. The only reason for the possibility that the inventory is larger under disruption is that the global firm only receives a proportion of inventory as he orders. When the cost of establishing inventory is small, the global firm anticipates the risk of a small inventory and thus intends to hedge more against supply disruption.

2.4.3 Strategic Inventory

We could call the inventory decision under demand uncertainty only as “safety stock”, which is obviously only driven by demand uncertainty. The difference between inventories driven

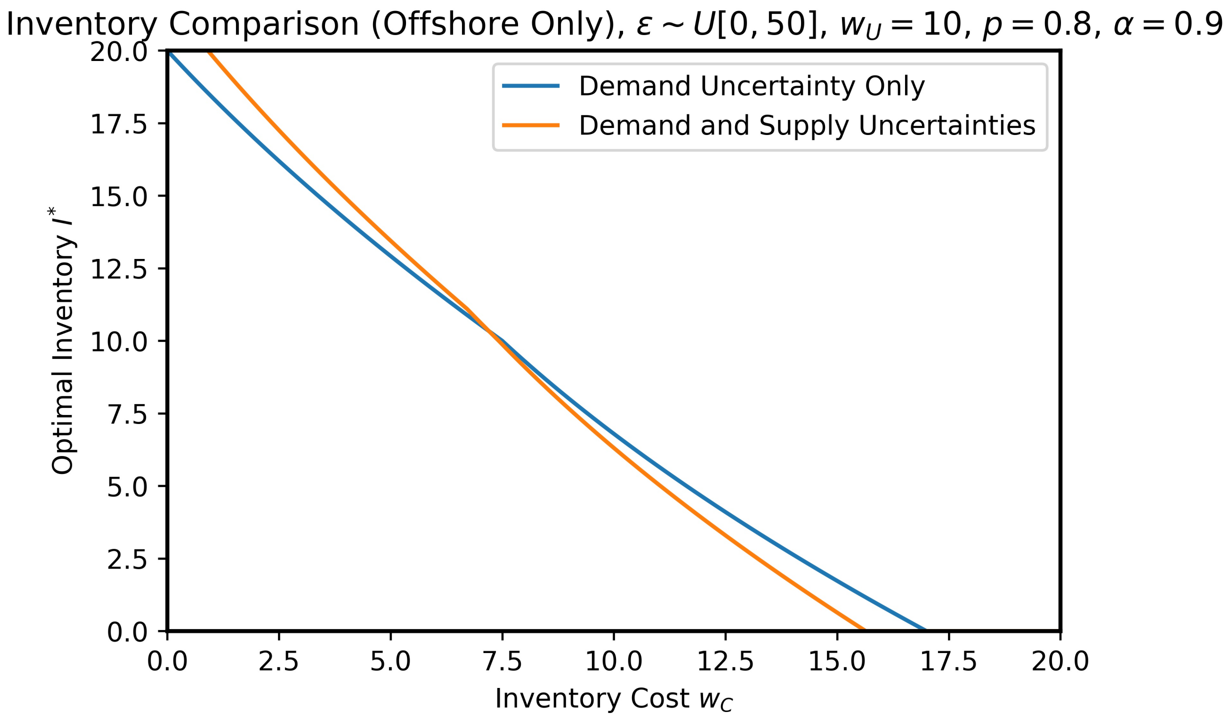


Figure 2.9: Inventory Comparison (Offshore Only)

by demand and supply disruption and the "safety stock" is called "strategic inventory". Then,

Proposition 2.4.3 *If $\epsilon \sim U[0, M]$ and $w_C < w_{O1}$, then the strategic inventory is always larger for the offshore only case rather than the flexibility case.*

Like the graph below, we can see when the inventory cost is small ($w_C < w_{FD4}$), flexibility cases degenerate to the offshore only cases with or without LT uncertainty because inventory is not that valuable due to the existence of local options. Then the strategic inventory value is the same for both flexibility and offshore only cases. When the inventory cost grows, the flexible global firm does not increase the strategic inventory much because it has another option: strategic capacity.

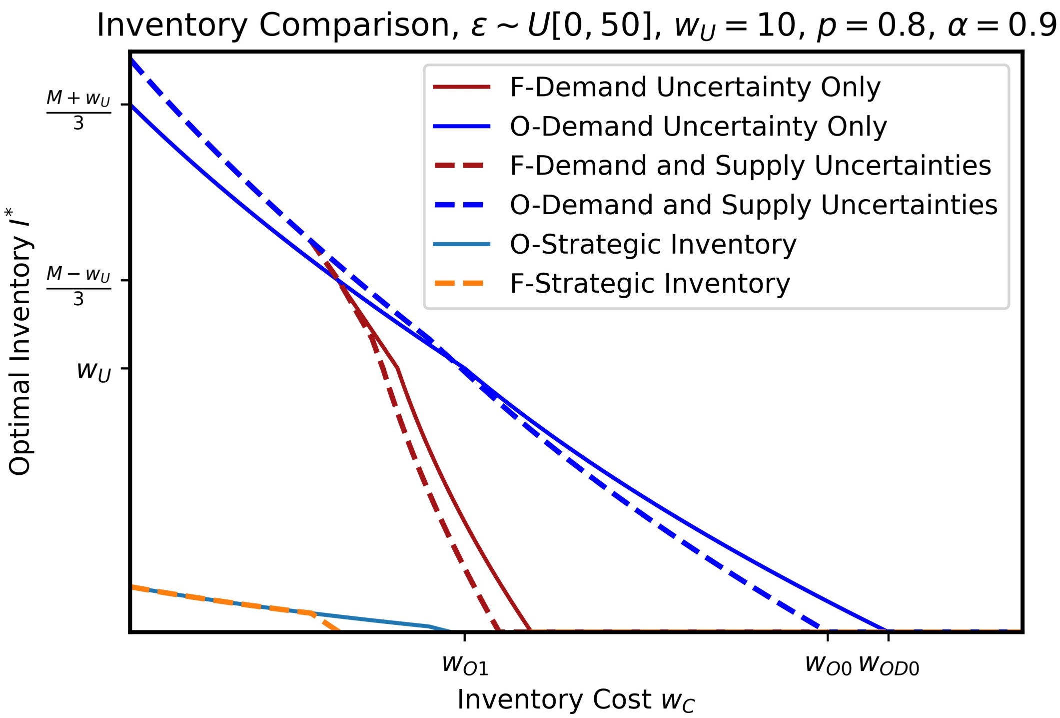


Figure 2.10: Inventory Comparison

Define "strategic capacity" as the expectation of the output quantity that is locally sourced and produced, i.e., $E[q_U^*]$. Obviously, strategic capacity only exists for the flexibility case. We have already known when the global firm switches to completely strategic capacity ($w_U > w_{F0}$ or $w_U > w_{FD0}$), and when the global firm switches out of strategic capacity completely ($w_U < w_{F2}$ or $w_U < w_{FD4}$). The only left one question is to compare the strategic capacity under demand and LT uncertainty with that one under demand uncertainty only, i.e.,

$$\int_{3I^*+w_U}^{M} \frac{\epsilon - w_U}{3} dG(\epsilon)$$

and

$$p \cdot \int_{3\alpha I^{**}+w_U}^{M} \frac{\epsilon - w_U}{3} dG(\epsilon) + (1-p) \cdot \int_{3I^{**}+w_U}^{M} \frac{\epsilon - w_U}{3} dG(\epsilon)$$

with uniform demand distribution $\epsilon \sim U[0, m]$, they become

$$\frac{(M - w_U - 3I^*)^2}{6M}$$

and

$$p \cdot \frac{(M - w_U - 3\alpha I^{**})^2}{6M} + (1-p) \cdot \frac{(M - w_U - 3I^{**})^2}{6M}$$

Due to the difficulty of substituting the optimal inventory I^* and I^{**}, we use the following numerical example showing that the strategic capacity is larger under two uncertainties when the inventory cost is medium. When the inventory cost is too small, then the inventory is too large. The strategic capacities are zero for both uncertain environments.

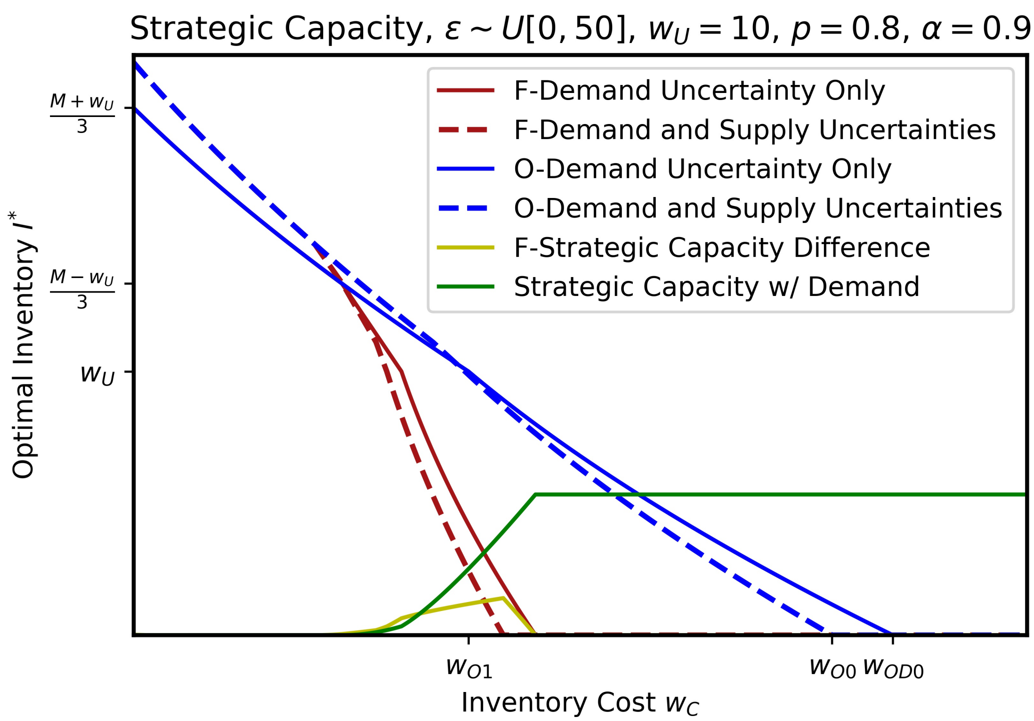


Figure 2.11: Strategic Capacity

Chapter 3

Selling Agri-Tech Products: Firm Strategy, Farmer Adoption, and Government Subsidy

3.1 Introduction

An unmanned aerial vehicle (UAV), commonly known as a drone, is an aircraft without a human pilot on board and a type of unmanned vehicle. It is now broadly used in agriculture, like observing, seeding, fertilizing, spraying pesticides, and so on. An agricultural drone is a type of UVAs applied to farming in order to help increase crop production and monitor crop growth. Sensors and digital imaging capabilities can give farmers a richer picture of their fields. This information is useful in improving crop yields and farm efficiency. UAVs let farmers see their fields from the sky. This bird's-eye view can reveal many issues, such as irrigation problems, soil variation, and pest and fungal infestations. Multispectral images show a near-infrared view as well as a visual spectrum view. The combination shows the farmer the differences between healthy and unhealthy plants, a difference not always clearly visible to the naked eye. Thus, these views can assist in assessing crop growth and production.

Moreover, this flying object is quite helpful in carrying and delivering things. Farmers have a tough job, but drones can make it easier. The average carrying capacity for professional agriculture UAVs is up to 220kg. After observing how their plants grow, farmers can send UAVs to carry fertilizer or pesticide and spray them to the target. Flying surely beats walking, not to mention UAVs can do the job more smoothly. UAVs have even been used to pollinate flowers and could one day prove helpful in compensating for the declining bee population. Additionally, the drone can survey crops for the farmer periodically to their liking. Weekly, daily, or even hourly, pictures can show the changes in the crops over time, thus showing possible trouble spots. Having identified these trouble spots, the farmer can attempt to improve crop management and production. This kind of job is tedious, but UAVs can handle it perfectly. According to a report from a French farming cooperative, OCEALIA Group [58], the increase in yield is more than 10%.

Many manufacturers produce UAVs. DJI Technology Cooperation is the world's largest drone manufacturer, accounting for over 70 percent of the drone market. It is a Chinese technology company headquartered in Shenzhen, Guangdong, with factories worldwide. It is known as a manufacturer of UAVs for aerial photography and videography. DJI also designs and manufactures camera gimbals, flight platforms, cameras, propulsion systems, camera stabilizers, and flight control systems. In 2015, DJI released the first agriculture drone octocopter Agras MG-1 for crop-dusting and other agriculture uses. The drone can carry aloft 2.6 gallons of liquid and can spray 7-10 acres an hour in automatic or manual mode. In 2020, the newest version Agras T20 improved the payload to 16L and the dispense rate to 15 kg per minute. Not to mention the real-time visual monitoring system and the omnidirectional digital radar. According to research by Meticulous Market Research Pvt. Ltd, the global market for agricultural drones is expected to reach $5.19 billion by 2025 [26]. This market has huge potential.

However, there are some problems with the UAV. The first one is that the price of UAV is high. The price ranges from 1,500 to over 20,000 US dollars for a commercial-grade spraying drone. So farmers may hesitate when they make decisions about buying a UAV.

The second problem is that UAVs have a high accident rate compared to traditional agriculture tools like tractors. There are even drone crash compilations on YouTube [28]. The reason may be that farmers do not use UAVs properly, which is related to their learning capability. As we all know, farmers need years of learning and training to handle trucks or tractors expertly. Not to mention drones are emerging tools for them. On the one hand, using UAVs increases output. On the other hand, it is hard to use. Firms can put effort into improving reliability. To be specific, UAV manufacturers can put effort into improving their products, teaching farmers to use them properly, or providing a service team. For instance, PrecisionHawk, a commercial drone and data company, has a professional team to help farmers with drone services. The flight team includes a pilot and visual observer. Hence, the output increase is promising.

Therefore, government plays an important role in the development of agri-technology products. It makes several subsidy schemes to incentivize farmers to buy agri-technology products or incentivize the firm to improve their products. According to [8], the Australian government offering AU$60 million (about 42 million US dollars) in grants to encourage smart farming. The grants will be provided to individuals and organizations in agriculture that are looking to trial or implement new technology products like drones in an effort to improve the condition of natural resources such as soils and vegetation, as well as output. However, it is not very clear how exactly the money will be spent. Should the government subsidize farmers directly or subsidize the manufacturing firm?

We aim to study the impact of new agri-technology products on agricultural operations. Our main research questions are:

1. What is farmers' reaction to the firm's pricing strategy?
2. How does the firm determine prices for agri-technology products and services? A natural question is whether the firm should provide service.
3. How do farmers' capability of learning agri-technology product influence the firm's strategy?
4. What subsidy scheme should the government adopt to improve social welfare?

3.2 Literature Review

The agriculture supply chain received considerable attention from the operations management literature. [25] is among the first papers to study crop planning problems. [41] study the farmland allocation decision between corn and soybeans in a multi-period framework, considering the crop rotation benefits. Recently, a lot of papers have examined the operational decisions of supply chain agents in the agricultural sector. The majority of papers in this literature focus on processors and studies their strategic (e.g., capacity investment) and operating (e.g., procurement and production planning) decisions. These papers consider idiosyncratic features of different agricultural industries, like citrus fruit in [35], palm oil in [10] and so on. [9] considers crop planning decisions in sustainable agriculture, i.e. how to allocate farmland among multiple crops in each growing season when the crops have rotation benefits. Finally, it comes up with not only an optimal policy but also a heuristic policy. [36] start to add government interventions to the agriculture supply chain setting.

Our paper is also related to the agricultural economics literature on subsidies. There are mainly three underlying goals of the government subsidy programs: (a) economical, (b) environmental, or (c) social. First, the economics literature that examines the economic benefits of manufacturer subsidy, without considering consumer subsidy, is vast. [51] makes a great summary of this research stream, the structure of the subsidy program is exogenously given, and the intent is to examine the impact of a specific structure on manufacturer profit and consumer welfare. In the meantime, economists also examine the impact of consumer-only subsidies, and make comparisons between consumer-only and manufacturer-only strategies. For example, [33] examines a situation in which two competing telecom firms need to decide whether to invest in broadband infrastructure in different areas with different population densities. By assuming that the infrastructure cost depends on the population density, he analyzes a two-stage game in which both firms first choose the areas they plan to invest in according to the population density and then select their competing price. By examining the equilibrium outcomes, he finds that the government can increase social welfare by offering consumer subsidies instead of firm subsidies.

Next, there is a growing body of research literature that examines the implications of subsidy programs that are intended to improve environmental sustainability. In the context of green technology adoption, [17] examines the impact of consumer subsidy on the manufacturer's response by incorporating demand uncertainty in a newsvendor setting. They show that the government can miss the desired adoption target level if it ignores demand uncertainty when designing consumer subsidies.

Moreover, there is another stream of research that focuses on subsidy programs that are intended to alleviate poverty or improve health quality in emerging markets. [7] study the food subsidies in the context of poverty alleviation. Also, [56] and [38] focus on the subsidies offered to malaria drug firms, and try to seek effective subsidies to help those poor people suffering from malaria get access to the drugs. That paper considers two kinds of subsidies that are offered to the malaria drug retailer: purchase subsidies and sales subsidies, and they argue that the donor should only subsidize purchases.

Unlike the aforementioned literature, our paper examines a different context: subsidy programs in the presence of new agriculture technology with a primary intention to improve social welfare. After the term Industry 4.0 was revived in 2011 by the German economic development agency, the fourth industrial revolution builds on the third, leveraging emerging technologies such as Additive Manufacturing, Advanced Robotics, Artificial Intelligence, Autonomous Vehicles, Blockchain, Internet of Things, etc. We focus on studying agri-technology products, especially drones.

There are also many ways to realize subsidy. Two commonly observed schemes are the input-based subsidies, which aim to reduce the input purchasing costs of farmers, and the output-based subsidies, which aim to lower farmers' output processing costs. [24] is one of the first papers to study the impact of an output price support program on the whole economy and finds that price support can increase the gross national product by 8% during a recession. [1] compare two output support schemes: the price loss coverage program and the agriculture risk coverage program. They find that the former dominates the later in terms of farmer revenue, consumer surplus, and government spending for a large range of parameter values. [64] compare the impact of input-based subsidy and output-based subsidy on farmer welfare and income inequality in developing economies. It finds that although both types of subsidies can reduce the aggregate income inequality in terms of the Gini coefficient, the input-based subsidy scheme can narrow the income gap between farmers under mild conditions, whereas the output-based scheme always widens this gap. In our paper we are going to examine four subsidy schemes. Some are input-based or output-based. Some are related to subsidizing technology improvement.

This paper also belongs to the emerging stream of literature on socially responsible operations. [13] study whether firms should prioritize the auditing of suppliers with low or high centrality in the setting of a supply network characterized by firms that source from multiple suppliers and suppliers that serve multiple firms. In equilibrium, despite the fact that auditing this supplier is better for the aggregate profit of the firms. They show that this inefficiency is corrected when the firms cooperate to jointly audit the suppliers and share the auditing cost in a fair manner.

3.3 Model

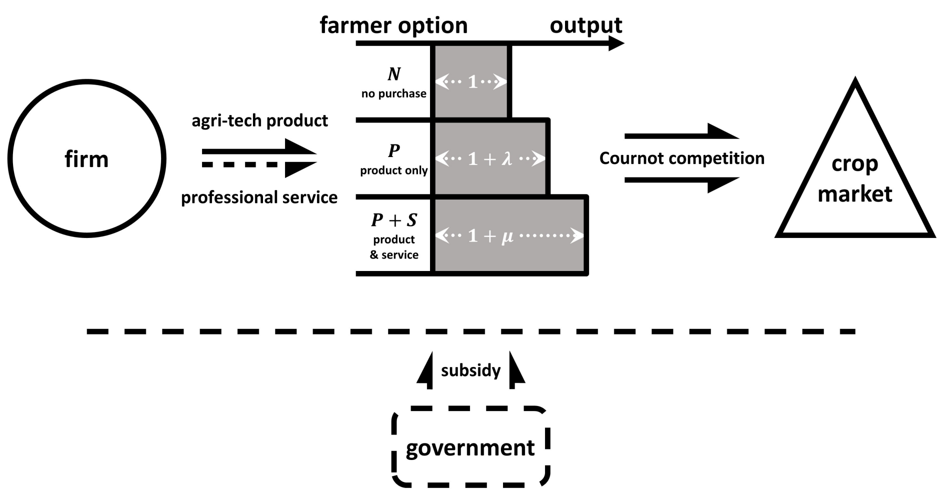


Figure 3.1: Illustration of the Setting

We consider a firm (she) that sells its agri-tech product (e.g., drones) to a large number of farmers (he). The farmers all grow one particular type of crop. Adopting the agri-tech product can enhance productivity, and the level of enhancement depends on the farmers' capability of utilizing the agri-tech product. The firm may also provide an assisting service team, who are professional in utilizing the agri-tech product, to the farmers. After the crops are harvested, the farmers compete in the crop market and receive their revenues. On top of the firm-and-farmers benchmark, we additionally consider the setting in which a government with a limited budget can subsidize the firm or the farmers. In the following, we describe the game in detail.

The Firm. The firm decides the price of the agri-tech product, p. Each unit of agri-tech product sold also incurs a production cost c for the firm. The firm can also provide professional services to the farmers to help them better utilize the agri-tech product. In practice, such practice may or may not be adopted by agri-tech companies. For example, in the drone market, PrecisionHawk is dedicated to such services to farmers, whereas DJI focuses on selling drones, with the provision of only some preliminary user manuals and embedded software. In the model, the firm also chooses whether to make such a service available to farmers; if the firm chooses to do so, she incurs a fixed cost f to maintain the professional service team. For each farmer who purchases such professional service, the firm charges a service fee s.

The Farmers. After the firm determines the product price, whether to provide the professional service and the service fee, all the farmers simultaneously decide on whether to purchase the agri-tech product and whether to purchase the service (if it is available).

We model the farmer population as a unit mass and each farmer is infinitesimal. This modeling approach is widely adopted in the agri-operations literature and well captures the reality that farmers are small, abundant, and fragmental enterprises. The farmers are homogeneous: they plant the same type of crop, and each of them has a unit size of land. In practice, these farmers are usually geographically in the same region; for example, all the grape farmers in Napa Valley, California, or all the litchi farmers in the Lingnan area of China.

A farmer i's crop output level, q_i, depends on the farmer's adoptions of the agri-tech product and the professional service:

$$q_i = \begin{cases} 1 & \text{if option } N\text{: not purchase the product,} \\ 1+\lambda & \text{if option } P\text{: purchase the product but not the professional service,} \\ 1+\mu & \text{if option } PS\text{: purchase both the product and the service.} \end{cases} \tag{3.1}$$

Here, λ represents farmers' capability to utilize the agri-tech product and μ represents such capability of the firm's professional team, with $0 < \lambda < \mu$. Note that we assume the farmers to have homogeneous capability; this assumption is based on that the farmers grow the same type of crop in the same region. As is demonstrated by Aramburu et al. (2019), there exists a “knowledge spillover effect” in agri-tech adoption programs—that is, farmers share information via social networks or help each other such that they achieve the same capability of a certain knowledge.

Let n_N be the number (also the proportion given the unit population size) of farmers who adopt option N, n_P be the number of farmers who adopt option P, and n_{PS} be the number of farmers who adopt option PS; $n_N + n_P + n_{PS} = 1$. Then farmers' total crop output is

$$q = 1 \cdot n_N + (1+\lambda) \cdot n_P + (1+\mu) \cdot n_{PS} = 1 + \lambda \cdot n_P + \mu \cdot n_{PS}. \tag{3.2}$$

The market price of the crop, r, is determined by the total crop output with

$$r = a - b \cdot q, \tag{3.3}$$

which follows the fashion of Cournot competition. We note that Cournot competition is standard to capture the competition on the output of a common good. Let a measure the total market size of the crop and b measure the intensity of farmers' competition. We assume that the market size a is large enough with $a > 4\mu + 1$.

