On the Value of Operational Flexibility in Anheuser Busch InBev’s Trailer Loading and Shipment Problem: Data-Driven Approaches and Reinforcement Learning

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On the Value of Operational Flexibility in Anheuser Busch InBev’s Trailer Loading and Shipment Problem: Data-Driven Approaches and Reinforcement Learning

by

Yunsi Yang

A dissertation presented to
The Graduate School
of Washington University in
partial fulfillment of the requirements for the degree of Doctor of Business Administration

May 2022
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Yunsi Yang

Washington University in St. Louis

May 2022
Dedicated to my parents and grandparents.
ABSTRACT OF THE DISSERTATION

On the Value of Operational Flexibility in Anheuser Busch InBev’s Trailer Loading and Shipment Problem: Data-Driven Approaches and Reinforcement Learning

by Yunsi Yang

Doctor of Business Administration in Supply Chain, Operations, and Technology

Washington University in St. Louis, 2022

Professor Panos Kouvelis, Chair

This paper introduces the Anheuser Busch Inbev (ABI)'s trailer shipment problem and suggests data-analytics methodologies to deal with it. The problem is to determine the proper weight of products loaded on a trailer (a transporting container) owned by ABI, which is delivered by a tractor (a motor vehicle) owned by third-party logistics (3PL) providers. ABI must meet a regulation that restricts the gross weight of the truck on the road. However, the challenge comes from the fact that the tractor weight is uncertain and unknown to ABI when it determines the load size on the trailer. We suggest machine learning based methodologies that compute the optimal load size (weight) of a trailer to minimize the associated cost. Furthermore, we propose a dynamic trailer assignment methodology using reinforcement learning, which further reduces the cost dramatically. Using the transaction-level shipping data obtained from ABI, the suggested methodologies are evaluated. This work introduces a general context of the trailer shipment
problem and suggests efficient data-analytics approaches to it, which can be widely applied in diverse industries associated with 3PL and has significant economic implications
1. Introduction

This paper introduces trailer shipment problem faced by ABI, and suggests data-analytics methodologies to effectively deal with it. The trailer shipment problem is to determine the proper weight of products loaded on a trailer, while meeting the gross weight limit regulation. The U.S. Department of Transportation (USDOT) imposes a regulation on the gross vehicle weight of commercial motor vehicles to reduce the risk of damaging roads and bridges (USDOT Regulatory Relief Fact Sheet 2014). As per the regulation, the gross vehicle weight is limited to 80,000 pounds at maximum for a commercial motor vehicle. This issue was initially brought to our attention by a consulting project for ABI. The company is a leading beverage manufacturer that manages more than ten production plants and distributes a large variety of products through multiple distribution centers throughout the United States. The company uses various transportation modes to deliver its products to customers and the majority of them is done by highway transportation (i.e., trucks). Consequently, the company is highly subject to the weight regulation, and thus checks the weight of every shipment before it leaves the plant to ensure the gross weight limit.

The major challenge of the trailer shipment problem is caused by the company’s logistics operations. ABI outsources most of the delivery operations to 3PL providers. 3PL is generally referred to as the use of other businesses for outsourcing distribution and fulfillment of products or services. As commonly recognized, outsourcing to 3PL providers helps improve operational efficiency, cost control, reliability, and speed (Ton and Wheelwright 2005). Companies can also benefit from economies of scale, capital savings, and financial risk reduction (Vasiliauskas and Jakubauskas 2007). For these reasons, many manufacturing companies strategically outsource their logistics tasks to 3PL providers. Approximately 80% of Fortune 500 companies and 96% of Fortune 100 companies rely on 3PL providers (Menner 2015). Furthermore, the 3PL market has seen fast growth. In the United State alone, the revenue of 3PL services reached $157.2 billion in 2014, increased by 7.4% compared to 2013 (Baskin 2015). This is even faster than the 2.8% growth of overall logistics spending (Baskin 2015).
In the logistics process, ABI and its 3PL providers take different responsibilities. A commercial motor vehicle consists of two parts: a tractor and a trailer. The tractor is the body of the motor vehicle, which has power to move, and is owned by the 3PL providers. The trailer is a container loaded with products, which is owned by ABI. Beverage products are loaded on trailers by ABI. Then, the 3PL providers later send tractors to ABI’s plants (breweries) to pick up and deliver the trailers to customers (wholesalers). Since it takes time to load products on a trailer, ABI pre-loads the trailers in advance, typically 4 to 48 hours earlier than the scheduled arrival time of tractors. This operating practice results in a challenge in the trailer shipment problem. The major difficulty is because of the fact that a tractor weight varies and is not predictable at the time when the company pre-loads the trailer. The randomness is caused by various factors such as physical difference of tractors, drivers’ equipment/belongings, fuel level, etc. Furthermore, it is practically impossible to ask 3PL provides to commit to a tractor type or share the tractor weight information in advance. Hence, it is challenging to pre-load trailers to exactly meet the maximum gross weight limit. There exists a trade-off in loading trailers. If the company pre-loads too much product, then it is likely to exceed the gross weight limit, leading to scaleback. Scaleback is a return of a tractor that hooks a loaded trailer to the loading area (the docking station) for adjustment (i.e., unloading product from the trailer). On the other hand, if the company pre-loads too little product, then it cannot fully utilize the trailer capacity and thus incurs some opportunity cost. This paper suggests data-analytics methodologies that minimize such mismatching costs.

