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Turbofan Engine Bypass Ratio as a Function of Thrust and Fuel Flow

Compiled by Andrew Dankanich
March 29th 2017

Spring 2017 MEMS 500 Independent Study at Washington University in St. Louis

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Acronyms and Engine Station Definitions

BPR	Bypass Ratio
TSFC	(Thrust) Specific Fuel Consumption
$\dot{m}_0$	Mass Flow Rate – Total
$\dot{m}_c$	Mass Flow Rate – Core
$\dot{m}_f$	Mass Flow Rate - Fuel
Pt3	Compressor Exit Pressure
Pt4	Turbine Entrance Pressure
Pt5	Turbine Exit Pressure
Tt4	Turbine Entrance Temperature
Tt5	Turbine Exit Temperature
Tt9	Core Exit Temperature
τ_λ	Thermal Limit Parameter
V9	Core Exit Velocity
V19	Fan Exit Velocity
M9	Core Exit Mach Number
M19	Fan Exit Mach Number

Abstract

Modern Turbofan engines can deliver high thrust without the high fuel consumption as compared to a turbojet engine. By trading the energy in the high velocity exhaust stream for power to drive a fan, the turbofan engine can process large amounts of air which yields a higher thrust per amount of fuel used. The amount of fuel used per thrust is called thrust specific fuel consumption (TSFC). A numerically lower value of TSFC is indicative that the engine uses less fuel to produce a given amount of thrust.

As bypass ratio (BPR) increases the overall efficiency of the engine increase which is a primary factor that yields lower TSFC for the turbofan engine. Additionally this report shows that a high bypass ratio engine can produce a greater amount of thrust while consuming the same amount of fuel as a lower BPR engine. Because of this, commercial airlines rely on the turbofan engine to deliver high efficiency and high thrust to carry people across the globe.

If a higher bypass ratio engine produces more thrust while consuming less fuel, why not stuff the largest diameter engine available on a plane and go flying? Several factors play into the sizing of a turbofan engine. Air flow distortion, weight, physical envelope, thrust output, fuel consumption and cost are some of the primary design impacts that the propulsion system can effect. The larger the engine the more thrust is generated as well as the more fuel is consumed though at lower TSFC. If the engine is too large, excess power becomes useless and wasteful for the aircraft. Plus, much like any other product, a more complex system can come with a higher price tag. A careful balance of these design parameters gives the commercial aircraft the best efficiency for the propulsion system in turn saving the airlines money.

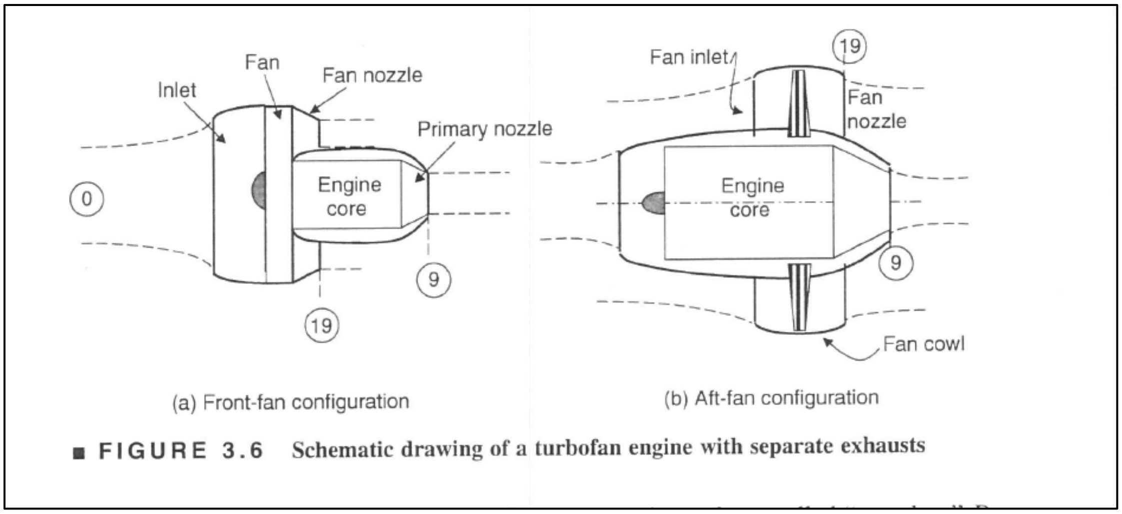
Introduction

TSFC can be used to “rank” the engine fuel efficiency and aide in the engine selection processes as an aircraft power plant. A large part of the cost of operating an airline is fuel; hence the desire for operators looking to turn a profit flying cargo or passengers to minimize this cost. According to airnav.com, Jet-A fuel costs were around $4.11 on September 22nd 2016 at the St. Louis Lambert airport.

Fuel usage is one of the largest factors in the cost of operating a commercial aircraft. The cost of fuel is based on a variety of economic, political and some technical factors, none of which are a primary topic at hand. However, the amount of fuel consumed to power the commercial aircraft can be diagnosed from solely a technical basis. It is from this angle that the primary variables that influence fuel costs for a commercial aircraft will be related and modeled.

Howstuffworks.com tells us that a Boeing 747 airplane can consume 1 gallon of fuel per second. Considering a 5 hour flight, that Boeing 747 can consume almost 20,000 gallons of fuel at $4.11 a gallon. It is easy to see how minimizing fuel consumption (minimizing TSFC) can have a significant impact on the costs of flying an airplane. The high bypass ratio turbofan engines are designed to take advantage of the conservation of momentum and produce high thrust at lower fuel consumption. So which bypass ratio produces the best fuel efficiency, and is there a limit to this variable?

Typical low bypass ratio engines have 1 or 2 stages of blades in the turbine, which are used to extract power to drive the compressor and the fan. High bypass turbofan engines have multiple turbine stages which work to extract more power to drive the large diameter fans. Figure 1 shows a general turbo fan engine that is applicable to this analysis and is sourced from Saeed Farokhi, Aircraft Propulsion (ref 3).



■ FIGURE 3.6 Schematic drawing of a turbofan engine with separate exhausts

Figure 1 Depiction of the type of turbofan engine used in this analysis. Sourced from Farokhi, Aircraft Propulsion

The primary goal of this analysis is to introduce the trends of a turbine engine when compared across an increasing BPR. The trends that are realized have an impact on fuel consumption and ultimately how much money is spent on fuel. Designing an aircraft is a balance between weight, thrust and drag. The propulsion system is a key component that touches all areas of aircraft design. As is a goal with any business, the airline companies strive to reduce cost wherever possible. Utilizing a high bypass ratio turbofan engine is one method to reduce fuel cost.

Methodology

Two excellent propulsion references were used to gather the equations pertinent to this analysis. Reference (3) and (4) yielded the primary equations necessary to execute the analysis. Before the equations were used, it was necessary to establish the initial conditions and methodology for how the analysis would be conducted. The general principles of how a turbojet and turbofan engine operates were known prior to establishing the methodology, but properly setting up any analysis is critical.

Two methods were identified as possible paths. In both methods free stream and altitude conditions were set. The first method, referred to as fixed core, was to establish an engine with a given core mass flow rate and fuel flow rate. The BPR would vary from 0 to 12 which would establish both the fan and the total mass flow rates for the engine. The concept was to continually put a larger fan on the front of a turbojet engine and observe the performance. The compressor performance was calculated along with burner and turbine properties. As the BPR increases, the engine produces more thrust with the same fuel flow. Because of the increasing thrust level, the initial thrust value (thrust produced by the BPR = 0 engine, or turbojet) is used as a baseline to calculate fuel savings.