A farmer i's utilize is formulated as

$$u_i = r \cdot q_i - p \cdot \mathbf{1}_{\{\text{option } P \text{ or } PS\}} - s \cdot \mathbf{1}_{\{\text{option } PS\}}, \tag{3.4}$$

	Explanation		Explanation
c	unit production cost of agri-tech product	p	price of the agri-tech product
f	fixed cost of maintaining service team	s	professional service fee
λ	farmers' capability to utilize the product		
μ	professions' capability to utilize the product		
a	size of the crop market	r	market price of the crop
b	competition intensity of the crop market	q	total output quantity of the crop
N	farmer option: not purchase product	n_N	number of farmers choosing N
P	farmer option: purchase product w/o service	n_P	number of farmers choosing P
PS	farmer option: purchase product with service	n_{PS}	number of farmers choosing PS

where r depends on all the farmers' decisions. We focus on the farmers' costs of adopting the agri-tech product and the professional service and, without loss of generality, normalize all the other cropping costs to zero.

3.4 Equilibrium Analysis

In the base model, we do not consider government subsidies and focus on the game between the firm and the farmers. Given that the firm is the first mover, we use backward induction to characterize the game—that is, first deriving the equilibrium of the subgame among farmers and then deriving the optimal decision for the firm.

3.4.1 The Farmers' Equilibrium

Given the price of the agri-tech product p and the service fee s (the service may not be offered by the firm, and we can let $s = +\infty$ for this case), each farmer has three options: Not to purchase the product (N), to purchase the product without service (P), and to purchase the product with service (PS). The farmers play a simultaneous game and, under Nash equilibrium, each farmer maximizes his own utility given all the other farmers' decisions fixed.

By Equation (4), a farmer's utility is affected by the other farmers' decisions through the market price of the crop: If the aggregate quantity produced by all the farmers is higher, then, by Equation (3), the crop price is lower and a farmer's marginal benefit from one unit of crop output decreases. Therefore, although the farmers are homogeneous, they may choose different strategies under equilibrium. This is because a larger number of agri-tech product (and professional service) adoption increases the total output quantity and thus decreases the crop price; then the benefit of adopting the product (service) diminishes as a larger number of farmers do so. Consequently, it is possible that only a portion of farmers adopt the product (service) under equilibrium. Then the potential outcomes of the equilibrium are: all the farmers choose the same option (N or P or PS), two options are chosen among the farmers (N and P, or N and PS, or P and PS), and all three options are chosen among the farmers (N and P and PS).

In the following, as a demonstration, we show how to characterize the equilibrium outcome when all the farmers choose either N or P. Following Equations (1) and (4), we can list any farmer i's utilities of choosing three options as follows:

$$\begin{cases} u_i(N) = r; \\ u_i(P) = r \cdot (1 + \lambda) - p; \\ u_i(PS) = r \cdot (1 + \mu) - p - s. \end{cases} \tag{3.5}$$

When either N or P is chosen by all the farmers, we must have

$$u_i(N) = u_i(P) > u_i(PS), \tag{3.6}$$

which, by algebraic deduction, implies

$$r = \frac{p}{\lambda} < \min\left\{\frac{p+s}{\mu}, \frac{s}{\mu - \lambda}\right\}. \tag{3.7}$$

In the above formulation, Equation (6) holds because, under equilibrium, all the farmers should be indifferent between options N and P. The rationale is from the definition of Nash equilibrium: No farmer should have the incentive to deviate from his decision. If $u_i(N) > u_i(P)$, a farmer with option P will deviate to N; if $u_i(N) < u_i(P)$, a farmer with option N will deviate to P; thus, the equilibrium maintains a stable structure that no farmer deviates only when $u_i(N) = u_i(P)$. In addition, no farmer should deviate to PS, and thus any farmer's utility should be higher than $u_i(PS)$.

With Equation (7), we can inversely obtain the total crop output through Equation (3):

$$q = \frac{a - r}{b} = \frac{\lambda a - p}{\lambda b}. \tag{3.8}$$

Note that the number of farmers who adopt option N, n_N, and the number of farmers who adopt option P, n_P, should satisfy both a total population of 1 and a total output of q:

$$\begin{cases} n_N + n_P = 1, \\ n_N + (1 + \lambda) \cdot n_P = (\lambda a - p)/(\lambda b). \end{cases} \tag{3.9}$$

These two facts can help us obtain the values of n_N and n_P:

$$\begin{cases} n_N = (\lambda b - a + b + \frac{p}{\lambda})/(\lambda b), \\ n_P = (a - b - \frac{p}{\lambda})/(\lambda b). \end{cases} \tag{3.10}$$

Finally, n_N and n_P should be both between 0 and 1, which gives rise to constraints on the price of the agri-tech product:

$$\lambda(a - b) - \lambda^2 b < p < \lambda(a - b). \tag{3.11}$$

In summary, for the possible equilibrium outcome that all the farmers choose either N or P, we are able to characterize the proportion of farmers with each choice as well as the conditions for the product price p and service fee s to generate such equilibrium outcome.

This approach can be applied to characterize all the potential equilibrium outcomes, with which we obtain:

Region	Characterization	Farmer Decision Proportion		
		n_N	n_P	n_{PS}
Ω_1	$0 \le s < (\mu - \lambda)k_\mu$, $0 \le p + s < \mu k_\lambda$	0	0	1
Ω_2	$(\mu - \lambda)k_\mu \le s < (\mu - \lambda)k_\lambda$, $\frac{s}{p} \ge \frac{\mu-\lambda}{\lambda}$	0	$\frac{s-(\mu-\lambda)k_\mu}{(\mu-\lambda)^2 b}$	$\frac{(\mu-\lambda)k_\lambda - s}{(\mu-\lambda)^2 b}$
Ω_3	$s \ge (\mu - \lambda)k_\lambda$, $0 \le p < \lambda k_\lambda$	0	1	0
Ω_4	$\lambda k_\lambda \le p < \lambda(a - b)$, $\frac{s}{p} \ge \frac{\mu-\lambda}{\lambda}$	$\frac{p-\lambda k_\lambda}{\lambda^2 b}$	$\frac{\lambda(a-b)-p}{\lambda^2 b}$	0
Ω_5	$\mu k_\mu \le p + s < \mu(a - b)$, $0 \le \frac{s}{p} < \frac{\mu-\lambda}{\lambda}$	$\frac{(p+s)-\mu k_\mu}{\mu^2 b}$	0	$\frac{\mu(a-b)-(p+s)}{\mu^2 b}$
Ω_6	$p \ge \lambda(a - b)$ or $p + s \ge \frac{\mu-\lambda}{\lambda}$	1	0	0

Table 3.1: Subgame Equilibrium

Note: $k_x = a - b - 2bx$ for any value x.

Proposition 3.4.1 *Given the agri-tech product's price p and the professional service fee s, the Nash equilibrium of the farmers' subgame is characterized in the above table.*

Figure 3.2 depicts regions Ω_1–Ω_6 that characterize the equilibrium outcomes of farmers' subgame. The characterization is based on the product price p and the service fee s, which are compared to thresholds determined by the farmers' market potential (a), competition intensity (b), and the capabilities of utilizing the product from the farmers (λ) and from the firm's professional service team (μ). Each region represents a stable status in which a certain portion of farmers make a certain decision, and no farmer intends to deviate from the current decision. In the following, we discuss these regions in detail.

In Ω_1, both p and s are sufficiently small such that all the farmers are willing to pay for both the agri-tech product and the service. Proceeded from Ω_1, if s gets higher but p is still low (Ω_2), then some farmers will give up the service and only purchase the product,

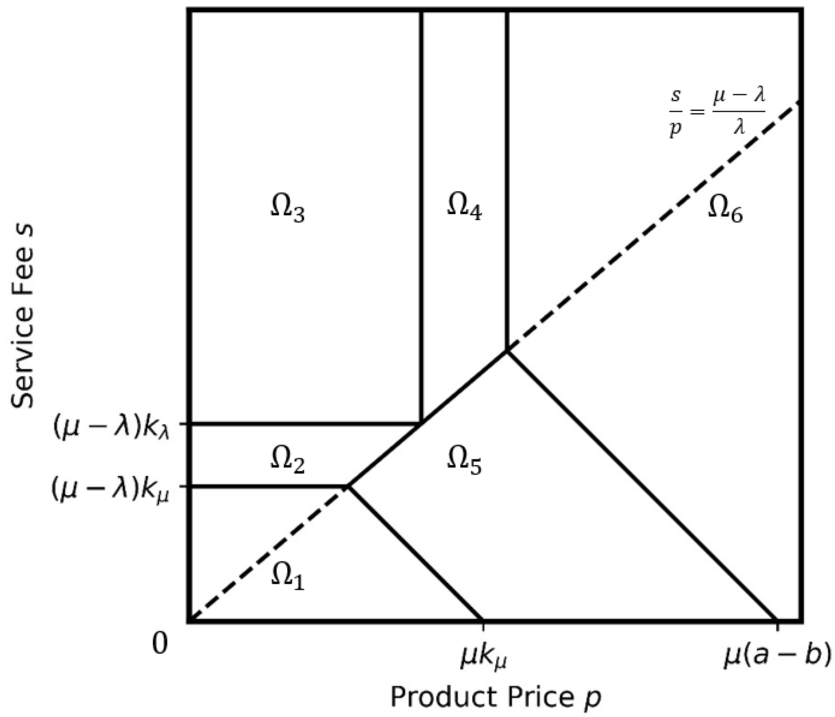


Figure 3.2: Parameter Regions Characterizing Farmers' Equilibrium
$k_x = a - b - 2bx$ for any value x.

whereas the others maintain the service; a further increase of s (Ω_3) makes all the farmers purchase the product only without service. Note that the boundaries between Ω_1 and Ω_2 and between Ω_2 and Ω_3 both increase in a and decrease in b, implying that a larger market potential and a less severe competition encourage more farmers to adopt the service. Back to the position of Ω_1, if p becomes higher but s is still low (Ω_5), then some farmers will give up both the product and the service because the service is only meaningful when associated with the product, whereas the other farmers still purchase both the product and the service since s is low; if p becomes significantly high (Ω_6), then no farmer purchases the product. The boundaries between Ω_1 and Ω_5 and between Ω_5 and Ω_6 also increase in a and decrease in b, implying that a larger market potential and a less severe competition encourage more farmers to purchase the product. Finally, for an intermediate p and a high s (Ω_4), a portion of farmers purchase the product only and the others purchase nothing.

We highlight the slash line of $s/p = (\mu - \lambda)/\lambda$, which segments the whole (p, s) parameter space by comparing the ratio s/p to the threshold $(\mu - \lambda)/\lambda$. It implies the comparison between the prices of service/product and their returns (the improvements in productivity). For the regions below this line, i.e., when s/p is lower than $(\mu - \lambda)/\lambda$, it is never optimal for any farmers to purchase a product only. This is because the service is relatively cheap compared to the agri-tech product and, if a farmer intends to purchase the product, he should additionally purchase the service to enhance the benefit. For the regions above this line, i.e., when s/p is higher than $(\mu - \lambda)/\lambda$, the farmers may purchase the product only and may also purchase both the product and the service. The co-existence of product-only option and product-and-service option in equilibrium is an outcome of market competition, where some farmers find that purchasing both can lead to a too-high overall output quantity that knocks down the market price to an unacceptable extent, whereas purchasing the product only can well balance the price-quantity trade-off in the crop market.[1]

From the firm's perspective, Proposition 1, by characterizing $n_P + n_{PS}$ (the total product sales) and n_{P+S} (the total service sales) under equilibrium, can be regarded as the demand

[1]In fact, there can be multiple equilibria, including one equilibrium in that some farmers choose N, some other farmers choose P, and the rest choose PS. The equilibrium we present is an outcome of refinement based on the firm's optimal strategy. We refer readers to the appendix for the details of this issue.

curves for the agri-tech product and the service. We can further examine the elasticity of such demand curves and obtain:

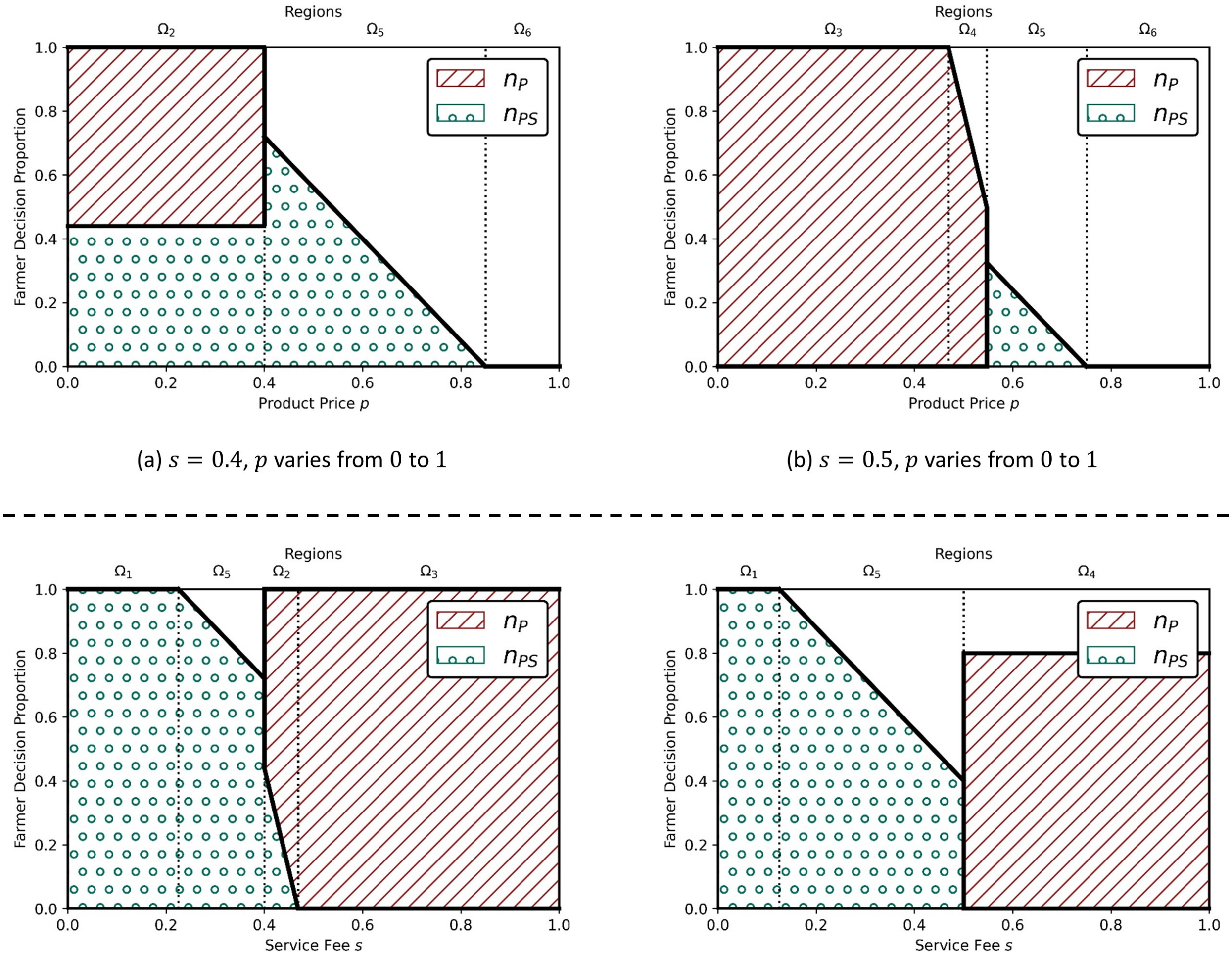
Proposition 3.4.2 *Under the equilibrium of farmers' subgame:*

(i) Given s, $n_P + n_{PS}$ decreases in p whereas n_{PS} may either increase or decrease in p;

(ii) Given p, n_{PS} decreases in s whereas $n_P + n_{PS}$ may either increase or decrease in s.

Proposition 3.4.2 implies that a more expensive agri-tech product leads to fewer farmers purchasing it ($n_P + n_{PS}$ decreases), but may promote the service (n_{PS} increases); symmetrically, a more expensive professional service leads to fewer farmers purchasing it (n_{PS} decreases), but may promote the product ($n_P + n_{PS}$ increases). For each statement, the first half is intuitive and meets the common cognition of a demand curve, whereas the second half, describing the cross elasticity between product and service, is of more interest. The results are because, as p (s) increases, the farmers' equilibrium outcome may alter with changes in regions characterized in Table 3.1 and Figure 3.2.

We elaborate on how farmers respond to varying p and s via the area charts in Figure 3.3. Much non-monotonicity is observed. For example, in Figure 3.3(a), we fix $s = 0.4$ and let p varies from 0 to 1. When p ranges from 0 to 0.4, some farmers purchase the product only and others additionally purchase the service (Ω_2). When p increases beyond 0.4, s/p falls below $(\mu - \lambda)/\lambda$ such that some farmers give up the product, whereas others jointly purchase the product and the service (Ω_5); this triggers a decrease in the total product purchase $n_P + n_{PS}$, but an increase in service purchase n_{PS}. As p further increases, fewer farmers adopt the



(a) $s = 0.4$, p varies from 0 to 1

(b) $s = 0.5$, p varies from 0 to 1

(c) $p = 0.4$, s varies from 0 to 1

(d) $p = 0.5$, s varies from 0 to 1

Figure 3.3: Area Charts of Farmers' Responses to Product Price and Service Fee
Parameters used to plot these figures are: $a = 5$, $b = 2.5$, $\lambda = 0.25$, and $\mu = 0.5$.

product-service bundle (n_{PS} decreases), until no farmer purchases the product (Ω_6). Figure 3.3(b) can be interpreted in a similar vein. For another example, In Figure 3.3(c), we fix $p = 0.4$ and let s varies from 0 to 1. Increasing s across 0.4 (region from Ω_5 to Ω_2) makes fewer farmers purchasing the service (n_{PS} decreases), but a lot more farmers beginning to purchase the product only (n_P increases), leading to an increase to total product purchase

($n_P + n_{PS}$ increases). As s further increases (in Ω_2), more farmers give up the service until all only purchase the product (Ω_3). Figure 3.3(d) can be interpreted in a similar vein.

3.4.2 The Firm's Decision

We now solve the firm's problem of determining the agri-tech product price, whether to offer the professional service, and the service fee. The firm's problem can be formulated as

$$\max_{p,s\geq 0} \quad \pi(p,s) = \left[n_P(p,s) + n_{PS}(p,s)\right] \cdot (p-c) + n_{PS}(p,s) \cdot s - \mathbb{1}_{\{n_{PS}(p,s)>0\}} \cdot f. \qquad (3.12)$$

In this formulation, $n_P(p,s)$ and $n_{PS}(p,s)$ represent the numbers of farmers who purchase the product only and who purchase both the product and the service, which are functions of p and s and characterized by Proposition 1. In case the firm does not offer the service, she can set $s = +\infty$ and thus $n_{PS}(p,s) = 0$. Therefore, $\left[n_P(p,s) + n_{PS}(p,s)\right] \cdot (p-c)$ is the profit from the product and $n_{PS}(p,s) \cdot s - \mathbb{1}_{\{n_{PS}(p,s)>0\}} \cdot f$ is the profit from the service. We note the difference between the two segments of profits: For the product, the firm incurs a variable cost for each agri-tech product sold, whereas for the service, the firm incurs a fixed cost to maintain the service team but not any additional cost for each service.

By solving the problem defined in Equation 3.12, we obtain:

Proposition 3.4.3 *The subgame perfect Nash equilibrium of the whole game and the corresponding firm profit and farmer welfare are characterized in table 3.2.*

In particular, the firm's optimal strategy is either to offer the agri-product only or to always bundle the agri-product and the professional service.

Region	Characterization	Firm Decision	
		p	s
Λ_1	$0 \le c < \lambda k_{2\lambda},\ 0 \le f < (\mu - \lambda)k_{\lambda+\mu}$ or $\lambda k_{2\lambda} \le c < \lambda(a-b),\ 0 \le f < \mu k_\mu - l_\lambda - c$ or $\lambda(a-b) \le c < \mu k_{2\mu},\ 0 \le c + f < \mu k_{2\mu}$	$p + s = \mu k_\mu$	
Λ_2	$0 \le c < \lambda k_{2\lambda},\ f \ge (\mu - \lambda)k_{\lambda+\mu}$	λk_λ	no service
Λ_3	$\lambda k_{2\lambda} \le c < \lambda(a-b),\ f \ge \mu k_\mu - l_\lambda - c$	$\frac{\lambda(a-b)+c}{2}$	no service
Λ_4	$\mu k_{2\mu} \le c < \mu(a-b),\ 0 \le f < l_\mu$	$p + s = \frac{\mu(a-b)+c}{2}$	
Λ_5	Otherwise	no price	no service

Region	Farmer Decision Proportion			Firm Profit	Farmer Welfare
	n_N	n_P	n_{PS}		
Λ_1	0	0	1	$\mu k_\mu - c - f$	k_μ
Λ_2	0	1	0	$\lambda k_\lambda - c$	k_λ
Λ_3	$\frac{2\lambda^2 b - \lambda(a-b)+c}{2\lambda^2 b}$	$\frac{\lambda(a-b)-c}{2\lambda^2 b}$	0	l_λ	$\frac{\lambda(a-b)+c}{2\lambda}$
Λ_4	$\frac{2\mu^2 b - \mu(a-b)+c}{2\mu^2 b}$	0	$\frac{\mu(a-b)-c}{2\mu^2 b}$	$l_\mu - f$	$\frac{\mu(a-b)+c}{2\mu}$
Λ_5	1	0	0	0	$a - b$

Note: $k_x = a - b - 2bx$ and $l_x = \frac{[(a-b)x-c]^2}{4bx^2}$ for any value x.

Table 3.2: Whole Game Nash Equilibrium

The graph below illustrates what the partition of cost space looks like. Along with the corollary showing the corresponding farmers' purchasing decisions in the following, we know the firm sets a bundle price such that all farmers buy both drones and service in the region Λ_1 where both costs are small. When the cost of producing a drone w_0 is still small but the service price goes large (Λ_2), the firm simply has no incentive to provide service at all. Then all farmers are induced to buy drones only. Since the selling price for a drone is not high due to its low production cost, and there is no service from the firm. We move from Λ_2 to Λ_3, the production cost increases. Hence the firm changes its pricing strategy such that part of the farmers buy drones only while the other part just quit. The new pricing strategy takes the production cost w_0 into consideration, which is different from the price in region Λ_2. Because the production cost is higher, the optimal selling price is also higher. It drives part of farmers to deviate from purchasing drones to not purchasing anything at all. On the other hand, if the service cost is small compared to the production cost (Λ_4), the firm also sets a bundle price anticipating farmers either buy a drone with service or buy nothing at all. When both costs are too high(Λ_5), the firm finds it is never profitable to sell anything.

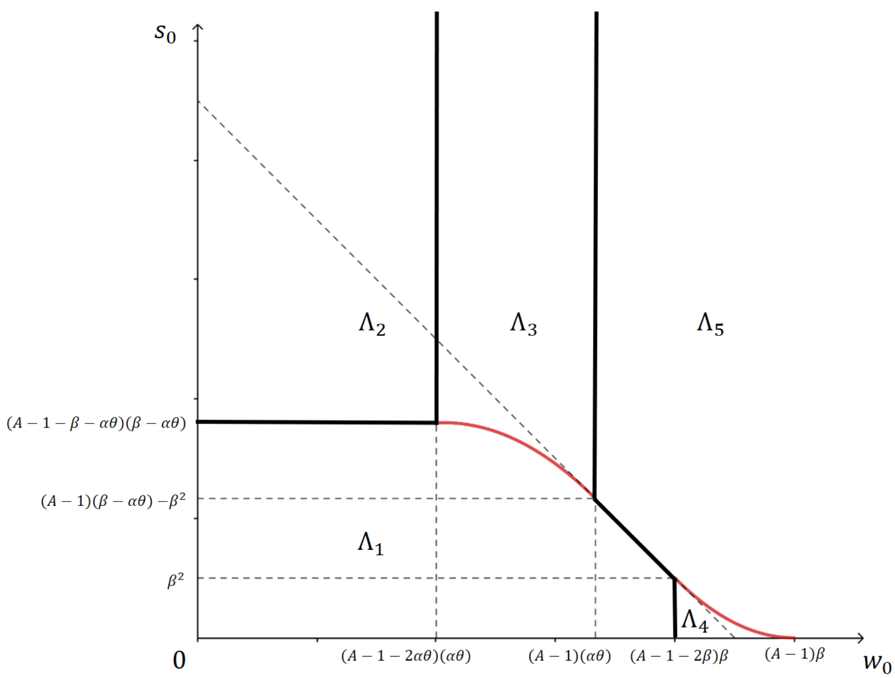


Figure 3.4: Partition of Cost Space ($A = 6$, $\alpha\theta = \frac{1}{3}$, $\beta = \frac{1}{2}$)

Corollary 3.4.1 *The corresponding farmers' purchasing behaviors are:*

Regions	Proportion of Farmers with θ buy		
	Nothing	Drone Only	Drone with Service
Λ_1	0	0	1
Λ_2	0	1	0
Λ_3	$\frac{-A+1+2\alpha\theta+\frac{w_0}{\alpha\theta}}{2\alpha\theta}$	$\frac{A-1-\frac{w_0}{\alpha\theta}}{2\alpha\theta}$	0
Λ_4	$\frac{-A+1+2\beta+\frac{w_0}{\beta}}{2\beta}$	0	$\frac{A-1-\frac{w_0}{\beta}}{2\beta}$
Λ_5	1	0	0

One interesting observation for the firm is that:

Proposition 3.4.4 *The firm sets prices to achieve either complete bundle selling or no bundle selling at all. The bundle selling happens when the service cost* s_0 *is small and the production cost of drones* w_0 *is relatively large.*

In other words, it is never optimal to let part of farmers buy drones only and other parts buy drones with service. Intuitively, when the firm is in a situation where all farmers buy drones while some of them also buy services. It can improve its profit by either decreasing the service price to induce all farmers to buy bundles or increasing the drone's price to make the service price relatively small.

