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Optical Interfference I

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Using Huygens' Principle to Predict Optical Interference

The Problem

Maxwell provided a formal approach to determining how light moved through an apparatus: Just solve a set of partial differential equations with the appropriate bounday conditions.

At the time Maxwell deduced his equations of motion for the electric and magnetic fields, however, the problem of understanding how light moves through optical instruments already had a long history. I. Newton had argued that a light beam was simply a stream of particles. C. Huygens* and Robert Hooke had argued that light was a wave phenomenon. In fact, Thomas Young showed in 1803 - the famous double slit experiment using light from the sun - that light *was* a wave phenomena. His argument - adapting Huygens understanding - is more intuitively attractive than Maxwell's formal analysis and in most cases agrees essentially with experiment. So I reproduce here a description of the double slit experiment using Huygens' argument.

The basic argument

A monchromatic wave will have an amplitude and a phase at all points in space. We identify the locus of points where the phase is the same as a *wave front* of the wave. This will be a point in a 1D space, a curve in a 2D space, and a surface in a 3D space. Huygens argued that we could deduce where a wave would go if we knew the wave's amplitude on some wave front at a particula time. Simply assume that spherical waves are emitted from all the points on the wave front with amplitudes that are proportional the amplitude of the wave on the wave front and a phase that is the same as the phase on the wave front.

If all these waves are superimposed at some observation point, the fact that the distances between the observation point and the different points on the wave front differ means that there is interference between the various waves. When this interference is resolved, we learn the amplitude and phase of the wave at the new observation point.

Huygens used this argument to deduce Snell's law, to design lenses for telescopes, etc. Fresnel used the same arguement to explain interference phenomena that he had observed.

* Christiaan Huygens (1629-1695) was a prominent Dutch mathematician and scientist.

Well, Eq.(3) describes a wave that goes through $\boldsymbol{r}$. Summing over all routes through the slit gives a wave

$$\int_{-\Delta/2}^{\Delta/2} dx \, \frac{1}{|x\,\hat{\imath} - \boldsymbol{R}_1|} \, \frac{1}{|\boldsymbol{R}_2 - x\,\hat{\imath}|} \, e^{\frac{2\pi i}{\lambda}\left\{|x\,\hat{\imath} - \boldsymbol{R}_1| + |\boldsymbol{R}_2 - x\,\hat{\imath}|\right\}} \tag{4}$$

In the case at hand, x will range over a distance $\sim \lambda$ while $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ will be $\gg \lambda$. This means that we can expand this result in x and neglect terms that are of the order of x/R_i, $i = 1, 2$.

Expanding

$$|\boldsymbol{R} - x\,\hat{\imath}| = \sqrt{(X - x)^2 + Y^2 + Z^2}$$

in x gives

$$|\boldsymbol{R} - x\,\hat{\imath}| = R - \frac{X}{R}\,x + \frac{R^2 - X^2}{2R^3}\,x^2 + \cdots$$

If $X = R\cos\theta$, this reduces to

$$|\boldsymbol{R} - x\,\hat{\imath}| = R - x\cos\theta + \frac{\sin^2\theta}{2R}\,x^2 + \mathcal{O}\left(\frac{1}{R^2}\right) \tag{5}$$