A second method, named thrust convergence, was investigated in order to limit the engine performance and remove the need to baseline the thrust level when calculating fuel savings, as is done in the fixed core method. In this case, the BPR is still varied but the thrust level becomes constant. To do this, a second layer of criteria was added into the computer routine which widely varies the airflow to the engine. Once the desired thrust level is achieved for the given BPR, the performance parameters are logged. It was later realized with method two that both the core airflow and the total airflow could be varied and it achieves the same result. It was also determined that the results matched method one, solidifying the overall approach in both methods.

The fuel savings results of both methods (fixed core and thrust convergence) are the same since TSFC is a value based on thrust (units are kg/sec/N or lb/lbf/hr) and in both methods the turbine temperature limit is the same. Regardless of the fact the core airflow is not fixed in method 2, the performance equations provide the same results because the compressor entrance conditions, compression ratio and turbine temperature are consistent. This became even more evident as the performance of each component was calculated.

Computing the compressor properties of the engine seemed straight forward. The analysis would utilize isentropic relationships with set efficiency factors; however, a critical method to setting the compressor performance was realized in Farokhi page 161 (ref 3). In this case, Eq (1) is used to set the compressor compression ratio, π_c, based on the free stream Mach number and the thermal limit parameter (see section VII on Turbine Temperature Limit) which also became a defined input.

$$\pi_c = \left[\frac{\sqrt{\tau_\lambda}}{1 + \frac{\gamma - 1}{2} {M_0}^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{1}$$

Utilizing this equation, the optimal compressor pressure ratio could be used to calculate maximum engine thrust at altitude. The original compressor pressure ratio was set at 40 which is close to the maximum operation of turbofan engines, however this ratio may not represent an optimum condition for an engine operating at altitude.

Since the fuel flow rate and engine performance are constant, it makes sense that T_{t4} and the fuel to air ratio for the core remain constant as well. The results of the analysis support this and with a constant T_{t4} limit, the engine is always running at the same performance level. This way, the amount of work produced by the core remains constant but the way the energy is converted is different. As the BPR increases, more mechanical work is needed to drive the fan. So the turbine needs to extract more energy from the hot gas flow and transmit the power through the shaft to the fan. This trade off can be seen by the decreasing trend in T_{t9} and V_9. As the turbine extracts more energy from the core, temperature and velocity decrease.

To realize any fuel savings the end has to consume less fuel. As BPR increases it is shown that TSFC decreases as well. The decreasing trend in TSFC means that for a given thrust level the engine consumes less fuel. Indecently, a lower TSFC also means for a given amount of fuel the turbofan engine is producing more thrust. In the fixed core analysis, it is shown that thrust levels increase with increasing BPR; however specific thrust (thrust per airflow) decreases due to the large amount of total mass flow processed by the engine. This trend is echoed in the thrust convergence method as well.

All in all, this analysis utilizes both a fixed core performance and fixed thrust convergence all while incorporating energy conservation principles to show the various trends of increasing BPR. A list of the primary input parameters and variable output parameters are show in Table 1. Mainly, because there are no convergence criteria, the fixed core method is able to produce cleaner plots and is used primarily in the analysis. The thrust convergence method provides a slightly varied analysis of the same parameters to solidify the results.

Table 1 List of Inputs and Outputs for the MATLAB Analysis

Inputs	Outputs
$\dot{m}_f$	T_{t9}
$\dot{m}_0$	V_9
τ_λ	TSFC
BPR	Fn
η	Fuel Cost

Calculating Thrust

Embedded in the equation for thrust is the tradeoff between the two different propulsive forces to produce thrust. The primary elements in the formula to calculate thrust are the momentum terms and the pressure area term. Eq (2) is the general form for the uninstalled net thrust.

$$F = \dot{m}_{exit}V_{exit} - \dot{m}_o V_o + (P_a - P_o)A_9 \tag{2}$$

The first two terms take into account the change in moment across the engine while the last term accounts for under or over expanded flow exiting the engine.

It would not be appropriate to simply conclude that a larger diameter fan will result in a larger $\dot{m}_{exit}$ thus a larger thrust. This is because the exit velocity of the exhaust gas is affected by the change in bypass ratio. A seemingly simple increase in mass flow through the fan affects temperatures and pressures throughout the engine. An expanded form of Eq (2) is show as Eq (3) with both low and high spools accounted for in the momentum exit term.

$$\begin{aligned} F = \dot{m}_{fan}V_{exit\,fan} + \dot{m}_{core}V_{exit\,core} - V_o\left(\dot{m}_{fan} + \dot{m}_{core}\right) + \left(P_{fan} - P_o\right)A_{fan} \\ + (P_{core} - P_o)A_{core} \end{aligned} \tag{3}$$

Here, “low spool” refers to the low pressure portion of the turbine engine that is the low pressure turbine (LPT) and the fan. Conversely, “high spool” refers to the high pressure portion of the engine that is the high pressure turbine (HPT) and the compressor. The expanded thrust formula, Eq (3), shows that both the velocity and area of the engine affect the thrust. Since the mass flow rate through the fan and the area of the fan are much larger than the core, this contribution will have a greater effect on thrust. However, the velocity of the core is greater than the fan, but because the mass flow rate is smaller, again the fan has a larger contribution to the thrust. Eq (3) is meant for use on a separate stream turbofan engine much like the one modeled in this analysis.

Fuel Flow Results

From the calculations it was shown that the turbojet engine consumed roughly 1.7 gallons of fuel per second while the BPR of 12 consumed less than a gallon a second (~0.9). In the fixed core method, the fuel flow rate is constant which shows that as the bypass ratio increases the engine is capable of producing greater thrust. If we assume the same airplane configuration, the excess thrust is not needed and the engine can be throttled back, hence consuming less fuel. This may not represent how a turbofan engine would be operated but it does solidify the fuel efficiency that can be realized through using a high bypass turbofan engine. This limitation in practical application is the primary reason the thrust convergence method was developed. By calculating engine performance around a common thrust level, it is shown that TSFC reduction is the same and hence the fuel savings are the same between methods.

30,000lbf was the baseline thrust that was established as the thrust that is generated by the engine at BPR = 0, or the turbojet engine. As was discussed in section V, with the same fuel flow, the thrust level increases with increasing BPR and each engine must be "baselined" to 30,000lbf. This process calculates a new fuel flow rate using the new, reduced, TSFC for each BPR. Since each increase in bypass ratio increased the thrust level, the TSFC subsequently decreased for each new engine. Thus the new TSFC yields a lower fuel flow rate at the baselined thrust level.

Initially, the fuel to air ratio was being calculated using equation (4):

$$f = \frac{\dot{m}_f}{\dot{m}_0} \tag{4}$$

Eq (4) is the proper equation for the overall fuel to air ratio, however, in the case of a turbofan engine with a split fan and core stream, it is important to realize that the denominator needs to be replaced with the core airflow only in order to back out a fuel flow rate. In this analysis the fuel to air ratio of the core remains the same and coupled with the thermal limit parameter, means the fuel to air ratio can be calculated using an energy balance approach across the burner. Eq (5) shows the equation used to calculate the fuel to air ratio for the core. This value also remains constant but is dependent on knowing T_{t4}.

$$f = \frac{(cp_c * Tt_4 - cp_c * Tt_3)}{(h_{pr} * \eta_b - cp_t * Tt_4)} \tag{5}$$

Eq (5) is a combination of turbine entrance properties and compressor exit properties. Since the engine is to be run on a turbine temperature limit, T_{t4} is known and the fuel to air ratio can be calculated.