From farmers' perspective, lower service price indeed attracts them to purchase service (Lemma 2). But higher product price does not necessarily results in more or less service purchasing (Lemma 1). Rather, relatively small service price, i.e. small service-to-product price ratio indicates (P1) is never optimal. By Lemma 3, then some of farmers choose to buy nothing while some of them buy bundles by Lemma 3. We achieve complete bundle selling.

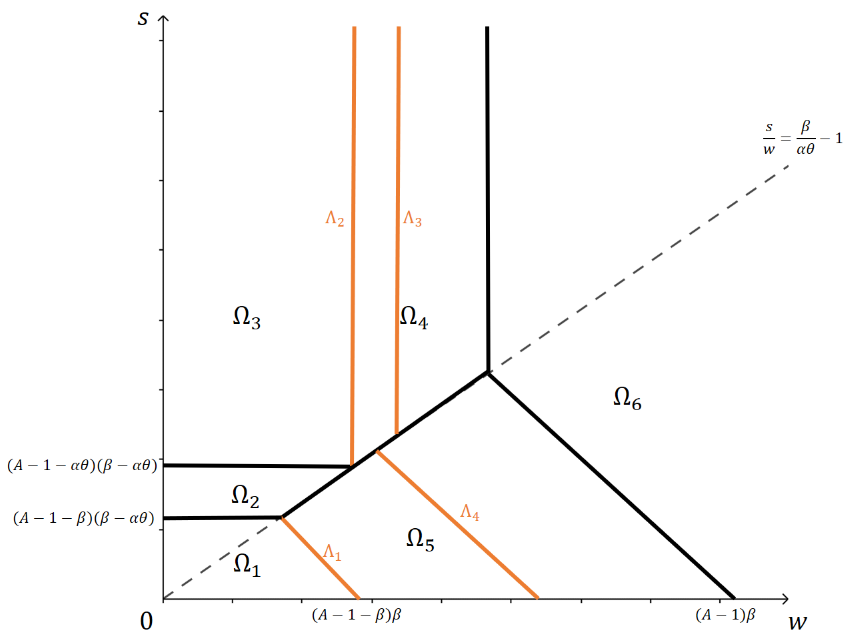


Figure 3.5: Farmers' Equilibria when Firm Makes Optimal Decisions

Four orange lines in the above graph are farmers' equilibria corresponding to the firm's four possible optimal pricing strategies. Like Proposition 3 says, farmers' possible equilibria after firm make its optimal selling strategies do not respond to service price only. In a sense, it accurately reflects the subordinate role of the service. It is also easy to see lines do not fall into region Ω_2 since the firm never allows the farmer equilibrium to be partial (P1) and partial (P2). In addition, the intersection point of the orange line representing Λ_4 with service-to-product price ratio line $\frac{s}{w} = \frac{\beta}{\alpha\theta} - 1$ is beneath the intersection point of Λ_3 with $\frac{s}{w} = \frac{\beta}{\alpha\theta} - 1$. Although the bundle cost in Λ_4 is larger that the product cost in Λ_3, i.e. $\frac{(A-1)\beta+w_0}{2} > \frac{(A-1)\alpha\theta+w_0}{2}$. The portion of the bundle price on the drone is actually always smaller than the drone price in Λ_3. The firm gives up some profit on the product while gaining extra profit by providing service and inducing farmers to buy the bundle.

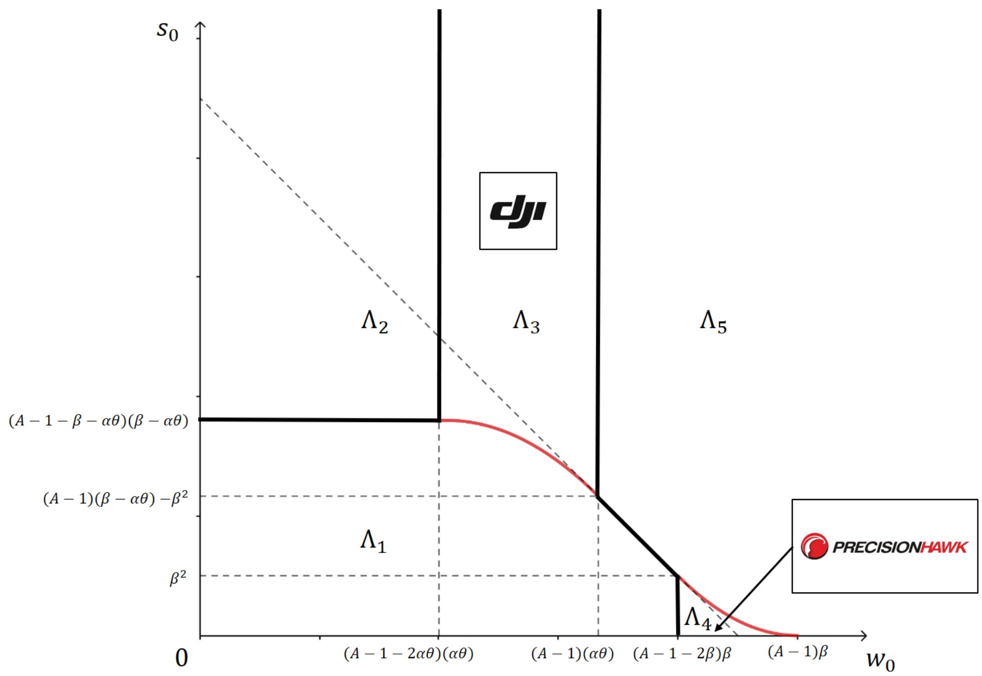


Figure 3.6: DJI vs. Precision Hawk

There are examples, in reality, verifying our model. The complete bundle selling strategy in the region Λ_4 is most likely from companies like Precision Hawk. Since it acts like a dealer, sells drones with the brand DJI and other original equipment manufacturers (OEM). The product cost is certainly higher than that of the OEMs. But the cost of supporting a professional team may be small. Precision Hawk collaborates with the largest drone pilot platform Droners.io. In the following example, the service price for hiring a droner in the central United States is 100 dollars per hour, which is relatively cheap compared to the retail price for an agricultural drone varying from \$1,500 to \$20,000.

No bundle selling strategy is most likely from DJI. On the one hand, we know the production cost should be lower than other dealers. On the other hand, the drone pilot platform is still in its early stages in China. The community is not as large as that in the US. So the cost to initializing a service team for DJI should be higher than Precision Hawk. Therefore, DJI focuses on developing agri-technology products while Precision Hawk seeks to gain profit by integrating service with products.

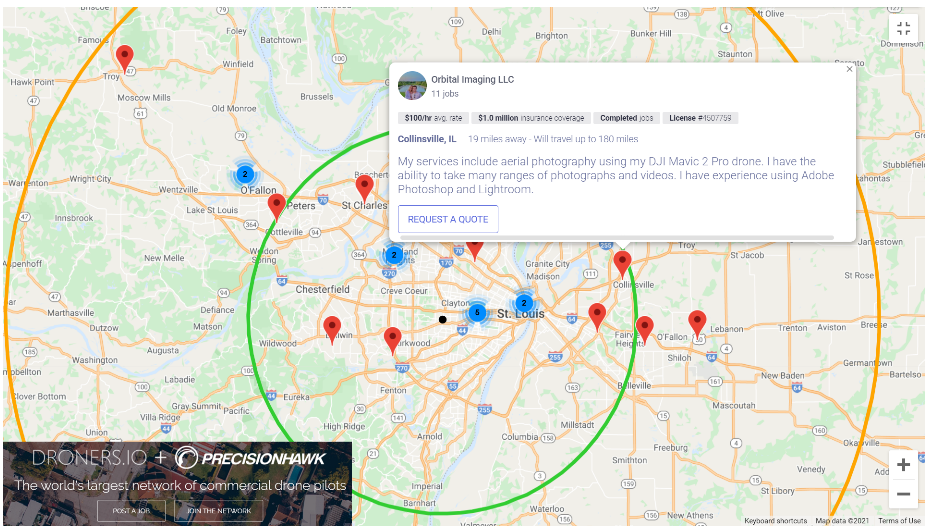


Figure 3.7: Droners.IO and Precision Hawk

3.4.3 Impact of Learning Capability

Recall the learning capability reflects the extent to which farmers can use the agri-technology product properly. We can easily see how farmers' learning capability affects firm's decision:

Proposition 3.4.5 *When farmers' learning capability θ increases,*

(1) region Λ_1 and Λ_5 shrink while region Λ_2 and Λ_3 expand.

(2) firm's selling price and optimal profit in region Λ_2 and Λ_3 increase. But the proportion of farmers who buy drones in Λ_3 decreases.

(3) farmers' aggregated profit in region Λ_2 and Λ_3 decreases.

The firm has less incentive to provide service if farmers are capable of using drones properly. The service is provided only if the service cost is small. Otherwise, the firm sets higher wholesale prices to gain more profits by free-riding farmers' high learning capability. As for farmers, the proportion of them who decide to buy drones only gets smaller since the product price increases.

In the following graph, solid black and red lines stand for the previous boundaries for the partition of cost space. After the learning capability increases from θ to θ', the boundaries change to solid blue lines, except for region Λ_4 since the boundary here stays the same. Complete bundle selling regions Λ_1 shrinks, hence the chance for the firm to achieve complete bundle selling gets smaller. Expansion of regions Λ_2 means the thresholds for both w_0 and s_0 to enter this region decreases. For certain cost parameters that guarantee bundle sells for farmers with lower learning capability, now it may result in cutting the service team if the learning capability increases. Region Λ_3 expands in a different way. While the threshold of service cost to reach this region decreases, the lower and upper bound of product cost both increase. The lower bound increases because Λ_2 "invades and occupies". In other words, for certain cost parameters that guarantee partial farmers are (P1) and others are (NP), now it may result in all farmers are (P1) if the learning capability increases. Because higher learning capacity enables farmers to produce more and hence gain more profit by using drones, despite the selling price of drones increases. The upper bound increases because Λ_3 occupies part of Λ_5. Thus it is less likely the firm will give up the market due to too high costs. Since the upper bound increases more than the lower bound, region Λ_3 expands overall.

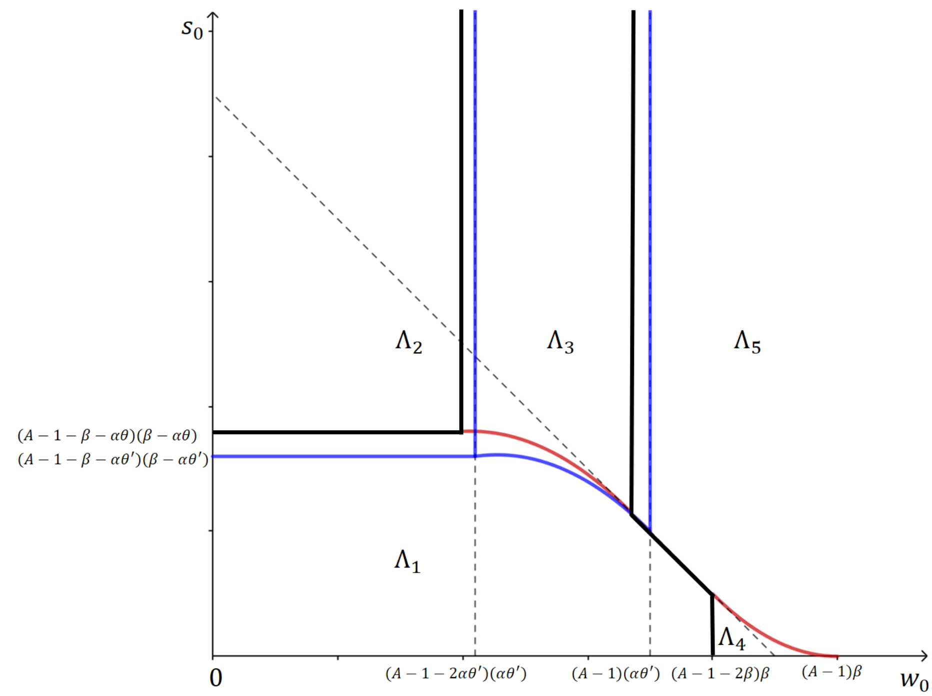


Figure 3.8: Partition of Cost Space ($A = 6$, $\alpha\theta = \frac{1}{3}$, $\alpha\theta' = \frac{7}{20}$, $\beta = \frac{1}{2}$)

What does not show in the graph is that the firm's optimal profit increases in regions Λ_2 and Λ_3 and stays the same in other regions. The total profit of farmers decreases in Λ_2 and Λ_3. Not to mention the propotion of farmers deciding to buy agri-technology products decreases. In reality, it is a quite dangerous signal. Because all efforts to improve farmers' education levels turn out to help the firm put more money in their pocket. In a sense, high learning capability may hurt farmers. Moreover, we can imagine if just a part of farmers increase their learning capability while others do not change. Then the firm still benefits from the higher learning capability of farmers. The low learning capability farmers suffer more than farmers with high learning capability due to competition disadvantage. Hence, our model assuming homogeneous learning capability farmers suffices to provide insights about the impact of learning capability. Will government subsidy schemes mitigate such harm? In the next section, we will explore the impact of different subsidy schemes.

3.5 Subsidy

To mitigate farmer poverty, and income inequality and encourage technology development, there are multiple possible subsidy schemes. Four commonly observed schemes are considered in this section.

Firstly, define farmers' welfare W_F as farmers' aggregated profit, no matter whether they buy drones or not. Naturally, a firm's profit is its welfare. Next, we define social welfare function as

$$W = \lambda \cdot W_F + (1 - \lambda) \cdot \pi$$

where the coefficient $\lambda \in [0, 1]$ measures to which extent the government cares for farmers. Because in reality the firm always occupies an advantageous position. It has resources, funding and power. A larger λ is common, which means that the government cares more about farmers than the firm. We say one subsidy scheme is better than the other one if and only if under certain budget constraints M, the social welfare is large.

3.5.1 Per-Unit Purchasing Subsidy ($\mathcal{P}$)

Suppose the government subsidizes farmers t_1 per drone they purchase. It is obvious that $t_1 \leq M$. The reason we introduce an extra parameter here is that the government can't observe exactly what proportion of farmers purchase drones. Hence it is impossible for the government to divide the budget evenly. Setting the scheme as subsidizing farmers fixed amount of money is more practicable.

Then farmers' possible profits are $\pi_0 = P \cdot 1$ if (NP), $\pi_1(\theta) = P \cdot (1 + \alpha \cdot \theta) - w$ if (P1), $\pi_2(\theta) = P \cdot (1 + \beta) - w - s$ if (P2). This results in a shift in firm's decision:

Corollary 3.5.1 (Per-Unit Purchasing Subsidy) *The firm's optimal profit and farmers' aggregated welfare are:*

Regions	Firm's Optimal Profit π	Farmers' aggregated welfare W_F
$\Lambda_1^{\mathcal{P}}$	$(A - 1 - \beta)\beta - w_0 - s_0 + t_1$	$A - 1 - \beta$
$\Lambda_2^{\mathcal{P}}$	$(A - 1 - \alpha\theta)(\alpha\theta) - w_0 + t_1$	$A - 1 - \alpha\theta$
$\Lambda_3^{\mathcal{P}}$	$\frac{1}{4}\left[A - 1 - \frac{w_0 - t_1}{\alpha\theta}\right]^2$	$\frac{(A-1)(\alpha\theta) + w_0 - t_1}{2(\alpha\theta)}$
$\Lambda_4^{\mathcal{P}}$	$\frac{1}{4}\left[A - 1 - \frac{w_0 - t_1}{\beta}\right]^2 - s_0$	$\frac{(A-1)\beta + w_0 - t_1}{2\beta}$
$\Lambda_5^{\mathcal{P}}$	0	$A - 1$

These regions are partitioned based on the costs of drones and service. Specifically,

Region	Characteristic
$\Lambda_1^{\mathcal{P}}$	$0 \leq w_0 \leq (A-1-2\alpha\theta)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta)$ and
	$0 \leq s_0 \leq (A-1-\beta-\alpha\theta)(\beta-\alpha\theta) - \frac{\lambda}{1-\lambda}(\beta-\alpha\theta)$,
	$(A-1-2\alpha\theta)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta) \leq w_0 \leq (A-1)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta)$ and
	$0 \leq s_0 \leq \frac{1}{4}\left[-\left(\frac{w_0-t_1}{\alpha\theta}\right)^2 + 2(A-1-2\alpha\theta)\cdot\frac{w_0-t_1}{\alpha\theta} - (A-1-2\beta)^2\right] - \frac{\lambda}{1-\lambda}\frac{(w_0-t_1)-(A-1-2\beta)(\alpha\theta)}{2\alpha\theta}$,
	$(A-1)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta) \leq w_0 \leq (A-1-2\beta)\beta + t_1 - \frac{\lambda}{1-\lambda}(2\beta)$ and
	$0 \leq s_0 \leq (A-1-\beta)\beta - w_0 + t_1 - \frac{\lambda}{1-\lambda}(2\beta)$
$\Lambda_2^{\mathcal{P}}$	$0 \leq w_0 \leq (A-1-2\alpha\theta)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta)$ and
	$s_0 \geq (A-1-\beta-\alpha\theta)(\beta-\alpha\theta) - \frac{\lambda}{1-\lambda}(\beta-\alpha\theta)$
$\Lambda_3^{\mathcal{P}}$	$(A-1-2\alpha\theta)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta) \leq w_0 \leq (A-1)(\alpha\theta) + t_1 - \frac{\lambda}{1-\lambda}(2\alpha\theta)$ and
	$s_0 \geq \frac{1}{4}\left[-\left(\frac{w_0-t_1}{\alpha\theta}\right)^2 + 2(A-1-2\alpha\theta)\cdot\frac{w_0-t_1}{\alpha\theta} - (A-1-2\beta)^2\right] - \frac{\lambda}{1-\lambda}\frac{(w_0-t_1)-(A-1-2\beta)(\alpha\theta)}{2\alpha\theta}$
$\Lambda_4^{\mathcal{P}}$	$(A-1-2\beta)\beta + t_1 - \frac{\lambda}{1-\lambda}(2\beta) \leq w_0 \leq (A-1)\beta + t_1 - \frac{\lambda}{1-\lambda}(2\beta)$ and
	$0 \leq s_0 \leq \frac{1}{4}\left[A-1-\frac{w_0-t_1}{\beta}\right]^2 - \frac{\lambda}{1-\lambda}\frac{(A-1)\beta-(w_0-t_1)}{2\beta}$
$\Lambda_5^{\mathcal{P}}$	otherwise

Compared to the partition graph in the benchmark, we can see there is a right shift of each boundary. This is because the subsidy for purchasing drones incentivizes the firm to set higher wholesale prices for most cases. Even a high production cost w_0 can be profitable now. Moreover, it is not surprising that the firm's optimal profit in each region is larger, while the farmer's aggregated welfare stays the same or maybe smaller. Although the subsidy aims to help farmers, actually the firm grasps all profits and makes farmers worse off at the same time.

Corollary 3.5.2 *The corresponding farmers' purchasing behaviors are:*

Regions	Proportion of Farmers with θ buy		
	Nothing	Drone Only	Drone With Service
$\Lambda_1^{\mathcal{P}}$	0	0	1
$\Lambda_2^{\mathcal{P}}$	0	1	0
$\Lambda_3^{\mathcal{P}}$	$\frac{-A+1+2\alpha\theta+\frac{w_0-t_1}{\alpha\theta}}{2\alpha\theta}$	$\frac{A-1-\frac{w_0-t_1}{\alpha\theta}}{2\alpha\theta}$	0
$\Lambda_4^{\mathcal{P}}$	$\frac{-A+1+2\beta+\frac{w_0-t_1}{\beta}}{2\beta}$	0	$\frac{A-1-\frac{w_0-t_1}{\beta}}{2\beta}$
$\Lambda_5^{\mathcal{P}}$	1	0	0

Also, the proportion of farmers who buy drones or drones with service increases in the region $\Lambda_3^{\mathcal{P}}$ and $\Lambda_4^{\mathcal{P}}$. On the one hand, Since the government subsidizes farmers' purchasing drones. On the other hand, only a part of farmers buy product or service in these two cost regions, the firm actually lowers the product price to attract more farmers to make a purchasing decision. However, more purchasing behavior means fiercer competition among farmers. We conclude:

Proposition 3.5.1 *Compared to the benchmark without subsidy, per-unit purchasing subsidy can't improve farmers' welfare. Instead, it may make farmers worse off. But it always benefits the firm.*

In the following graph, we not only show the solid black and red boundary shift under the per-unit purchasing subsidy scheme but also include the solid blue boundaries for the social planner. The former represents what happens if this subsidy scheme is implemented. While the latter one represents the government's optimal goal, which is ideal but not practical.

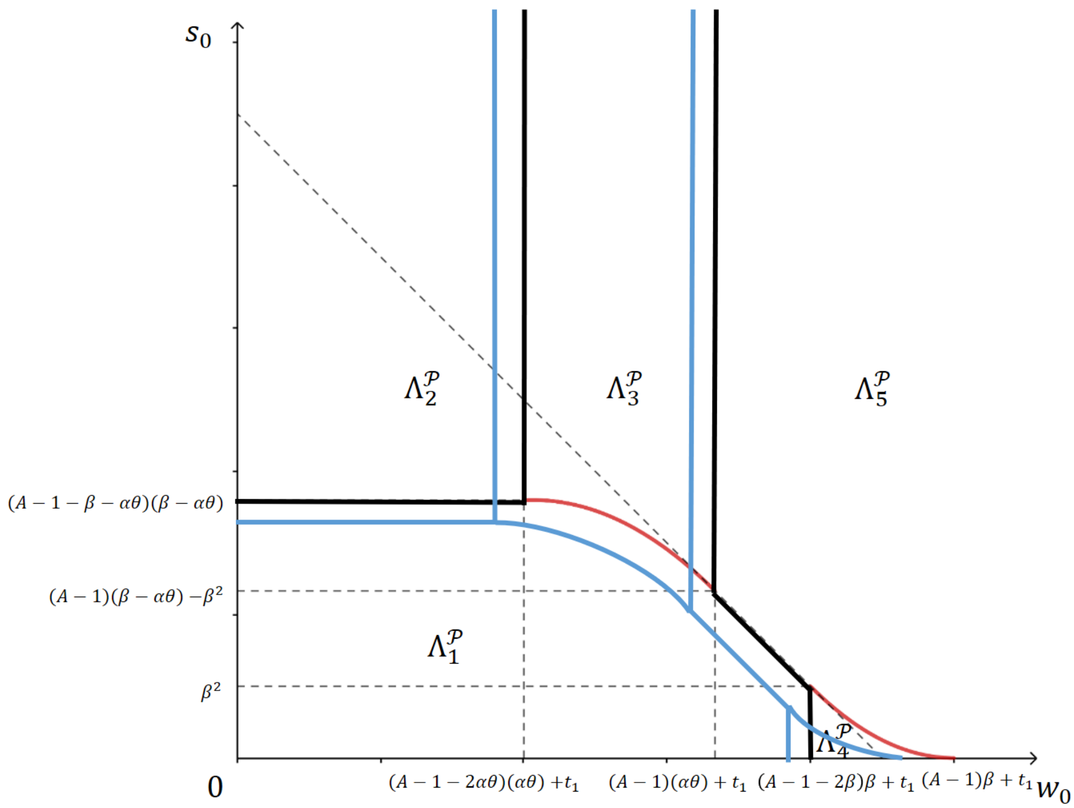


Figure 3.9: Partition of Cost Space under Scheme $\mathcal{P}$

Since the government tries to maximize the total social welfare $W = \lambda \cdot W_F + (1-\lambda)\cdot \pi$. When $\lambda \neq 0$, the objectives of the social planner and the firm are not aligned. The solid blue lines partition the cost space such that each region has the largest total social welfare for certain cost parameters. The exact partition is in the Appendix. An interesting fact about the new partition is that it depends on the coefficient λ. With greater λ, comes smaller $\Lambda_1^{\mathcal{P}}$ and $\Lambda_4^{\mathcal{P}}$, which means it is harder to firm to achieve complete bundle selling if the government cares more about farmers. Too small λ can even make $\Lambda_1^{\mathcal{P}}$ and $\Lambda_4^{\mathcal{P}}$ disappear. Hence, the scheme $\mathcal{S}$ benefits drone companies focusing on producing products, like DJI. But it harms companies mainly providing service, like Precision Hawk. In short, what is good for the firm may not be good for the social planner, and vice versa.

