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Big Gulp Gets Even Bigger: Effects of Apparent Mass and Momentum Flux on the Forces during Baleen Whale Feeding

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ABSTRACT: The feeding behavior of the *Balaenoptera physalus*, or more commonly known as fin whales, is some of the most extreme and somewhat curious feeding techniques among mammals and other aquatic vertebrate animals. The specific process includes a lunge deep into the ocean, and large gulp of water to catch many krill at one dive. These dives are relatively short due to the high energy expenditure, most likely due to the high drag exerted on the animal during the process. However, this drag is even more significant than previously thought. Due to the effects of apparent mass and momentum flux, the overall force is much greater than previously modeled. By mathematically modeling these effects during the lunge feeding process, researchers can have a more accurate understanding of additional inertial factors of ellipsoids, thus giving a better understanding in airship design.

INTRODUCTION

The study of fluid mechanics dives deep to reveal behavior of some of the Earth's largest and extreme animals: rorqual whales. The lunge-feeding process can be interpreted as the largest mechanical behavior, while extreme in nature and energy expensive. These large and mysterious animals dive to deep depths, in order to accelerate enough to open their mouths or "gape" open to almost perpendicular to the roof of their mouths. The large gulps of water are the only method able to capture a large volume of krill, to combat the large energy it takes to complete this process.

The lunge feeding process consists of five major events to completion as follow: "(1) accelerating the body, (2) lowering the mandibles and presenting the floor of the mouth of the oncoming flow, (3) generating dynamic pressure that expands the buccal cavity, (4) closing the mouth around a large volume of water, and (5) expelling this volume through baleen plates located on the roof of the mouth, thereby retaining the prey inside of the buccal cavity" (Goldbogen and Pyenson 290). The five step process is demonstrated in Fig. 1, outlining the five main shapes that the whale takes, as it opens and closes its mouth.

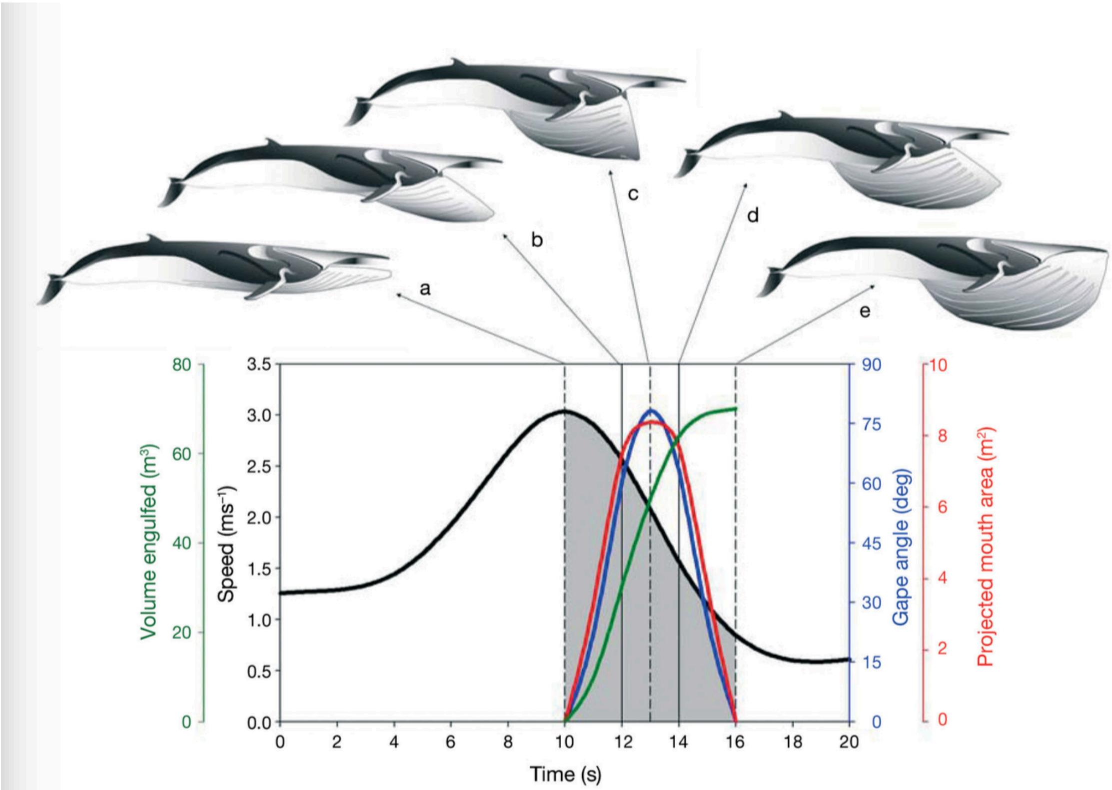

Figure 1. *Balaenoptera physalus.* Five step process making up lunge feeding incorporated from the Marine Ecology Progress Series article "Big gulps require high drag" (Goldbogen et. al 295). The following schematic represents: (a) the mouth beginning to open, (b) the ventral grooves begin the expand due to water entering the mouth, (c) the maximum opening for best exposure to water entering, (d) the ventral grooves are almost nearly expanded, (e) the ventral grooves are fully expanded and the mouth is closed at full capacity.

The process of lunge feeding is related to diving of birds in flight as both involve the use of a streamlined body. The similarities reflect the importance of minimization of drag for evolutionary success. However, the drag created in lunge diving can be greater than initially expected due to the effect of the apparent mass acting on the whale. In order to maximize food intake, the whales must expand. However, in order to minimize the drag, the bodies of rorqual in a bloated and expanded shape, the bodies must be streamlined. Their shape has proven to be highly optimal in lowering the high expended energy of the process. The whales make many dives in order to gather enough krill for their hearty diet, and so an efficient dive is crucial to survival.

To compute the energy expenditure and drag on the whales, the lunging process was mechanically modeled. The results of the modeling demonstrated that the drag is significant larger than initially estimated form studies and observations. The kinematic data were incorporated from Goldbogen's work with high-resolution digital tags and morphological data of the engulfment apparatus to quantify the speed, acceleration, and net engulfment volume during the process, and we will use this data.

MATERIALS AND METHODS

Mechanics of the body during lunge feeding:

The speed and acceleration of the whales were taken using the high resolution tags creating Fig. 2 below, adapted from Goldbogen's work:

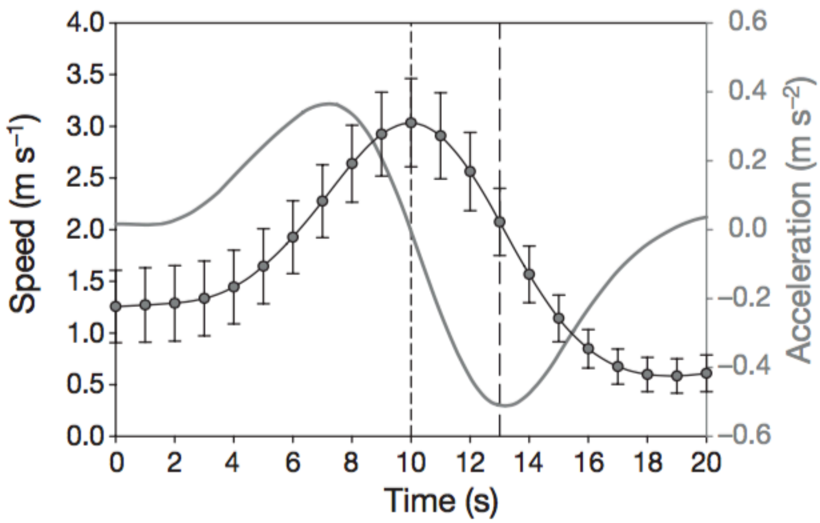

Figure 2. *Balaenoptera physalus.* Kinematics of the body during a lunge. Average speed of the body (black line) calculated for 50 lunges performed by 7 fin whales (see Goldbogen et al. 2006). Error bars represent 2 standard deviations about the mean. Acceleration of the body (gray line) is calculated from the change in speed over each 1 s interval. The vertical, closely dashed line represents the moment when the mouth opens at maximum speed, and the vertical, widely dashed line marks the moment of greatest deceleration, which should occur at maximum gape (Goldbogen et. al 291).