With the fuel to air ratio known, the modified version of Eq (4) can be used to calculate the fuel flow rate.

$$\dot{m}_f = f * \dot{m}_{core} \tag{6}$$

However, as the problem was set up to operate on a constant fuel to air ratio and constant core air flow, the fuel flow rate is constant as well. This is where the "adjusted" fuel flow rate and the "baseline" thrust can be merged. Equation (7) shows how the adjusted fuel flow rate is calculated:

$$\dot{m}_{f\ adjusted} = TSFC * F_{Net\ BPR=0} \tag{7}$$

In Eq (7) TSFC decreases with each increase in BPR and as a result, the adjusted fuel flow rate decreases as well. From this the fuel savings can be realized and plotted against the bassline fuel flow rate. This result can be seen in Section IX. This is one of the primary difference areas of the fixed core and the thrust convergence methods. In the thrust convergence method, core flow is not constant and results in a lower fuel flow rate for each BPR increase. However, fuel to air ratio is constant since the fuel flow rate decreases with a decreasing core flow rate. This keeps TSFC the same in both methods.

Turbine Temperature Limit Effect on Engine Performance

During the initial set up of this project, Eq (8) was used to calculate the turbine exit temperature for each BPR. It was found that this method is not the most applicable for the way this problem is set up.

$$T_{t4} = \frac{1}{c_{pt}(1+f)} [c_{pc} T_{t3} + f\eta_b h_{pr}] \tag{8}$$

When Eq (8) was used to calculated turbine temperature, the thrust levels of BPR 2, 3, 4 and 5 were less than BPR = 0. This yielded a higher fuel consumption and higher TSFC and required some thinking to diagnose. The reason the thrust levels dipped is because the fuel to air ratio was being incorrectly calculated. Back in section VI, the fallacy of using Eq (4) was introduced and that the fuel to air ratio is correctly calculated using Eq (5). This allows the fuel to air ratio to be calculated on the core properties only, whereas Eq (4) is applicable to turbojet engine only. Furthermore, because the fuel to air ratio was changing, Eq (8) was also changing. The variation in the fuel to air ratio as well as T_{t4} allowed the thrust levels to dip once the engine BPR increased from 0.

This error brought to light that the core was not being run to a limit for each BPR. In Farokhi (ref 3), the principles of a turbine temperature limit and how this plays into the engine operating performance are introduced and related to the compressor pressure ratio, which was originally set at 40, the upper end of what a turbofan engine is capable of operating at.

Since the evaluation of each BPR is at a constant compressor compression ratio, the turbine temperature should also be a constant across the different engines; in other words, the various BPR's should be run to a turbine temperature limit. Running to a turbine temperature limit ensures each engine is producing max thrust, which is one of the original assumptions for the fixed core method.

Farokhi (ref 3) defines the thermal limit parameter as the following:

$$\tau_\lambda = \frac{h_{t4}}{h_0} \tag{9}$$

The thermal limit parameter is important in how the engine will operate. The higher the value of the thermal limit parameter the hotter the engine will run. For this analysis, the thermal limit parameter is set at 8 which yields a T_{t4} of around 1600 Kelvin. Farokhi (ref 3) also shows how to extract the optimum compressor pressure ratio utilizing the thermal limit parameter with Farokhi equation 2.75 or Eq (1) that was introduced in section IV.

From Eq (1) a compressor pressure ratio of about 23 is calculated based on a flight Mach of 0.88. Both equations are used in this analysis and provide important operating parameters for the engine.

The turbine entrance temperature (or burner exit temperature) is also a critical component in calculating the exit temperature of the turbine, T_{t5}. To calculate the turbine exit temperature, another energy balance is conducted with Eq (10).

$$T_{t5} = T_{t4} - \frac{c_{pc} * (T_{t3} - T_{t2}) + BPR * c_{pc} * (T_{t13} - T_{t2})}{c_{pt}(1+f) * \eta_m} \qquad (10)$$

There are a few components to Eq (10) including compressor temperatures, fan inlet temperature, BPR and fuel to air ratio. From this equation, it can be seen how a larger BPR will reduce Tt5 which aligns with the overall knowledge that the core has to extract more work to drive a larger fan. The results in Section IX support this relationship, showing a reducing turbine exit temperature.

Fuel Consumption

With the performance of the engine calculated, the specific fuel consumption can be extracted for each BPR with Eq (11). It is important to realize here that with the fixed core method, the net thrust is increasing for each engine but the fuel flow rate is constant, thus TSFC is decreasing for each engine. In the thrust convergence method, fuel flow decreases for each engine because the core airflow is allowed to decrease in order to maintain a constant net thrust. The result is TSFC decreases in the same manner for both methods.

$$TSFC = \frac{\dot{m}_f}{F_{Net}} \tag{11}$$

After TSFC is calculated for each engine, the amount of fuel savings can be calculated. An important step in the fixed core method is to re-baseline the fuel flow rate based on the initial thrust level of the turbojet engine (BPR = 0). This procedure was outline in Section VI with Eq (7). With the thrust level set, the "dollar per second" fuel consumption can be calculated with Eq (12). In the case of the thrust convergence method, the adjusted fuel flow rate is not applicable and the regular fuel flow rate is used.

$$Fuel\ Consumption\ in\ (\frac{\$}{second}) = \left(\frac{\dot{m}_{f\ adjusted}}{3.78541\ \frac{L}{Gal} 0.804 \frac{kg}{L}}\right) 4.14 \frac{\$}{Gal} \tag{12}$$

With each of these parameters defined and calculated, the analysis can be executed and the results are show in Section IX.

Results

The primary purpose of this analysis is to show the fuel savings potential as the BPR increases. As such, the results are plotted with BPR as the dependent variable. The first set of plots is shown in Figure 2 and contains the fuel to air ratio, net thrust and specific thrust. Both net thrust and specific thrust follow the anticipated trends that turbofan engines exhibit as BPR increases. The initial engine at a BPR = 0 produces 30,000 lbf of thrust while the BPR = 12 turbofan engine produces upwards of 60,000 lbf of thrust. Since the compressor performance and turbine temperature are fixed, the fuel to air ratio falls out as a constant, 0.0307.