One may notice the boundary shift creates challenges for comparison because of inter-boundary cases. We mainly focus on comparison between regions with the same subscript, like $\Lambda_1^{\mathcal{P}}$ and $\Lambda_1^{\mathcal{GP}}$. It suffices to provide interesting results and profound insights.

Region	Characteristic
$\Lambda_1^{\mathcal{GP}}$	$0 \le w_0 \le (A-1-2\alpha\theta)(\alpha\theta)+t_1$ and $0 \le s_0 \le (A-1-\beta-\alpha\theta)(\beta-\alpha\theta)$,
	or $(A-1-2\alpha\theta)(\alpha\theta)+t_1 \le w_0 \le (A-1)(\alpha\theta)+t_1$ and
	$0 \le s_0 \le \frac{1}{4}\left[-\left(\frac{w_0-t_1}{\alpha\theta}\right)^2 + 2(A-1-2\alpha\theta)\cdot\frac{w_0-t_1}{\alpha\theta} - (A-1-2\beta)^2\right]$
	or $(A-1)(\alpha\theta)+t_1 \le w_0 \le (A-1-2\beta)\beta+t_1$ and $0 \le s_0 \le (A-1-\beta)\beta-w_0+t_1$
$\Lambda_2^{\mathcal{GP}}$	$0 \le w_0 \le (A-1-2\alpha\theta)(\alpha\theta)+t_1$ and $s_0 \ge (A-1-\beta-\alpha\theta)(\beta-\alpha\theta)$
$\Lambda_{G3}^{\mathcal{P}}$	$(A-1-2\alpha\theta)(\alpha\theta)+t_1 \le w_0 \le (A-1)(\alpha\theta)+t_1$ and
	$s_0 \ge \frac{1}{4}\left[-\left(\frac{w_0-t_1}{\alpha\theta}\right)^2 + 2(A-1-2\alpha\theta)\cdot\frac{w_0-t_1}{\alpha\theta} - (A-1-2\beta)^2\right]$
$\Lambda_4^{\mathcal{GP}}$	$(A-1-2\beta)\beta+t_1 \le w_0 \le (A-1)\beta+t_1$ and $0 \le s_0 \le \frac{1}{4}\left[A-1-\frac{w_0-t_1}{\beta}\right]^2$
$\Lambda_5^{\mathcal{GP}}$	otherwise

3.5.2 Per-Unit Selling Subsidy ($\mathcal{S}$)

Suppose the government subsidizes the firm t_2 per drone it sells. We can show:

Proposition 3.5.2 *From the welfare perspective, the per-unit selling subsidy scheme is equivalent to the per-unit purchasing subsidy scheme.*

No matter how the government subsidizes drones, the firm always keeps the efficient prices unchanged. It either increases prices under $\mathcal{P}$ to force farmers to spend all subsidy they receive, or simply take the subsidy directly into pockets under $\mathcal{S}$. The firm acts like a "leech". To explore the underlying reasons, notice that the firm can anticipate farmers' responses based on its own costs. But the government can't. Also, subsidy schemes are common knowledge for all players. It is similar to the first mover advantage in literature. Therefore, $\mathcal{P}$ and $\mathcal{S}$ are not effective subsidy schemes if the government cares more about farmers. However, they could be good schemes if the government wants to help the firm to thrive.

3.5.3 Service Subsidy ($\mathcal{V}$)

Suppose the government subsidizes the firm t_3 if it provides service. Farmers' problem stays the same. However, the firm has an incentive to provide service even when facing a high service cost.

Corollary 3.5.3 (Service Subsidy) *The firm's optimal profit and farmers' aggregated welfare are:*

Regions	Firm's Optimal Profit π	Farmers' aggregated welfare W_F
$\Lambda_1^{\mathcal{Y}}$	$(A-1-\beta)\beta - w_0 - s_0 + t_3$	$A-1-\beta$
$\Lambda_2^{\mathcal{Y}}$	$(A-1-\alpha\theta)(\alpha\theta) - w_0$	$A-1-\alpha\theta$
$\Lambda_3^{\mathcal{Y}}$	$\frac{1}{4}\left[A-1-\frac{w_0}{\alpha\theta}\right]^2$	$\frac{(A-1)(\alpha\theta)+w_0}{2(\alpha\theta)}$
$\Lambda_4^{\mathcal{Y}}$	$\frac{1}{4}\left[A-1-\frac{w_0}{\beta}\right]^2 - s_0 + t_3$	$\frac{(A-1)\beta+w_0}{2\beta}$
$\Lambda_5^{\mathcal{Y}}$	0	$A-1$

These regions are partitioned based on the costs of drones and service. Specifically,

Region	Characteristic
$\Lambda_1^{\mathcal{Y}}$	$0 \le w_0 \le (A-1-2\alpha\theta)(\alpha\theta)$ and $0 \le s_0 \le (A-1-\beta-\alpha\theta)(\beta-\alpha\theta)+t_3$,
	or $(A-1-2\alpha\theta)(\alpha\theta) \le w_0 \le (A-1)(\alpha\theta)$ and
	$0 \le s_0 \le \frac{1}{4}\left[-\left(\frac{w_0}{\alpha\theta}\right)^2 + 2(A-1-2\alpha\theta)\cdot\frac{w_0}{\alpha\theta} - (A-1-2\beta)^2\right] + t_3$
	or $(A-1)(\alpha\theta) \le w_0 \le (A-1-2\beta)\beta$ and $0 \le s_0 \le (A-1-\beta)\beta - w_0 + t_3$
$\Lambda_2^{\mathcal{Y}}$	$0 \le w_0 \le (A-1-2\alpha\theta)(\alpha\theta)$ and $s_0 \ge (A-1-\beta-\alpha\theta)(\beta-\alpha\theta)+t_3$
$\Lambda_3^{\mathcal{Y}}$	$(A-1-2\alpha\theta)(\alpha\theta) \le w_0 \le (A-1)(\alpha\theta)$ and
	$s_0 \ge \frac{1}{4}\left[-\left(\frac{w_0}{\alpha\theta}\right)^2 + 2(A-1-2\alpha\theta)\cdot\frac{w_0}{\alpha\theta} - (A-1-2\beta)^2\right] + t_3$
$\Lambda_4^{\mathcal{Y}}$	$(A-1-2\beta)\beta \le w_0 \le (A-1)\beta$ and $0 \le s_0 \le \frac{1}{4}\left[A-1-\frac{w_0}{\beta}\right]^2 + t_3$
$\Lambda_5^{\mathcal{Y}}$	otherwise

Compared to the partition graph in the benchmark, we can see there an upward shift of each boundary. It is because the subsidy of service enables the firm to provide service even when facing a high service cost.

Corollary 3.5.4 *The corresponding farmers' purchasing behaviors are:*

Regions	Proportion of Farmers with θ buy		
	Nothing	Drone Only	Drone With Service
$\Lambda_1^{\mathcal{V}}$	0	0	1
$\Lambda_2^{\mathcal{V}}$	0	1	0
$\Lambda_3^{\mathcal{V}}$	$\frac{-A+1+2\alpha\theta+\frac{w_0}{\alpha\theta}}{2\alpha\theta}$	$\frac{A-1-\frac{w_0}{\alpha\theta}}{2\alpha\theta}$	0
$\Lambda_4^{\mathcal{V}}$	$\frac{-A+1+2\beta+\frac{w_0}{\beta}}{2\beta}$	0	$\frac{A-1-\frac{w_0}{\beta}}{2\beta}$
$\Lambda_5^{\mathcal{V}}$	1	0	0

It seems farmers' welfare does not change in each region. However, we should remember that the region has shifted. For example, when $(w_0, s_0) \in [0, (A-1-2\alpha\theta)(\alpha\theta)] \times [(A-1-\beta-\alpha\theta)(\beta-\alpha\theta), (A-1-\beta-\alpha\theta)(\beta-\alpha\theta)+t_3]$. Farmers' welfare is $A-1-\alpha\theta$ in benchmark but is $A-1-\beta$ under $\mathcal{V}$. It tells us

Proposition 3.5.3 *Compared to the benchmark without subsidy, service subsidy does not change farmers' welfare for most cost parameters. However, it may make farmers worse off in certain cases. But it always benefits the firm.*

(Put in Appendix) The following graph provides the partition of cost space $\Lambda_i^{\mathcal{V}}$.

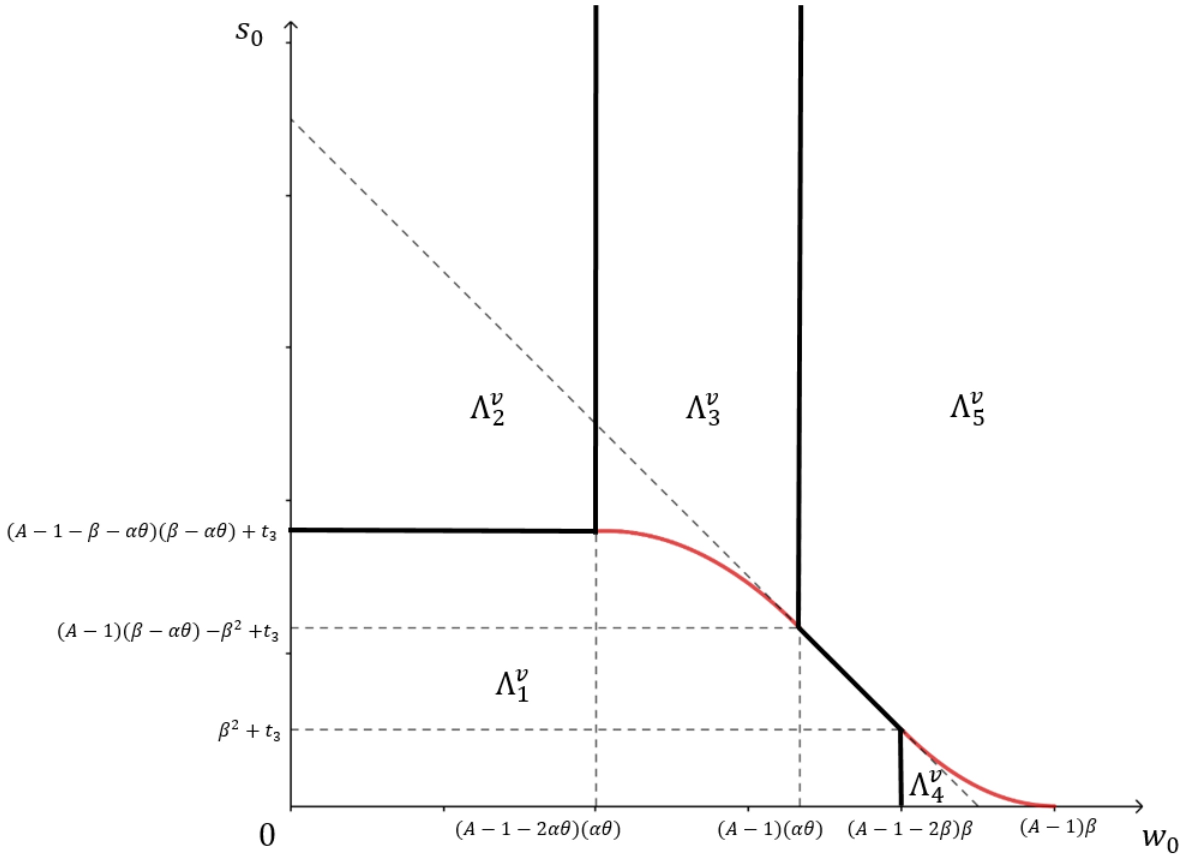


Figure 3.10: Partition of Cost Space under Scheme $\mathcal{V}$

3.5.4 Output Subsidy ($\mathcal{O}$)

Suppose the government subsidizes farmers t_4 per unit output. Then farmers' possible profits are $\pi_0 = (P+t_4)\cdot 1$ if (NP), $\pi_1(\theta) = (P+t_4)\cdot(1+\alpha\cdot\theta)-w$ if (P1), $\pi_2(\theta) = (P+t_4)\cdot(1+\beta)-w-s$ if (P2). Without further ado,

Corollary 3.5.5 *The firm's optimal profit and farmers' aggregated welfare are:*

Regions	Firm's Optimal Profit π	Farmers' aggregated welfare W_F
$\Lambda_1^{\mathcal{O}}$	$(A+t_4-1-\beta)\beta - w_0 - s_0$	$A+t_4-1-\beta$
$\Lambda_2^{\mathcal{O}}$	$(A+t_4-1-\alpha\theta)(\alpha\theta) - w_0$	$A+t_4-1-\alpha\theta$
$\Lambda_3^{\mathcal{O}}$	$\frac{1}{4}\left[A+t_4-1-\frac{w_0}{\alpha\theta}\right]^2$	$\frac{(A+t_4-1)(\alpha\theta)+w_0}{2(\alpha\theta)}$
$\Lambda_4^{\mathcal{O}}$	$\frac{1}{4}\left[A+t_4-1-\frac{w_0}{\beta}\right]^2 - s_0$	$\frac{(A+t_4-1)\beta+w_0}{2\beta}$
$\Lambda_5^{\mathcal{O}}$	0	$A+t_4-1$

These regions are partitioned based on costs of drones and service. Specifically,

Region	Characteristic
$\Lambda_1^{\mathcal{O}}$	$0 \le w_0 \le (A+t_4-1-2\alpha\theta)(\alpha\theta)$ and $0 \le s_0 \le (A+t_4-1-\beta-\alpha\theta)(\beta-\alpha\theta)$,
	or $(A+t_4-1-2\alpha\theta)(\alpha\theta) \le w_0 \le (A+t_4-1)(\alpha\theta)$ and
	$0 \le s_0 \le \frac{1}{4}\left[-\left(\frac{w_0}{\alpha\theta}\right)^2 + 2(A+t_4-1-2\alpha\theta)\cdot\frac{w_0}{\alpha\theta} - (A+t_4-1-2\beta)^2\right]$
	or $(A+t_4-1)(\alpha\theta) \le w_0 \le (A+t_4-1-2\beta)\beta$ and $0 \le s_0 \le (A+t_4-1-\beta)\beta - w_0$
$\Lambda_2^{\mathcal{O}}$	$0 \le w_0 \le (A+t_4-1-2\alpha\theta)(\alpha\theta)$ and $s_0 \ge (A+t_4-1-\beta-\alpha\theta)(\beta-\alpha\theta)$
$\Lambda_3^{\mathcal{O}}$	$(A+t_4-1-2\alpha\theta)(\alpha\theta) \le w_0 \le (A+t_4-1)(\alpha\theta)$ and
	$s_0 \ge \frac{1}{4}\left[-\left(\frac{w_0}{\alpha\theta}\right)^2 + 2(A+t_4-1-2\alpha\theta)\cdot\frac{w_0}{\alpha\theta} - (A+t_4-1-2\beta)^2\right]$
$\Lambda_4^{\mathcal{O}}$	$(A+t_4-1-2\beta)\beta \le w_0 \le (A+t_4-1)\beta$ and $0 \le s_0 \le \frac{1}{4}\left[A+t_4-1-\frac{w_0}{\beta}\right]^2$
$\Lambda_5^{\mathcal{O}}$	otherwise

Compared to the partition graph in the benchmark, we can see the change is equivalent to the expansion of market size A. Both farmers and the firm are able to take advantage of it.

Corollary 3.5.6 *The corresponding farmers' purchasing behaviors are:*

Regions	Proportion of Farmers with θ buy		
	Nothing	Drone Only	Drone With Service
$\Lambda_1^{\mathcal{O}}$	0	0	1
$\Lambda_2^{\mathcal{O}}$	0	1	0
$\Lambda_3^{\mathcal{O}}$	$\frac{-A-t_4+1+2\alpha\theta+\frac{w_0}{\alpha\theta}}{2\alpha\theta}$	$\frac{A+t_4-1-\frac{w_0}{\alpha\theta}}{2\alpha\theta}$	0
$\Lambda_4^{\mathcal{O}}$	$\frac{-A-t_4+1+2\beta+\frac{w_0}{\beta}}{2\beta}$	0	$\frac{A+t_4-1-\frac{w_0}{\beta}}{2\beta}$
$\Lambda_5^{\mathcal{O}}$	1	0	0

Still, when we look at $(w_0, s_0) \in [0, (A-1-2\alpha\theta)(\alpha\theta)] \times [(A-1-\beta-\alpha\theta)(\beta-\alpha\theta), (A+t_4-1-\beta-\alpha\theta)(\beta-\alpha\theta)]$. Farmers' welfare is $A-1-\alpha\theta$ in benchmark but is $A+t_4-1-\beta$ under $\mathcal{O}$. It tells us

Proposition 3.5.4 *Although output subsidy has a chance to benefit farmers and firm at the same time, it may hurt farmers' welfare if the budget is too small ($M < \beta-\alpha\theta$).*

The following table and graph provide the partition of cost space $\Lambda_i^{\mathcal{O}}$.

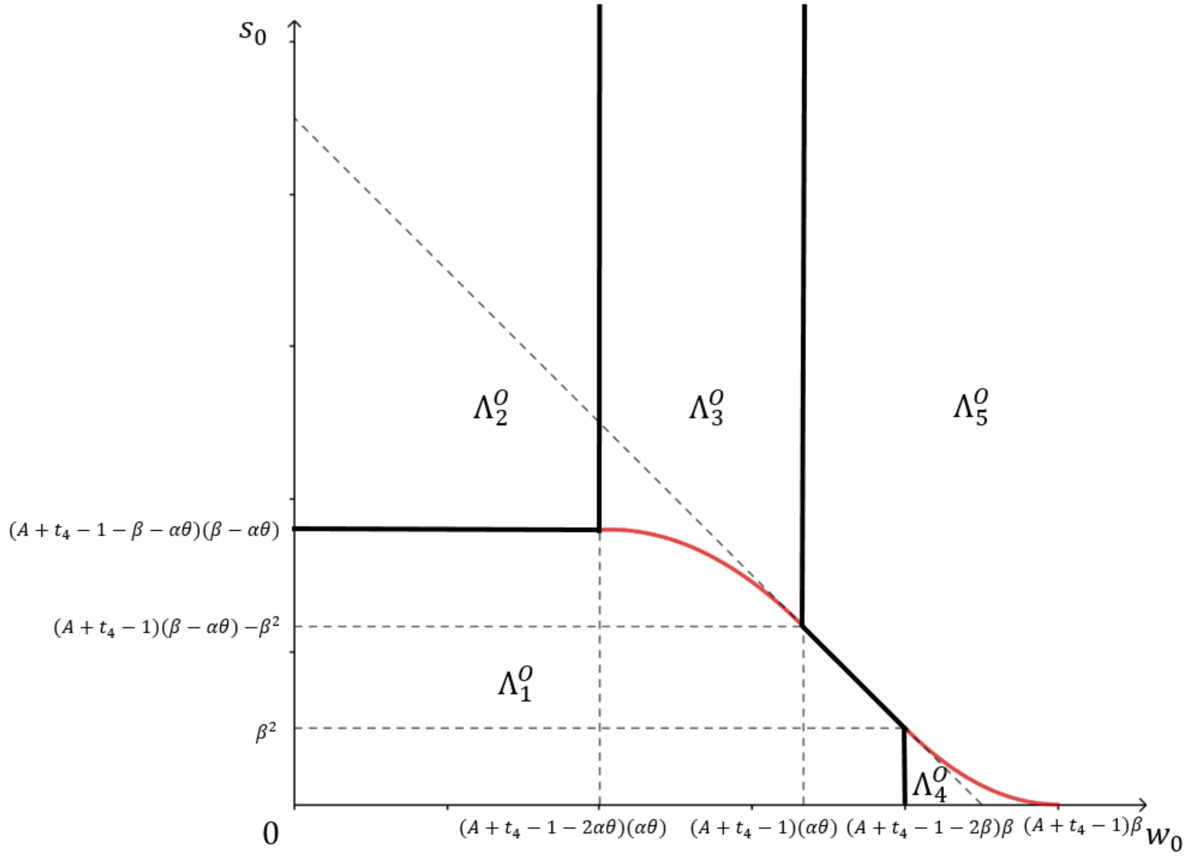


Figure 3.11: Partition of Cost Space under Scheme $\mathcal{O}$

3.6 Comparison

We have discussed how subsidy affects firm's and farmers' welfare within the same subsidy scheme. It is vital to study how different subsidy schemes affect welfare since it is the social planner's objective to choose the best scheme. We use notation $W_i^{G\mathcal{J}}$ to represent the social welfare in region $i \in \{1,2,3,4,5\}$ for the government under subsidy scheme $\mathcal{J} \in \{\mathcal{P}, \mathcal{V}, \mathcal{O}\}$. WLOG, we assume all subsidy scheme reaches its maximum budget, i.e. $t_i = M$ for $i \in \{1,2,3,4,5\}$.

Proposition 3.6.1 (Scheme $\mathcal{P}$ vs. $\mathcal{V}$) *Comparing per-unit purchasing subsidy and service subsidy,*

(1) $W_i^{G\mathcal{P}} = W_i^{G\mathcal{V}}$ *for* $i \in \{1,5\}$.

(2) $W_2^{G\mathcal{P}} > W_2^{G\mathcal{V}}$ *always holds.*

(3) $W_3^{G\mathcal{P}} > W_3^{G\mathcal{V}}$ *if and only if* $A - 1 - \frac{w_0}{\alpha\theta} + \frac{M}{2(\alpha\theta)} > \frac{\lambda}{1-\lambda}$.

(4) $W_4^{G\mathcal{P}} > W_4^{G\mathcal{V}}$ *if and only if* $A - 1 - \frac{w_0}{\beta} - \frac{\beta}{2} + \frac{M}{2\beta} > \frac{\lambda}{1-\lambda}$.

Since region 1 and 5 in the cost partition space for both subsidy schemes are two extreme cases, either all farmers buy drones with service or no farmers make any purchasing decisions. Subsidizing per-unit purchasing or subsidizing services are the same. Either the firm provides a bundle price including both product subsidy and service subsidy, or the subsidy does not improve social welfare at all. However, when the farmers buy agri-technology products only, subsidizing per-unit purchasing always dominates subsidizing service. Since there is no service to subsidize. The trick parts are region 3 and 4. In $\Lambda_3^{\mathcal{P}}$, farmers are worse off while the firm is better off. In $\Lambda_3^{\mathcal{V}}$, both farmers and the firm have the same welfare as in the benchmark because no service is provided. If the government cares more for the firm, i.e. a small λ, it is more likely the scheme $\mathcal{P}$ is preferred. If the budget is adequate, scheme $\mathcal{P}$ is also preferred. Since even a small weighting $1 - \lambda$ on the firm can be amplified by a huge budget. In $\Lambda_4^{\mathcal{P}}$, still farmers are worse off while the firm is better off. But in $\Lambda_4^{\mathcal{V}}$, farmers have the same welfare as in the benchmark while the firm is better off because service is provided. Since only partial farmers need the service, the boost on the firm is smaller than scheme $\mathcal{P}$. That is why the structure in region 4 is similar to that in region s: large budget or small coefficient λ lead to subsidy scheme $\mathcal{P}$.