Inertia Factors that Affect Kinematic Modeling

In this study, the Baleen whales are to be modelled as ellipsoids as seen in Fig. 3. The average body length of the whales is approximately 20 meters for the adult, which is divided into two segments to more easily model the expansion in the buccal portion. This model alleviates the differentiation in individual body shapes, while proving to incorporate the main importance, which is the problem of the potential flow of the fluid about the ellipsoid.

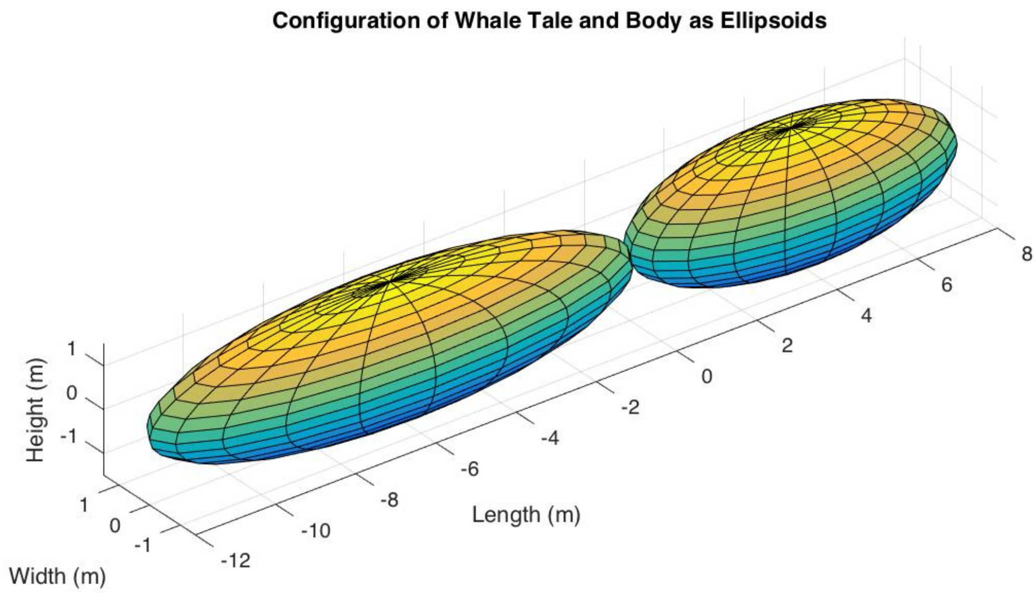

Figure 3. *Balaenoptera physalus*. Model of the body of a Baleen fin whale created using MATLAB. The dimensions of the whale are incorporated from the Marine Ecology Progress Series November 2007 Issue, "Big Gulps require high drag for fin whale lunge feeding" by Jeremy Goldbogen and Nicholas D. Pyenson. The part of the body that expands during the feed is modeled as the origin, with the tail a separate ellipsoid due to remaining constant during the process.

The other dimensions and constants incorporated into the model can be found in Table 1. These are used to parametrize the whale body in our calculations. Thus, our results are for the whales that were measured by the previous investigators. Thus, the entire mass (both of both the whale and of engulfed water) can be represented by summation of the aft portion (subscript A) and the forward portion (subscript F):

(1) $M_T = \frac{4\pi}{3} * a_A b_A c_A + \frac{4\pi}{3} * a_F b_F c_F$

where: $a_A = 12$ m, $b_A = c_A = 1$ m, $a_F = 8$ m, and $b_F = c_f$. The parameter $c_F = b_f$ begin at 1 m but are allowed to change with time in order to accommodate the water and krill engulfed by the whale.

Table 1. Parameters incorporated into the model. All morphological and physiological parameters correspond to an adult fin whale where the body length (L) = 20 m. Plus or minus symbols represent one standard deviation

Parameter	Symbol	Value	Reference
Average adult body length	L	20 m	Lockyer (1976)
Average adult body mass	M	50 000 kg	Lockyer (1976)
Length of ventral grooves	L_V	8 m (0.4L)	Orton & Brodie (1987
Radius of the body	R	1.5 m	Lockyer & Waters (1986)
Body speed	U	see Fig. 1	Goldbogen et al. (2006)
Projected mouth area as a function of gape angle	S_M	see Fig. 3	Lambertsen et al. (1995)
Baleen filter area	A_B	3.0 m^2	Kawamura (1980)
Baleen fringe diameter	L_F	Range = 2.5×10^{-2} to 1.4×10^{-1} cm; Average = 7.5×10^{-2} cm	Kawamura (1980)
Baleen plate spacing	L_P	Average = 0.6 ± 0.2 cm	USNM 504258, 504243; MVZ 124428; UCMP 85366
Prey density (krill)	P_D	Average = 0.15 kg m^{-3}	Croll et al. (2005)
Daily energetic demand (krill)	Γ	901 ± 258 kg d^{-1}	Croll et al. (2006)
Average foraging dive duration and surface recovery time	($T_D + T_S$)	9 min	Croll et al. (2001); Acevedo-Gutierrez et al. (2002); Goldbogen et al. (2006)
Duration between consecutive lunges at depth	T_L	Average = 30 s	Goldbogen et al. (2006)
Number of lunges per dive	N_o	Average = 4	Goldbogen et al. (2006)

Table 1. incorporated parameters from the Marine Ecological Progress Series November 2007 Issue, "Big Gulps require high drag for fin whale lunge feeding" by Jeremy Goldbogen and Nicholas D. Pyenson. All of these parameters correspond to an adult fin whale, tagged from field data from the previous experiment in Goldbogen's work.

Due to the growing interest in airship and ellipse design, terms have been developed to most practically express the characteristics of this type of motion. L. B. Tuckerman expresses inertia factors to represent special cases, such as the elliptic cylinder, prolate spheroid, and oblate spheroid. The fin whale follows the dimensions of prolate spheroid, to most accurately describe the shape of the animal.

The additional inertia of the translational potential flow of the fluid (K_1) is proportional to the following coefficient:

(2) $K_1 = \frac{4\pi}{3} abck_1$

where $\frac{4\pi}{3} abc$ is the volume of the ellipsoid where $a > b = c$,

(3) $k_1 = \frac{\alpha_0}{2-\alpha_0}$

where α_0 is defined as:

(4) $\alpha_0 = \sqrt{\frac{1-e^2}{e^3}} \log(\frac{1+e}{1-e} - 2e)$

and where the eccentricity e is further defined as:

(5) $e = \sqrt{1 - \frac{b^2}{a^2}}$

The eccentricity in Eq. (4) is defined as the eccentricity of the central section normal to the intermediate (b) and minimum (c) axes of the ellipsoids. These coefficients are sufficient to calculate the Tuckerman coefficients used to determine the effect of apparent mass during the lunge feeding process.

Mass Distribution

Table 1 refers to the dimensions used in the main calculations below. The average mass of the fin whale for adult baleens is 50,000 kg (Goldbogen et. al 291). However, due to the nature of the feeding, the whale takes in significant about of water, increasing the volume up to 82 m^3 averaging at about 71m^3. Therefore, the mass distribution throughout the main stages of the lunge feeding can be modeled by:

(6) $M_T = M_B + 68{,}000 \sin(\frac{\pi t}{12})$

where M_B represents the starting mass of the whale body and M_T represents the total mass distribution during the lunge feeding process.

The method by which the forces were calculated in previous work is a direct application of Newton's Law, as follows:

(7) $F_C = M_T * a(t)$

where F_c is the classical force calculated, M_T is the mass distribution and a(t) is the acceleration with respect to time. Equation (7) is what we use to calculate the classical value of force that past researchers have used.

However, in addition to the kinetic energy of the mass of the whale, there is also kinetic energy in the water that that must move around the whale body. This is not just the water displaced by the whale but all of the water that must move to make the streamlines around the whale. In hydrodynamics, this is referred to as apparent mass. The equation to model the force including the effect of the apparent mass coefficient is as follows:

(8) $F_T = M_T[1 + k_1] * a(t)$

where k_1 is the Tuckerman coefficient discussed above. The Tuckerman term gives the effect of apparent mass. Equation (8) is what we use to calculate the force on the whale including consideration of apparent mass alone on the force distribution of the whale.