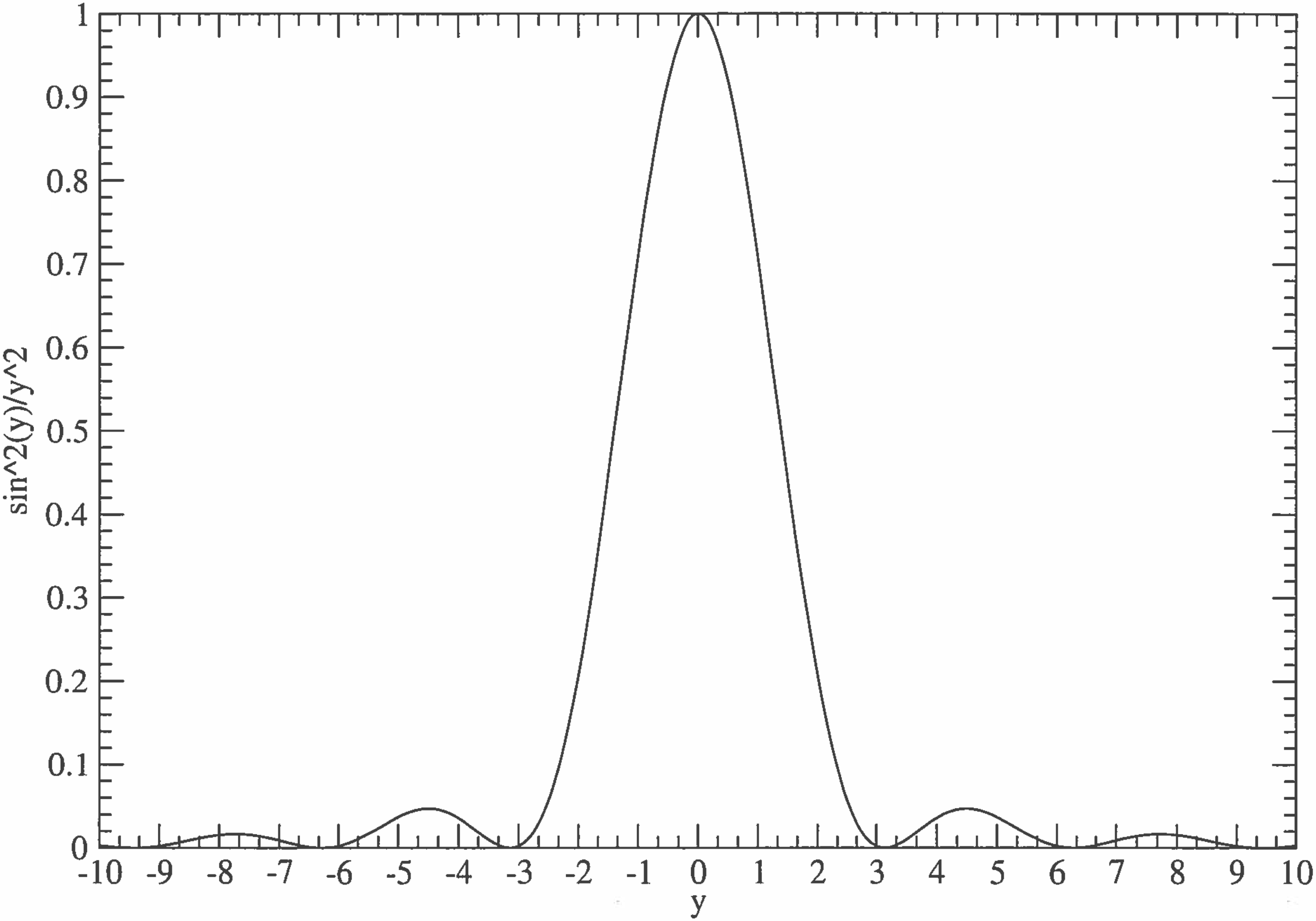
Neglecting terms of the order or x/R_i, Eq.(4) reduces to

$$\frac{e^{\frac{2\pi i}{\lambda}(R_1 + R_2)}}{R_1\,R_1} \int_{-\Delta/2}^{\Delta/2} dx \, e^{-2\pi i\, x\, (\cos\theta_1 + \cos\theta_2)/\lambda} \tag{6}$$

The intensity of the light at $\boldsymbol{R}_2$ is proportional to the square of the field intensity at $\boldsymbol{R}_2$. With the complex representation for the field in Eq.(6), the intensity is simply the expression in Eq.(6) times it's complex conjugate. After evaluating the integrals explicitly, we get

$$I(\boldsymbol{R}_2) = \frac{\mathcal{C}}{R_1^2\,R_2^2} \iint_{-\Delta/2}^{\Delta/2} dx\,dx' \, e^{-2\pi i (x - x')(\cos\theta_1 + \cos\theta_2)/\lambda} \tag{7}$$

$$= \frac{\mathcal{C}\,\Delta^2}{R_1^2\,R_2^2} \frac{\sin^2(\pi\,b\,\Delta)}{(\pi\,b\,\Delta)^2} \tag{8}$$


sin^2(y)/y^2
y
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10