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Radiometer Science

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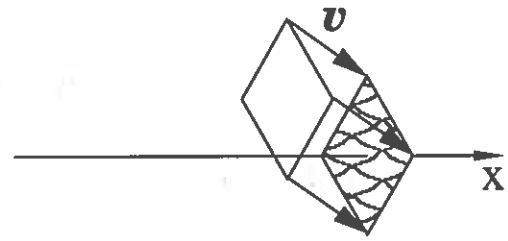
A Chem120 view of the Radiometer

A. The Momentum Flux in an Ideal Gas



Imagine a vane of area $\mathcal{A}$ perpendicular to the x-axis located in an ideal gas. In a time τ, all molecules in the volume $\tau\, v_x \times \mathcal{A}$ moving with $v_x > 0$ will hit the vane. If the vane stops the molecules, each molecule will transfer a momentum $m\, v_x$ to the vane.

The density $\rho = N/V$ atoms per unit volume in the gas is the same at all $\boldsymbol{r}$ in the gas so the number of collisions will be

$$\rho\, \tau\, v_x \times \mathcal{A}$$

and the transferred momentum will be

$$P_x = \rho\, \tau\, m\, v_x^2\, \mathcal{A}. \tag{1}$$

Well, this would be the case if all the molecules were moving with the same velocity. They aren't, of course. Maxwell first gave an argument that the distribution on v_x values would be *normal*. That is, the distribution on v_x would have the form

$$f(v_x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-v_x^2/2\sigma^2}.$$

The definition of the absolute thermodynamic temperature T and Boltzmann's constant k_B make

$$\sigma^2 = \frac{k_B T}{m}$$

If we use this $f(v_x)$, we can *average* (1) and get

$$\begin{aligned}\bar{P}_x =& \rho\, \tau\, m\, \mathcal{A} \int_0^\infty v_x^2\, f(v_x)\, dv_x = \frac{1}{2}\rho\, \tau\, m\, \mathcal{A} \int_{-\infty}^{\infty} v_x^2\, f(v_x)\, dv_x \\ =& \frac{1}{2}\rho\, \tau\, m\, \mathcal{A}\, \sigma^2 = \frac{1}{2}\rho\, \tau\, k_B T\, \mathcal{A}\end{aligned} \tag{2}$$

B. Application to an elastic wall

If, rather than being stopped by the vane, the molecules bounce back perfectly, then the momentum transferred to the vane would be twice the result (2),

$$\Delta \text{ momentum}_x = \rho \, \tau \, A \, k_B T$$

The force on the vane will just be the rate of transfer of momentum,

$$F_{vane} = \rho \, A \, k_B T$$

The pressure on the wall is just the force per unit area,

$$p = \rho \, k_B T$$

[i.e., $p\,V = N\,k_B T$!]

B. Application to a real wall

We suppose that the wall itself has a *temperature* T_{wall}. If $T_{wall} = T$ of the gas, the *principle*† *of detailed balance occuring* says that the flux of the molecules that bounce back must balance that given in (2).

C. If the vane temperature is different from the gas temperature

In this case, there will be a momentum

$$\bar{P}_x = \frac{1}{2}\rho \, \tau \, k_B T \, A$$

transferred from the gas to the vane and a momentum

$$\bar{P}'_x = -\frac{1}{2}\rho \, \tau \, k_B T_{vane} \, A$$

lost to the bounced molecules. Thus

$$\Delta \text{ momentum}_x = \rho \, \tau \, A \, k_B (T - T_{vane})$$

† For every process occuring in an equilibrium system, there must be a corresponding *mechanically reversed* process.

and force on the vane will be

$$F_{vane} = \rho \mathcal{A}\, k_B (T - T_{vane}). \tag{3}$$

There is a contribution like (3) to each side of a vane, of course (with opposite signes). The resulting force drives the motion seen in the radiometer.