Washington University in St. Louis

Washington University Open Scholarship

Mechanical Engineering and Materials Science
Independent Study

Mechanical Engineering & Materials Science

12-19-2016

The Effects of Rarefaction and Thermal Non-equilibrium on a Blunt Body and a Bicone in Hypersonic Flow and their Shape Optimization for Reducing both Drag and Heat Transfer

Samuel Gardner

Washington University in St Louis

Ramesh K. Agarwal

Washington University in St. Louis

Follow this and additional works at: <https://openscholarship.wustl.edu/mems500>

Recommended Citation

Gardner, Samuel and Agarwal, Ramesh K., "The Effects of Rarefaction and Thermal Non-equilibrium on a Blunt Body and a Bicone in Hypersonic Flow and their Shape Optimization for Reducing both Drag and Heat Transfer" (2016). *Mechanical Engineering and Materials Science Independent Study*. 18.
<https://openscholarship.wustl.edu/mems500/18>

This Final Report is brought to you for free and open access by the Mechanical Engineering & Materials Science at Washington University Open Scholarship. It has been accepted for inclusion in Mechanical Engineering and Materials Science Independent Study by an authorized administrator of Washington University Open Scholarship. For more information, please contact digital@wumail.wustl.edu.

The Effects of Rarefaction and Thermal Non-equilibrium on a Blunt Body and a Bicone in Hypersonic Flow and their Shape Optimization for Reducing both Drag and Heat Transfer

Samuel T. Gardner* and Ramesh K. Agarwal †

Washington University in St. Louis, St. Louis, MO. 63130

Design of space vehicles pose many challenging problems due to their hypersonic speeds as they need to travel through different flow regimes due to changes in the density of the atmosphere with altitude. Some of the key characteristics associated with hypersonic flow are extremely high temperatures and heat transfer to the wall of the spacecraft. At these temperatures, the assumption of thermal equilibrium is no longer valid and the effect of rotational non-equilibrium must be included in the modeling of diatomic gas flow. This thesis employs the Navier-Stokes equations, which are modified to include a rotational non-equilibrium relaxation model to analyze the heat transfer, drag, and shock standoff distance for hypersonic flow past an axisymmetric blunt body and a bicone for various levels of rarefaction – including the rotational non-equilibrium effect. The customized flow solver, ZLOW, is used to calculate the numerical solutions for laminar viscous hypersonic flow past a blunt body and a bicone at Knudsen numbers Kn in continuum-transition regime with and without rotational non-equilibrium. The effects of rarefaction in the continuum-transition regime are modeled by applying the Maxwellian velocity slip and temperature jump boundary conditions on the surface. The effects of the rotational non-equilibrium terms are discussed in this thesis for both the continuum ($Kn \approx 0$) and slip flow regime ($Kn \leq 0.1$). In addition, both the blunt body and bicone are optimized in hypersonic, rarefied flow with rotational non-equilibrium by using a multi-objective genetic algorithm (MOGA) for reduction of both drag and heat transfer.

Nomenclature

Kn	Knudsen Number	N_A	Avogadro's number
λ	Mean Free Path	P	Pressure
ρ	Mass Density	U_S	Flow velocity
u	Bulk Velocity	U_W	Wall velocity
E	Total Energy	σ	Momentum accommodation coefficient
e_r	Rotational energy	T_0	Flow temperature
e_{tran}	Translational energy	T_W	Wall temperature
k	Thermal conductivity	α	Thermal accommodation coefficient
τ	Rotational relaxation time	Pr	Prandtl number
Z_R	Average number of collisions	γ	Adiabatic index
T_t	Translational temperature	$NS1T$	Navier-Stokes without thermal non-equilibrium
T_r	Rotational temperature	$NS2T$	Navier-Stokes with thermal non-equilibrium
R	Universal gas constant		
d	Average molecule diameter		

*Undergraduate Research Assistant, Student member AIAA

†William Palm professor of Engineering, Fellow AIAA

I. Introduction

Hypersonic vehicles, such as reentry vehicles, experience high heat loads and increased drag during flight. Reentry vehicles fly at high altitudes where the air is rarefied due to the reduced atmospheric density. Rarefied flow is associated with flow outside of the continuum regime, which many commercial flow solvers can not accurately predict. This makes accurate simulations and meaningful results difficult to obtain; however, getting these results is essential to the design of hypersonic vehicles due to the extreme cost associated with testing in that environment, as can be seen by the high cost associated with NASA's X series of vehicles.¹