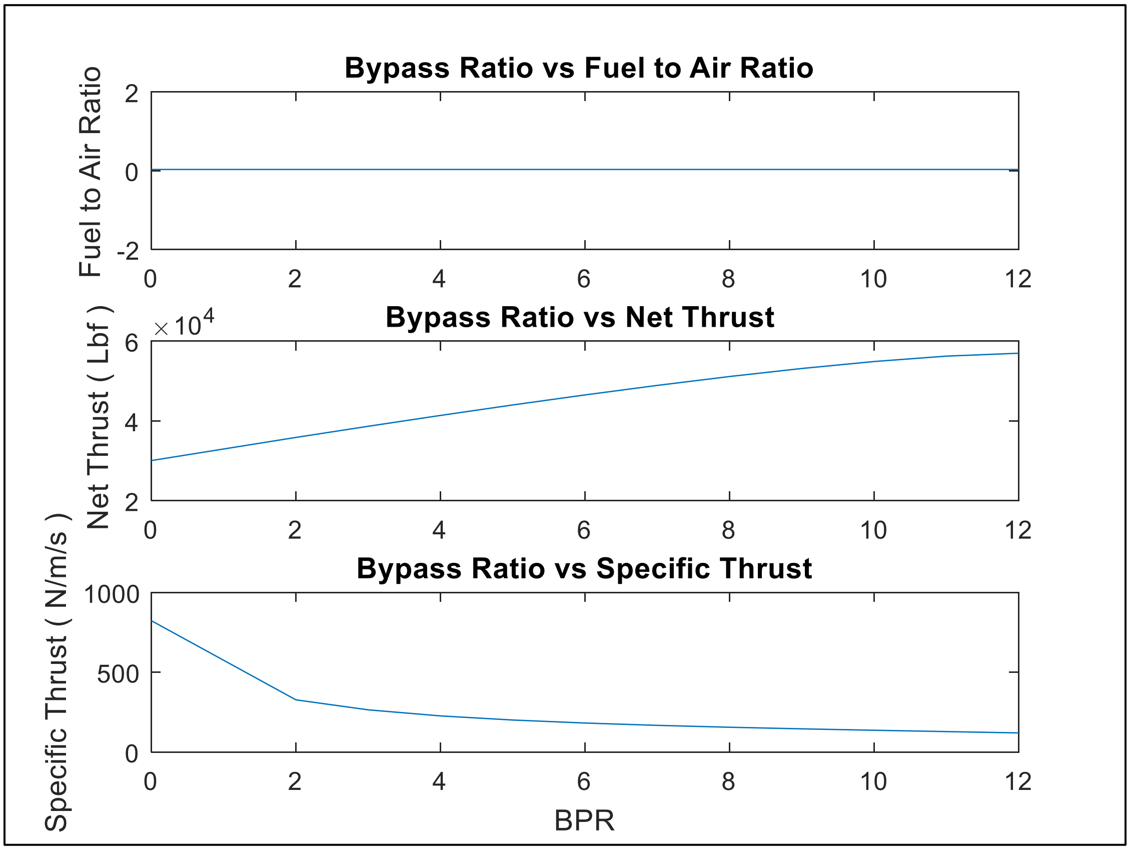


Figure 2 Fuel to Air ratio, Net thrust and Specific Thrust of the Fixed Core method

The simpler trends of the analysis are the air flow rates through the engine. As BPR increases, the overall diameter of the engine increases. A larger diameter hole can pass a higher amount of air which is shown as increasing trends in total airflow and fan flow in Figure 3.

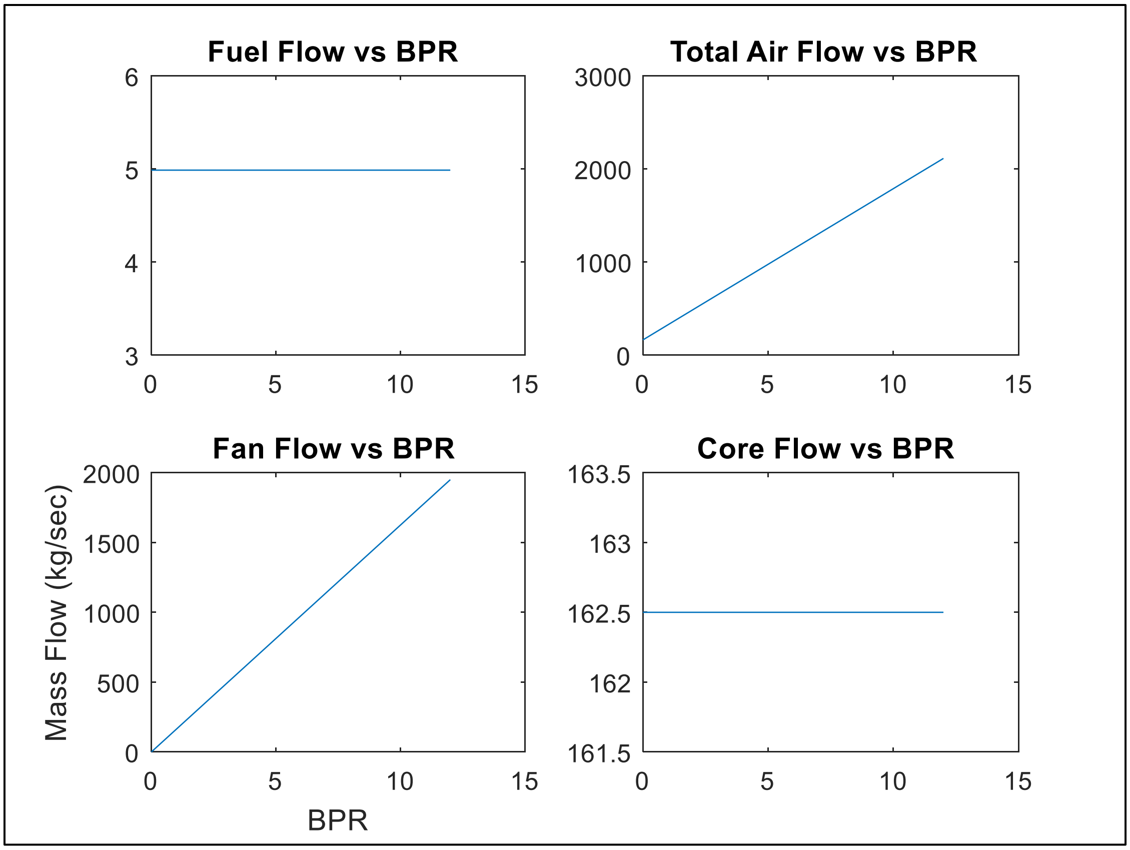


Figure 3 Mass flows for the Fixed Core method

Figure 3 also shows the results of the constant core analysis in which the air flow through the core and the fuel flow are fixed, allowing the net thrust to increase as shown in Figure 2. On the contrary, Figure 4 and Figure 5 show the trends of the thrust convergence method which modulates core flow and fuel flow to keep thrust at a constant level. The way the thrust convergence method was set up made it difficult to converge on exactly 30,000 lbf of thrust, which is why Figure 4 shows some variation in the net thrust; but these variations are on the order of single pounds of thrust.

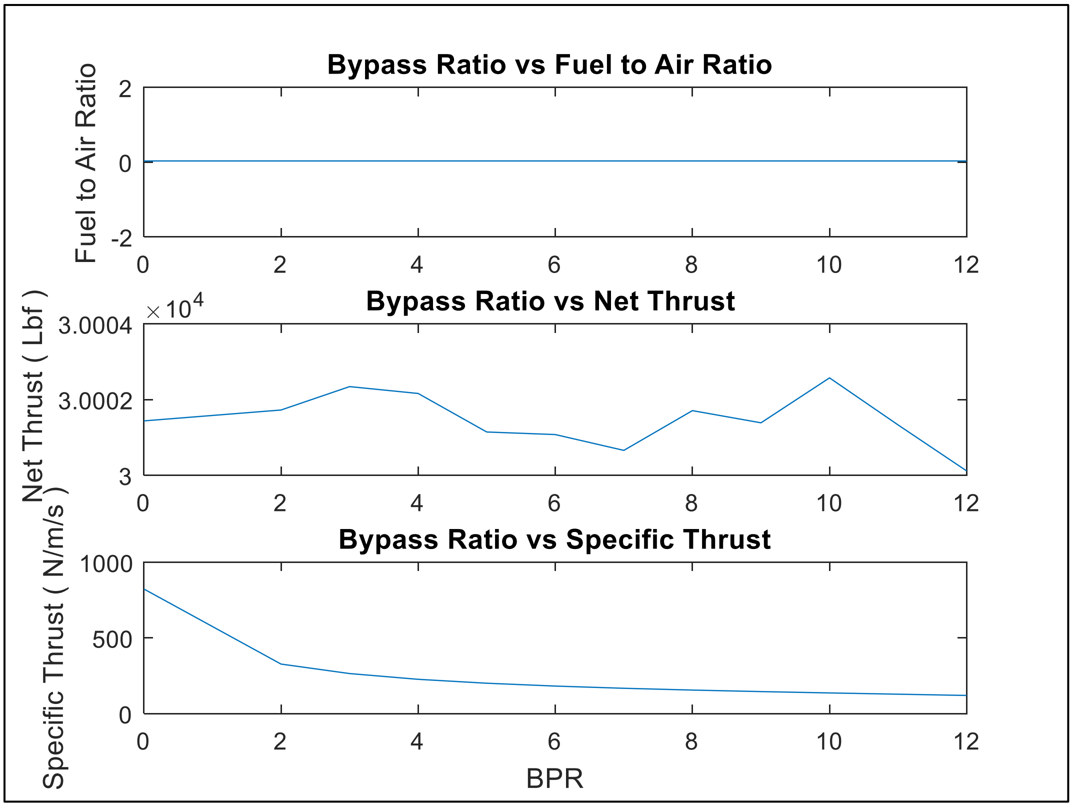


Figure 4 Fuel to Air ratio, Net thrust and Specific thrust for the Thrust Convergence Method