However, apparent mass is not the only effect that must be considered. One must also consider the kinetic energy of the water that is engulfed within the expanding body cavity of the whale. The correct manner to include the effect of the water that enters the whale body is to consider that the necessary consumed power for the whale to dive (force times velocity) must be equal to the time rate of change of kinetic energy of the system. The time rate of change of the total kinetic energy of a whale that increases its mass through capturing water is:

$$(9)\ v * F_T = \frac{dT}{dt} = \frac{d\left[\frac{1}{2} * M_T * v^2\right]}{dt} = (M_T * v * \frac{dv}{dt}) + \frac{1}{2} * v^2 * \frac{dM_T}{dt}$$

and

$$(10)\qquad F_T = M_T * a(t) + \frac{1}{2} * \frac{dM_T}{dt} * v$$

where $\frac{dM_T}{dt}$ is the time derivative of the total mass of whale and engulfed water, $\alpha(t)$ is the acceleration, dv/dt, and v is the velocity of the whale during the lunge feed. /the first term in Eq. (9) is the classic Newton's affect, while the second term is the effect of the added momentum transfer of the engulfed water. Equation (9) is what is used to compute the impact that the momentum addition has on the force distribution, mostly when the speed is the highest at the widest point of the gape at the four second mark. Fig. 14 shows the deviation from the classical force distribution.

The complete equation for force on the whale, which includes both apparent mass and the addition of momentum flux from the water is given below.

$$(11)\qquad F_T = M_T[1 + k_1] * a(t) + \frac{1}{2} * (\frac{dM_T}{dt}[1 + k_1] + \left[\frac{dk_t}{dt} * M_t\right]) * v$$

Equation (11) is used to calculate the total force necessary to propel a whale during its lunging dive. Equation (11) is the correct equation, against with other approximations must be compared.

NUMERICAL RESULTS

Mass Distribution

The change in whale total mass during a typical lunging process is represented in Fig. 4, which demonstrates the non-linear manner in which the mass increases as the water enters the buccal cavity of the whale. The blue dots are from the data, and the red curve is a smooth fit of the data so that we will be able both to interpolate and take derivatives of M_T with respect to time analytically.

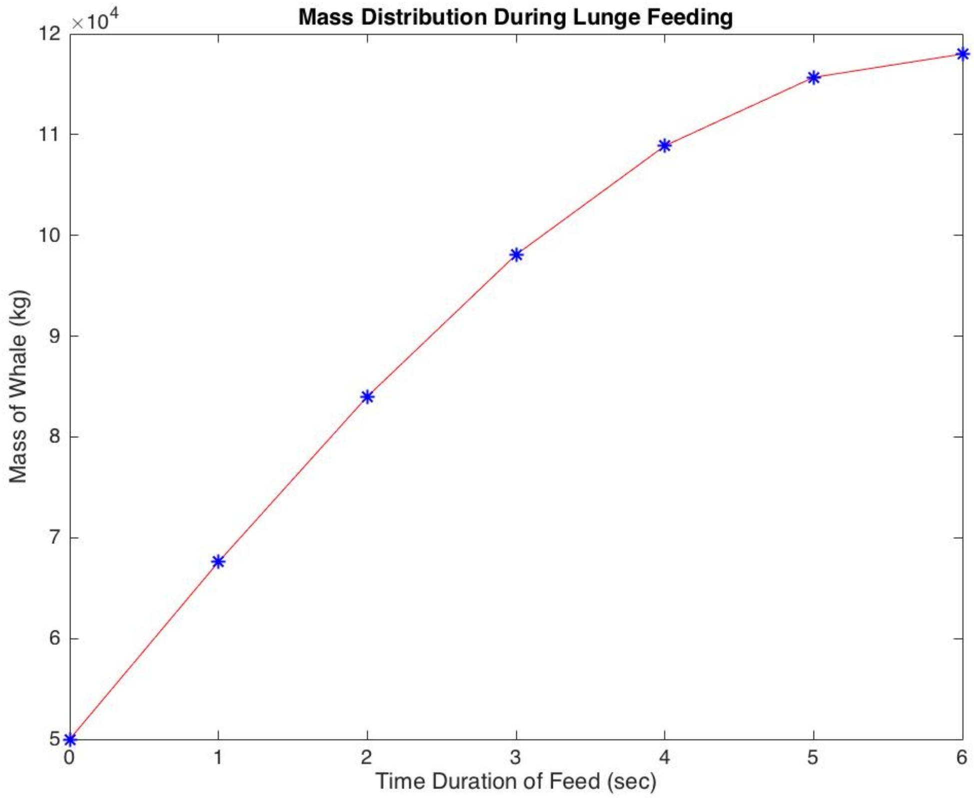

Figure 4. Total mass of whale from the beginning of the opening stage to maximum fill capacity.

Inertia Factors of Ellipsoids: Tuckerman Coefficients

In order to determine the effects of the apparent mass on total force, each part of the Tuckerman coefficients was carefully calculated and modeled to change as the mass and diameter of the whale changed. The first item to compute is the eccentricity. Since we have the total mass as a function of time in Fig. 4, we can break that mass up into two virtual ellipsoids as shown in Eq. (1). The value of $c_F = b_F$ can be determined at any time in order to give the corresponding mass in Fig. 4. With that information, the eccentricity of either the aft or forward body can be determined from Eq. (5). That eccentricity is shown in Fig. 5. From that, the Tuckerman constant α_0 can be found from Eq. (4). This is shown in Fig. 6. Finally, the Tuckerman factor for each part follows directly (due to the corresponding mass). The Tuckerman constant for the forward body is shown in Fig. 7.

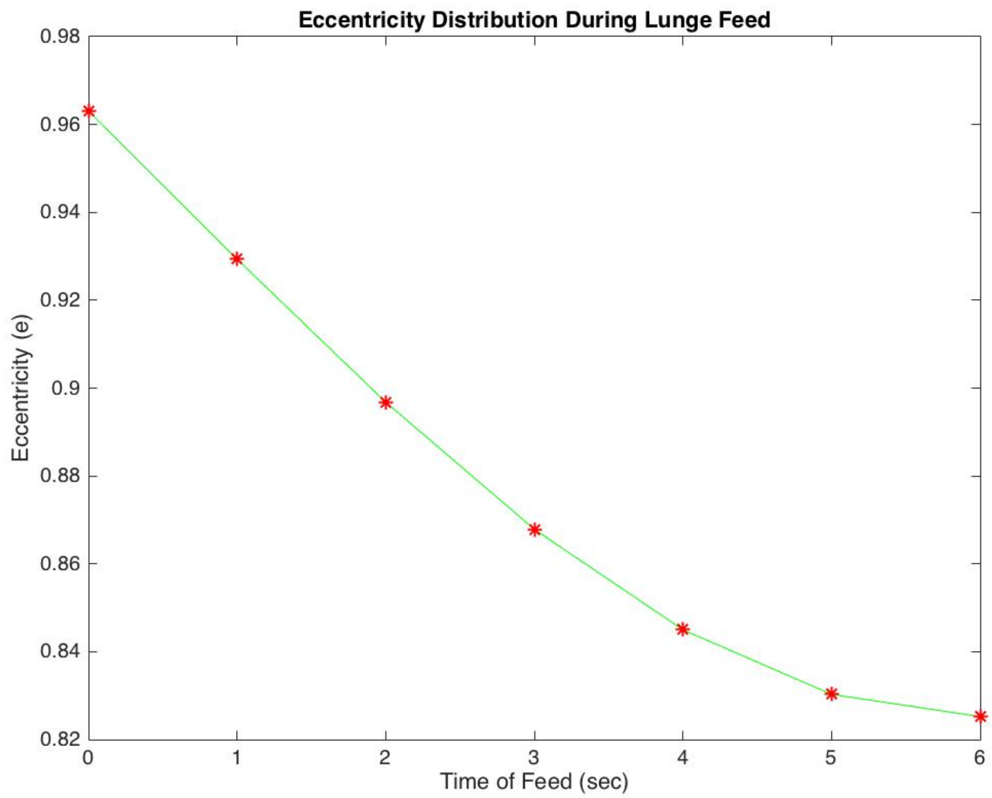

Fig. 5 Eccentricity distribution during the five phases of the lunge feeding from the beginning opening phase to the capacity filling throughout the six second time frame.