In this paper the custom FORTRAN flow solver, ZLOW, is used to study the effects of thermal non-equilibrium in hypersonic rarefied flow. The flow solver employs the Navier-Stokes equations with a translational/rotational thermal non-equilibrium model to analyze viscous, laminar, hypersonic rarefied flow. The effects of thermal non-equilibrium on heat transfer and drag are studied for the axisymmetric blunt body and bicone. The results are from the inclusion of the translational/rotational non-equilibrium terms are discussed for both the continuum regime ($Kn \approx 0$) and the slip flow regime ($Kn \leq 0.1$).

In addition to the effects of thermal non-equilibrium, the axisymmetric blunt body and bicone are optimized using a genetic algorithm to reduce the drag and maximum heat transfer. The blunt body is optimized including the effects of rarefaction ($Kn = 0.01$) and thermal non-equilibrium and is compared to results obtained in literature.² The bicone is optimized for the first time in literature.

II. Hypersonic Flow Models

A. Thermal Rotational Non-equilibrium

The 3-D Navier-Stokes equations for a diatomic gas in rotational thermal non-equilibrium in conservation law form and Cartesian coordinates are expressed as:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} + \frac{\partial \mathbf{E}_v}{\partial x} + \frac{\partial \mathbf{F}_v}{\partial y} + \frac{\partial \mathbf{G}_v}{\partial z} = \mathbf{S} \quad (1)$$

where

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ (\rho E + p)u \\ \rho e_r u \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ (\rho E + p)v \\ \rho e_r v \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ (\rho E + p)w \\ \rho e_r w \end{bmatrix} \\ \mathbf{E}_v &= \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \\ \tau_{zx} \\ q_{tran}^x + q_r^x + u_i \tau_{xi} \\ q_r^x \end{bmatrix}, \quad \mathbf{F}_v = \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{zy} \\ q_{tran}^y + q_r^y + u_i \tau_{yj} \\ q_r^y \end{bmatrix}, \quad \mathbf{G}_v = \begin{bmatrix} 0 \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{zz} \\ q_{tran}^z + q_r^z + u_i \tau_{zk} \\ q_r^z \end{bmatrix} \\ \mathbf{Q} &= \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \\ \rho e_r \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\rho R}{Z_{RT}}(T_t - T_r) \end{bmatrix} \end{aligned} \quad (2)$$

Of particular interest in Eq. 2 are the thermal non-equilibrium terms:

$$e_{tran} = \frac{2}{3}RT_t, \quad e_r = RT_r, \quad k_{tran} = \frac{15}{4}\mu R, \quad k_r = \mu R \quad (3)$$

Parker's formulas⁵ are used to model the rotational relaxation time, τ , and the average number of collisions, Z_R , in the energy exchange that occurs between translational and rotational energies.

$$\tau = \frac{\mu}{p} \quad (4)$$

$$Z_R = \frac{Z_R^\infty}{1 + \frac{\pi^{3/2}}{2} \left(\frac{T_{ref}}{T_t} \right)^{1/2} + \left(\frac{\pi^2}{4} + \pi \right) \left(\frac{T_{ref}}{T_t} \right)} \quad (5)$$

The constants are obtained from Parker's formulas: $Z_R^\infty = 15.7$ and $T_{ref} = 80 \text{ K}$.⁵

B. Rarefaction

As the atmospheric density is reduced the flow becomes rarefied. The degree of rarefaction is characterized by the Knudsen number:

$$Kn = \frac{\lambda}{L} \quad (6)$$

Where λ is the mean free path of the flow and L is the characteristic length associated with the geometry. The mean free path is the average distance traveled by a particle in the flow between collisions. The mean free path is determined by:

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P} \quad (7)$$

Where R is the universal gas constant, T is the free stream temperature, d is the average molecule diameter, N_A is Avogadro's number and P is the free stream pressure.