Proposition 3.6.2 (Scheme $\mathcal{P}$ vs. $\mathcal{O}$) *Comparing per-unit purchasing subsidy and service subsidy,*

(1) $W_1^{G\mathcal{P}} > W_1^{G\mathcal{O}}$ *if and only if* $1-\beta > \frac{\lambda}{1-\lambda}$.

(2) $W_2^{G\mathcal{P}} > W_2^{G\mathcal{O}}$ *if and only if* $1-\lambda > \frac{\lambda}{1-\lambda}$.

(3) $W_3^{G\mathcal{P}} > W_3^{G\mathcal{O}}$ *if and only if* $\frac{1-\alpha\theta}{1+\alpha\theta} \cdot \left[A - 1 - \frac{w_0}{\alpha\theta} + \frac{M}{2} + \frac{M}{2(\alpha\theta)}\right] > \frac{\lambda}{1-\lambda}$.

(4) $W_4^{G\mathcal{P}} > W_4^{G\mathcal{O}}$ *if and only if* $\frac{1-\beta}{1+\beta} \cdot \left[A - 1 - \frac{w_0}{\beta} + \frac{M}{2} + \frac{M}{2\beta}\right] > \frac{\lambda}{1-\lambda}$.

(5) $W_5^{G\mathcal{P}} < W_5^{G\mathcal{O}}$ *always holds.*

Still, scheme $\mathcal{P}$ is preferred if the social planner cares more for the firm, i.e. small λ, except for region 5. Since the firm quits the market if both costs are too high. Improving the market size is always good for social welfare. The results for region 3 and 4 are similar to the comparison between scheme $\mathcal{P}$ and $\mathcal{V}$. But in regions 1 and 2, output subsidy has a chance to dominate per-unit purchasing subsidy.

Proposition 3.6.3 (Scheme $\mathcal{V}$ vs. $\mathcal{O}$) *Comparing per-unit purchasing subsidy and service subsidy,* *(1)* $W_1^{G\mathcal{V}} > W_1^{G\mathcal{O}}$ *if and only if* $1-\beta > \frac{\lambda}{1-\lambda}$.

(2) $W_i^{G\mathcal{V}} < W_i^{G\mathcal{O}}$ *always holds for* $i \in \{2,3,5\}$.

(3) $W_4^{G\mathcal{V}} > W_4^{G\mathcal{O}}$ *if and only if* $A - 1 - \frac{w_0}{\beta} - \frac{M}{2} + \frac{\lambda}{1-\lambda} < 2$.

Subsidy scheme $\mathcal{O}$ dominates $\mathcal{V}$ for most cases. Only when the budget is adequate or λ is small, the reverse may happen. But this may lead to the dominance of scheme $\mathcal{P}$. Hence, we see the limitation of the service subsidy scheme because the service team may not even exist in some cases.

Generally, if the government cares more for farmers, then output subsidy should be the preference unless the budget is too tight. If the government cares more for the firm, then a per-unit purchasing subsidy or per-unit selling subsidy should be the top priority. The condition for the service subsidy to be the best scheme is harsh. But it does not mean we should abandon such an option. The reality is much more complicated because the objective of the government may not be social welfare alone. Employment rate, the satisfaction of customers, etc. could be social planner's considerations. Especially for developed countries like the USA, the service sector is a major source of economic growth. Improving service is as important as encouraging manufacturing.

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Appendix A

Flexibility Value of Reshoring Capacity under Import Cost Uncertainty and Domestic Competition

A.1 Proofs

Proof of Lemma 1: Since the stage 3 problem is a linear problem. The optimal sourcing and production decision depend on the comparison among the three costs, $w_C(1+t_F)$, $w_C(1+t_R)+\delta$ and w'_U. (1) When $w_C(1+t_F) < w_C(1+t_R)+\delta$ and $w_C(1+t_F) < w'_U$, the optimal way to satisfy the output target is to source and produce in country C, i.e., $q^*_C = Q$, $q^*_{UC} = 0$, and $q^*_{UU} = 0$. The first condition implies $t_F < t_R + \frac{\delta}{w_C}$, and the second condition implies $t_F < \frac{w'_U}{w_C} - 1$. (2) When $w_C(1+t_R)+\delta < w_C(1+t_F)$ and $w_C(1+t_R)+\delta < w'_U$, the optimal way to satisfy the output target is to source in country C and produce in country U. Since the production in country U is subject to the capacity K_U, $q^*_{UC} = \min\{Q, K_U\}$ and any remaining quantity has to be satisfied using the production in country C, i.e., $q^*_C = (Q-K_U)^+$. The onshore sourcing and production option is never used, i.e., $q^*_{UU} = 0$. (3) When $w'_U < w_C(1+t_F)$ and $w'_U < w_C(1+t_R)+\delta$, the optimal way to satisfy the output target is to source and produce in country U. Since the production in country U is subject to the capacity K_U, $q^*_{UU} = \min\{Q, K_U\}$ and any remaining quantity has to be satisfied using the production in country C, i.e., $q^*_C = (Q-K_U)^+$. Finally, we have $q^*_{UC} = 0$. ■

Proof of Lemma 2: After solving the stage 3 problem, we know $EC(Q, K_U) = m_A \cdot Q$ if $0 \le Q < K_U$ and $EC(Q, K_U) = m_A \cdot K_U + m_B \cdot (Q - K_U)$ if $Q \ge K_U$. It is easy to check that the objective function in stage-2 problem $(\epsilon - Q)Q - EC(Q, K_U)$ is continuous. The first order derivative $[(\epsilon - Q)Q - EC(Q)]'$ is $\epsilon - m_A - 2Q$ if $0 < Q < K_U$, and is $\epsilon - m_B - 2Q$ if $Q \ge K_U$. So the objective function is piece-wise concave.

(1) When $\epsilon \in [0, m_A)$, notice $\epsilon - m_B < \epsilon - m_A < 0$. Then $[(\epsilon - Q)Q - EC(Q)]'$ is always negative. So the optimal output target quantity is $Q^m = 0$. (2) When $\epsilon \in [m_A, 2K_U + m_A)$, notice $0 < \epsilon - m_A < 2K_U$. Let $\epsilon - m_A - 2Q = 0$, the solution $Q = \frac{\epsilon - m_A}{2}$ lies in the range $0 < Q < K_U$. And $\epsilon - m_B - 2Q$ is always negative for $Q \ge K_U$. So the optimal output target quantity is $Q^m = \frac{\epsilon - m_A}{2}$. (3) When $\epsilon \in [2K_U + m_A, 2K_U + m_B)$, notice $\epsilon - m_A - 2Q > 0 > \epsilon - m_B - 2Q$ always holds. The objective function increases in the range $0 < Q < K_U$ and decreases in the range $Q \ge K_U$. So the optimal output target quantity is $Q^m = K_U$. (4) When $\epsilon \in [2K_U + m_B, \infty)$, notice $\epsilon - m_B \ge 2K_U$. Let $\epsilon - m_B - 2Q = 0$, the solution $Q = \frac{\epsilon - m_B}{2}$ lies in the range $Q \ge K_U$. And $\epsilon - m_A - 2Q$ is always positive for $0 < Q < K_U$. So the optimal output target quantity is $Q^m = \frac{\epsilon - m_B}{2}$. ■

Proof of Proposition 1: From equation (3), we take first order derivative over K_U:

$$[R(K_U) - C_U K_U]' = \int_{2K_U+m_A}^{2K_U+m_B} (\epsilon - 2K_U - m_A) dG(\epsilon) + \int_{2K_U+m_B}^{\infty} (m_B - m_A) dG(\epsilon) - C_U.$$

The second order derivative is $[R(K_U) - C_U K_U]'' = \int_{2K_U+m_A}^{2K_U+m_B} (-2) dG(\epsilon) < 0$. So, the stage-1 objective function is concave in K_U. Let

$$C_0^m = R'(0) = \int_{m_A}^{m_B} (\epsilon - m_A) dG(\epsilon) + \int_{m_B}^{\infty} (m_B - m_A) dG(\epsilon) = (m_B - m_A) - \int_{m_A}^{m_B} G(\epsilon) d\epsilon.$$

The optimal capacity $K_U^m = 0$ for $C_U > C_0^m$ because $[R(K_U) - C_U K_U]'$ is always negative for this range, which means the objective function is always decreasing. Otherwise, when $0 \le C_U \le C_0^m$, the optimal capacity K_U^m solves $[R(K_U) - C_U K_U]' = 0$. We obtain equation (4). ■

Proof of Proposition 2: From section 4.2, firm L's best response function is $Q_L(Q) = \frac{\epsilon - w_U' - Q}{2}$. The global firm solves the profit maximization problem $\max_{Q \ge 0} (\epsilon - (Q + Q_L)) Q - EC(Q, K_U)$, where $EC(Q, K_U)$ is given by $EC(Q, K_U) = m_A \cdot Q$ if $0 \le Q < K_U$ and $EC(Q, K_U) = m_A \cdot K_U + m_B \cdot (Q - K_U)$ if $Q \ge K_U$ (in stage 3 the global firm's problem is identical to the monopoly model). The best response function can be derived as the following:

$$Q(Q_L) = \begin{cases} \frac{\epsilon - m_A - Q_L}{2}, & \text{if } \frac{\epsilon - m_A - Q_L}{2} < K_U \\ K_U, & \text{if } \frac{\epsilon - m_B - Q_L}{2} \le K_U \le \frac{\epsilon - m_A - Q_L}{2} \\ \frac{\epsilon - m_B - Q_L}{2}, & \text{if } \frac{\epsilon - m_B - Q_L}{2} > K_U. \end{cases}$$

When $Q_L(Q)$ intersects with the third case of $Q(Q_L)$, the equilibrium is $(Q^{d,3}, Q_L^{d,3}) = (\frac{\epsilon - 2m_B + w_U'}{3}, \frac{\epsilon - 2w_U' + m_B}{3})$. Nonnegativity and $Q^{d,3} > K_U$ require that $\epsilon > \max\{2w_U' - m_B, 2m_B - w_U', 3K_U + 2m_B - w_U'\}$. Assuming $w_U' > m_B$ (so $2w_U' - m_B > 2m_B - w_U'$), when $K_U' < w_U' - m_B$, we have $3K_U + 2m_B - w_U' < 2w_U' - m_B$, and the competition region starts from $\epsilon = 2w_U' - m_B$. This corresponds to case (i).

When $K_U' > w_U' - m_B$, the equilibrium $(Q^{d,3}, Q_L^{d,3})$ emerges when $\epsilon > 3K_U + 2m_B - w_U'$. When $Q_L(Q)$ intersects with the second case of $Q(Q_L)$, the equilibrium is $(Q^{d,2}, Q_L^{d,2}) = (K_U, \frac{\epsilon - w_U' - K_U}{2})$. Nonnegativity and $\frac{\epsilon - m_B - Q_L^{d,2}}{2} \le K_U \le \frac{\epsilon - m_A - Q_L^{d,2}}{2}$ require that $\epsilon > w_U' + K_U$ and $3K_U + 2m_A - w_U' < \epsilon < 3K_U + 2m_B - w_U'$. When $K_U' < w_U' - m_A$, we have $3K_U + 2m_A - w_U' <$

$w'_U + K_U$, and the competition region starts from $\epsilon = w'_U + K_U$. Equilibrium $(Q^{d,2}, Q_L^{d,2})$ emerges when $\epsilon \in [w'_U + K_U, 3K_U + 2m_B - w'_U]$, and equilibrium $(Q^{d,3}, Q_L^{d,3})$ emerges when $\epsilon \in (3K_U + 2m_B - w'_U, \infty)$. This corresponds to case (ii).

Finally, when $Q_L(Q)$ intersects with the first case of $Q(Q_L)$, the equilibrium is $(Q^{d,1}, Q_L^{d,1}) = (\frac{\epsilon - 2m_A + w'_U}{3}, \frac{\epsilon - 2w'_U + m_A}{3})$. Nonnegativity and $Q^{d,1} < K_U$ require that $\epsilon > \max\{2m_A - w'_U, 2w'_U - m_A, 3K_U + 2m_A - w'_U\}$. When $K'_U > w'_U - m_A$, we have $3K_U + 2m_A - w'_U > 2w'_U - m_A$, and the competition region starts from $\epsilon = 2w'_U - m_A$. Equilibrium $Q^{d,1}, Q_L^{d,1})$ emerges when $\epsilon \in [2w'_U - m_A, 3K_U + 2m_A - w'_U]$, equilibrium $(Q^{d,2}, Q_L^{d,2})$ emerges when $\epsilon \in [3K_U + 2m_A - w'_U, 3K_U + 2m_B - w'_U]$, and equilibrium $(Q^{d,3}, Q_L^{d,3})$ emerges when $\epsilon \in (3K_U + 2m_B - w'_U, \infty)$. This corresponds to case (iii). ■

Proof of Proposition 3: (1) First, when $w'_U \le m_B$, we always have $K_U \ge [w'_U - m_B]^+ = 0$. For the global firm's capacity decision problem in stage 1, there are two cases to discuss.

(i). When $0 < K_U \le w'_U - m_A$, then we have:

$$\begin{aligned} R(K_U) = &\int_{m_A}^{2K_U + m_A} \frac{(\epsilon - m_A)^2}{4} dG(\epsilon) + \int_{2K_U + m_A}^{K_U + w'_U} \left[-K_U^2 + (\epsilon - m_A)K_U\right] dG(\epsilon) + \\ &\int_{K_U + w'_U}^{3K_U + 2m_B - w'_U} \left[-\frac{K_U^2}{2} + \frac{\epsilon - 2m_A + w'_U}{2} K_U\right] dG(\epsilon) + \\ &\int_{3K_U + 2m_B - w'_U}^{\infty} \left[\frac{(\epsilon - 2m_B + w'_U)^2}{9} + (m_B - m_A)K_U\right] dG(\epsilon). \end{aligned} \tag{A.1}$$

$$\begin{aligned} \frac{\partial[R(K_U) - C_U K_U]}{\partial K_U} = &\int_{2K_U + m_A}^{K_U + w'_U} [\epsilon - m_A - 2K_U]\, dG(\epsilon) + \int_{K_U + w'_U}^{3K_U + 2m_B - w'_U} \left[\frac{\epsilon - 2m_A + w'_U}{2} - K_U\right] dG(\epsilon) \\ &+ \int_{3K_U + 2m_B - w'_U}^{\infty} (m_B - m_A) dG(\epsilon) - C_U. \end{aligned}$$

$$\begin{aligned} \frac{\partial^2[R(K_U) - C_U K_U]}{\partial K_U^2} = &- \int_{2K_U + m_A}^{K_U + w'_U} 2dG(\epsilon) - \int_{K_U + w'_U}^{3K_U + 2m_B - w'_U} dG(\epsilon) - \frac{1}{2} K_U \cdot g(K_U + w'_U) \\ &+ \frac{3}{2} K_U \cdot g(3K_U + 2m_B - w'_U). \end{aligned}$$

Stage-1 objective function is concave in K_U given the assumption that the pdf $g(\cdot)$ is a non-increasing function, since

$$
\begin{aligned}
-\int_{K_U+w'_U}^{3K_U+2m_B-w'_U} dG(\epsilon) &\le -\left[(3K_U+2m_B-w'_U)-(K_U+w'_U)\right]\cdot g(3K_U+2m_B-w'_U)\\
&= -\,(2K_U+2m_B-2w'_U)\cdot g(3K_U+2m_B-w'_U),
\end{aligned}
$$

and thus,

$$
\begin{aligned}
\frac{\partial^2[R(K_U)-C_UK_U]}{\partial K_U^2} \le &-(K_U+2m_B-2w'_U)\cdot g(3K_U+2m_B-w'_U)-\frac{1}{2}K_U\cdot[g(K_U+w'_U)\\
&-g(3K_U+2m_B-w'_U)] \le 0.
\end{aligned}
$$

Let

$$
C_0^d = R'(0) = \int_{m_A}^{w'_U}(\epsilon-m_A)dG(\epsilon)+\int_{w'_U}^{2m_B-w'_U}\frac{\epsilon-2m_A+w'_U}{2}dG(\epsilon)+\int_{2m_B-w'_U}^{\infty}(m_B-m_A)dG(\epsilon).
$$

Since $\frac{\partial[R(K_U)-C_UK_U]}{\partial K_U}$ is decreasing in K_U, the optimal capacity $K_U^d = 0$ for $C_U \ge C_0^d$; otherwise, when $C_U < C_0^d$, the optimal capacity K_U^d satisfies the first order condition:

$$
\begin{aligned}
FOC_2^d: &\int_{2K_U^d+m_A}^{K_U^d+w'_U}\left[\epsilon-m_A-2K_U^d\right]dG(\epsilon)+\int_{K_U^d+w'_U}^{3K_U^d+2m_B-w'_U}\left[\frac{\epsilon-2m_A+w'_U}{2}-K_U^d\right]dG(\epsilon)\\
&+\int_{3K_U^d+2m_B-w'_U}^{\infty}(m_B-m_A)dG(\epsilon) = C_U.
\end{aligned}
$$

Check the constraint $0 < K_U^d \le w'_U - m_A$, we have the threshold:

$$
C_2^d := R'(w'_U-m_A) = \int_{2w'_U-m_A}^{2w'_U+2m_B-3m_A}\frac{\epsilon-w'_U}{2}dG(\epsilon)+\int_{2w'_U+2m_B-3m_A}^{\infty}(m_B-m_A)dG(\epsilon).
$$

(ii). If $K_U > w'_U - m_A$, then we have:

$$
\begin{aligned}
R(K_U) = &\int_{m_A}^{2w'_U-m_A}\frac{(\epsilon-m_A)^2}{4}dG(\epsilon)+\int_{2w'_U-m_A}^{3K_U+2m_A-w'_U}\frac{(\epsilon-2m_A+w'_U)^2}{9}dG(\epsilon)+\\
&\int_{3K_U+2m_A-w'_U}^{3K_U+2m_B-w'_U}\left[-\frac{K_U^2}{2}+\frac{\epsilon-2m_A+w'_U}{2}\cdot K_U\right]dG(\epsilon)+\\
&\int_{3K_U+2m_B-w'_U}^{\infty}\left[\frac{(\epsilon-2m_B+w'_U)^2}{9}+(m_B-m_A)K_U\right]dG(\epsilon),
\end{aligned}
\tag{A.2}
$$

$$\frac{\partial[R(K_U)-C_UK_U]}{\partial K_U}=\int_{3K_U+2m_A-w'_U}^{3K_U+2m_B-w'_U}\left[\frac{\epsilon-2m_A+w'_U}{2}-K_U\right]dG(\epsilon)+\int_{3K_U+2m_B-w'_U}^{\infty}(m_B-m_A)dG(\epsilon)-C_U,$$

$$\begin{aligned}\frac{\partial^2[R(K_U)-C_UK_U]}{\partial K_U^2}=&\frac{3}{2}K_U\cdot\left[g(3K_U+2m_B-w'_U)-g(3K_U+2m_A-w'_U)\right]-\int_{3K_U+2m_A-w'_U}^{3K_U+2m_B-w'_U}dG(\epsilon)\\ \le&0.\end{aligned}$$

The optimal capacity K_U^d satisfies the first order condition:

$$FOC_1^d:\int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U}\left[\frac{\epsilon-2m_A+w'_U}{2}-K_U^d\right]dG(\epsilon)+\int_{3K_U^d+2m_B-w'_U}^{\infty}(m_B-m_A)dG(\epsilon)=C_U.$$

(2) When $w'_U>m_B$, we always have $[w'_U-m_B]^+>0$. There are three cases to discuss. If $K_U\le w'_U-m_B$, then the objective function is the same as the single-firm model. The reshoring threshold and capacity are characterized in Proposition 1. Check the constraint $0<K_U^d\le w'_U-m_B$, we have the threshold:

$$C_1^d:=R'(w'_U-m_B)=\int_{m_A-2m_B+2w'_U}^{2w'_U-m_B}(\epsilon-m_A+2m_B-2w'_U)dG(\epsilon)+\int_{2w'_U-m_B}^{\infty}(m_B-m_A)dG(\epsilon).$$

If $w'_U-m_B<K_U\le w'_U-m_A$, then $R(K_U)$ is the same as (A.1), and FOC_2^d is the characterizing equation. If $K_U>w'_U-m_A$, then $R(K_U)$ is the same as (A.2), and FOC_1^d is the characterizing equation. ■

Proof of Proposition 4: There are three first order conditions determining the global firm's optimal reshoring capacity decisions: FOC^m, FOC_1^d and FOC_2^d. We prove them one by one.

(1) Taking derivative over μ_R for FOC^m using Implicit Function Theorem, we have:

$$\frac{\partial K_U^m}{\partial\mu_R}=\frac{[1-G(2K_U^m+m_B)]\cdot\frac{\partial m_B}{\partial\mu_R}-[1-G(2K_U^m+m_A)]\cdot\frac{\partial m_A}{\partial\mu_R}}{2[G(2K_U^m+m_B)-G(2K_U^m+m_A)]}.$$

The denominator is always larger than 0. By the fact that $\frac{\partial m_B}{\partial \mu_R} = 0$ and $\frac{\partial m_A}{\partial \mu_R} > 0$, we have:

$$\frac{\partial K_U^m}{\partial \mu_R} = \frac{-[1 - G(2K_U^m + m_A)] \cdot \frac{\partial m_A}{\partial \mu_R}}{2[G(2K_U^m + m_B) - G(2K_U^m + m_A)]} < 0.$$

(2) Taking derivative over μ_R for FOC_1^d, we have:

$$\frac{\partial K_U^d}{\partial \mu_R} = \frac{-[1-G(3K_U^d+2m_A-w_U')+K_U^d \cdot g(3K_U^d+2m_A-w_U')] \cdot \frac{\partial m_A}{\partial w_C}}{[G(3K_U^d+2m_B-w_U')-G(3K_U^d+2m_A-w_U')]-\frac{3K_U^d}{2} \cdot [g(3K_U^d+2m_B-w_U')-g(3K_U^d+2m_A-w_U')]} < 0.$$

(3) Taking derivative over μ_R for FOC_2^d, we have:

$$\frac{\partial K_U^d}{\partial \mu_R} = \frac{-[1-G(2K_U^d+m_A)] \cdot \frac{\partial m_A}{\partial w_C}}{[G(3K_U^d+2m_B-w_U')+G(K_U^d+w_U')-2G(2K_U^d+m_A)]-\frac{3K_U^d}{2} \cdot g(3K_U^d+2m_B-w_U')+\frac{K_U^d}{2} \cdot g(K_U^d+w_U')} < 0.$$

Finally, it can be easily shown that $\frac{\partial C_0^m}{\partial \mu_R} < 0$ and $\frac{\partial C_0^d}{\partial \mu_R} < 0$ since they are special cases when $K_U^m = 0$ and $K_U^d = 0$. ■

Proof of Proposition 5: There are three first order conditions determining the global firm's optimal reshoring capacity decision: FOC^m, FOC_1^d and FOC_2^d. We prove them one by one.