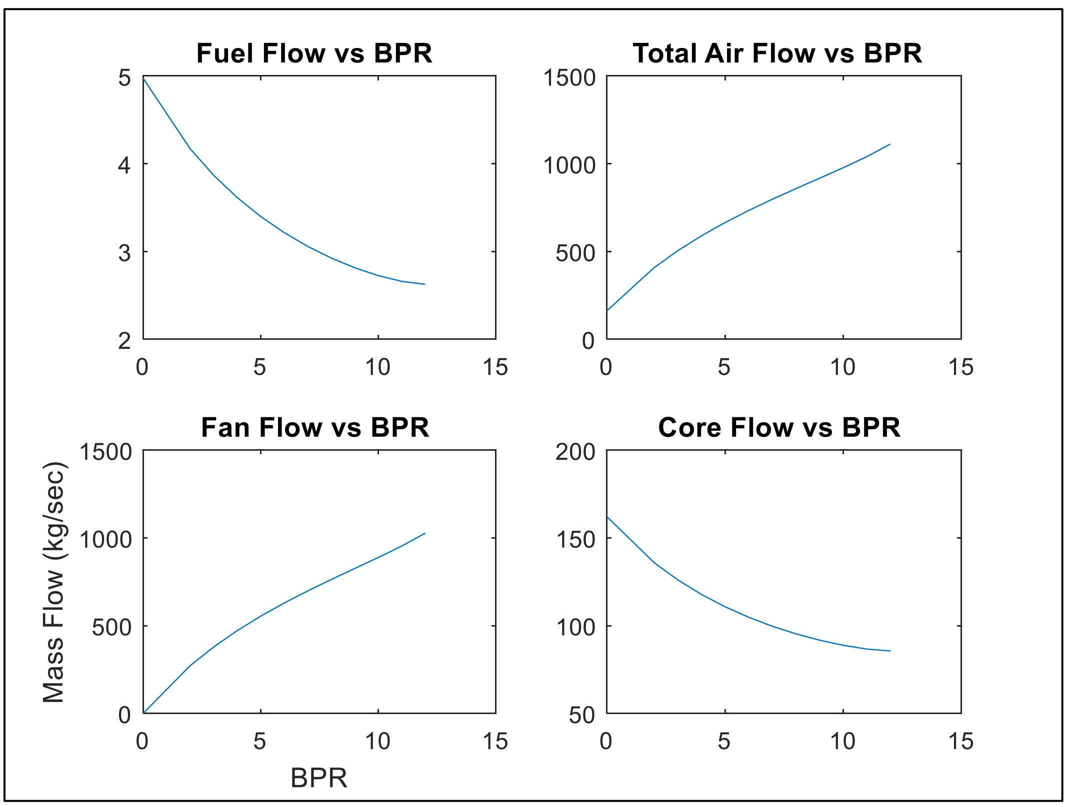


Figure 5 Mass Flow Rates for the Thrust Convergence Method

One of the primary areas where energy conservation principles are applied is the turbine. For each step up in BPR the turbine has to extract more work to drive the larger fan. In application, this can be accomplished by adding stages to the turbine to increase the work output by the low spool of the engine. This concept is represented in Figure 6 and where the exit temperature, pressure and velocity decrease as BPR increases.

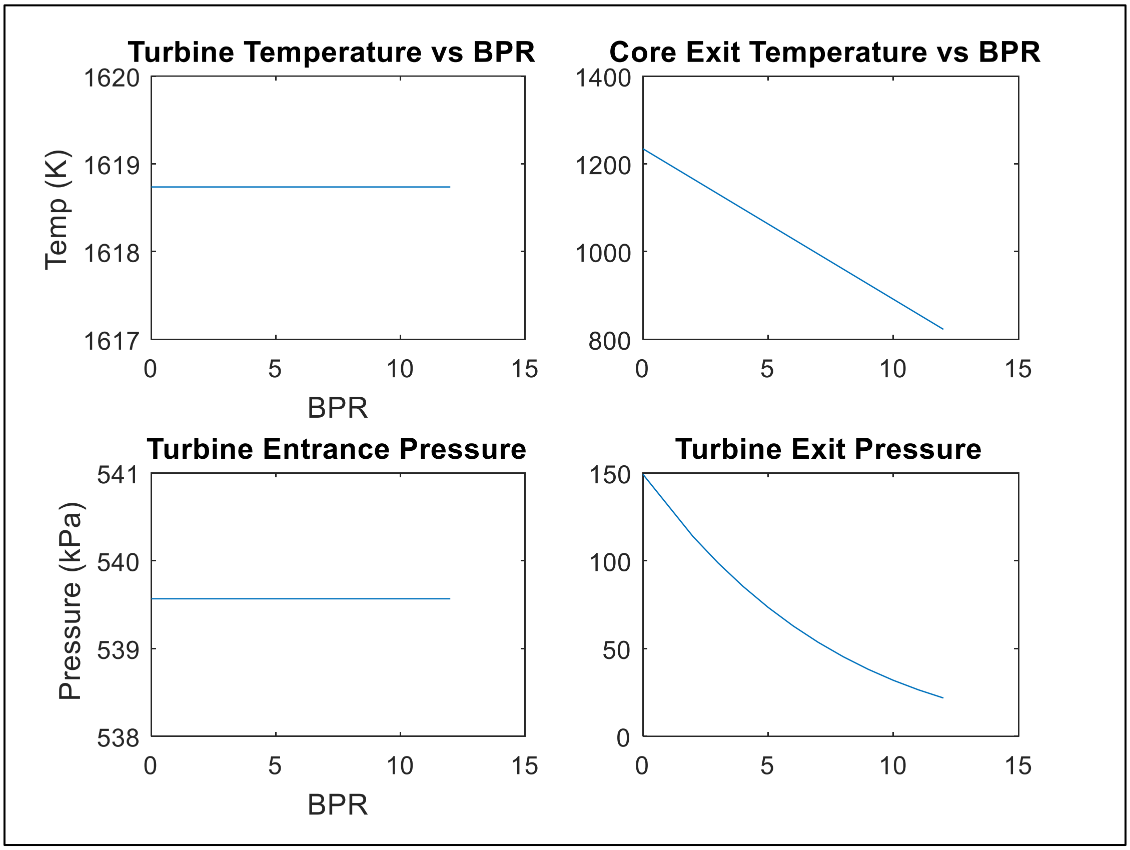


Figure 6 Tt4, Tt5, Pt4 and Pt5; Turbine Entrance and Exit.