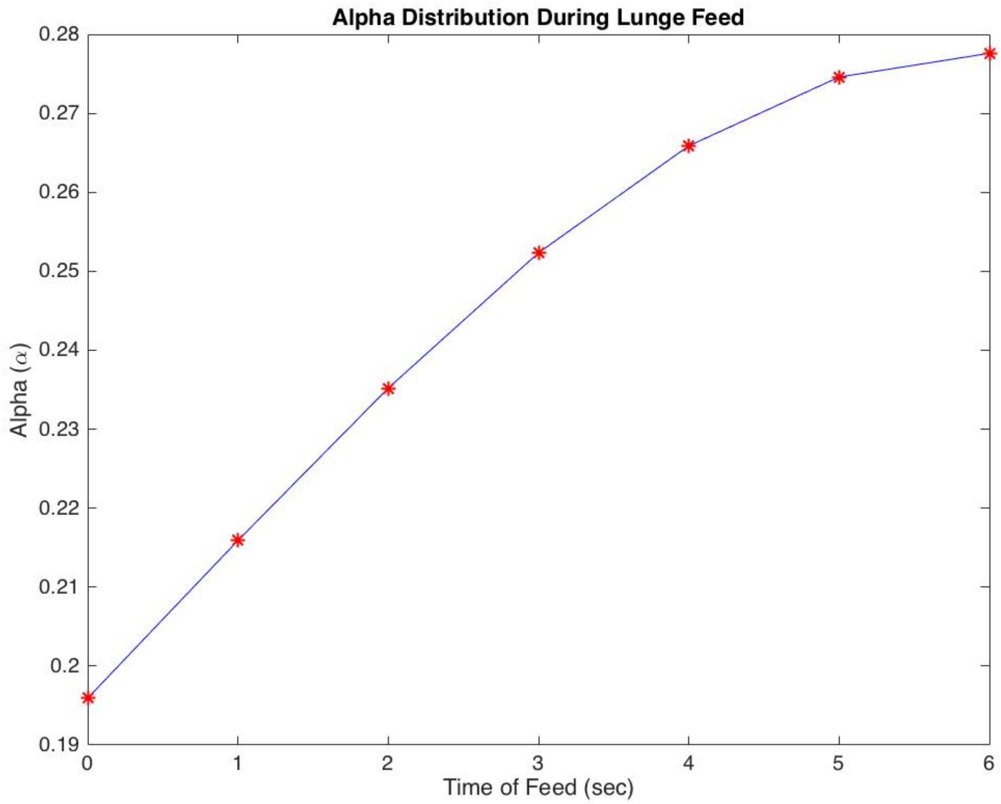

Figure 6. Alpha-0 distribution during the five phases of the lunge feeding from the beginning opening phase to the capacity filling throughout the six second time frame.

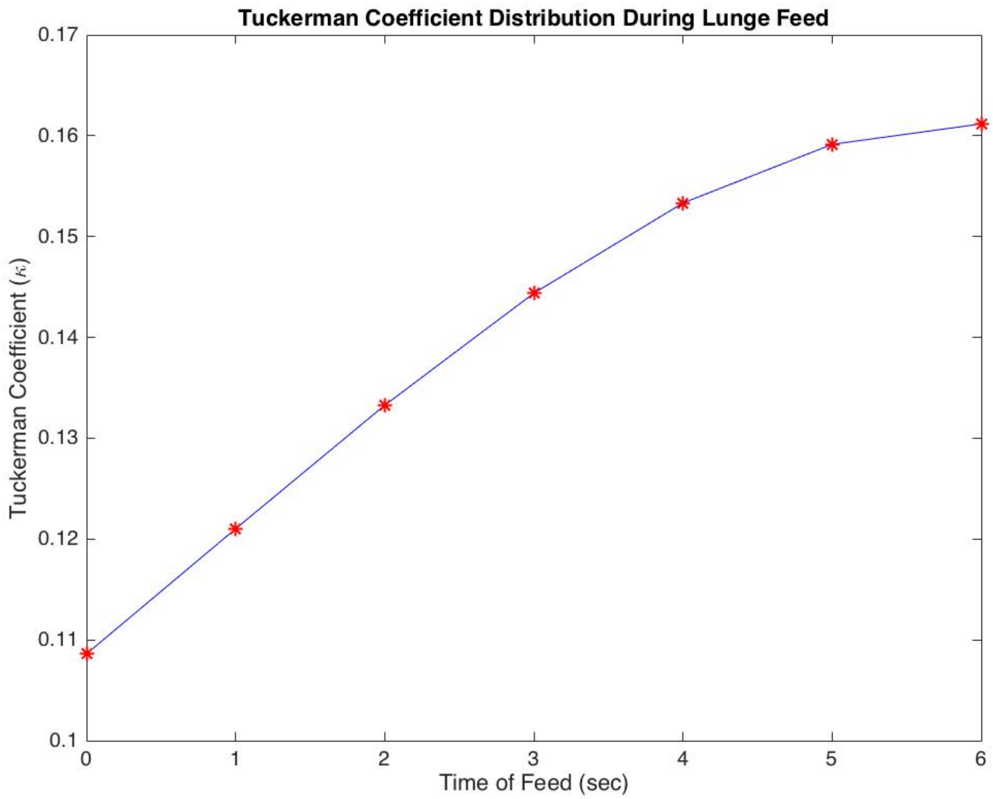

Figure 7. Tuckerman coefficient distribution during the five phases of the lunge feeding from the beginning opening phase to the capacity filling throughout the six second time frame.

Apparent Mass Consideration

Figures 4-7 show how the added apparent mass of the whale can by up to 16% of the actual whale mass. However, apparent mass term is added on top of the mass that is added by the engulfed krill and water. The effect of apparent mass is therefore magnified during the lunge. Figures 8-9 show the increase of total mass (and total volume) of the whale as krill is engulfed along with the added apparent mass (and apparent volume) when we consider the Tuckerman apparent mass terms. The actual whale mass increases from 2 kg to 8 kg during the lung. However, the total mass (with apparent mass) increases to 10 kg. Thus, the apparent mass is about equal to the original mass of the whale. It is noteworthy that the increased actual mass increases the eccentricity (adding to the apparent mass effect), and it also results in a momentum flux due to the time rate of change of mass. We have treated this momentum flux theoretically already in this paper, and will later develop it numerically, as well.

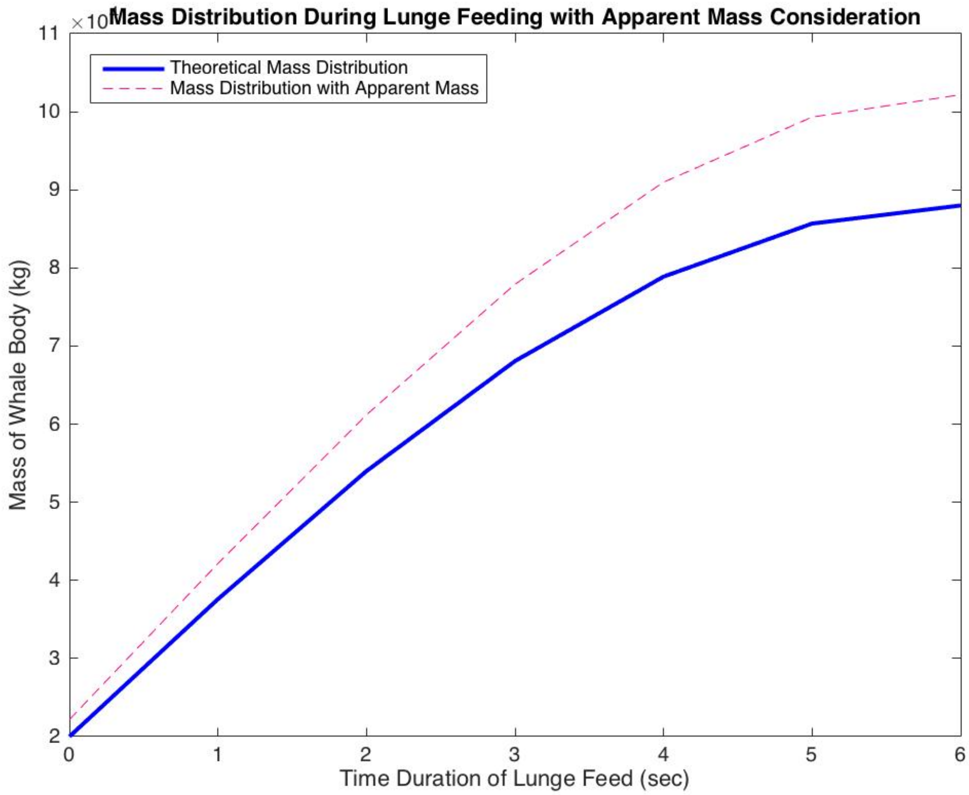

Figure 8. Mass of whale during the lunge feed including original mass, mass of engulfed water, and apparent mass. The dashed line represents the significant change in mass than previously calculated in the classical model represented by the blue line.