The flow regime is determined by the degree of rarefaction. For low Knudsen numbers ($Kn \approx 0$) the flow is in the continuum regime. Many flow equations, such as the Navier-Stokes, are only valid in the continuum regime. As rarefaction increases ($Kn \geq 0.01$) the flow enters the transitional regime. A subsection of the transitional regime is the slip regime, $Kn \leq 0.1$. In the slip regime the equations used to model continuum flow can be modified to obtain accurate solutions.⁴ This is done by adding Maxwell's slip boundary conditions at the wall.

Maxwell's boundary conditions are comprised of velocity slip and temperature jump terms.³ The velocity slip boundary condition can be expressed as:

$$U_s - U_w = \left(\frac{2 - \sigma}{\sigma} \right) \lambda \frac{\partial u_x}{\partial n} \quad (8)$$

Where U_s is the velocity of the flow, U_w is the velocity at the wall, σ is the momentum accommodation coefficient, λ is the local mean free path, and $\frac{\partial u_x}{\partial n}$ is the velocity gradient normal to the wall.

The temperature jump condition is expressed similarly:

$$T_0 - T_W = \left(\frac{2 - \alpha}{\alpha} \right) \frac{2\gamma}{(\gamma + 1)Pr} \lambda \frac{\partial T}{\partial n} \quad (9)$$

Where T_0 is temperature of the flow, T_W is the wall temperature, α is the thermal accommodation coefficient, γ is the adiabatic index, and Pr is the Prandtl number.

III. Genetic Algorithm Overview

Genetic algorithms are a class of stochastic optimization algorithms inspired by evolution, where the fittest individuals survive and reproduce to create new populations. In a genetic algorithm a set of individuals are iterated over, combining traits from the best performers until a unique, optimized shape is converged upon. A genetic algorithm follows five steps:

- Initialization: Randomly population size N is generated
- Evaluation: Each individual is assigned a fitness based on optimization parameters

- Natural Selection: Remove individuals with poor fitness
- Reproduction: Create a new set of individuals
- Convergence Check

A multi-objective genetic algorithm determines the fitness of an individual based on two or more objective parameters. In some cases, such as optimizing hypersonic flow for reduction of drag and maximum heat transfer, the optimization parameters are in conflict with each other. When this is the case the weighting of the objective parameters creates a unique solution. The steps for the shape optimization process are detailed in figure 1.

Figure 1: Genetic algorithm flowchart

The shape of an individual is generated using Bezier curves. Bezier curves are parametric curves that are defined by a finite number of control points. Bezier curves are ideal for genetic algorithms due to the ease of sharing curve characteristics through the exchange of control point information. In the initialization step the algorithm creates a random set of control points that are parameterized and loaded into a mesh generating script. The evaluation step takes the individuals and runs the CFD simulation to obtain the objective values. Once all of the individuals of a generation have been simulated, they are given a fitness value based on their performance in the weighted objective functions relative to the other individuals of the generation. The natural selection step takes the top half of the generation and labels them survivors. The survivors are allowed to reproduce to populate a new generation by using a skewed random draw to select two survivors that share information using a crossover and mutation function. Once the new generation has been populated the process repeats until convergence. Convergence is generally defined as the objective values having an improvement of less than 1% across three consecutive generations. Each weighted function converges to a single unique optimized geometry.

IV. Results

A. Axisymmetric Blunt Body NS1T vs NS2T

In this section the results of computations for hypersonic flow past the 3-D, axisymmetric blunt body in air at four Knudsen numbers (0.002, 0.01, 0.05, 0.10) using the Navier-Stokes equations with (NS2T) and without (NS1T) rotational non-equilibrium are presented. Maxwell's slip boundary conditions are employed for all cases. The blunt body geometry and mesh can be found in figure 2. The leading edge radius is 6.35mm and a segmented (35x105x10)x400 mesh is used based on the mesh validation study presented by Seager.²

The free stream flow conditions were taken from Lobb's experiments and are detailed below:

$$Ma = 7.1, T_{\infty} = 293K, T_W = 1000K, L = 0.0127m, \gamma = 1.4, Pr = 0.72 \quad (10)$$

Figure 2: Detailed mesh and geometry for the axisymmetric blunt body

The free stream density and pressures were calculated using Eqs. 6 and 7 based on the desired Knudsen numbers. Table 1 details these conditions.