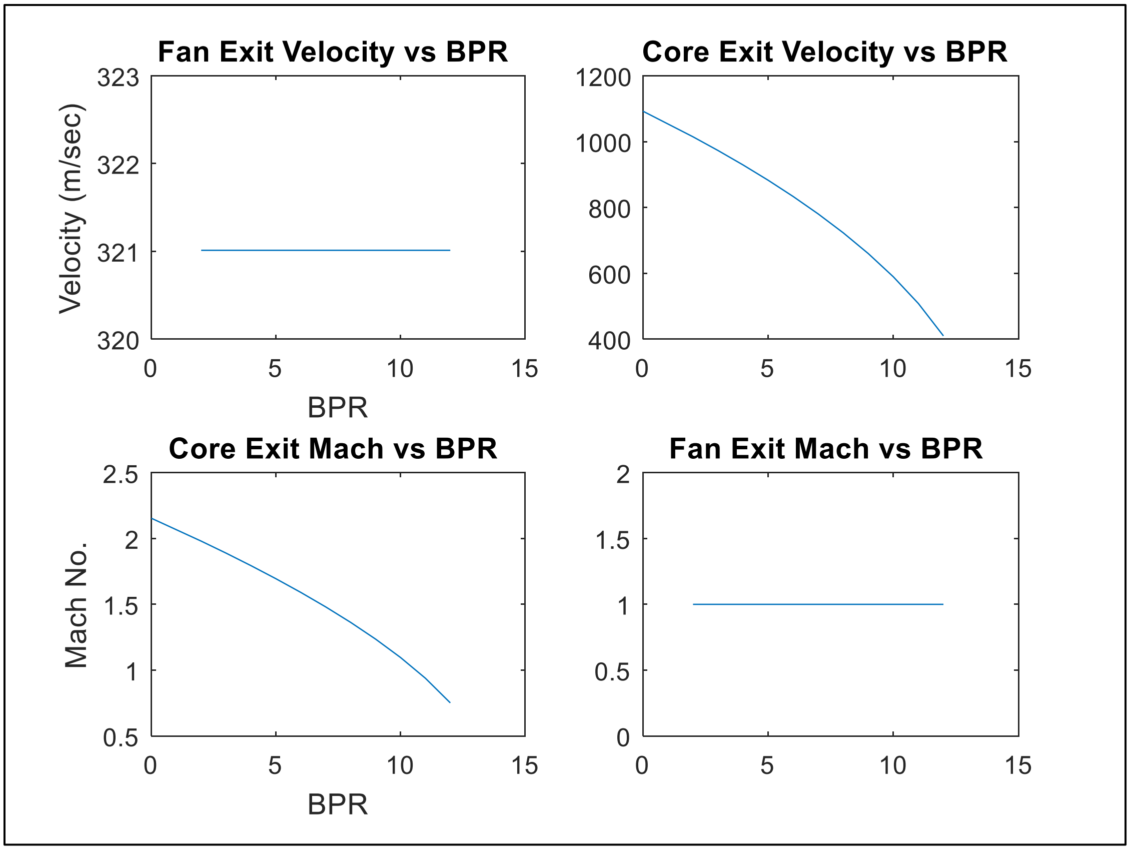


Figure 7 V9, V19, M9 and M19; Fan and Core Exit Conditions

The analysis wraps up with Figure 8 and Figure 9 which show TSFC and the fuel consumption cost for the engines. As expected, as the BPR increases the TSFC decreases. The TSFC calculated for the turbojet configuration is around 1.316, which is comparable to some industry quotes of other turbojet engines; namely the Olympus 593 (Concorde engine) which is advertised at 1.195 from reference (5). At the highest BPR analyzed, 12 (which is higher than most production turbofan engines), the TSFC drops to 0.695 which is slightly high when compared to industry turbofan engines. This could be due, in-part, to the component efficiencies that were assumed in this analysis, as well as the flight conditions. The other drivers that can affect the engine performance are the compression ratio of the engine and the turbine temperature limit.

The math shows the fuel savings that are realized with turbofan engines. As Figure 9 shows, the turbojet engine cost 6.78 $/sec which is 1.65 gallons a second at $4.11 a gallon. This is compared to 3.98 $/sec, just under a gallon per second, with the BPR = 8 (close to industry). Over a 5 hour flight that yields a fuel cost of $122,040 vs $71,640, which easily adds up over the many flights conducted by the commercial airlines.