Figure 9 gives the corresponding volume and apparent volume of the whale due to both apparent mass and engulfed mass.

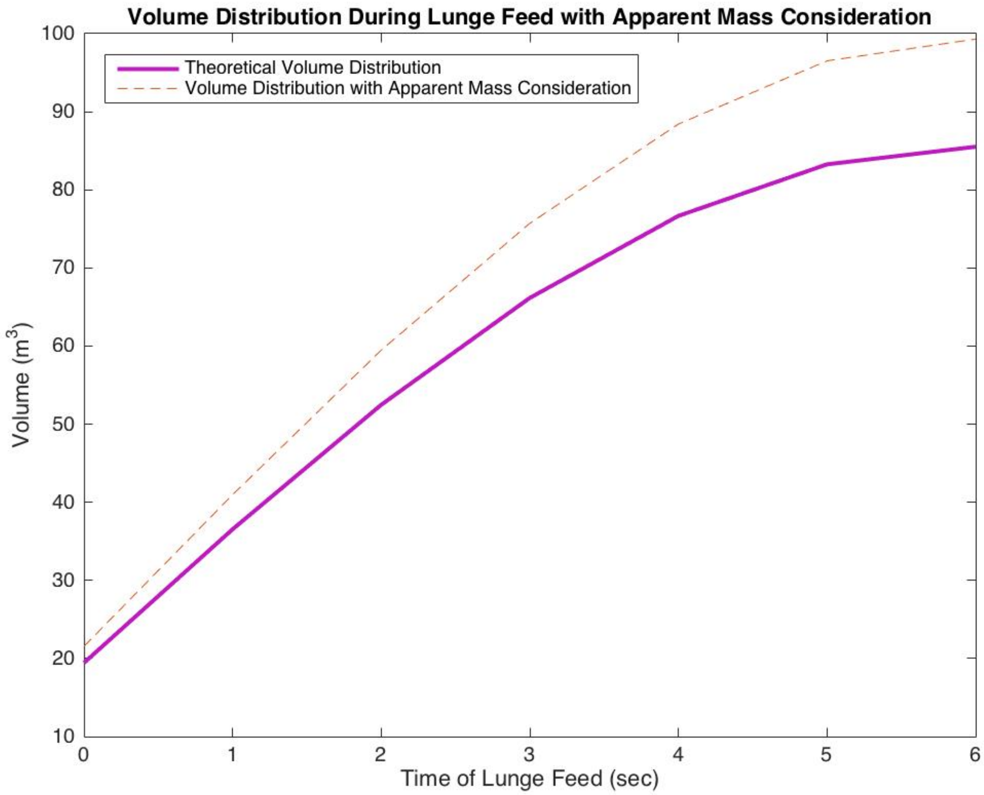

Figure 9. Volume distribution during the lunge feed similar to Fig. 8 (above) to demonstrate the significant impact that the engulfed mass and apparent mass on the volume distribution during the lunge feed. The total volume of the whale (including engulfed water and krill) can be seen in the dashed line. The classical model is the solid line.

In previous studies, the mass of the system did not include the mass of the engulfed volume. Because the engulfed mass was left out, the calculation of the drag was performed with classical kinematics. Although the added mass initially has minimal effect on the system, the engulfment volume demonstrates significant deviation as the feeding continues. Therefore, the calculation should take into account the effects that the engulfed mass of the water creates to add additional inertia to the system and more accurately model the process.

Whale acceleration

The final piece of numerical data that is needed is the acceleration of the whale during lunge feeding. From the data provided by Goldbogen in Fig. 2, we have been able to formulate the acceleration distribution is modeled below by interpolating the data taken from the tagged whales to create Fig. 10 below. With this acceleration, we are ready to compute the forces during whale feeding from the various models—either Eq. (7), Eq. (8), Eq. (9-10), or Eq. (11).

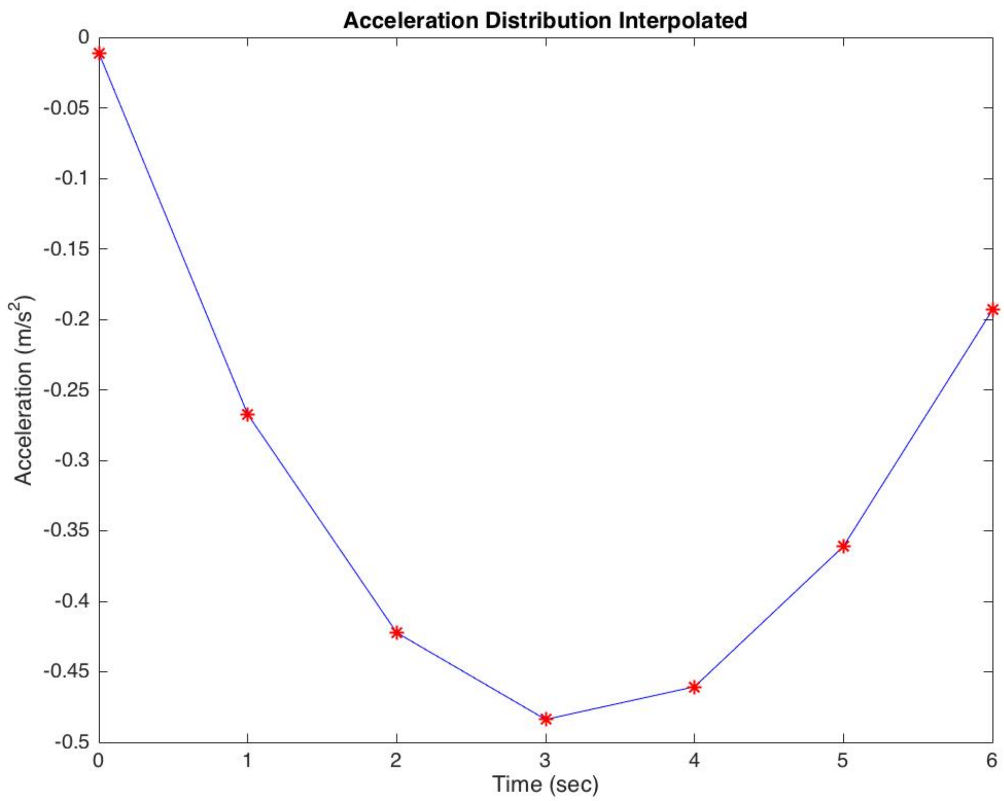

Figure 10. Acceleration found by use of extraction of points of data from Table 2 to come to the equation a(t)= =-0.0014*t.^3 +0.055*t.^2 -.31*t -0.011 to use for the force distribution throughout the lunge feed. The data can be found in appendix A.

KINEMATIC RESULTS

Figure 12 gives the computed force based on the classical kinematic modeling approach, Eq. (7). The results we have computed here are consistent with those reported by the previous investigators who used the classical method. Thus, Fig. 11 is a good starting point before taking into account the Tuckerman coefficients and momentum flux.