Table 1: Free stream parameters for blunt body simulations

Blunt Body Free Stream Parameters			
Knudsen Number	Pressure (Pa)	Mean Free Path (m)	Characteristic Length
0.002	245.595	2.54E-5	.0127
0.01	49.854	1.27E-4	.0127
0.05	9.824	6.35E-4	.0127
0.1	4.912	1.27E-3	.0127

Figure 3 shows the comparison of Mach contours obtained using the NS1T and NS2T equations. In all contour figures the NS1T results are on the top half of the contour plot and the NS2T results are on the bottom half. From figure 3 it is seen that the inclusion of rotational non-equilibrium has a large effect on the prediction of bow shock structure, especially as rarefaction increases. The NS2T equations predict a much more diffuse bow shock structure with a larger standoff distance.

To analyze the degree of thermal non-equilibrium present in the NS2T case the translational and rotational temperatures are plotted at the stagnation line in figure 4. It is seen that the shock imparts the thermal non-equilibrium. As expected, there is a large spike in translational temperature immediately after the shock. In the continuum regime the rotational temperature equilibrates shortly after the shock region. As the degree of rarefaction increases, the molecules in the flow decrease which decreases the number of collisions. Since the translational and rotational energies depend on collisions to equilibrate, the thermal non-equilibrium exists to a larger degree with the decreased flow density. At $Kn = 0.01$ the flow equilibrates in the boundary layer, $Kn = 0.05$ reaches equilibrium at the wall and $Kn = 0.1$ never returns to equilibrium.

The effect that thermal equilibrium has on the surface properties drag and heat transfer are detailed in figures 5 and 6. It can be seen that a lower pressure coefficient is predicted in the NS2T case. This effect is amplified as rarefaction increases. The heating coefficient, shown in 6 shows a similar trend. The heating coefficient increases as rarefaction increases; however, the NS2T predicts a lower heating coefficient across the leading edge for each case. Similarly to the pressure coefficient, this effect is amplified as rarefaction

Figure 3: Mach field for Mach 7.1 flow about an axisymmetric blunt body

Figure 4: Translational and rotational temperatures at the stagnation line for Mach 7.1 flow about an axisymmetric blunt body with thermal non-equilibrium

increases.

Figure 5: Pressure coefficient on the surface of an axisymmetric blunt body in Mach 7.1 flow

The results for drag coefficient, total drag, maximum heat transfer and standoff distance for each case are detailed in tables 2 and 3 for the NS1T and NS2T cases respectively. Table 4 shows the difference in predicted values for the parameters of interest. It is seen that in the continuum regime NS1T and NS2T predict similar results, with the largest difference coming in the shock standoff distance. As rarefaction increases the predicted values begin to diverge. Thermal non-equilibrium has the largest effect on the predictions for maximum heat transfer and shock wave standoff distance. At the limit of the modified Navier-Stokes validity, $Kn = 0.10$, NS2T predicts a shock wave standoff distance 33.8% larger than NS1T and maximum heat flux 7.6% lower. It can be determined from this that the inclusion of thermal non-equilibrium terms are significant for modeling flow in the slip regime.

B. Axisymmetric Bicone NS1T vs NS2T

In this section the results of computations for hypersonic flow past the 3-D, axisymmetric bicone in Nitrogen at four Knudsen numbers (0.0013, 0.01, 0.057, 0.10) using the Navier-Stokes equations with (NS2T) and without (NS1T) rotational non-equilibrium terms are presented. Maxwell's slip boundary conditions are employed for all cases. The bicone geometry and mesh can be found in figure 7.

The free stream flow conditions, obtained from literature,⁶ are listed below:

$$Ma = 15.6, T_{\infty} = 42.6, T_W = 297, L = 0.09208m, \gamma = 1.4, Pr = 0.72 \quad (11)$$

Figure 6: Heat transfer on the surface of an axisymmetric blunt body in Mach 7.1 flow

Table 2: Blunt Body Results with Thermal Equilibrium - NS1T

NS1T - Thermal Equilibrium Results						
Kn	P_{∞} (Pa)	Density ($\frac{kg}{m^3}$)	Cd	Drag (N)	Qmax ($\frac{W}{m^2}$)	Standoff (m)
0.002	245.595	2.92E-3	0.9622	1.0563	1.48E+06	8.67E-4
0.01	49.85	5.84E-4	1.161	0.2549	6.10E+05	1.004E-3
0.05	9.824	1.17E-4	1.517	0.06662	2.69E+05	1.2444E-3
0.1	4.912	5.839E-05	1.722	0.037809	1.79E+05	1.39E-3

