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# Radiometer Science

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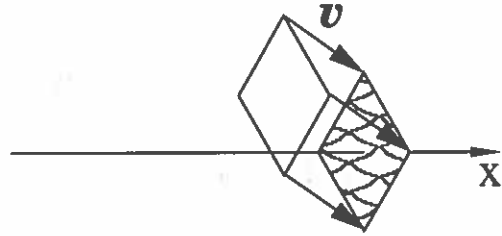
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## A Chem120 view of the Radiometer

### A. The Momentum Flux in an Ideal Gas

Imagine a vane of area  $\mathcal{A}$  perpendicular to the  $x$ -axis located in an ideal gas. In a time  $\tau$ , all molecules in the volume  $\tau v_x \times \mathcal{A}$  moving with  $v_x > 0$  will hit the vane. If the vane stops the molecules, each molecule will transfer a momentum  $m v_x$  to the vane.



The density  $\rho = N/V$  atoms per unit volume in the gas is the same at all  $r$  in the gas so the number of collisions will be

$$\rho \tau v_x \times \mathcal{A}$$

and the transferred momentum will be

$$P_x = \rho \tau m v_x^2 \mathcal{A}. \quad (1)$$

Well, this would be the case if all the molecules were moving with the same velocity. They aren't, of course. Maxwell first gave an argument that the distribution on  $v_x$  values would be *normal*. That is, the distribution on  $v_x$  would have the form

$$f(v_x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-v_x^2/2\sigma^2}.$$

The definition of the absolute thermodynamic temperature  $T$  and Boltzmann's constant  $k_B$  make

$$\sigma^2 = \frac{k_B T}{m}$$

If we use this  $f(v_x)$ , we can *average* (1) and get

$$\begin{aligned} \bar{P}_x &= \rho \tau m \mathcal{A} \int_0^{\infty} v_x^2 f(v_x) dv_x = \frac{1}{2} \rho \tau m \mathcal{A} \int_{-\infty}^{\infty} v_x^2 f(v_x) dv_x \\ &= \frac{1}{2} \rho \tau m \mathcal{A} \sigma^2 = \frac{1}{2} \rho \tau k_B T \mathcal{A} \end{aligned} \quad (2)$$

**B. Application to an elastic wall**

If, rather than being stopped by the vane, the molecules bounce back perfectly, then the momentum transferred to the vane would be twice the result (2),

$$\Delta \text{ momentum}_x = \rho \tau \mathcal{A} k_B T$$

The force on the vane will just be the rate of transfer of momentum,

$$F_{\text{vane}} = \rho \mathcal{A} k_B T$$

The pressure on the wall is just the force per unit area,

$$p = \rho k_B T$$

[i.e.,  $pV = N k_B T$ !]

**B. Application to a real wall**

We suppose that the wall itself has a *temperature*  $T_{\text{wall}}$ . If  $T_{\text{wall}} = T$  of the gas, the *principle† of detailed balance occurring* says that the flux of the molecules that bounce back must balance that given in (2).

**C. If the vane temperature is different from the gas temperature**

In this case, there will be a momentum

$$\bar{P}_x = \frac{1}{2} \rho \tau k_B T \mathcal{A}$$

transferred from the gas to the vane and a momentum

$$\bar{P}'_x = -\frac{1}{2} \rho \tau k_B T_{\text{vane}} \mathcal{A}$$

lost to the bounced molecules. Thus

$$\Delta \text{ momentum}_x = \rho \tau \mathcal{A} k_B (T - T_{\text{vane}})$$

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† For every process occurring in an equilibrium system, there must be a corresponding *mechanically reversed* process.

and force on the vane will be

$$F_{vane} = \rho \mathcal{A} k_B (T - T_{vane}). \quad (3)$$

There is a contribution like (3) to each side of a vane, of course (with opposite signs). The resulting force drives the motion seen in the radiometer.