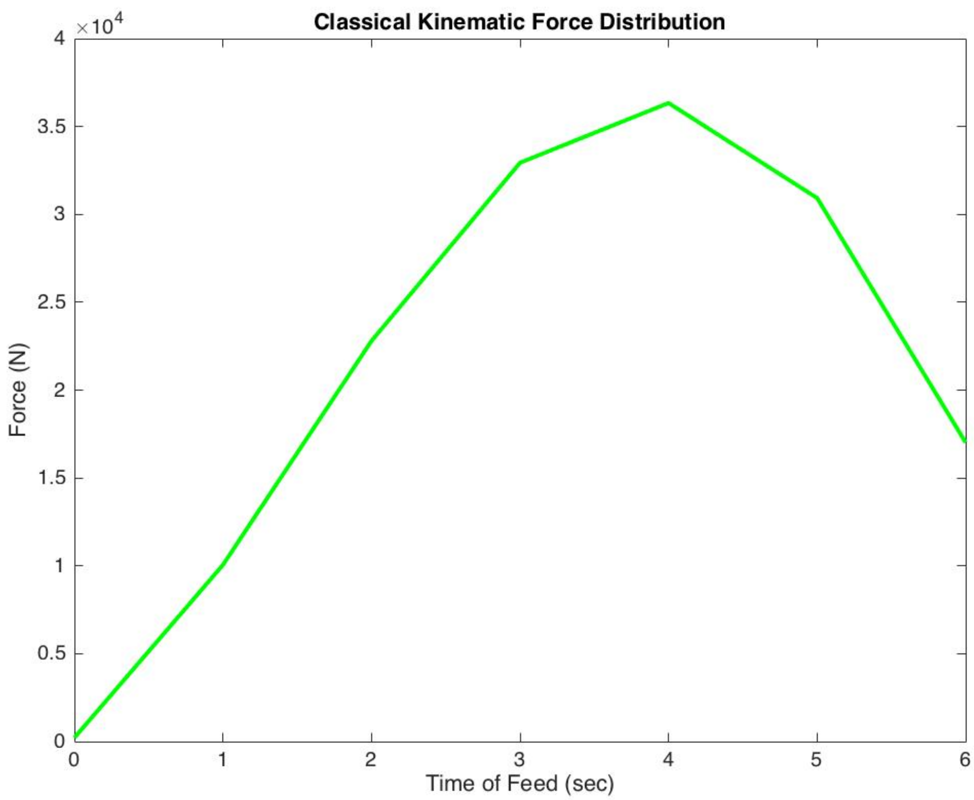

Figure 11. The drag force on the whale during the lunge feed from calculation and modeling of Goldbogen. Kinematics of the body during a lunge from time the start of the mouth opening at lowest point of the dive.

The next step in the comparisons is to add the Tuckerman apparent mass effect to the computations, Eq. (8). We already saw that the added apparent mass is significant, showing a 16% increase in apparent mass, Figs. 7-8. The added apparent mass thus already demonstrates significant deviation from the classical force modelling from previous studies. Figure 12 displays the significant variance in whale force when one incorporates the Tuckerman coefficients, Eq. (8).

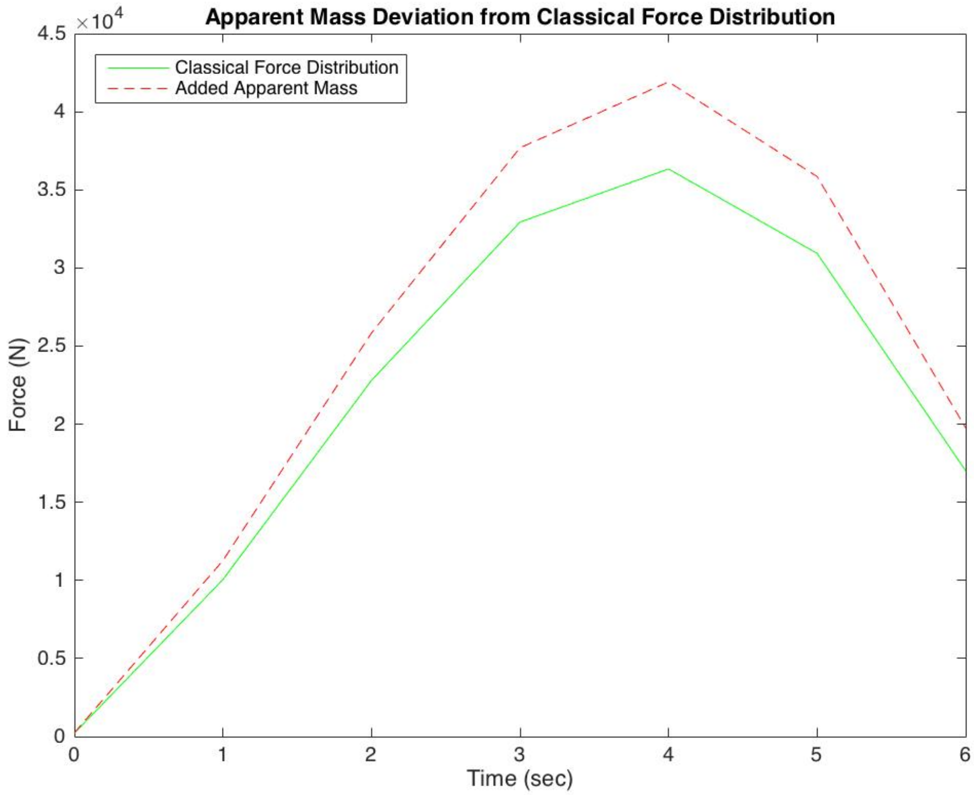

Figure 12. The apparent mass added into the classical kinematic equation shows significant deviation from the classical method alone. This model represents these two methods with the dashed line representing the added apparent mass and the solid line representing without the apparent mass taken into place.

Figure 12 demonstrates the most significant impact occurs at around the maximum gape, when the mouth is almost fully open. The drag force on the whale is upwards of 4.0kN which is substantially higher than the previously calculated 3.5kN from the classical model. Therefore, the Tuckerman coefficients are useful in more accurately modeling the ellipsoidal shape of the whale.

Figure 13 models the classical kinematic behavior of the whale with the inclusion of momentum flux, including the effect that it has on the overall drag force on the whale, Eq. (9).

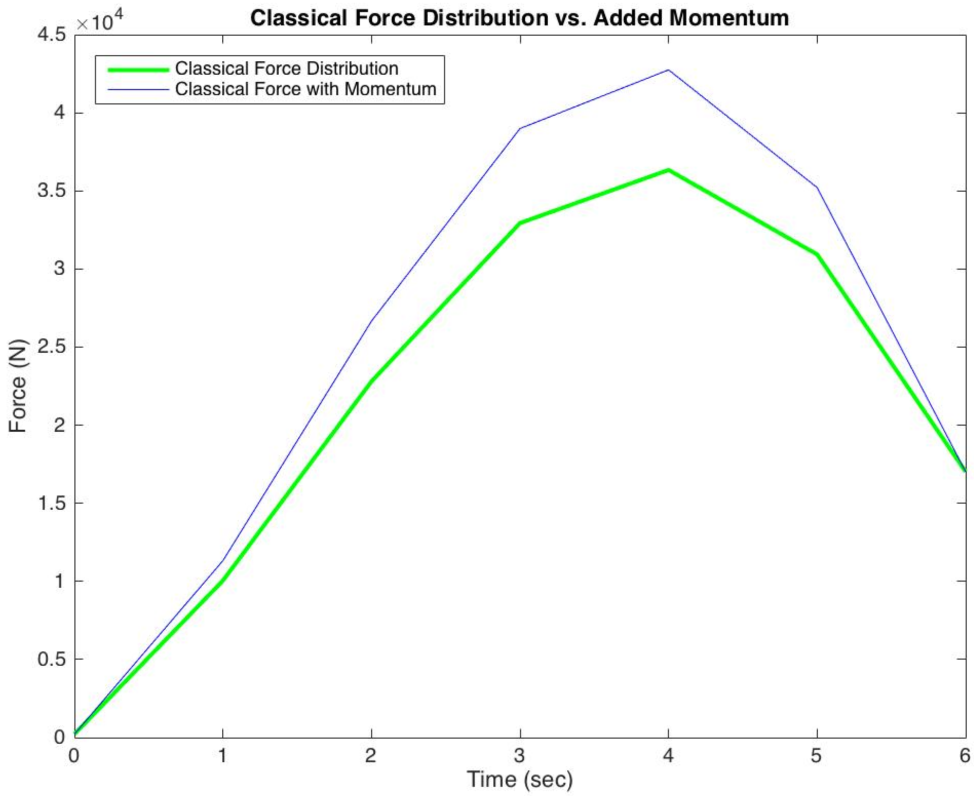

Figure 13. Classical force distribution versus the force distribution with added momentum. This does not take into account the apparent mass addition. The additional momentum causes the drag force on the whale to go up to about 42kN at the largest gape of the mouth, when it is fully extended.