(1) Taking derivative over μ_F for FOC^m using Implicit Function Theorem, we have:

$$\frac{\partial K_U^m}{\partial \mu_F} = \frac{[1 - G(2K_U^m + m_B)] \cdot \frac{\partial m_B}{\partial \mu_F} - [1 - G(2K_U^m + m_A)] \cdot \frac{\partial m_A}{\partial \mu_F}}{2[G(2K_U^m + m_B) - G(2K_U^m + m_A)]}. \tag{A.3}$$

By the fact that $\frac{\partial m_B}{\partial \mu_F} > \frac{\partial m_A}{\partial \mu_F} > 0$, $\frac{\partial K_U^m}{\partial \mu_F} > 0$ is equivalent to

$$\frac{\partial m_A / \partial \mu_F}{\partial m_B / \partial \mu_F} < \frac{1 - G(2K_U^m + m_B)}{1 - G(2K_U^m + m_A)} =: \Theta_{K_U}^m.$$

(2) Taking derivative over μ_F for FOC_1^d, we have:

$$\frac{\partial K_U^d}{\partial \mu_F} = \frac{[1-G(3K_U^d+2m_B-w_U')+K_U^d \cdot g(3K_U^d+2m_B-w_U')] \cdot \frac{\partial m_B}{\partial \mu_F} - [1-G(3K_U^d+2m_A-w_U')+K_U^d \cdot g(3K_U^d+2m_A-w_U')] \cdot \frac{\partial m_A}{\partial \mu_F}}{[G(3K_U^d+2m_B-w_U')-G(3K_U^d+2m_A-w_U')]-\frac{3K_U^d}{2} \cdot [g(3K_U^d+2m_B-w_U')-g(3K_U^d+2m_A-w_U')]}.$$

Therefore, $\frac{\partial K_U^m}{\partial \mu_F} > 0$ is equivalent to

$$\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} < \frac{1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')}{1 - G(3K_U^d + 2m_A - w_U') + K_U^d \cdot g(3K_U^d + 2m_A - w_U')}.$$

(3) Taking derivative over μ_F for FOC_2^d, we have:

$$\frac{\partial K_U^d}{\partial \mu_F} = \frac{\left[1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')\right] \cdot \frac{\partial m_B}{\partial \mu_F} - \left[1 - G(2K_U^d + m_A)\right] \cdot \frac{\partial m_A}{\partial \mu_F}}{\left[G(3K_U^d + 2m_B - w_U') + G(K_U^d + w_U') - 2G(2K_U^d + m_A)\right] - \frac{3K_U^d}{2} \cdot g(3K_U^d + 2m_B - w_U') + \frac{K_U^d}{2} \cdot g(K_U^d + w_U')}.$$

Therefore, $\frac{\partial K_U^m}{\partial \mu_F} > 0$ is equivalent to

$$\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} < \frac{1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')}{1 - G(2K_U^d + m_A)}.$$

So, the threshold $\Theta_{K_U}^d$ takes the following form: when $w_U' \leq m_B$,

$$\Theta_{K_U}^d := \begin{cases} \dfrac{1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')}{1 - G(3K_U^d + 2m_A - w_U') + K_U^d \cdot g(3K_U^d + 2m_A - w_U')}, & \text{if } C_U \in [0, C_2^d) \\ \dfrac{1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')}{1 - G(2K_U^d + m_A)}, & \text{if } C_U \in [C_2^d, C_0^d) \end{cases}$$

When $w_U' > m_B$,

$$\Theta_{K_U}^d := \begin{cases} \dfrac{1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')}{1 - G(3K_U^d + 2m_A - w_U') + K_U^d \cdot g(3K_U^d + 2m_A - w_U')}, & \text{if } C_U \in [0, C_2^d) \\ \dfrac{1 - G(3K_U^d + 2m_B - w_U') + K_U^d \cdot g(3K_U^d + 2m_B - w_U')}{1 - G(2K_U^d + m_A)}, & \text{if } C_U \in [C_2^d, C_1^d) \\ \dfrac{1 - G(2K_U^d + m_B)}{1 - G(2K_U^d + m_A)}, & \text{if } C_U \in [C_1^d, C_0^m) \end{cases} \quad \blacksquare$$

Proof of Proposition 6: From the proof of Proposition 5, we know it suffices to prove the monopoly model and the duopoly model when $w_U' \leq m_B$, because they cover all possible FOCs characterizing the optimal capacity. We prove them one by one.

(1) For the monopoly model, the global firm's expected output quantity is

$$E[Q^m] = \int_{m_A}^{2K_U^m+m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2K_U^m+m_A}^{2K_U^m+m_B} K_U^m dG(\epsilon) + \int_{2K_U^m+m_B}^{\infty} \frac{\epsilon - m_B}{2} dG(\epsilon).$$

By the fact that $\frac{\partial m_B}{\partial \mu_R} = 0$, $\frac{\partial m_A}{\partial \mu_R} > 0$, and $\frac{\partial K_U^m}{\partial \mu_R} < 0$, we have:

$$\begin{aligned}
\frac{\partial E[Q^m]}{\partial \mu_R} = & \int_{m_A}^{2K_U^m+m_A} \left(-\frac{1}{2} \cdot \frac{\partial m_A}{\partial \mu_R}\right) dG(\epsilon) + \int_{2K_U^m+m_A}^{2K_U^m+m_B} \frac{\partial K_U^m}{\partial \mu_R} dG(\epsilon) + \\
& \int_{2K_U^m+m_B}^{\infty} \left(-\frac{1}{2} \cdot \frac{\partial m_B}{\partial \mu_R}\right) dG(\epsilon) < 0.
\end{aligned}$$

(2) For the duopoly model with $w'_U \le m_B$ and $C_U \in [0, C_2^d)$, we have $K_U^d > w'_U - m_A$, and

$$\begin{aligned}
E[Q^d] = & \int_{m_A}^{2w'_U-m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\epsilon - 2m_A + w'_U}{3} dG(\epsilon) + \\
& \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} K_U^d dG(\epsilon) + \int_{3K_U^d+2m_B-w'_U}^{\infty} \frac{\epsilon - 2m_B + w'_U}{3} dG(\epsilon) \\
E[Q_L^d] = & \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\epsilon - 2w'_U + m_A}{3} dG(\epsilon) + \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\epsilon - w'_U - K_U^d}{2} dG(\epsilon) \\
& + \int_{3K_U^d+2m_B-w'_U}^{\infty} \frac{\epsilon - 2w'_U + m_B}{3} dG(\epsilon) \\
E[Q^d + Q_L^d] = & \int_{m_A}^{2w'_U-m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{2\epsilon - m_A - w'_U}{3} dG(\epsilon) \\
& + \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\epsilon + K_U^d - w'_U}{2} dG(\epsilon) + \int_{3K_U^d+2m_B-w'_U}^{\infty} \frac{2\epsilon - m_B - w'_U}{3} dG(\epsilon).
\end{aligned}$$

By the fact that $\frac{\partial m_B}{\partial \mu_R} = 0$, $\frac{\partial m_A}{\partial \mu_R} > 0$, and $\frac{\partial K_U^m}{\partial \mu_R} < 0$, we have:

$$\begin{aligned}
\frac{\partial E[Q^d]}{\partial \mu_R} = & -\frac{1}{2} \int_{m_A}^{2w'_U-m_A} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) - \frac{2}{3} \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) + \\
& \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) < 0 \\
\frac{\partial E[Q_L^d]}{\partial \mu_R} = & \frac{1}{3} \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) - \frac{1}{2} \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) > 0 \\
\frac{\partial E[Q^d + Q_L^d]}{\partial \mu_R} = & -\frac{1}{2} \int_{m_A}^{2w'_U-m_A} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) - \frac{1}{3} \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) + \\
& \frac{1}{2} \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) < 0.
\end{aligned}$$

(3) For the duopoly model with $w_U' \leq m_B$ and $C_U \in [C_2^d, C_0^d)$, we have $0 < K_U^d \leq w_U' - m_A$, and

$$E[Q^d] = \int_{m_A}^{2K_U^d+m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2K_U^d+m_A}^{3K_U^d+2m_B-w_U'} K_U^d dG(\epsilon) + \int_{3K_U^d+2m_B-w_U'}^{\infty} \frac{\epsilon - 2m_B + w_U'}{3} dG(\epsilon)$$

$$E[Q_L^d] = \int_{K_U^d+w_U'}^{3K_U^d+2m_B-w_U'} \frac{\epsilon - w_U' - K_U^d}{2} dG(\epsilon) + \int_{3K_U^d+2m_B-w_U'}^{\infty} \frac{\epsilon - 2w_U' + m_B}{3} dG(\epsilon)$$

$$\begin{aligned} E[Q^d + Q_L^d] = & \int_{m_A}^{2K_U^d+m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2K_U^d+m_A}^{K_U^d+w_U'} K_U^d dG(\epsilon) \\ & + \int_{K_U^d+w_U'}^{3K_U^d+2m_B-w_U'} \frac{\epsilon + K_U^d - w_U'}{2} dG(\epsilon) + \int_{3K_U^d+2m_B-w_U'}^{\infty} \frac{2\epsilon - m_B - w_U'}{3} dG(\epsilon). \end{aligned}$$

By the fact that $\frac{\partial m_B}{\partial \mu_R} = 0$, $\frac{\partial m_A}{\partial \mu_R} > 0$, and $\frac{\partial K_U^m}{\partial \mu_R} < 0$, we have:

$$E[Q^d] = -\frac{1}{2} \int_{m_A}^{2K_U^d+m_A} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) + \int_{2K_U^d+m_A}^{3K_U^d+2m_B-w_U'} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) < 0$$

$$E[Q_L^d] = -\frac{1}{2} \int_{K_U^d+w_U'}^{3K_U^d+2m_B-w_U'} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) > 0$$

$$E[Q^d + Q_L^d] = -\frac{1}{2} \int_{m_A}^{2K_U^d+m_A} \frac{\partial m_A}{\partial \mu_R} dG(\epsilon) + \int_{2K_U^d+m_A}^{K_U^d+w_U'} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) + \frac{1}{2} \int_{K_U^d+w_U'}^{3K_U^d+2m_B-w_U'} \frac{\partial K_U^d}{\partial \mu_R} dG(\epsilon) < 0. \quad \blacksquare$$

Proof of Proposition 7: It suffices to prove for the monopoly model and the duopoly model when $w_U' \leq m_B$, because they cover all possible FOCs characterizing the optimal capacity. We prove them one by one.

(1) For the monopoly model, taking derivative over μ_F, we have:

$$\frac{\partial E[Q^m]}{\partial \mu_F} = \int_{m_A}^{2K_U^m+m_A} \left(-\frac{1}{2} \cdot \frac{\partial m_A}{\partial \mu_F}\right) dG(\epsilon) + \int_{2K_U^m+m_A}^{2K_U^m+m_B} \frac{\partial K_U^m}{\partial \mu_F} dG(\epsilon) + \int_{2K_U^m+m_B}^{\infty} \left(-\frac{1}{2} \cdot \frac{\partial m_B}{\partial \mu_F}\right) dG(\epsilon).$$

By equation (A.3), we have $\frac{\partial E[Q^m]}{\partial \mu_F} = -\frac{1-G(m_A)}{2} \cdot \frac{\partial m_A}{\partial \mu_F} < 0$.

(2) For the duopoly model with $w'_U \le m_B$ and $C_U \in [0, C_2^d)$, we have $K_U^d > w'_U - m_A$. Taking derivative for the optimal output quantities over μ_F, we have:

$$\begin{aligned}
\frac{\partial E[Q^d]}{\partial \mu_F} =& -\frac{1}{2}\int_{m_A}^{2w'_U-m_A} \frac{\partial m_A}{\partial \mu_F} dG(\epsilon) - \frac{2}{3}\int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\partial m_A}{\partial \mu_F} dG(\epsilon) + \\
& \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\partial K_U^d}{\partial \mu_F} dG(\epsilon) - \frac{2}{3}\int_{3K_U^d+2m_B-w'_U}^{\infty} \frac{\partial m_B}{\partial \mu_F} dG(\epsilon) \\
\frac{\partial E[Q_L^d]}{\partial \mu_F} =& \frac{1}{3}\int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\partial m_A}{\partial \mu_F} dG(\epsilon) - \frac{1}{2}\int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\partial K_U^d}{\partial \mu_F} dG(\epsilon) + \\
& \frac{1}{3}\int_{3K_U^d+2m_B-w'_U}^{\infty} \frac{\partial m_B}{\partial \mu_F} dG(\epsilon) \\
\frac{\partial E[Q^d + Q_L^d]}{\partial \mu_F} =& -\frac{1}{2}\int_{m_A}^{2w'_U-m_A} \frac{\partial m_A}{\partial \mu_F} dG(\epsilon) - \frac{1}{3}\int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \frac{\partial m_A}{\partial \mu_F} dG(\epsilon) + \\
& \frac{1}{2}\int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \frac{\partial K_U^d}{\partial \mu_F} dG(\epsilon) - \frac{1}{3}\int_{3K_U^d+2m_B-w'_U}^{\infty} \frac{\partial m_B}{\partial \mu_F} dG(\epsilon).
\end{aligned}$$

We can express $\frac{\partial K_U^d}{\partial \mu_F}$ as $\frac{\partial K_U^d}{\partial \mu_F} = \mathcal{B} \cdot \frac{\partial m_B}{\partial \mu_F} - \mathcal{A} \cdot \frac{\partial m_A}{\partial \mu_F}$, where

$$\begin{aligned}
\mathcal{A} =& \frac{\left[1-G(3K_U^d+2m_A-w'_U)+K_U^d \cdot g(3K_U^d+2m_A-w'_U)\right]}{\left[G(3K_U^d+2m_B-w'_U)-G(3K_U^d+2m_A-w'_U)\right]-\frac{3K_U^d}{2}\cdot\left[g(3K_U^d+2m_B-w'_U)-g(3K_U^d+2m_A-w'_U)\right]} \\
\mathcal{B} =& \frac{\left[1-G(3K_U^d+2m_B-w'_U)+K_U^d \cdot g(3K_U^d+2m_B-w'_U)\right]}{\left[G(3K_U^d+2m_B-w'_U)-G(3K_U^d+2m_A-w'_U)\right]-\frac{3K_U^d}{2}\cdot\left[g(3K_U^d+2m_B-w'_U)-g(3K_U^d+2m_A-w'_U)\right]}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial E[Q^d + Q_L^d]}{\partial \mu_F} =& \left[\mathcal{B} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} dG(\epsilon) - \frac{1}{3}\int_{3K_U^d+2m_B-w'_U}^{\infty} dG(\epsilon)\right] \cdot \frac{\partial m_B}{\partial \mu_F} - \left[\mathcal{A} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U}\right. \\
& \left. dG(\epsilon) + \frac{1}{2}\int_{m_A}^{2w'_U-m_A} dG(\epsilon) + \frac{1}{3}\int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} dG(\epsilon)\right] \cdot \frac{\partial m_A}{\partial \mu_F}.
\end{aligned}$$

So $E[Q^d + Q_L^d]$ increases in μ_F if $\frac{\partial m_A/\partial \mu_F}{\partial m_B/\partial \mu_F} < \Theta_{Q+Q_L}^d$, and

$$\Theta_{Q+Q_L}^d := \frac{\mathcal{B} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \left(\frac{1}{2}\right) dG(\epsilon) - \int_{3K_U^d+2m_B-w'_U}^{\infty} \left(\frac{1}{3}\right) dG(\epsilon)}{\mathcal{A} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \left(\frac{1}{2}\right) dG(\epsilon) + \int_{m_A}^{2w'_U-m_A} \left(\frac{1}{2}\right) dG(\epsilon) + \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \left(\frac{1}{3}\right) dG(\epsilon)}.$$

Similarly,

$$\Theta_Q^d := \frac{\mathcal{B} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} dG(\epsilon) - \int_{3K_U^d+2m_B-w'_U}^{\infty} \left(\frac{2}{3}\right) dG(\epsilon)}{\mathcal{A} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} dG(\epsilon) + \int_{m_A}^{2w'_U-m_A} \left(\frac{1}{2}\right) dG(\epsilon) + \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \left(\frac{2}{3}\right) dG(\epsilon)}$$

$$\Theta_{Q_L}^d := \frac{\mathcal{B} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \left(\frac{1}{2}\right) dG(\epsilon) - \int_{3K_U^d+2m_B-w'_U}^{\infty} \left(\frac{1}{3}\right) dG(\epsilon)}{\mathcal{A} \cdot \int_{3K_U^d+2m_A-w'_U}^{3K_U^d+2m_B-w'_U} \left(\frac{1}{2}\right) dG(\epsilon) + \int_{2w'_U-m_A}^{3K_U^d+2m_A-w'_U} \left(\frac{1}{3}\right) dG(\epsilon)}.$$

It is straightforward that $\Theta_{Q+Q_L}^d < \Theta_Q^d < \Theta_{Q_L}^d < \Theta_{K_U}^d$.

(3) For the duopoly model with $w'_U \le m_B$ and $C_U \in [C_2^d, C_0^d)$, the same argument follows. The proof is omitted here due to page limitation. ■

Proof of Proposition 8: Substituting the optimal capacity into the profit expression $\Pi(K_U^j)$, $(j = m, d)$ and $\Pi_L(K_U^j)$, we have the optimal profit. We prove for the three first order conditions determining the global firm's optimal capacity for the monopoly and duopoly modelsone by one.

(1) For the monopoly model, taking derivative over μ_R and μ_F, we have:

$$\frac{\partial \Pi(K_U^m)}{\partial \mu_R} = -\left[\int_{m_A}^{2K_U^m+m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2K_U^m+m_A}^{\infty} K_U^m dG(\epsilon)\right] \cdot \frac{\partial m_A}{\partial \mu_R} < 0$$

$$\frac{\partial \Pi(K_U^m)}{\partial \mu_F} = -\left[\int_{m_A}^{2K_U^m+m_A} \frac{\epsilon - m_A}{2} dG(\epsilon) + \int_{2K_U^m+m_A}^{\infty} K_U^m dG(\epsilon)\right] \cdot \frac{\partial m_A}{\partial \mu_F} - \left[\int_{2K_U^m+m_B}^{\infty} \left(\frac{\epsilon - m_B}{2} - K_U^m\right) dG(\epsilon)\right] \cdot \frac{\partial m_B}{\partial \mu_F} < 0.$$

(2) For the duopoly model with $w'_U \le m_B$ and $C_U \in [0, C_2^d)$,

$$\begin{aligned}
\frac{\partial \Pi(K_U^d)}{\partial \mu_R} =& \left[\int_{m_A}^{2w'_U - m_A} \left(-\frac{\epsilon - m_A}{2} \right) dG(\epsilon) + \int_{2w'_U - m_A}^{3K_U^d + 2m_A - w'_U} \left(-\frac{4(\epsilon - 2m_A + w'_U)}{9} \right) dG(\epsilon) + \right. \\
& \left. + \int_{3K_U^d + 2m_A - w'_U}^{\infty} \left(-K_U^d \right) dG(\epsilon) \right] \cdot \frac{\partial m_A}{\partial \mu_R} < 0 \\
\frac{\partial \Pi(K_U^d)}{\partial \mu_F} =& \left[\int_{m_A}^{2w'_U - m_A} \left(-\frac{\epsilon - m_A}{2} \right) dG(\epsilon) + \int_{2w'_U - m_A}^{3K_U^d + 2m_A - w'_U} \left(-\frac{4(\epsilon - 2m_A + w'_U)}{9} \right) dG(\epsilon) + \right. \\
& \left. \int_{3K_U^d + 2m_A - w'_U}^{\infty} \left(-K_U^d \right) dG(\epsilon) \right] \cdot \frac{\partial m_A}{\partial \mu_F} + \\
& \left[\int_{3K_U^d + 2m_B - w'_U}^{\infty} \left(-\frac{4(\epsilon - 2m_B + w'_U)}{9} + K_U^d \right) dG(\epsilon) \right] \cdot \frac{\partial m_B}{\partial \mu_F} \\
& < 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi_L(K_U^d)}{\partial \mu_R} =& \frac{\partial m_A}{\partial \mu_R} \cdot \left[\int_{2w'_U - m_A}^{3K_U^d + 2m_A - w'_U} \frac{2(\epsilon + m_A - 2w'_U)}{9} dG(\epsilon) \right] - \\
& \frac{\partial K_U^d}{\partial \mu_R} \cdot \left[\int_{3K_U^d + 2m_A - w'_U}^{\infty} \frac{\epsilon - K_U^d - w'_U}{2} dG(\epsilon) \right] \\
& > 0 \\
\frac{\partial \Pi_L(K_U^d)}{\partial \mu_F} =& \frac{\partial m_A}{\partial \mu_F} \cdot \left[\int_{2w'_U - m_A}^{3K_U^d + 2m_A - w'_U} \frac{2(\epsilon + m_A - 2w'_U)}{9} dG(\epsilon) \right] - \\
& \frac{\partial K_U^d}{\partial \mu_F} \cdot \left[\int_{3K_U^d + 2m_A - w'_U}^{3K_U^d + 2m_B - w'_U} \frac{\epsilon - K_U^d - w'_U}{2} dG(\epsilon) \right] \\
& + \left[\int_{3K_U^d + 2m_B - w'_U}^{\infty} \frac{2(\epsilon + m_B - 2w'_U)}{9} dG(\epsilon) \right] \cdot \frac{\partial m_B}{\partial \mu_F}
\end{aligned}$$

(3) For the duopoly model with $w'_U \le m_B$ and $C_U \in [C_2^d, C_0^d)$ the proof is similar to case (2), and is omitted due to page limitation.

