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Representation of solutions to one-dimensional Schrödinger and perturbed Bessel equations in terms of Neumann series of Bessel functions

Abstract

The new representations of solutions to the one-dimensional SchrÃűdinger equation $-y'' + q(x)y = \omega^2 y$ and to the perturbed Bessel equation $-y'' + \frac{\ell(\ell+1)}{x^2} + q(x)y = \omega^2 y$ are introduced.

For the first equation a pair of linearly independent solutions has the form

$$c(\omega, x) = \cos \omega x + 2\sum_{n=0}^{\infty}(-1)^n \beta_{2n}(x) j_{2n}(\omega x)$$

and

$$s(\omega, x) = \sin \omega x + 2\sum_{n=0}^{\infty}(-1)^n \beta_{2n+1}(x) j_{2n+1}(\omega x),$$

where j_n are the spherical Bessel functions and the coefficients β_j can be calculated by the simple recursive procedure.

For the second equation the regular solution u_ℓ satisfying the asymptotics $u_\ell(\omega, x) \sim x^{\ell+1}$, $x \to 0$ has the form

$$u_\ell(\omega, x) = \frac{2^{\ell+1}\Gamma(\ell + 3/2)}{\sqrt{\pi}\omega^\ell} x j_\ell(\omega x) + \sum_{n=0}^{\infty}(-1)^n \beta_n(x) j_{2n}(\omega x),$$

where the coefficients β_n can be obtained by a similar recursive procedure.

The representations are based on the expansion of the integral kernels of the transmutation operators into Fourier-Legendre series and the recent results obtained by the author jointly with V. V. Kravchenko. It is shown that the partial sums of the series approximate the solutions uniformly with respect to ω. Convergence rate estimates and representations for the derivatives of the solutions are given.

The talk is based on the results obtained with V. V. Kravchenko, Luis J. Navarro and R. Castillo-Perez.

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