Figure 14 gives the total force acting on the whale taking into account the added apparent mass and momentum flux during the feeding can be modeled below. It compares the four methods of force calculation to demonstrate the drastic effect that apparent mass and momentum flux both cause the whale during the feeding process. One can see that the peak force on the whale with both of the added effects is 58kN as opposed to the classical estimate of 33 kN. This represents a 75% increase in max force and over a 50% increase in expended energy.

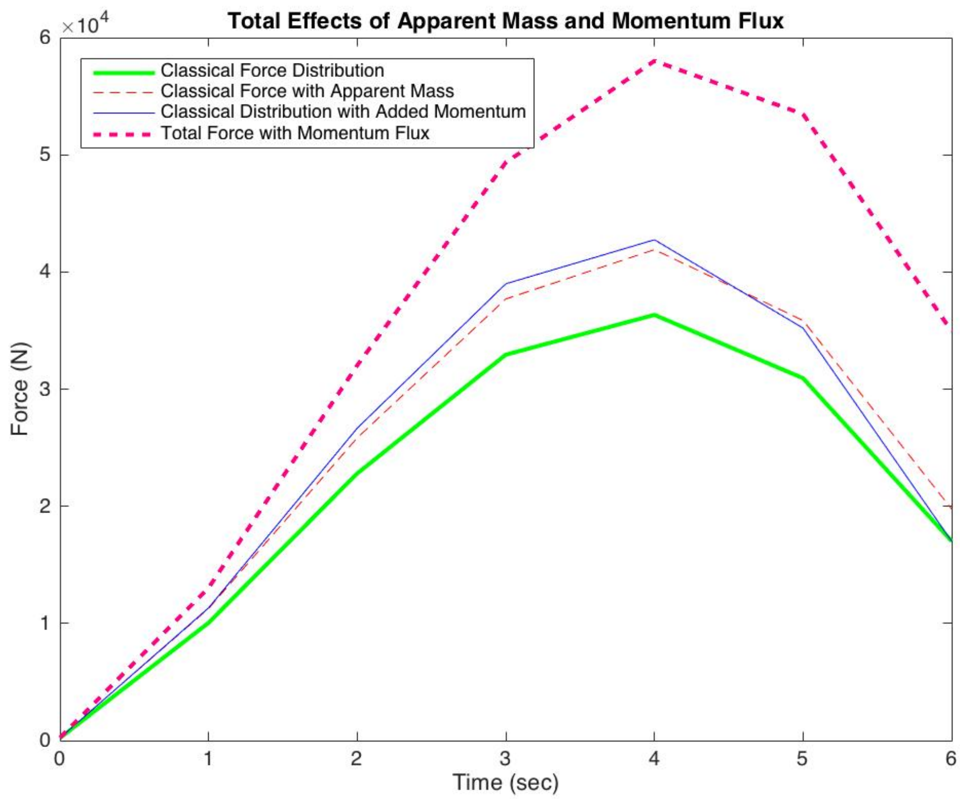

Figure 14. Comparison graph to represent the classical force distribution, the classical with added momentum, and classical with just apparent mass, and the total force on the whale due to apparent mass and momentum flux effects. The drag force increases to almost twice the amount of force previously calculated on the whale with the kinematic approach.

DISCUSSION

This study demonstrates that the effects of apparent mass and momentum flux create a larger drag force on the fin whale *Balaenoptera physalus* lunge feeding. This presents the first mathematical model that demonstrates the significance the Tuckerman coefficients hold, for the whales as ellipsoidal. These coefficients are used to determine that the apparent mass has a significant effect during the lunge feed process. The process that we used to determine this combines kinematic data and mathematical modelling to create a better and more accurate kinematic analysis of these creatures.

By calculating and integrating the Tuckerman coefficients into the mass distribution of the whales, we found that eccentricity decreases to 0.825, the alpha increases to 0.228, the Tuckerman coefficient increases to 0.16. Therefore, the apparent inertia added to the whale increases to 99.30, and increases the volume of the body up to 83.27kg^3. The final mass of the whale due to the effects of apparent mass is 102,182.92 kg, a 1.7x the whales original body weight. Therefore, this extra weight distribution plays an important role in determining the correct force distribution across the lunge feed.

Due to the change in mass distribution the Tuckerman coefficients determine that there is a significant change to the drag force than was previously calculated for fin whales. From Fig. 13, the force with the addition of the apparent mass is 41,907N at its highest point, as opposed to the 36,336N previously calculated. The apparent mass increases the drag force by 15%. This is a significant increase from the previously known drag force.

The momentum flux addition is the most significant change to the drag force increasing the force to upwards of 58,000 N. This is much higher than previous models, thus demanding that these effects must be taken into account when modelling fin whale feeding.

CONCLUSION

Due to the energetic demand that this puts on the fin whales, it begs the question how these animals are equipped to handle such forces so frequently in their ecological behavior. However, this process continues to be the most favorable type of feeding for more than just fin whales. The deep dives that fin whales embark are also used by mink whales, blue whales, and even killer whales. Therefore, this process can be argued to be favored among many different types of whales. The amount of drag force that these animals undergo is an incredible feat, which can teach us a lot about our own engineering practices in airship design and aerodynamics. The model discussed in this study allows a more accurate model as the drag is even more significant than previously thought. Due to the effects of apparent mass and momentum flux, the overall force is much greater than previously modelled by over 15%. By mathematically modelling these effects during the lunge feeding process, there is a more accurate understanding of additional inertial factors of ellipsoids, which can then be applied to many other factors to model more efficient and more accurate aerodynamically and hydro-dynamically favorable designs.

Acknowledgments. This project would not be possible without the constant and driving support of Professor David Peters. Dr. Peters singlehandedly, lead the way in the process, and guiding me to these results every step of the way. I sincerely want to thank him, I and appreciate all of the time he has spent working with me on this project, through all of the trials and tribulations. He is truly and man of wisdom, and inspiration for me to really “dive deep” into my work.