Substitute $\frac{\partial K_U^d}{\partial \mu_F}$ into the expressions above. The threshold is:

$$\Lambda_L^d := \begin{cases} \dfrac{\mathcal{B} \cdot \int_{3K_U^d + 2m_A - w'_U}^{3K_U^d + 2m_B - w'_U} \frac{\epsilon - K_U^d - w'_U}{2} dG(\epsilon) - \int_{3K_U^d + 2m_B - w'_U}^{\infty} \frac{2(\epsilon + m_B - 2w'_U)}{9} dG(\epsilon)}{\mathcal{A} \cdot \int_{3K_U^d + 2m_A - w'_U}^{3K_U^d + 2m_B - w'_U} \frac{\epsilon - K_U^d - w'_U}{2} dG(\epsilon) + \int_{2w'_U - m_A}^{3K_U^d + 2m_A - w'_U} \frac{2(\epsilon + m_A - 2w'_U)}{9} dG(\epsilon)}, & \text{if } C_U \in [0, C_2^d) \\ \dfrac{\mathcal{B} \cdot \int_{3K_U^d + 2m_A - w'_U}^{3K_U^d + 2m_B - w'_U} \frac{\epsilon - K_U^d - w'_U}{2} dG(\epsilon) - \int_{3K_U^d + 2m_B - w'_U}^{\infty} \frac{2(\epsilon + m_B - 2w'_U)}{9} dG(\epsilon)}{\mathcal{A} \cdot \int_{3K_U^d + 2m_A - w'_U}^{3K_U^d + 2m_B - w'_U} \frac{\epsilon - K_U^d - w'_U}{2} dG(\epsilon)}, & \text{if } C_U \in [C_2^d, C_0^d) \quad \blacksquare \end{cases}$$

Proof of Proposition 9: When $w'_U \le m_B$,

$$
\begin{aligned}
C_0^d &= \int_{m_A}^{w'_U} (\epsilon - m_A) dG(\epsilon) + \int_{w'_U}^{2m_B - w'_U} \frac{\epsilon - 2m_A + w'_U}{2} dG(\epsilon) + \int_{2m_B - w'_U}^{\infty} (m_B - m_A) dG(\epsilon) \\
C_0^m &= \int_{m_A}^{m_B} (\epsilon - m_A) dG(\epsilon) + \int_{m_B}^{\infty} (m_B - m_A) dG(\epsilon) \\
&= \int_{m_A}^{w'_U} (\epsilon - m_A) dG(\epsilon) + \int_{w'_U}^{m_B} (\epsilon - m_A) dG(\epsilon) + \int_{m_B}^{2m_B - w'_U} (m_B - m_A) dG(\epsilon) + \\
&\quad \int_{2m_B - w'_U}^{\infty} (m_B - m_A) dG(\epsilon)
\end{aligned}
$$

For each integral interval, C_0^m is larger or equal to C_0^d. Hence $C_0^d \le C_0^m$. ∎

Proof of Proposition 10: When $w'_U > m_B$ and $C_U \in (C_2^d, C_1^d)$, K_U^d solves FOC_2^d and K_U^m solves FOC^m, where

$$
\begin{aligned}
FOC_2^d : &\int_{2K_U^d + m_A}^{K_U^d + w'_U} \left[\epsilon - m_A - 2K_U^d\right] dG(\epsilon) + \int_{K_U^d + w'_U}^{3K_U^d + 2m_B - w'_U} \left[\frac{\epsilon - 2m_A + w'_U}{2} - K_U^d\right] dG(\epsilon) \\
&+ \int_{3K_U^d + 2m_B - w'_U}^{\infty} (m_B - m_A) dG(\epsilon) = C_U \\
FOC^m : &\int_{2K_U^m + m_A}^{2K_U^m + m_B} (\epsilon - 2K_U^m - m_A) dG(\epsilon) + \int_{2K_U^m + m_B}^{\infty} (m_B - m_A) dG(\epsilon) = C_U
\end{aligned}
$$

Substitute K_U^m into the LHS of FOC_2^d. Since values of each integral interval increases, we have the new LHS of FOC_2^d is larger than C_U. Hence $K_U^d > K_U^m$. ∎

Proof of Proposition 11: Assuming $\epsilon \sim U[0, M]$, we substitute $dG(\epsilon) = \frac{1}{M} d\epsilon$ into the optimal capacities. For the monopoly model, we have:

$$
K_U^m = \begin{cases}
\frac{M - m_A - \sqrt{2MC_U}}{2}, & \text{if } C_U < \frac{(m_B - m_A)^2}{2M} \\
\frac{2M - m_B - m_A - \frac{2MC_U}{m_B - m_A}}{4}, & \text{if } \frac{(m_B - m_A)^2}{2M} < C_U < \frac{(m_B - m_A)(2M - m_B - m_A)}{2M} \\
0, & \text{if } C_U > \frac{(m_B - m_A)(2M - m_B - m_A)}{2M}.
\end{cases}
$$

For the duopoly model with $w_U' \le m_B$, we have

$$K_U^d = \begin{cases} \frac{2(M-2m_A+w_U')-\sqrt{(M-2m_A+w_U')^2+12MC_U}}{3}, \\ \qquad\qquad \text{if } C_U \in \left[0, \frac{(m_B-m_A)(M-3m_A+m_B+w_U')}{3M}\right) \\ \frac{M-m_A-m_B+w_U'-\frac{MC_U}{m_B-m_A}}{2}, \\ \qquad\qquad \text{if } C_U \in \left[\frac{(m_B-m_A)(M-3m_A+m_B+w_U')}{3M}, \frac{(m_B-m_A)(M+m_A-m_B-w_U')}{M}\right) \\ 2(m_B-m_A) - \sqrt{4(m_B-m_A)^2 - 2(m_B-m_A)(M-m_A-m_B+w_U') + (w_U'-m_A)^2 + 2MC_U}, \\ \qquad\qquad \text{if } C_U \in \left[\frac{(m_B-m_A)(M+m_A-m_B-w_U')}{M}, \frac{2(m_B-m_A)(M-m_A-m_B+w_U')-(w_U'-m_A)^2}{2M}\right) \\ 0, \\ \qquad\qquad \text{if } C_U \in \left[\frac{2(m_B-m_A)(M-m_A-m_B+w_U')-(w_U'-m_A)^2}{2M}, \infty\right) \end{cases}$$

For the duopoly model with $w_U' > m_B$, we have:

$$K_U^d = \begin{cases} \frac{2(M-2m_A+w_U')-\sqrt{(M-2m_A+w_U')^2+12MC_U}}{3}, \\ \qquad\qquad \text{if } C_U \in \left[0, \frac{(m_B-m_A)(M-3m_A+m_B+w_U')}{3M}\right) \\ \frac{M-m_A-m_B+w_U'-\frac{MC_U}{m_B-m_A}}{2}, \\ \qquad\qquad \text{if } C_U \in \left[\frac{(m_B-m_A)(M-3m_A+m_B+w_U')}{3M}, \frac{(m_B-m_A)(M+m_A-m_B-w_U')}{M}\right) \\ 2(m_B-m_A) - \sqrt{4(m_B-m_A)^2 - 2(m_B-m_A)(M-m_A-m_B+w_U') + (w_U'-m_A)^2 + 2MC_U}, \\ \qquad\qquad \text{if } C_U \in \left[\frac{(m_B-m_A)(M+m_A-m_B-w_U')}{M}, \frac{(m_B-m_A)(2M-m_A+3m_B-4w_U')}{2M}\right) \\ \frac{M-\frac{m_A+m_B}{2}-\frac{MC_U}{m_B-m_A}}{2}, \\ \qquad\qquad \text{if } C_U \in \left[\frac{(m_B-m_A)(2M-m_A+3m_B-4w_U')}{2M}, \frac{(m_B-m_A)(2M-m_A-m_B)}{2M}\right) \\ 0, \\ \qquad\qquad \text{if } C_U \in \left[\frac{(m_B-m_A)(2M-m_A-m_B)}{2M}, \infty\right) \end{cases}$$

Denote $C_3^d := \frac{(m_B-m_A)(M-3m_A+m_B+w_U')}{3M}$ and $C_1^m := \frac{(m_B-m_A)^2}{2M}$. It is straightforward to verify that: $C_1^m < C_3^d < C_2^d < C_0^d < C_0^m$ for $w_U' \le m_B$, and $C_1^m < C_3^d < C_2^d < C_1^d < C_0^m$ for $w_U' > m_B$. Therefore, we need to compare K_U^d and K_U^m in 6 intervals. To see clearly why we have to compare w_U' and $\frac{m_A+m_B}{2}$, we start from $[C_3^d, C_2^d)$.

(1) In the range $C_U \in [C_3^d, C_2^d)$, $K_U^d < K_U^m$ is equivalent to

$$\frac{M - m_A - m_B + w'_U - \frac{MC_U}{m_B - m_A}}{2} < \frac{2M - m_B - m_A - \frac{2MC_U}{m_B - m_A}}{4} \quad \Leftrightarrow \quad w'_U < \frac{m_A + m_B}{2},$$

which is the reason why we have to compare w'_U and $\frac{m_A+m_B}{2}$.

(2) When $C_U \in [0, C_1^m)$, it is quite straightforward that $K_U^d < K_U^m$ always holds.

(3) When $C_U \in [C_1^m, C_3^d)$, there exists a unique solution I_1 to $K_U^d = K_U^m$ if and only if $w'_U > \frac{m_A+m_B}{2}$, where I_1 is defined as $I_1 := \frac{-B_1+\sqrt{B_1^2-4A_1C_1}}{4A_1}$ with $A_1 = \frac{12M^2}{(m_B-m_A)^2}$,$B_1 = \frac{4M(2M-13m_A+3m_B+8w'_U)}{m_B-m_A} - 64M$, and $C_1 = (2M - m_B - m_A)^2 - (6m_A - 2m_B - 4w'_U)^2$.

(4) When $C_U \in [C_2^d, C_0^d)$ and $w'_U \le m_B$, there also exists a unique solution I_2 to $K_U^d = K_U^m$ if and only if $w'_U > \frac{m_A+m_B}{2}$, where I_2 is defined as $I_2 := \frac{-B_2+\sqrt{B_2^2-4A_2C_2}}{4A_2}$ where $A_2 = \frac{4M^2}{(m_B-m_A)^2}$, $B_2 = -\frac{4M(2M+3m_A-5m_B)}{m_B-m_A} - 2M$, and $C_1 = ((2M+3m_A-5m_B))^2 - 4(m_B-m_A)^2 + 2(m_B-m_A)(M - m_A - m_B + w'_U) - (w'_U - m_A)^2$. And $K_U^d > K_U^m$ always holds when $C_U \in [C_2^d, C_1^d)$ and $w'_U > m_B$.

(5) When $C_U \in [C_0^d, C_0^m)$ and $w'_U \le m_B$, $0 = K_U^d < K_U^m$ always holds. And $K_U^d = K_U^m > 0$ always holds when $C_U \in [C_1^d, C_0^m)$ and $w'_U > m_B$.

(6) When $C_U \in [C_0^m, \infty)$, $K_U^d = K_U^m = 0$. ∎

A.2 Impact of Section 301 Tariffs

This appendix reports the impact of section 301 tariffs on domestic production and imports from China in the directly affected industries as defined in the United States International Trade Commission report (USITC, 2023). For each industry, we report the domestic U.S. gross output, nontariff-inclusive U.S. imports from China, and the estimated tariff impact on these two values from USITC (2023). The combined output is calculated as the sum of domestic U.S. gross output and U.S. imports from China. The domestic U.S. gross output in the absence of tariffs is calculated as (actual domestic U.S. gross output)/(1+tariff effect measured as percentage changes). The U.S. imports from China in the absence of tariffs is calculated similarly. The impact of section 301 tariffs on the combined output is calculated by taking the difference between the total output in the absence of tariffs and the actual output when the tariffs were in place.

Audio and Video Equipment:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	3	3.7	4.8	8.1
Nontariff-inclusive U.S. imports from China ($B)	13.6	12.8	12	12.2
Combined output ($B)	16.6	16.5	16.8	20.3
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	0.2	2.8	7.3	6.4
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−1.3	−15.8	−33.4	−37.8
Domestic U.S. gross output in the absence of tariffs ($B)	2.99	3.60	4.47	7.61
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	13.78	15.20	18.02	19.61
Combined output in the absence of tariffs ($B)	16.77	18.80	22.49	27.23
Impact of tariffs on combined output (% change)	−1.03	−12.24	−25.30	−25.44

Average impact of tariffs on combined output: −16%.

Cut and Sew Apparel Manufacturing:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	10.4	11.1	9.2	12.4
Nontariff-inclusive U.S. imports from China ($B)	25	22.5	13.3	17.2
Combined output ($B)	35.4	33.6	22.5	29.6
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	0	1.9	6.2	6.3
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−0.2	−14.7	−39.9	−39.1
Domestic U.S. gross output in the absence of tariffs ($B)	10.40	10.89	8.66	11.67
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	25.05	26.38	22.13	28.24
Combined output in the absence of tariffs ($B)	35.45	37.27	30.79	39.91
Impact of tariffs on combined output (% change)	−0.14	−9.85	−26.93	−25.83

Average impact of tariffs on combined output: -15.69%.

Computer Equipment:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	36.2	34.2	35.8	39
Nontariff-inclusive U.S. imports from China ($B)	61.3	54.2	60	68.9
Combined output ($B)	97.5	88.4	95.8	107.9
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	0.3	1.3	1.5	1.2
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−1.3	−5.9	−6.7	−5.3
Domestic U.S. gross output in the absence of tariffs ($B)	36.09	33.76	35.27	38.54
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	62.11	57.60	64.31	72.76
Combined output in the absence of tariffs ($B)	98.20	91.36	99.58	111.29
Impact of tariffs on combined output (% change)	−0.71	−3.24	−3.80	−3.05

Average impact of tariffs on combined output: −2.7%.

Other Miscellaneous Manufacturing:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	68.3	68.2	65.4	68
Nontariff-inclusive U.S. imports from China ($B)	37.8	36.3	34.8	48.4
Combined output ($B)	106.1	104.5	100.2	116.4
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	0.1	0.9	2.5	2.4
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−0.3	−5.1	−12.3	−11.7
Domestic U.S. gross output in the absence of tariffs ($B)	68.23	67.59	63.80	66.41
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	37.91	38.25	39.68	54.81
Combined output in the absence of tariffs ($B)	106.15	105.84	103.49	121.22
Impact of tariffs on combined output (% change)	−0.04	−1.27	−3.17	−3.98

Average impact of tariffs on combined output: -2.12%.

Household and Institutional Furniture and Kitchen Cabinets:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	40.7	38.5	39.9	44.7
Nontariff-inclusive U.S. imports from China ($B)	18.2	15	13.1	15.8
Combined output ($B)	58.9	53.5	53	60.5
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	1.2	6.4	7.4	7.5
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−3.8	−19.7	−25.4	−25.4
Domestic U.S. gross output in the absence of tariffs ($B)	40.22	36.18	37.15	41.58
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	18.92	18.68	17.56	21.18
Combined output in the absence of tariffs ($B)	59.14	54.86	54.71	62.76
Impact of tariffs on combined output (% change)	−0.40	−2.49	−3.13	−3.60

Average impact of tariffs on combined output: −2.40%.

Other Electrical Equipment and Components:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	51.7	52.2	50.1	58.5
Nontariff-inclusive U.S. imports from China ($B)	15.1	11.9	11	14.5
Combined output ($B)	66.8	64.1	61.1	73
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	1.4	5.7	7	7
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−9.4	−33.1	−39.4	−37.7
Domestic U.S. gross output in the absence of tariffs ($B)	50.99	49.39	46.82	54.67
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	16.67	17.79	18.15	23.27
Combined output in the absence of tariffs ($B)	67.65	67.17	64.97	77.95
Impact of tariffs on combined output (% change)	−1.26	−4.57	−5.96	−6.35

Average impact of tariffs on combined output: -4.54%.

Other General Purpose Machinery:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	109.5	112	101.3	111.9
Nontariff-inclusive U.S. imports from China ($B)	12.5	10.9	10.9	12.7
Combined output ($B)	122	122.9	112.2	124.6
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	1.3	3.9	5.4	5.3
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−19.3	−42.1	−47.1	−47.6
Domestic U.S. gross output in the absence of tariffs ($B)	108.09	107.80	96.11	106.27
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	15.49	18.83	20.60	24.24
Combined output in the absence of tariffs ($B)	123.58	126.62	116.71	130.50
Impact of tariffs on combined output (% change)	−1.28	−2.94	−3.87	−4.52

Average impact of tariffs on combined output: −3.15%.

Semiconductors and Other Electronic Components:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	116.7	117.7	122.1	130.9
Nontariff-inclusive U.S. imports from China ($B)	24.3	8.9	9.2	8.8
Combined output ($B)	141	126.6	131.3	139.7
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	1.2	5.9	7.8	6.4
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−15.4	−66.2	−70.6	−72.3
Domestic U.S. gross output in the absence of tariffs ($B)	115.32	111.14	113.27	123.03
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	28.72	26.33	31.29	31.77
Combined output in the absence of tariffs ($B)	144.04	137.47	144.56	154.80
Impact of tariffs on combined output (% change)	−2.11	−7.91	−9.17	−9.75

Average impact of tariffs on combined output: -7.24%.

Plastic Products:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	206.6	206.2	199.3	207
Nontariff-inclusive U.S. imports from China ($B)	15.9	15.6	17.7	21.1
Combined output ($B)	222.5	221.8	217	228.1
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	0.3	1.6	2.8	2.8
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−3.9	−19.6	−25.3	−23.7
Domestic U.S. gross output in the absence of tariffs ($B)	205.98	202.95	193.87	201.36
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	16.55	19.40	23.69	27.65
Combined output in the absence of tariffs ($B)	222.53	222.36	217.57	229.02
Impact of tariffs on combined output (% change)	−0.01	−0.25	−0.26	−0.40

Average impact of tariffs on combined output: −0.23%.

Motor Vehicle Parts:

Variable	2018	2019	2020	2021
Domestic U.S. gross output ($B)	275.5	268.5	228.7	250.2
Nontariff-inclusive U.S. imports from China ($B)	15.2	12.2	9.5	12.3
Combined output ($B)	290.7	280.7	238.2	262.5
Estimated impact of section 301 tariffs on domestic U.S. gross output (% change)	0.3	2	2.8	3
Estimated impact of section 301 tariffs on U.S. imports from China (% change)	−9.6	−42.5	−50.3	−50.1
Domestic U.S. gross output in the absence of tariffs ($B)	274.68	263.24	222.47	242.91
Nontariff-inclusive U.S. imports from China in the absence of tariffs ($B)	16.81	21.22	19.11	24.65
Combined output in the absence of tariffs ($B)	291.49	284.45	241.59	267.56
Impact of tariffs on combined output (% change)	−0.27	−1.32	−1.40	−1.89

Average impact of tariffs on combined output: -1.22%.

Appendix B

Selling Agri-Tech Products: Firm Strategy, Farmer Incentives, and Government Subsidy

B.1 Proofs

Proposition B.1.1 *Given the firm's product price p, and service fee s. We characterize farmers' responses as the equilibrium proportion of their purchasing strategies in different price regions. Specifically,*

Regions and Characterizations	n_N	n_P	n_{P+S}
$\Omega_1 : 0 \leq s < (\mu - \lambda)(a - b - \mu b)$ and $0 \leq p + s < \mu(a - b - \mu b)$	0	0	1
$\Omega_2 : (\mu - \lambda)(a - b - \mu b) \leq s < (\mu - \lambda)(a - b - \lambda b)$ and $\frac{s}{p} \geq \frac{\mu-\lambda}{\lambda}$	0	$\frac{s-(\mu-\lambda)(a-b-\mu b)}{(\mu-\lambda)^2 b}$	$\frac{(\mu-\lambda)(a-b-\lambda b)-s}{(\mu-\lambda)^2 b}$
$\Omega_3 : s \geq (\mu - \lambda)(a - b - \lambda b)$ and $0 \leq p < \lambda(a - b - \lambda b)$	0	1	0
$\Omega_4 : \lambda(a - b - \lambda b) \leq p < \lambda(a - b)$ and $\frac{s}{p} \geq \frac{\mu-\lambda}{\lambda}$	$\frac{p-\lambda(a-b-\lambda b)}{\lambda^2 b}$	$\frac{\lambda(a-b)-p}{\lambda^2 b}$	0
$\Omega_5 : \mu(a - b - \mu b) \leq p + s < \mu(a - b)$ and $0 \leq \frac{s}{p} < \frac{\mu-\lambda}{\lambda}$	$\frac{(p+s)-\mu(a-b-\mu b)}{\mu^2 b}$	0	$\frac{\mu(a-b)-(p+s)}{\mu^2 b}$
$\Omega_6 : p \geq \lambda(a - b)$ or $p + s \geq \mu(a - b)$	1	0	0

Proof of Proposition 1.

By the definition of a farmer i's crop output level q_i and utility u_i, we have

$$\begin{aligned} u_i(N) &= r \\ u_i(P) &= r \cdot (1 + \lambda) - p \\ u_i(P + S) &= r \cdot (1 + \mu) - p - s \end{aligned}$$

It is straightforward that a farmer i with capability λ prefers purchasing the product but not the professional service than purchasing nothing if and only if $u_i(P) > u_i(N)$, i.e. $r > \frac{p}{\lambda}$. Similarly, $u_i(P + S) > u_i(N)$ is equivalent to $r > \frac{p+s}{\mu}$. And $u_i(P + S) > u_i(P)$ is equivalent to $r > \frac{s}{\mu-\lambda}$. The order of these three price thresholds is determined by the firm's prices p and s.

(1) If $\frac{s}{p} > \frac{\mu-\lambda}{\lambda}$, then $\frac{p}{\lambda} < \frac{p+s}{\mu} < \frac{s}{\mu-\lambda}$. As the following graph suggests, when market price $r < \frac{p}{\lambda}$, strategy N is the best for all farmers. When $r > \frac{s}{\mu-\lambda}$, strategy P+S is the best for all farmers. In the middle when $\frac{p}{\lambda} < r < \frac{s}{\mu-\lambda}$, strategy P dominates other two strategies. When $r = \frac{p}{\lambda}$, there is no difference between strategy N and P since $u_i(P) = u_i(N)$. So a part of farmers adopts strategy N while others adopt strategy P, i.e. $n_{P+S} = 0$ and $n_N + n_P = 1$. The total crop output is $q = 1 + \lambda \cdot n_P$. Notice that the market price is not given before the farmer's purchasing happens. Instead, the market price is the result of farmers' purchasing strategies. Although the production quantity of each individual farmer is too small to influence the market price. The total output quantity is large

enough to affect the market price. The equilibrium market price $\tilde{r}$ follows

$$\tilde{r} = a - b \cdot (1 + \lambda \cdot n_P).$$

Following [55], we assume the farmers can form rational expectations about the market clearing price such that in equilibrium, $r = \tilde{r}$, i.e.

$$\frac{p}{\lambda} = a - b \cdot (1 + \lambda \cdot n_P),$$

Hence

$$n_P = \frac{p - \lambda(a - b - \lambda b)}{\lambda^2 b}.$$

Feasible n_P should lie between $[0,1]$, which requires

$$p \in [\lambda(a - b - \lambda b), \lambda(a - b)].$$

To make sure every point in the price space belongs to only one region, we adopt the principle that every interval is close on the left and open on the right. Since the boundary fits any adjacent regions. Define region

$$\Omega_4 : \lambda(a - b - \lambda b) \le p < \lambda(a - b) \text{ and } \frac{s}{p} > \frac{\mu - \lambda}{\lambda}.$$

For $p \ge \lambda(a - b)$, we have $n_N = 1$. And for $p < \lambda(a - b - \lambda b)$, $n_N = 0$.

Similar argument is also valid for $p = \frac{s}{\mu - \lambda}$. Therefore, $n_N = 0$ and

$$n_{P+S} = \frac{(\mu - \lambda)(a - b - \lambda b) - s}{(\mu - \lambda)^2 b}$$

Feasible region satisfying $n_{P+S} \in [0,1]$ is $s \in [(\mu - \lambda)(a - b - \mu b), (\mu - \lambda)(a - b - \lambda b)]$. Define region

$$\Omega_2 : (\mu - \lambda)(a - b - \mu b) \le s < (\mu - \lambda)(a - b - \lambda b) \text{ and } \frac{s}{p} > \frac{\mu - \lambda}{\lambda}.$$

For $s \ge (\mu - \lambda)(a - b - \lambda b)$, $n_{P+S} = 0$. And for $s < (\mu - \lambda)(a - b - \mu b)$, $n_{P+S} = 1$.

(2) If $\frac{s}{p} < \frac{\mu - \lambda}{\lambda}$, then $\frac{s}{\mu - \lambda} < \frac{p+s}{\mu} < \frac{p}{\lambda}$. When market price $r < \frac{p+s}{\mu}$, strategy N is the best for all farmers. When $r > \frac{p+s}{\mu}$, strategy P+S is the best for all farmers.

When $r = \frac{p+s}{\mu}$, there is no difference between strategy N and P+S since $u_i(N) = u_i(P + S)$. So a part of farmers adopts strategy N while others adopts strategy P+S, i.e. $n_P = 0$ and $n_N + n_{P+S} = 1$. Following the rational expectations in the previous case, we have

$$\frac{p+s}{\mu} = a - b\cdot(1+\mu\cdot n_{P+S}),$$

and

$$n_{P+S} = \frac{\mu(a-b)-(p+s)}{\mu^2 b}.$$

Feasible region satisfying $n_{P+S} \in [0,1]$ is $p+s \in [\mu(a-b-\lambda b), \mu(a-b)]$. Define region

$$\Omega_5 : \mu(a-b-\mu b) \le p+s < \mu(a-b) \text{ and } 0 \le \frac{s}{p} < \frac{\mu-\lambda}{\lambda}.$$

For $p+s \ge \mu(a-b)$, $n_{P+S} = 0$. And for $p+s < \mu(a-b-\mu b)$, $n_{P+S} = 1$.

(3) If $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$, then $\frac{p}{\lambda} = \frac{p+s}{\mu} = \frac{s}{\mu-\lambda}$, the market price should make all farmers indifferent among strategies N, P and P+S, i.e.

$$u_i(N) = u_i(P) = u_i(P+S)$$

In addition, since

$$\frac{p}{\lambda} = a - b\cdot(1+\lambda\cdot n_P + \mu\cdot n_{P+S}),$$

and $(1+\lambda\cdot n_P+\mu\cdot n_{P+S}) \in [1, 1+\mu]$, we know that $p \in [\lambda(a-b-\mu b), \lambda(a-b)]$. The line $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$ with constraint $p \in [\lambda(a-b-\mu b), \lambda(a-b)]$ is the boundary of regions Ω_i, $i \in \{2,4\}$. Later when we solve the firm's optimization problem in Proposition 2, we can see it is reasonable to let the region Ω_2 and Ω_4 include the boundary $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$. In other words, it is possible farmers' three strategies coexist in equilibrium. But we can further refine them to only two strategies exist in the same time. And it does not affect firm's decision at all.

For other regions, define

$$\begin{aligned}
\Omega_1 &: \quad 0 \le s < (\mu-\lambda)(a-b-\mu b) \text{ and } 0 \le p+s < \mu(a-b-\mu b)\\
\Omega_3 &: \quad s \ge (\mu-\lambda)(a-b-\lambda b) \text{ and } 0 \le p < \lambda(a-b-\lambda b)\\
\Omega_6 &: \quad p \ge \lambda(a-b) \text{ or } p+s \ge \mu(a-b)
\end{aligned}$$

The above six regions cover the whole price space. It is straightforward to check that $n_{P+S} = 1$ in region Ω_1, $n_P = 1$ in region Ω_3, and $n_N = 1$ in region Ω_6.