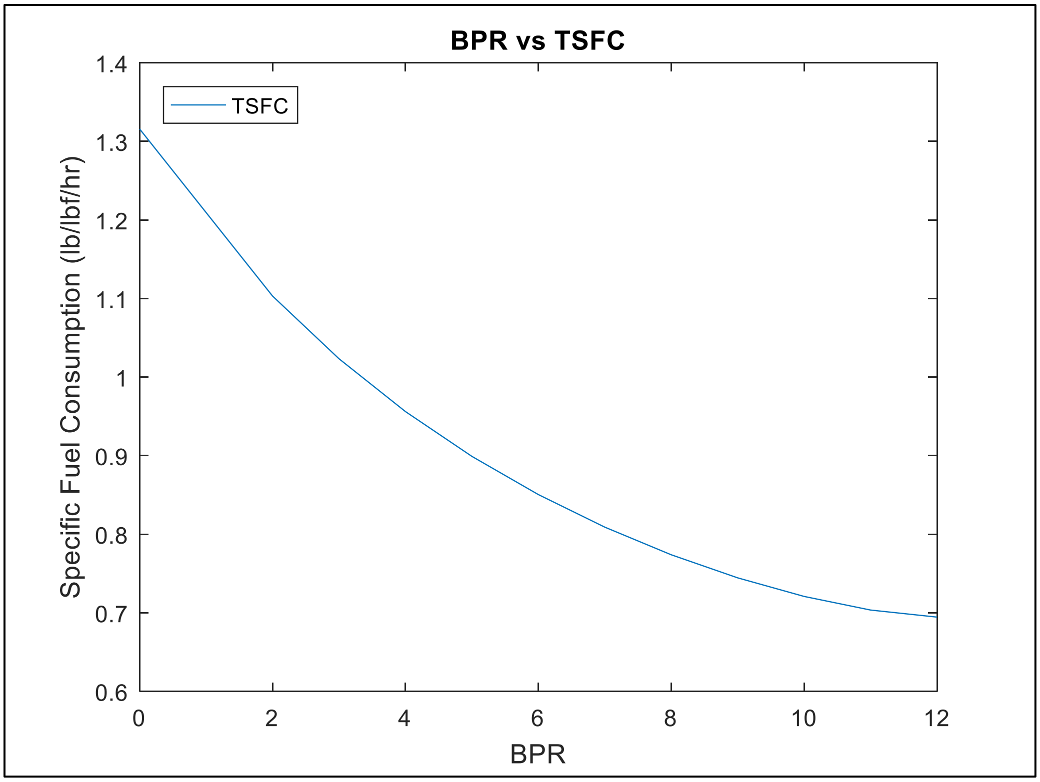


Figure 8 Thrust Specific Fuel Consumption

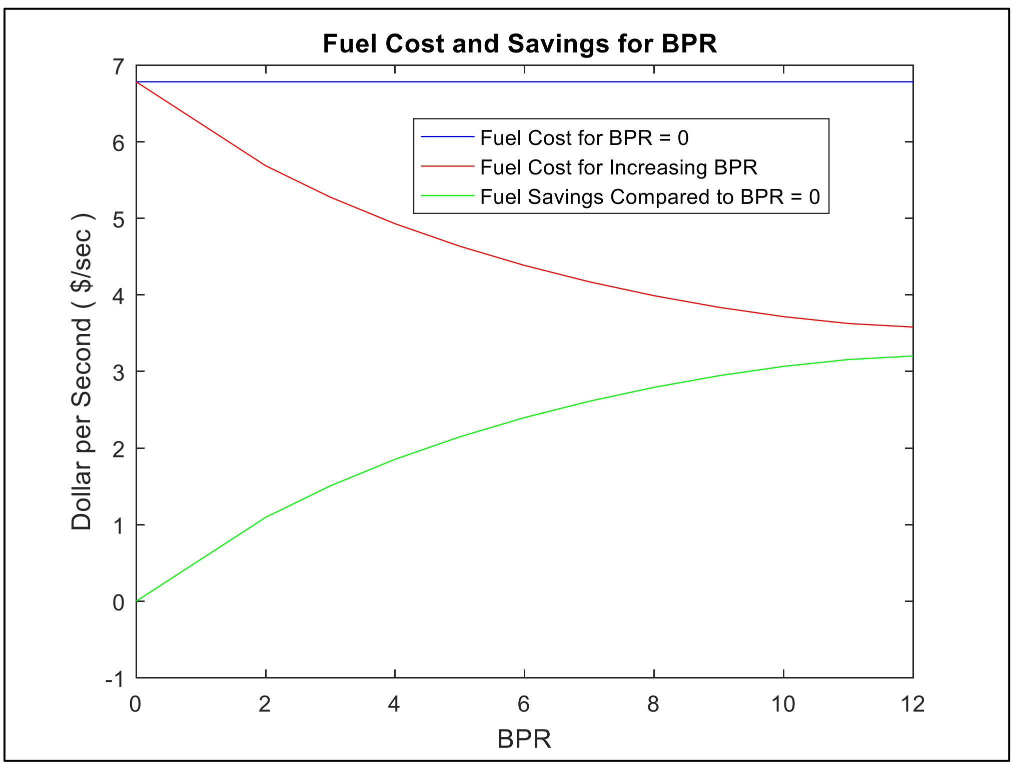


Figure 9 Fuel Cost and Savings

Conclusion

This analysis incorporated the principle equations governing the performance of a turbine engine and through a computer simulation, varied the bypass ratio to show how the specific fuel consumption decreases as bypass ratio increases. Due to this phenomenon, brought on by the increase in propulsive efficiency, it was shown that fuel costs can be severely impacted. This impact is in a positive way resulting in fuel costs savings that any airline operator can capitalize on.

Some of the early turbo fan engines dates back to the 40's when German engineers tested prototype engines (ref 8). However, research and new designs are ever evolving. Future air breathing propulsion systems for the commercial airlines may grow even larger, producing more thrust at higher efficiencies, much like the Trent 1000 and GE GEnx are doing now.

The current high efficiency turbofan engines vary between 8 and 10 bypass ratio and employ various methods to achieve an even greater propulsive efficiency. The Trent 1000 engine produced by Rolls-Royce is a three shaft engine which utilizes a high-pressure, intermediate-pressure and low-pressure turbine (ref 7). Additionally, the General Electric GEnx engines are running an overall pressure ratio over 50 at the top of climb and are around a BPR of 9 (ref 6). No TSFC numbers are published but the company boasts a 15% improved fuel efficiency claim, some of which could be due to the advanced materials being used and the improved component efficiencies. The primary customer of these engine programs is the Boeing 787, which is one the most recent, newly developed domestic commercial aircraft.

All in all, utilizing the increased propulsive efficiency and the lower TSFC of a high bypass turbo fan engine, commercial airlines can (and are) saving tens of thousands of dollars per flight. Another positive outcome of using high bypass ratio engines is the ability to transport more passengers due to the increased thrust levels. These savings and increased efficiencies have a large impact on the global economy, environment and how people travel. Each of these focus areas stand to improve even further as turbo fan engines advance over time.