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APPENDIX A

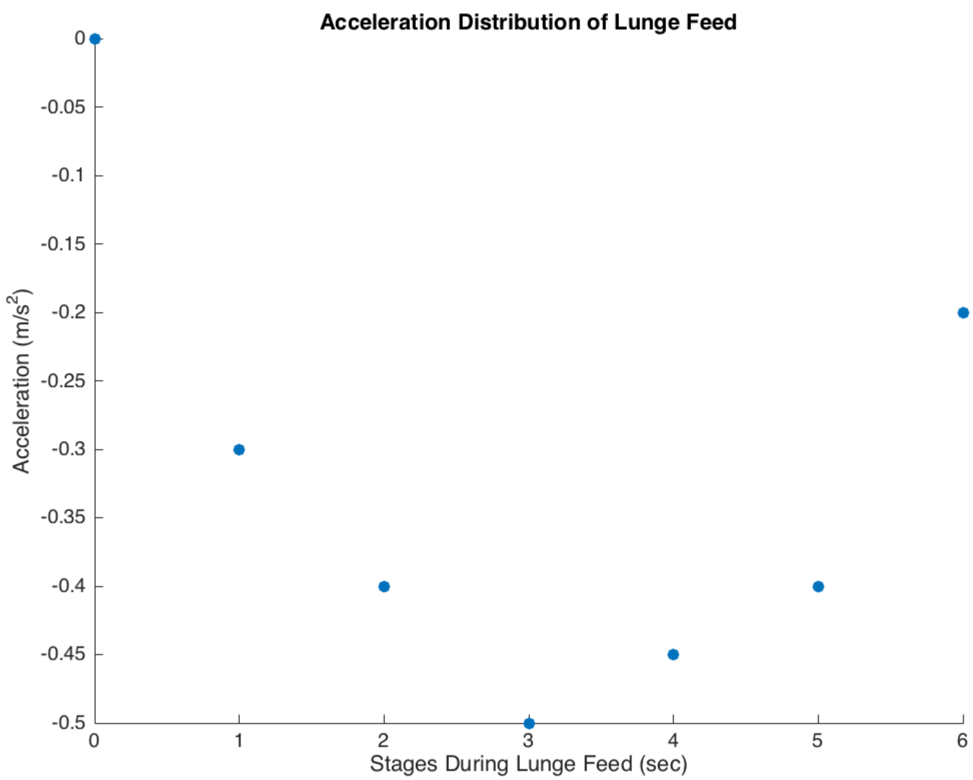
MATLAB Code Used to Create the Acceleration Model:

```
Acceleration=[0;-.3;-.4;-0.5;-0.45;-.4;-.2];
Time=[0;1;2;3;4;5;6];
Stages={'Stage 1:Closed'; 'Stage 2: 30 Degrees';'Stage
3:60
 Degrees';'Stage 3: Fullest Opening'; 'Stage 4:60
Degrees';'Stage 5:
 30 Degrees';'Stage 6:Closed But Full'};
T= table(Time,Acceleration,'RowNames',Stages)
x=Time;
y=Acceleration;
scatter(x,y,'filled')
title('Acceleration Distribution of Lunge Feed');
xlabel('Stages During Lunge Feed (sec)');
ylabel('Acceleration (m/s^2)');
figure
time=0:1:6;
acceleration=-0.0014*x.^3 +0.055*x.^2 -.31*x -0.011;
x=time;
y=acceleration;
xmarkers=x;
ymarkers=y;
plot(x,y,'b',xmarkers,ymarkers,'r*')
title('Acceleration Distribution Interpolated');
xlabel('Time (sec)');
ylabel('Acceleration (m/s^2)');
T=

Stage 1:Closed
Stage 2: 30 Degrees
Stage 3:60 Degrees
Stage 3: Fullest Opening
Stage 4:60 Degrees
Stage 5: 30 Degrees
Stage 6:Closed But Full
Time    Acceleration
____    ____________
0             0
1          -0.3
```

```
2         -0.4
3         -0.5
4        -0.45
5         -0.4
6         -0.2
```

1		



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APPENDIX B

MATLAB Code Used to create the force models above:

```
t=0:1:6;
totallength=20;
snootlength=8;
taillength=totallength-snootlength;
totalmassstart=50000;
acceleration=-0.0014*t.^3 +0.055*t.^2 -.31*t -0.011;
```

```
speed=-0.0014/4*t.^4+0.055/3*t.^3-.31/2*t.^2 -0.011*t;
massflowrate=(17000*(pi)/3)*cos(((pi)*t)/12);
format long g
tailmass=30000;
bodymass=20000+68000*sin(((pi)*t)/12);
totalmassdist=tailmass+bodymass;
volumedist=bodymass/1029;
x=t;
y=volumedist;
plot(x,y,'Linewidth',2,'Color',[.75,.1,.75])
hold on
a=4;
b=sqrt((volumedist.*3)/(4*(pi)*a));
bodyvoldist=(4/3)*(pi)*a*b.^2
e=sqrt(1-(b.^2/a^2))
alpha=((1-e.^2)/e.^3)*(log((1+e)/(1-e))-2.*e)
k=alpha./(2-alpha)
k_dot=(1./(2-(alpha.^2)));
apparentinertia=k.*bodyvoldist+volumedist
x=t;
y=apparentinertia;
plot(x,y, '--')
title('Volume Distribution During Lunge Feed with
Apparent Mass
 Consideration');
xlabel('Time of Lunge Feed (sec)');
ylabel('Volume (m^3)');
legend('Theoretical Volume Distribution','Volume
Distribution with
 Apparent Mass Consideration','Location','northwest')
hold off
figure

x=t;
y=bodymass;
plot(x,y,'b','Linewidth',2)
hold on
x=t;
y=k.*bodymass+bodymass
plot(x,y,'--','Color',[1,0,.5])
title('Mass Distribution During Lunge Feeding with
Apparent Mass
 Consideration');
```

```
xlabel('Time Duration of Lunge Feed (sec)')
ylabel('Mass of Whale Body (kg)');
legend('Theoretical Mass Distribution','Mass
Distribution with
 Apparent Mass','Location','northwest');
```

1

```
hold off

figure
x=t;
y=b;
xmarkers=t;
ymarkers=b;
plot(x,y,'r',xmarkers,ymarkers,'b*');
title('Radial and Lateral Expansion Distribution of the
Whale');
xlabel('Time (sec)');
ylabel('Length (m)');
figure
x=t;
y=e;
xmarkers=t;
ymarkers=e;
plot(x,y,'g',xmarkers,ymarkers,'r*');
title('Eccentricity Distribution During Lunge Feed');
xlabel('Time of Feed (sec)');
ylabel('Eccentricity (e)');
figure
x=t;
y=alpha;
xmarkers=t;
ymarkers=alpha;
plot(x,y,'b',xmarkers,ymarkers,'r*')
title('Alpha Distribution During Lunge Feed');
xlabel('Time of Feed (sec)');
ylabel('Alpha (\alpha)');
figure
x=t;
y=k;
xmarkers=t;
```

```
ymarkers=k;
plot(x,y,'b--p',xmarkers,ymarkers)
title('Tuckerman Coefficient Distribution During Lunge
Feed');
xlabel('Time of Feed (sec)');
ylabel('Tuckerman Coefficient (\kappa)');
figure
x=t;
forceclassical=abs((bodymass.*acceleration));
y=forceclassical;
plot(x,y,'g','Linewidth',2)
title('Classical Kinematic Force Distribution');
xlabel('Time of Feed (sec)');
ylabel('Force (N)');
figure
x=t;
forceclassical=abs((bodymass.*acceleration))
```

2

```
y=forceclassical;
plot(x,y,'g','Linewidth',2)
hold on
x=t;
forceapp=abs((bodymass.*(1+k).*acceleration))
y=forceapp;
plot(x,y, 'r--')
title('Force Distribution with Apparent Mass
Consideration');
xlabel('Time (sec)');
ylabel('Force (N)');
legend('Classical Force Distribution','Added Apparent
 Mass','Location','northwest');
hold off
figure
x=t;
forceclassical=abs((bodymass.*acceleration));
y=forceclassical;
plot(x,y,'g','Linewidth',2)
x=t;
hold on
classical_withmdot=forceclassical+abs((.5*speed.*(massf
```

```
lowrate)));
y=classical_withmdot;
plot(x,y, 'b');
title('Classical Force Distribution vs. Added
Momentum');
xlabel('Time (sec)');
ylabel('Force (N)');
legend('Classical Force Distribution','Classical Force
with
 Momentum','Location','northwest');
hold off
figure
forceclassical=abs((bodymass.*acceleration));
y=forceclassical;
plot(x,y,'g','Linewidth',2)
title('Classical Kinematic Force Distribution');
xlabel('Time of Feed (sec)');
ylabel('Force (N)');
hold on
x=t;
forceapp=abs(((1+k).*bodymass.*acceleration));
y=forceapp;
plot(x,y, 'r--')
title('Force Distribution with Apparent Mass
Consideration');
xlabel('Time (sec)');
ylabel('Force (N)');
x=t;
classical_withmdot=forceclassical+abs(.5*speed.*(massfl
owrate))
y=classical_withmdot;
plot(x,y, 'b');
x=t;
```

3

```
totalforce=forceapp+abs(.5*speed.*(massflowrate.*(1+k)+
k.*bodymass))
x=t;
y=totalforce;
plot(x,y,'--','Linewidth',2,'Color',[1,0,.5])
title('Total Effects of Apparent Mass and Momentum
```

```
Flux');
xlabel('Time (sec)');
ylabel('Force (N)');
legend('Classical Force Distribution','Classical Force
with Apparent
 Mass','Classical Distribution with Added
Momentum','Total Force with
 Momentum Flux','Location','northwest');
hold off
```

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