Table 3: Blunt Body Results with Thermal non-equilibrium - NS2T

NS2T - Thermal non-equilibrium Results						
Kn	P_{∞} (Pa)	Density ($\frac{kg}{m^3}$)	Cd	Drag (N)	Qmax ($\frac{W}{m^2}$)	Standoff (m)
0.002	245.595	2.9196E-3	0.9579	1.0516	1.44E+06	9.183E-4
0.01	49.85	5.839E-4	1.134	0.2494	5.96E+05	1.08E-3
0.05	9.824	1.168E-4	1.447	0.06354	2.59E+05	1.549E-3
0.1	4.912	5.839E-05	1.667	0.03660	1.65E+05	1.86E-3

Table 4: Percent Difference Between NS1T and NS2T Simulations for Given Knudsen Numbers

Percent Difference of Objective Values			
Knudsen Number	Drag	Qmax	Standoff Distance
0.002	0.45	2.29	-5.92
0.01	2.15	2.33	-7.57
0.05	4.61	3.69	-24.45
0.1	3.19	7.63	-33.82

Figure 7: Axisymmetric Bicone Geometry and Mesh

The free stream density and pressures were calculated using Eqs.6 and 7 based on the desired Knudsen numbers. Table 5 details these conditions for each case analyzed.

Table 5: Free stream parameters for bicone simulations

Bicone Free Stream Parameters			
Knudsen Number	Pressure (Pa)	Mean Free Path (m)	Characteristic Length
0.0013	2.227	1.20E-4	.09208
0.01	0.8148	9.21E-4	.09208
0.057	0.143	5.25E-3	.09208
0.1	0.07985	9.21E-3	.09208

Figure 8 details the Mach contours for the axisymmetric bicone obtained using the NS1T and NS2T equations. As was the case with the blunt body, NS1T is the top contour and NS2T is the bottom contour. Figure 8 shows that the shock remains attached to the fore cone in all cases, as expected for a sharp nosed hypersonic geometry. When the flow reaches the corner region of the bicone, a bow shock forms. This bow shock becomes more pronounced as Knudsen number increases. Similarly to the blunt body, the shock becomes much thicker and further from the wall as rarefaction increases.

Figure 8: Mach field for Mach 15.6 flow about an axisymmetric bicone

The effects of thermal non-equilibrium are highest along the surface, as can be seen in figure 9. The leading tip of the bicone produces a very large temperature inequality that increases in magnitude as rarefaction increases. It is also worth noting that the NS1T predicts a lower translational temperature than NS2T along the surface of the bicone in all cases.

Figure 10 details the pressure coefficient across the surface of the bicone. For flow in or near the continuum regime there is little difference in the pressure coefficient predicted by NS1T and NS2T. At higher degrees

of rarefaction the predictions from NS1T and NS2T begin to diverge. NS2T predicts a much lower peak pressure at the fore cone aft cone juncture. This region is of the most interest as it is the largest contribution of drag.

The heating coefficient is plotted in figure 11 and is the largest indicator of the thermal non-equilibrium effects. Even when flow remains in the continuum regime, the leading point heat transfer rate is lower in the NS2T model than the NS1T. As rarefaction increases into the slip regime the leading edge plays a much smaller role in the maximum heat transfer rates. The bicone corner becomes the dominant region for heat transfer as the flow progresses into the slip flow regime. In this region the NS2T equations again predict a lower value of heating coefficient than the NS1T equations.

Figure 9: Surface translational and rotational temperature for Mach 15.6 flow about an axisymmetric bicone with thermal non-equilibrium

The parameters of interest for the axisymmetric bicone simulations are listed in tables 6 and 7 for the NS1T and NS2T cases respectively. The effects of thermal non-equilibrium make the largest impact at the limit of the validity of the equations, $Kn = 0.1$. The effects of thermal non-equilibrium on drag are less prominent than with the blunt body; however, the effects of heat transfer are much greater than the blunt body. At the largest Knudsen number the NS2T equations predict a maximum heat flux 13.6% lower than NS1T equations. From Table 8 it can be determined that the effects of thermal non-equilibrium are significant and should be considered when dealing with flows in the slip region of the transition flow regime.