Appendix A – Matlab Code

```
%%%%%%%%%%%% MEMS 500 Independent Study %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%% Andrew Dankanich %%%%% Fall 2016 / Spring 2017%%%%%%%%%%%%%%%
clear;
close all;
clc;


%%%%% Flight Conditions and Free Stream Constants%%%%%%%%%%%%%%%%%%%%%%%%%%

M0 = 0.88; % Free Stream Mach Number
gc = 32.2 ; %constant lbf to lbm
R = 287; %kJ/kg universal gas constant
g = 1.4; % Gamma for Air
alt = 35000; %Feet, This is not directly used, but coincides with T0 and P0
rec = 0.96; % Inlet Recovery
T0 = 233; % K Free stream temperature at 35k
P0 = 15; % kPa Free stream pressure at 35k
a0 = sqrt(g*R*T0); % m/s
Pt0 = P0 * (1+((g-1)/2)*M0^2)^(g/(g-1)); % lbf/ft^2
Tt0 = T0 * (1+((g-1)/2)*M0^2); % R
mft0 = sqrt(g)*M0*(1+((g-1)/2)*M0^2)^-((g+1)/(2*(g-1)));
u0 = M0*sqrt(g*R*T0); %Free Stream Velocity
den0 = P0/(R*T0); %Free Stream Density

%%%%%%%%%%%%%%%%%%%%%%%%%%%

bpr = [0, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]; %Various Bypass Ratios
sz = length(bpr);

n = 0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%

for j = bpr;
  BPR = j;
  mdotc = 162.5; % kg/s CORE AIRFLOW ONLY. This remains constant for all BPR
and through "guess and check" yields around 30,000lbf for the turbojet
configuration (BPR = 0)
  mdotfan = BPR*mdotc; % Calculate Fan mass flow
  mdot0 = mdotfan + mdotc; % Total Engine Inlet Airflow


  %%%%%%%% Station 2 and 3 Compressor Inlet and Exit %%%%%%%%%%%%%%%%%%%%%

  tau_a = 8; % Thermal Limit Parameter, See definition in Burner Section
  %pic = 40;
  %Compressor Pressure Ratio: From Farohki, equation 4.74 page 161
  pic = ((sqrt(tau_a)/(1+((g-1)/2)*M0^2)))^(g/(g-1)); %
  etac = 0.9; % Compressibility Efficiency factor of the Compressor
  rec = .995; %Inlet Recovery
  Pt2 = Pt0*rec ;
  Tt2 = Tt0;
  Pt3 = Pt2*pic;
  Tt3 = Tt2*(1+((1/etac)*((pic^((g-1)/g))-1)));
```

```
%%%%%%%% Station 13 and 19 Fan Properties %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
pifan = 1.6; % Using a Typical Single Stage Fan value between 1.4-1.6
Pt13 = Pt2*pifan; %
Pt19 = Pt13*.95; %Account for a Small pressure loss across the Fan
tau_r = Tt0/T0;
tau_fan = pifan^((g-1)/g);
Tt13 = Tt2*tau_fan; %
V19_a0_fan = sqrt((2/(g-1))*((tau_r*tau_fan)-1));
P19 = Pt19/((1+(g-1)/2)^(g/(g-1)));
M19 = (((Pt19/P19)^((g-1)/g))-1)/((g-1)/2);
T19 = Tt13/((Pt19/P19)^((g-1)/g));
a19 = sqrt(g*R*T19);
V19 = a19*M19;

%%%%%%%% Station 4 Burner Exit/Turbine Inlet %%%%%%%%%%%%%%%%%%%%%%%%%%%%

g_t = 1.33; %Ratio of specific heats for the Turbine
g_c = g; %Ratio of specific heats for the compressor is the same as air
cpt = (g_t/(g_t-1))*R; % Metric Unit value should be ~1156
cpc = (g/(g-1))*R; % Metric Unit value should be ~1004
eta_b = .95; %Burner efficiency
pib = 0.95; % Pressure Ratio Across the burner
hpr = 42000; % kJ/kg
Pt4 = Pt3*pib; %

%Now we need to set the "Thermal Limit Parameter" IE Turbine Temp Limit
% tau_a = ht4 / h0 % This is the definition of the Thermal Limit Parameter
tau_a = 8; %This can be adjusted and is a driving factor in Engine
Performance
% tau_a of 8 means Tt4 is ~1600 K if T0 is 233k
Tt4 = (cpc*T0*tau_a)/cpt; % This becomes a constant Temp Limit for all
BPR's
f = (cpt*Tt4 - cpc*Tt3)/(hpr*10^3*eta_b - cpt*Tt4); %Need to convert hpr
from kJ to J with 10^3. Realize that fuel to air ratio becomes constant as
well.
mdot4 = mdotc*(1+f); % This is the core air flow and fuel flow
mdotfuel = f*mdotc;

%%%%%%%% Station 5 Turbine Exit %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
eta_m = .95; % Mechanical efficiency of the Turbine
eta_t = .85; % Flow efficiency of the turbine
%Energy Balance across the Turbine for Tt5.
Tt5 = Tt4 - ((cpc*(Tt3-Tt2) + BPR*cpc*(Tt13-Tt2))/((1+f)*cpt*eta_m));
Pt5 = Pt4*((Tt5/Tt4)^(g_t/(eta_t*(g_t-1))));

%%%%%%%% Station 9 Core Exit %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Assuming an Ideal expansion through the Nozzle
Pt9 = Pt5; %Assume Ideal Nozzle
Tt9 = Tt5; %Station 9 we assume same as turbine exit
P9 = P0; % Assume ideally expanded
%%Assume the Core is Choked for Cruise Condition IE M = 1
M9 = sqrt((((Pt9/P9)^((g-1)/g))-1)*(2/(g-1)));
T9 = Tt9/(1+(g-1)/2*M9^2);
mdot9 = mdot4;
V9 = M9*sqrt(g*R*T9);
V9_a0_core = V9/a0;
```

```
  % Thrust contribution from the Core ONLY
  cfg = 1; % Nozzle coefficient
  Fgcore = mdot9*gc*V9*cfg;


 %%%%%%%% Overall Engine Thrust %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

 %Specific Thrust
 Fn_mdot = (a0/(1+BPR))*(V9_a0_core - M0+BPR*(V19_a0_fan - M0)); % N/m/s

 %Net Thrust
 Fn = (Fn_mdot * mdot0)*.224809; %lbf (converting from Newton to lbf)
 Fn_Metric = (Fn_mdot * mdot0); %Newtons  or kg(m/s^2)

 % Thrust Specific Fuel Consumption
  tsfc = mdotfuel / Fn_Metric; % kg/N/s
  tsfc_english = ((mdotfuel*2.20462) / Fn)*3600 ; % lb/lbf/hr (converting kg
to lbm and seconds to hour)

 %%%%%%%% Parameters for Storage %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 n=n+1;
 BPR_Plot(1,n) = BPR;
 mdot0_Plot(1,n) = mdot0;
 mdotfan_Plot(1,n) = mdotfan;
 mdotfuel_Plot(1,n) = mdotfuel; %
 mdotc_Plot(1,n) = mdotc; %
 f_a_ratio(1,n) = f; %
 Thrust_Net(1,n) = Fn;
 Thrust_Net_Metric(1,n) = Fn_Metric;
 Spec_Thrust(1,n) = Fn_mdot;
 TSFC(1,n) = tsfc;
 TSFC_English(1,n) = tsfc_english;
 V9_Plot(1,n) = V9;
 Tt4_Plot(1,n) = Tt4;
 Tt5_Plot(1,n) = Tt5;
 Tt9_Plot(1,n) = Tt9;
 M9_Plot(1,n) = M9;
 Pt4_Plot(1,n) = Pt4;
 Pt5_Plot(1,n) = Pt5;
 V19_Plot(1,n) = V19;
 M19_Plot(1,n) = M19;



%%%%%%%% Calculate a new Fuel Flow Rate %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate a new fuel flow rate for each BPR
% Use the respective TSFC and the thrust value for engine BPR = 0

if n==sz %n needs to match the BPR array count to enter this segment

%Since each engine was evaluated at Max Thrust, if we throttle the engine/s
%to reduce air flow such that the engine is producing the same net thrust
%as the first BPR configuration (BPR=0), it can be shown that the higher
%BPR engines can still produce the same net thrust at a lower fuel flow
%rate.
% 1 N = .224809 lbf

mdotfuel_new = TSFC.*Thrust_Net_Metric(1,1);%*.224809; $ This is the Adjusted
Fuel Flow rate
```

```
%Fuel Costs as a function of BPR
%Cost of Jet Fuel per gallon as of 9/22/16 $4.14 /gallon
%Jet fuel is 0.804 kg/L or 6.71 lb/gallon
%1 US Gallon is 3.78541 Liter
%
Fuel_Cost = (4.14*(mdotfuel_Plot./(.804*3.78541))); % $/sec Dollar per second
%
Fuel_Cost_HBPR = (4.14*(mdotfuel_new./(.804*3.78541))); % $/sec
Savings = Fuel_Cost-Fuel_Cost_HBPR; % $/sec

end


%%%%%%%% Plotting %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


if n == sz
figure
subplot(311)
plot(BPR_Plot,f_a_ratio)
title('Bypass Ratio vs Fuel to Air Ratio ')
subplot(312)
plot(BPR_Plot, Thrust_Net)
title('Bypass Ratio vs Net Thrust ')
subplot(313)
plot(BPR_Plot, Spec_Thrust)
title('Bypass Ratio vs Specific Thrust')
subplot(311)
ylabel('Fuel to Air Ratio')
subplot(312)
ylabel(' Net Thrust ( Lbf )')
subplot(313)
xlabel('BPR')
ylabel('Specific Thrust ( N/m/s )')


figure
plot(BPR_Plot, Fuel_Cost, 'blue', BPR_Plot, Fuel_Cost_HBPR, 'red',  BPR_Plot,
Savings, 'green')
title('Fuel Cost and Savings for BPR ')
legend('Fuel Cost for BPR = 0','Fuel Cost for Increasing BPR','Fuel Savings
Compared to BPR = 0')
legend('Location','NorthWest')
xlabel('BPR')
ylabel('Dollar per Second ( $/sec )')

figure
subplot(221)
plot(BPR_Plot(1,2:n),V19_Plot(1,2:n))
title('Fan Exit Velocity vs BPR ')
subplot(222)
plot(BPR_Plot, V9_Plot)
title('Core Exit Velocity vs BPR ')
subplot(223)
plot(BPR_Plot, M9_Plot)
title('Core Exit Mach vs BPR ')
```

```
subplot(224)
plot(BPR_Plot(1,2:n), M19_Plot(1,2:n))
title('Fan Exit Mach vs BPR ')
subplot(221)
xlabel('BPR')
ylabel('Velocity (m/sec)')
subplot(223)
xlabel('BPR')
ylabel('Mach No.')

figure
plot(BPR_Plot, TSFC_English)
title('BPR vs TSFC')
legend('TSFC')
legend('Location','NorthWest')
xlabel('BPR')
ylabel('Specific Fuel Consumption (lb/lbf/hr)')

figure
subplot(221)
plot(BPR_Plot, mdotfuel_Plot)
title('Fuel Flow vs BPR ')
subplot(222)
plot(BPR_Plot, mdot0_Plot)
title('Total Air Flow vs BPR ')
subplot(223)
plot(BPR_Plot, mdotfan_Plot)
title('Fan Flow vs BPR ')
subplot(224)
plot(BPR_Plot, mdotc_Plot)
title('Core Flow vs BPR ')
subplot(223)
xlabel('BPR')
ylabel('Mass Flow (kg/sec)')

figure
subplot(221)
plot(BPR_Plot,Tt4_Plot)
title('Turbine Temperature vs BPR ')
subplot(222)
plot(BPR_Plot, Tt9_Plot)
title('Core Exit Temperature vs BPR ')
subplot(223)
plot(BPR_Plot, Pt4_Plot)
title('Turbine Entrance Pressure' )
subplot(224)
plot(BPR_Plot, Pt5_Plot)
title('Turbine Exit Pressure' )
subplot(223)
xlabel('BPR')
ylabel(' Pressure (kPa)')
subplot(221)
xlabel('BPR')
ylabel('Temp (K)')

end
end
```

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