By

$$n_P + n_{P+S} = \begin{cases} 1 & \text{in } \Omega_1, \Omega_2, \Omega_3 \\ \dfrac{\lambda(a-b)-p}{\lambda^2 b} & \text{in } \Omega_4 \\ \dfrac{\mu(a-b)-(p+s)}{\mu^2 b} & \text{in } \Omega_5 \end{cases}$$

we can easily prove Corollary 1. Further, we seek to strength the results by considering two lemma. It seems that low product price and low service fee favor framers' purchasing decision. But the detail is rather subtle. Notice n_{P+S} measures the number of farmers purchasing service and $n_P + n_{P+S}$ measures the number of farmers purchasing service. We first consider the agri-tech product price elasticity of $n_P + n_{P+S}$ and n_{P+S}.

Lemma B.1.1 *(i) For any given service fee s, the product price elasticity:*

$$\frac{\partial(n_P + n_{P+S})/(n_P + n_{P+S})}{\partial p/p} = \begin{cases} -\dfrac{p}{\lambda(a-b)-p} & \textit{in } \Omega_4 \\ -\dfrac{p}{\mu(a-b)-(p+s)} & \textit{in } \Omega_5 \\ 0 & \textit{Otherwise} \end{cases}$$

(ii)

$$\frac{\partial n_{P+S}/n_{P+S}}{\partial p/p} = \begin{cases} -\dfrac{p}{\mu(a-b)-(p+s)} & \textit{in } \Omega_5 \\ 0 & \textit{Otherwise} \end{cases}$$

However, if $(\mu-\lambda)(a-b-\lambda b) < s < (\mu-\lambda)(a-b)$, n_{P+S} jumps from $\frac{\mu(a-b)-(p+s)}{\mu^2 b}$ to 0 when p decreases across the line $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$.

Part (i) tells us the product price elasticity of $n_P + n_{P+S}$ is non-positive, which means the number of farmers purchasing agri-tech product decreases with product price in most cases. Notice $\left|-\frac{p}{\lambda(a-b)-p}\right| \geq \frac{a-b-\lambda b}{\lambda b}$ in Ω_4, and $\left|-\frac{p}{\mu(a-b)-(p+s)}\right| \geq \frac{\lambda(a-b-\mu b)}{\mu^2 b}$ in Ω_4. Applying assumption $a > \max\left\{(1+2\lambda)b, \left(1+\mu+\frac{\mu^2}{\lambda}\right)b\right\}$ indicates a highly elastic relationship since $\left|\frac{\partial(n_P+n_{P+S})/(n_P+n_{P+S})}{\partial p/p}\right| > 1$ in region Ω_4 and Ω_5. Hence, the number of farmers purchasing agri-tech product changes more than the product price. Meanwhile, we observe perfectly inelasticity in other regions because $n_P + n_{P+S}$ does not respond to a price change at all.

Part (ii) tells us the number of farmers purchasing professional service decreases with product price in Ω_5. However, one crossing boundary example (Ω_5 to Ω_4) shows that it is not always true that lower product price price encourages service purchasing. Intuitively, low product price favors product purchasing and low service price favors service purchasing, which are pretty obvious. However, the effect of low product price on service purchasing is complicated. On one hand, low product price allows more farmers to possess a drone which makes it more possible for them to take a step ahead, i.e. purchasing service. On the other hand, high product price makes the service price relatively low. Farmers may directly purchase both agri-tech product and service. The same argument is valid for low service price.

Lemma B.1.2 *(i) For any given product price p, the service price elasticity:*

$$\frac{\partial(n_P + n_{P+S})/(n_P + n_{P+S})}{\partial s/s} = \begin{cases} -\dfrac{s}{\mu(a-b)-(p+s)} & \text{in } \Omega_5 \\ 0 & \text{Otherwise} \end{cases}$$

However, if $\lambda(a-b-\mu b) < p < \lambda(a-b-\lambda b)$, $n_P + n_{P+S}$ jumps from $\frac{s-(\mu-\lambda)(a-b-\mu b)}{(\mu-\lambda)^2 b}$ to 0 when s decreases across the line $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$.

(ii)

$$\frac{\partial n_{P+S}/n_{P+S}}{\partial s/s} = \begin{cases} -\dfrac{s}{(\mu-\lambda)(a-b-\lambda b)-s} & \text{in } \Omega_2 \\ -\dfrac{s}{\mu(a-b)-(p+s)} & \text{in } \Omega_5 \\ 0 & \text{Otherwise} \end{cases}$$

$n_P + n_{P+S}$ always decreases in p for given s while n_{P+S} may either increase or decrease in p. In region Ω_5, n_{P+S} always decreases but it increases between Ω_4 and Ω_5. We can also show that n_{P+S} may either increase in p between region Ω_2 and Ω_5.

Similarly, n_{P+S} always decreases in s for given p while $n_P + n_{P+S}$ may either increase or decrease in s

Proposition B.1.2 *The firm's optimal pricing decisions and optimal profits depend on the production cost and service cost, specifically,*

Region	Characteristic	p^*	s^*	π^*
Λ_1	$0 \le c < \lambda k_{2\lambda}$ and $0 \le f < (\mu-\lambda)k_{\lambda+\mu}$, $\lambda k_{2\lambda} \le c < \lambda(a-b)$ and $0 \le f < -\frac{c^2}{4\lambda^2 b} + \frac{k_{2\lambda}\cdot c}{2\lambda b} - \frac{k_{2\mu}^2}{4b}$, $\lambda(a-b) \le c < \mu k_{2\mu}$ and $0 \le c+f < \mu k_\mu$	$(p+s)^* = \mu k_\mu$		$\mu k_\mu - c - f$
Λ_2	$0 \le c < \lambda k_{2\lambda}$ and $f \ge (\mu-\lambda)k_{\lambda+\mu}$	λk_λ	NA	$\lambda k_\lambda - c$
Λ_3	$\lambda k_{2\lambda} \le c < \lambda(a-b)$ and $f \ge -\frac{c^2}{4\lambda^2 b} + \frac{k_{2\lambda}\cdot c}{2\lambda b} - \frac{k_{2\mu}^2}{4b}$	$\frac{\lambda(a-b)+c}{2}$	NA	$\frac{(\lambda(a-b)-c)^2}{4\lambda^2 b}$
Λ_4	$\mu k_{2\mu} \le c < \mu(a-b)$ and $0 \le f < \frac{(\mu(a-b)-c)^2}{4\mu^2 b}$	$(p+s)^* = \frac{\mu(a-b)+c}{2}$		$\frac{(\mu(a-b)-c)^2}{4\mu^2 b} - f$
Λ_5	Otherwise	NA	NA	0

Note: $k_{2\lambda} = a-b-2\lambda b$, $k_{2\mu} = a-b-2\mu b$, $k_{\lambda+\mu} = a-b-(\lambda+\mu)b$

Proof of Proposition 2 Since the firm's problem is

$$\max_{p,s\ge 0} \quad \pi(p,s) = (n_P + n_{P+S})\cdot(p-c) + n_{P+S}\cdot s - f\cdot \mathbb{1}_{\{n_{P+S}>0\}}$$

and by Proposition 1, different combinations of price decisions p and s lead to different farmers' responses. We first calculate the local optimal solution in each price regions. Then compare all regions to derive the global optimal solution.

(1) In Ω_1, the firm's problem becomes:

$$\begin{aligned}\max_{p,s\ge 0} \quad & 1\cdot(p-c)+s-f \\ \text{s.t.} \quad & 0 \le s < (\mu-\lambda)(a-b-\mu b) \\ & 0 \le p+s < \mu(a-b-\mu b)\end{aligned}$$

Since $p+s < \mu(a-b-\mu b)$, the objective function is always less than $\mu(a-b-\mu b)-c-f$. And the value is attained when $p+s = \mu(a-b-\mu b)$, which belongs to the region Ω_5. Notice farmers are indifferent on the boundary. The objective function in region Ω_5 on the boundary $p+s = \mu(a-b-\mu b)$ is the same as that in region Ω_1. Therefore, we can exclude region Ω_1 as the possible region where the global optimal solution lies. Because it is always dominated by region Ω_5.

(2) In Ω_2, we first discuss the scenario on the boundary $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$. By Proposition 1, farmers' three possible strategies perhaps coexist. we have

$$\frac{s}{\mu-\lambda} = \frac{p}{\lambda} = a - b\cdot(1+\lambda\cdot n_P + \mu\cdot n_{P+S}),$$

(a) If $n_{P+S} = 0$, then no service should be provided by the firm. The objective function is at most $1 \cdot (p-c)$, which is the same as that in region Ω_3. Therefore, we can exclude the boundary $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$ as the possible region where the global optimal solution lies. Because it is always dominated by region Ω_5.

(b) If $n_{P+S} > 0$, then the firm provides service hence incurs the fixed cost f no matter how many farmers purchase it. The objective

$$\begin{aligned}\pi(p,s) =&(1-n_N)\cdot(p-c) + (1-n_N-n_P)\cdot s - f \\ \leq& 1\cdot(p-c) + (1-n_P)\cdot s - f\end{aligned}$$

The equality is attained if and only if $n_N = 0$. We refine the firm considers maximizing its profit for the best possible case. So we exclude the cases when $n_N > 0$ on the boundary $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$. By $n_P + n_{P+S} = 1$ we have $n_{P+S} = \frac{(\mu-\lambda)(a-b-\lambda b)-s}{(\mu-\lambda)^2 b}$.

Hence, the firm's problem becomes:

$$\begin{aligned}\max_{p,s\geq 0} \quad & 1\cdot(p-c) + \frac{(\mu-\lambda)(a-b-\lambda b)-s}{(\mu-\lambda)^2 b}\cdot s - f \\ \text{s.t.} \quad & (\mu-\lambda)(a-b-\mu b) \leq s < (\mu-\lambda)(a-b-\lambda b) \\ & \frac{s}{p} \geq \frac{\mu-\lambda}{\lambda}\end{aligned}$$

By the second constraint, we know $p \leq \frac{\lambda}{\mu-\lambda}\cdot s$. Hence,

$$\pi(p,s) \leq \pi\left(\frac{\lambda}{\mu-\lambda}\cdot s, s\right) = \frac{-s^2 + (\mu-\lambda)(a-b)s}{(\mu-\lambda)^2 b} - c - f$$

The maximum is $\pi_2^* = \mu(a-b-\mu b)-c-f$ when $p^* = \lambda(a-b-\mu b)$ and $s^* = (\mu-\lambda)(a-b-\mu b)$.

(3) In Ω_3, the firm's problem becomes:

$$\begin{aligned}\max_{p\geq 0} \quad & 1\cdot(p-c) \\ \text{s.t.} \quad & s \geq (\mu-\lambda)(a-b-\lambda b) \\ & 0 \leq p < \lambda(a-b-\lambda b)\end{aligned}$$

Similar to region Ω_1, since $p < \lambda(a-b-\lambda b)$, the objective function is always less than $\lambda(a-b-\lambda b)-c$. And the value is attained when $p = \lambda(a-b-\lambda b)$, which belongs to the region Ω_4. Notice farmers are indifferent on the boundary. The objective function in region Ω_4 on the boundary $p + s = \mu(a - b - \mu b)$ is the same as that in region Ω_3. Therefore, we can exclude region Ω_3 as the possible region where the global optimal solution lies. Because it is always dominated by the region Ω_4.

(4) In Ω_4, following the similar argument in case (2), we discuss the scenario on the boundary $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$. By Proposition 1, farmers' three possible strategies perhaps coexist. we have

$$\frac{s}{\mu - \lambda} = \frac{p}{\lambda} = a - b \cdot (1 + \lambda \cdot n_P + \mu \cdot n_{P+S}),$$

(a) If $n_{P+S} = 0$, then no service should be provided by the firm. The objective function is the same as that in the inner area of Ω_4.

(b) If $n_{P+S} > 0$, then the firm provides service hence incurs the fixed cost f no matter how many farmers purchase it. The objective

$$\begin{aligned}\pi(p, s) =& (1 - n_N) \cdot (p - c) + (1 - n_N - n_P) \cdot s - f \\ \leq& 1 \cdot (p - c) + (1 - n_P) \cdot s - f\end{aligned}$$

The equality is attained if and only if $n_N = 0$. We refine the firm considers maximizing its profit for the best possible case. So we exclude the cases when $n_N > 0$ on the boundary $\frac{s}{p} = \frac{\mu-\lambda}{\lambda}$. By $n_P + n_{P+S} = 1$ we have $n_{P+S} = \frac{(\mu-\lambda)(a-b-\lambda b)-s}{(\mu-\lambda)^2 b}$. Hence, the firm's objective becomes $1\cdot(p-c)+\frac{(\mu-\lambda)(a-b-\lambda b)-s}{(\mu-\lambda)^2 b}\cdot s-f$, which is the same as that in the region Ω_2. In addition, the constraints $\lambda(a - b - \lambda b) \leq p < \lambda(a - b)$ in Ω_4 indicates the it is always dominated by the region Ω_2. We exclude the case when $n_{P+S} > 0$.

Hence, the firm's problem becomes:

$$\begin{aligned}\max_{p\geq 0} \quad & \frac{\lambda(a - b) - p}{\lambda^2 b} \cdot (p - c) \\ \text{s.t.} \quad & \lambda(a - b - \lambda b) \leq p < \lambda(a - b) \\ & \frac{s}{p} \geq \frac{\mu - \lambda}{\lambda}\end{aligned}$$

It is straightforward to obtain

$$\begin{cases} \pi_3^* = \lambda(a - b - \lambda b) - c & \text{if } 0 \leq c < \lambda(a - b - 2\lambda b) \\ \pi_4^* = \dfrac{(\lambda(a - b) - c)^2}{4\lambda^2 b} & \text{if } \lambda(a - b - 2\lambda b) \leq c < \lambda(a - b) \end{cases}$$

and the corresponding pricing decision is $p^* = \lambda(a-b-\lambda b)$ and $p^* = \frac{\lambda(a-b)+c}{2}$ respectively.

(5) In Ω_5, the firm's problem becomes:

$$\begin{aligned}
\max_{p,s\geq 0} \quad & \frac{\mu(a-b)-(p+s)}{\mu^2 b}\cdot(p-c)+\frac{\mu(a-b)-(p+s)}{\mu^2 b}\cdot s - f \\
\text{s.t.} \quad & \mu(a-b-\mu b)\leq p+s<\mu(a-b) \\
& 0\leq \frac{s}{p}<\frac{\mu-\lambda}{\lambda}
\end{aligned}$$

It is straightforward to obtain

$$\begin{cases}
\pi_1^* = \mu(a-b-\mu b)-c-f & \text{if } 0\leq c<\mu(a-b-2\mu b) \\
\pi_5^* = \dfrac{(\mu(a-b)-c)^2}{4\mu^2 b}-f & \text{if } \mu(a-b-2\mu b)\leq c<\mu(a-b)
\end{cases}$$

and the corresponding pricing decision is $(p+s)^* = \mu(a-b-\mu b)$ and $(p+s)^* = \frac{\mu(a-b)+c}{2}$ respectively.

(6) In Ω_6, the firm's optimal profit is 0. Since the prices are too high, no farmers make any purchasing decisions.

To compare the above 5 local optima, firstly notice that $\pi_1^* = \pi_2^*$. And $\pi_5^* \geq \pi_1^*$ always holds, but π_5^* is feasible if and only if $\mu(a-b-2\mu b)\leq c<\mu(a-b)$. Similarly, $\pi_4^* \geq \pi_3^*$ always holds, but π_4^* is feasible if and only if $\lambda(a-b-2\lambda b)\leq c<\lambda(a-b)$. Notice

$$\mu(a-b-2\mu b)\geq\lambda(a-b)\Leftrightarrow a\geq\frac{2\mu^2 b}{\mu-\lambda}+b$$

which always holds if we assume the market size a is large enough. We only need to compare π_1^* vs. π_3^*, π_1^* vs. π_4^*, π_1^* vs. 0 and π_5^* vs. 0.

$$\begin{aligned}
&\pi_1^*>\pi_3^*\Leftrightarrow f<(\mu-\lambda)(a-b-\mu b-\lambda b) \\
&\pi_1^*>\pi_4^*\Leftrightarrow f<-\frac{c^2}{4\lambda^2 b}+\frac{(a-b-2\lambda b)c}{2\lambda b}-\frac{(a-b-2\mu b)^2}{4b} \\
&\pi_1^*>0\Leftrightarrow c+f<\mu(a-b-\mu b) \\
&\pi_5^*>0\Leftrightarrow f<\frac{(\mu(a-b)-c)^2}{4\mu^2 b}
\end{aligned}$$

Therefore, $\pi_1^* = \mu(a-b-\mu b) - c - f$ is the global optimal solution when the cost parameters fall into the cost region

$$\Lambda_1 : 0 \le c < \lambda(a-b-2\lambda b) \text{ and } 0 \le f < (\mu-\lambda)(a-b-\mu b-\lambda b),$$
$$\lambda(a-b-2\lambda b) \le c < \lambda(a-b) \text{ and } 0 \le f < -\frac{c^2}{4\lambda^2 b} + \frac{(a-b-2\lambda b)c}{2\lambda b} - \frac{(a-b-2\mu b)^2}{4b}$$
$$\lambda(a-b) \le c < \mu(a-b-2\mu b) \text{ and } 0 \le c + f < \mu(a-b-\mu b)$$

$\pi_3^* = \lambda(a-b-\lambda b) - c$ is the global optimal solution when the cost parameters fall into the cost region

$$\Lambda_2 : 0 \le c < \lambda(a-b-2\lambda b) \text{ and } f \ge (\mu-\lambda)(a-b-\mu b-\lambda b)$$

$\pi_4^* = \frac{(\lambda(a-b)-c)^2}{4\lambda^2 b}$ is the global optimal solution when the cost parameters fall into the cost region

$$\Lambda_3 : \lambda(a-b-2\lambda b) \le c < \lambda(a-b) \text{ and } f \ge -\frac{c^2}{4\lambda^2 b} + \frac{(a-b-2\lambda b)c}{2\lambda b} - \frac{(a-b-2\mu b)^2}{4b}$$

$\pi_5^* = \frac{(\mu(a-b)-c)^2}{4\mu^2 b} - f$ is the global optimal solution when the cost parameters fall into the cost region

$$\Lambda_4 : \mu(a-b-2\mu b) \le c < \mu(a-b) \text{ and } 0 \le f < \frac{(\mu(a-b)-c)^2}{4\mu^2 b}$$

Otherwise, the firm makes zero profit.

It shows Λ_1-Λ_5 that characterize the firm's optimal decisions based on unit production cost and fixed cost of maintaining a service team.

In addition, define farmers' welfare as $\sum_i u_i$, then we have

Region	n_N^*	n_P^*	n_{P+S}^*	Firm's Profit	Farmers' Welfare	q^*	r^*
Λ_1	0	0	1	$\mu k_\mu - c - f$	k_μ	$1 + \mu$	k_μ
Λ_2	0	1	0	$\lambda k_\lambda - c$	k_λ	$1 + \lambda$	k_λ
Λ_3	$1 - \frac{\lambda(a-b)-c}{2\lambda^2 b}$	$\frac{\lambda(a-b)-c}{2\lambda^2 b}$	0	l_λ	$\frac{\lambda(a-b)+c}{2\lambda}$	$\frac{\lambda(a+b)-c}{2\lambda b}$	$\frac{\lambda(a-b)+c}{2\lambda}$
Λ_4	$1 - \frac{\mu(a-b)-c}{2\mu^2 b}$	0	$\frac{\mu(a-b)-c}{2\mu^2 b}$	$l_\mu - f$	$\frac{\mu(a-b)+c}{2\mu}$	$\frac{\mu(a+b)-c}{2\mu b}$	$\frac{\mu(a-b)+c}{2\mu}$
Λ_5	1	0	0	0	$a - b$	1	$a - b$

where

$$-\frac{c^2}{4\lambda^2 b} + \frac{(a-b-2\lambda b)c}{2\lambda b} - \frac{(a-b-2\mu b)^2}{4b} = -\frac{(\lambda(a-b)-c)^2}{4\lambda^2 b} - c + \mu(a-b-\mu b) = \mu k_\mu - l_\lambda - c$$

Apparently, the fixed cost of maintaining a service team f has no impact on farmers' welfare. But farmers' welfare increases as the unit production cost c increases. Because high unit production cost induces the firm to set a high selling price of the agri-tech product. Thus fewer farmers tend to purchase the product, which leads to a mild competition in the crop market. The market price of the crop is high. The positive effect of high market price dominates the negative effect of low total output quantity. Therefore, the farmers' welfare increases.

There are other ways to partition the parameter regions that characterize the firm's optimal decisions. If we base on the market size and competition intensity, then

Region	Characteristic	Farmers' Welfare
Λ_1	$(1+2\mu)b+\frac{c}{\mu} \le a < b+\frac{c}{\lambda}$, $a \ge (1+\lambda+\mu)\,b+\frac{f}{\mu-\lambda}$ and $\lambda^2a^2-2\lambda(2\lambda\mu b+\lambda b+c)a+(4\mu^2+4\mu+1)\lambda^2b^2+(4\lambda c+4\lambda f+2c)\lambda b+c^2<0$	k_μ
Λ_2	$(1+2\lambda)b+\frac{c}{\lambda} \le a < (1+\lambda+\mu)\,b+\frac{f}{\mu-\lambda}$	k_λ
Λ_3	$b+\frac{c}{\mu} \le a < \min\{b+\frac{c}{\lambda},(1+2\mu)b+\frac{c}{\mu}\}$, $\max\{b+\frac{c}{\lambda},(1+2\mu)b+\frac{c}{\mu}\}a < (1+2\lambda)b+\frac{c}{\lambda}$ and $\lambda^2a^2-2\lambda(2\lambda\mu b+\lambda b+c)a+(4\mu^2+4\mu+1)\lambda^2b^2+(4\lambda c+4\lambda f+2c)\lambda b+c^2\ge 0$	$\frac{\lambda(a-b)+c}{2\lambda}$
Λ_4	$b+\frac{c}{\lambda} \le a < \min\{(1+2\lambda)b+\frac{c}{\lambda},(1+2\mu)b+\frac{c}{\mu}\}$	$\frac{\mu(a-b)+c}{2\mu}$
Λ_5	Otherwise	$a-b$

If we base on farmers' capability and profession's capability to utilize the agri-tech product, then

Region	Characteristic	Farmers' Welfare
Λ_1	$\mu_3 \le \mu < \mu_4$ and $0 \le \lambda < \max\left\{\lambda_2, \frac{a-b-\sqrt{k_{2\mu}^2+4bf}}{2b}\right\}$	k_μ
Λ_2	$0 \le \mu < \mu_3$ and $\lambda_2 \le \lambda < \lambda_3$, $\mu_3 \le \mu < \mu_4$ and $\lambda \ge \max\left\{\lambda_2, \frac{a-b-\sqrt{k_{2\mu}^2+4bf}}{2b}\right\}$, $\mu \ge \mu_4$ and $\lambda_2 \le \lambda < \lambda_3$	k_λ
Λ_3	$0 \le \mu < \mu_3$ and $\lambda_1 \le \lambda < \lambda_2$, $\mu \ge \mu_4$ and $\lambda_1 \le \lambda < \lambda_2$, $\lambda \ge \lambda_3$	$\frac{\lambda(a-b)+c}{2\lambda}$
Λ_4	$\mu_1 \le \mu < \mu_2$ and $0 \le \lambda < \lambda_1$	$\frac{\mu(a-b)+c}{2\mu}$
Λ_5	Otherwise	$a-b$

where

$$
\begin{aligned}
\lambda_1 = \mu_1 &= \frac{c}{a-b} \\
\lambda_2 = \mu_2 &= \frac{a-b-\sqrt{(a-b)^2-8bc}}{4b} \\
\lambda_3 = \mu_4 &= \frac{a-b+\sqrt{(a-b)^2-8bc}}{4b} \\
\mu_3 &= \frac{a-b-\sqrt{(a-b)^2-4b(c+f)}}{2b}
\end{aligned}
$$