Figure 10: Pressure coefficient for Mach 15.6 flow about an axisymmetric bicone

Table 6: Bicone results with thermal equilibrium

Thermal Equilibrium Results					
Kn	P_∞ (Pa)	Density ($\frac{kg}{m^3}$)	Cd	Drag (N)	Q_{max} ($\frac{W}{m^2}$)
0.0013	2.227	1.76E-4	2.209	30.942	1.28E5
0.01	0.815	6.45E-5	2.246	11.511	6.25E4
0.057	0.143	1.13E-5	2.321	2.0876	1.55E4
0.1	0.0799	6.32E-6	2.392	1.2014	9.44E3

Table 7: Bicone results with thermal equilibrium

Thermal non-equilibrium results					
Kn	P_∞ (Pa)	Density ($\frac{kg}{m^3}$)	Cd	Drag (N)	Q_{max} ($\frac{W}{m^2}$)
0.0013	2.227	1.76E-4	2.209	30.942	1.30E5
0.01	0.815	6.45E-5	2.236	11.460	6.25E4
0.057	0.143	1.13E-5	2.314	2.0813	1.36E4
0.1	0.0799	6.32E-6	2.351	1.1808	8.16E3

Figure 11: Surface heating coefficient for Mach 15.6 flow about an axisymmetric bicone

Table 8: Percent difference between bicone 1T and 2T simulations for given Knudsen numbers

Percent Difference of Objective Values		
Knudsen Number	Drag	Qmax
0.0013	0	-2.12
0.01	0.445	0.077
0.057	0.302	12.47
0.1	1.71	13.61

C. Optimization of Axisymmetric Blunt Body

The axisymmetric blunt body is optimized at $Kn = 0.01$ using the modified Navier-Stokes equations with Maxwell's slip boundary conditions with rotational non-equilibrium. The optimization was done with evenly weighted importance on the reduction of drag and maximum heat transfer. The results were then compared to the evenly weighted results obtained by Seager to analyze how the addition of rarefaction and thermal non-equilibrium change the optimization results.

Figure 12 details the optimization results from this study (blue line), the original geometry (green line) and the optimization obtained in the literature. The inclusion of rarefaction and thermal non-equilibrium produced a slightly larger leading edge than the result from literature. Table 9 shows the percent reduction in the objective values from this optimization. It is expected that the drag will benefit more than the heat transfer for the blunt body due to the large leading radius of the original geometry. Drag was reduced by 16.22% and heat transfer was reduced by 7.78%. The optimization obtained in the literature found a reduction of 22% for drag and 13% reduction in heat transfer. These results suggest that when rarefaction and thermal non-equilibrium are included in the model the optimization becomes less effective.

Figure 12: Optimized blunt body geometry vs literature

D. Optimization of Axisymmetric Bicone

The optimization of a 3-D axisymmetric bicone is a novel problem that has not been investigated prior to this study. Due to the relative complexity of the geometry and the increased computational resources

Figure 13: Optimized blunt body geometry vs original geometry Mach contours

Figure 14: Optimized blunt body geometry vs original geometry pressure coefficient

Figure 15: Optimized blunt body geometry vs original geometry heating coefficient

Table 9: Blunt body optimization objective value improvements

Blunt Body Optimization Percent Improvement			
Objective Value	Original	Optimized	Improvement (percent)
C_D	1.136	0.952	16.22
Qmax	5.96E5	5.50E5	7.78

required for the simulation and optimization of the bicone in comparison to the blunt body, the bicone is only optimized for an evenly weighted case of reducing drag and heat transfer in the continuum regime with thermal non-equilibrium.

The free stream parameters used in the optimization algorithm are the same as was detailed previously in table 7 for $Kn = 0.0013$ including thermal non-equilibrium. The optimized geometry can be seen in figure 16. The blue line is the optimized shape, the red line is the initial geometry and the green line are the bezier control points. The leading edge of the optimized bicone has become rounded, which helps reduce the heat transfer. It was seen in figure 11 that the other primary area of high heat transfer is right after the corner, this region of the optimized shape has become rounded as well. Figure 19 details how heat flux is improved by the optimized geometry. The original geometry had a peak heat transfer rate at the leading edge. The rounding of the leading edge of the bicone shows the heat flux is only one third of the value of the original geometry.

The optimization algorithm obtained a 4.21% reduction in drag and 48.18% reduction in maximum heat transfer. It is expected for the sharp leading edge of the bicone to have higher potential for the reduction of heat transfer than drag coefficient. The results, which have never before been presented in literature, show that bicone geometry can have significant reduction in the heat transfer in order to make it a more desirable geometry for hypersonic vehicle design.

Figure 16: Optimized bicone geometry

Figure 17: Optimized bicone geometry vs original geometry Mach contours

Figure 18: Optimized bicone geometry vs original geometry pressure coefficient

Figure 19: Optimized bicone geometry vs original geometry heating coefficient

Table 10: Bicone optimization objective value improvements

Bicone Optimization Percent Improvement			
Objective Value	Original	Optimized	Improvement (percent)
Drag	30.94	29.65	4.21
Qmax	1.30E5	6.76E4	48.18

V. Conclusion

This study had three primary objectives:

- Quantify the effect of both rarefaction and thermal translational/rotational non-equilibrium on the simulation of a blunt body and bicone in hypersonic flow
- Analyze the effect of rarefaction and thermal non-equilibrium on the optimization of a blunt body in hypersonic flow for the reduction of drag and heat transfer
- Optimize the bicone in hypersonic flow for the reduction of drag and heat transfer

The Navier-Stokes equations were modified to include Maxwell's slip boundary conditions, in order to extend viability through the slip regime, and rotational non-equilibrium terms. The simulations were run in a custom Fortran solver, ZLOW. The blunt body and bicone were run with and without thermal non-equilibrium terms for varying degrees of rarefaction. It was determined that the magnitude of thermal non-equilibrium increased as the degree of rarefaction increased, making it an important consideration to the computation of hypersonic rarefied flow.

The blunt body was optimized for the reduction of drag and maximum heat flux using a multi-objective genetic algorithm and was compared to results obtained in the literature. The optimization obtained in this study predicted a slightly larger leading edge radius than the results in literature. An important note is that the effectiveness of the optimization process was reduced with the increased rarefaction.

The primary contribution of this study was the optimization of the 3-D axisymmetric bicone in hypersonic thermal non-equilibrium for the reduction of both drag and heat transfer. This geometry optimization has not been presented previously in literature. The optimization was done for flow in the continuum regime. The optimization algorithm was able to obtain a 48% reduction in maximum heat flux and a 4% reduction in drag. Future work will be done to create a pareto optimal front for the bicone and to investigate how rarefaction changes the effectiveness of the optimization process.

Acknowledgments

The first author gratefully acknowledges the NASA Space Grant for their generosity in support of this research. The work of both Wenwen Zhao and Christopher Seager is recognized specially, their work and guidance were instrumental to the completion of this research.

References

- ¹Harsha, P.T., Keel, L.C., Castrogiovanni, A., and Robert T. Sherrill, "X-43A Vehicle Design and Manufacture." *AIAA Paper No. 2005-3334*, 2005.
- ²Seager, C., Agarwal, R.K., "Shape Optimization of an Axisymmetric Blunt Body in Hypersonic Flow for Reducing Drag and Heat Transfer." *Journal of Thermophysics and Heat Transfer*, Online, Sept.2015.
- ³Maxwell, J.C., "On Stresses in Rarefied Gases Arising from Inequalities of Temperature." *Philosophical Transactions of the Royal Society of London*, 170:231-256, Jan. 1879.
- ⁴Lofthouse, A., "Nonequilibrium Hypersonic Aerothermodynamics Using the Direct Simulations Monte Carlo and Navier-Stokes Models." Ph.D Thesis, University of Michigan, 2008.
- ⁵Parker, J.G., "Rotational and Vibrational Relaxation in Diatomic Gases." *Physics of Fluids*, 2:449-462, 1959.
- ⁶Zhao, W., Chen, W., and Agarwal, R., "COMputation of Rarefied Hypersonic Flows Using Modified Form of Conventional Burnett Equations." *Journal of Spacecraft and Rockets*, 52:789-803, Feb. 2015.