The trailer shipment problem has a significant economic impact for ABI. The company delivers a large number of shipments through a huge logistics network everyday. One of the company’s breweries in California is investigated in this paper. The brewery is a mid-size facility in terms of a shipment volume, which daily handles about 30 to 40 containers. The shipping cost includes fees paid to the 3PL providers for delivery, the reworks, and the opportunity costs. Since ABI has multiple plants each of which delivers a large number of trailers, the mismatching cost is significant for the company. In this paper, we demonstrate that ABI can considerably reduce the cost by using data-analytics methodologies that efficiently deal with the trailer shipment problem.
<table>
<thead>
<tr>
<th>Transportation Mode</th>
<th>Value (billion $)</th>
<th>Tons (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highway (trucking)</td>
<td>10,132 (↑62.5%)</td>
<td>8,060 (↑2.8%)</td>
</tr>
<tr>
<td>Rail</td>
<td>473 (↑52.2%)</td>
<td>1,629 (↓13.1%)</td>
</tr>
<tr>
<td>Water</td>
<td>302 (↑237.6%)</td>
<td>576 (↓15.4%)</td>
</tr>
<tr>
<td>Air</td>
<td>451 (↑70.1%)</td>
<td>5 (↑28.9%)</td>
</tr>
<tr>
<td>Pipeline</td>
<td>543 (↑263.9%)</td>
<td>636 (↓7.2%)</td>
</tr>
</tbody>
</table>

Source: U.S. Department of Transportation, National Transportation Statistics 2018.

Table 1  U.S. Domestic Freight for 2012 (% change, 2002-2012)

Even though the trailer shipment problem is motivated by ABI’s logistics operations, the problem itself is quite general and thus can be adopted by many other companies in different industries. In fact, a lot of companies rely on 3PL providers for delivering their products to customers. Such companies are also subject to the gross weight limit regulation, and thus need to deal with the trailer shipment problem. Besides, trailer shipment problem is a crucial issue in highway transportation that is the most widely used transportation mode in various industries. According to National Transportation Statistics 2018 by the U.S. Department of Transportation, highway transportation (i.e., trucking) is dominant logistics mode in the U.S. Table 1 compares item values and tons of major transportation modes for the U.S. domestic freight in 2012. Over 10 trillion in goods were transported through trucking in 2012, which accounts for 85% of the total value of goods transported in the year (Figure 1). In terms of weight, more than 8 trillion tons of goods were transported via trucking in 2012, which accounts for 75% of the total weight of goods transported in the year (Figure 1). Highway transportation has seen a huge growth—it has increased by 62.5% (in value of goods) compared to 2002. Such high demand drove the cost to rapidly increase. The motor carrier cost increased by 7.8%, reaching $641 billion for U.S. businesses in 2017 (Phillips 2018). Considering the extensive use of the highway transportation (trucking) in diverse industries, the trailer shipment problem has high economic implications.
In this paper, we propose to solve the trailer shipment problem in two stages from static to dynamic. In the static stage, we apply nonparametric data-analytics methodologies, Sample Average Approximation (SAA) and Empirical Risk Minimization (ERM), on historical data to optimize trailer weight load. In the dynamic stage, we train Reinforcement Learning (RL) model to use results from the static stage and real-time information to dynamic assign trailers. SAA is a well-known nonparametric data-driven approach to solving stochastic optimization problems. This approach uses the empirical distribution of tractor weight based upon the sample data and approximates the expected mismatching cost to be minimized by determining the optimal weight of a loaded trailer. ERM is a machine learning principle that incorporates influential features that affect the uncertain factor (i.e. tractor weight), and utilizes the relationship in estimating the expected mismatching cost (empirical risk) to be minimized. Hence, the optimal trailer weight is given as a function of the influential factors. ERM enables better prediction of uncertainty of tractor weight in the problem. Analyzing the data obtained from ABI, we identify delivery distance of shipments as an influential feature that is significantly related to tractor weight. After the above methodologies determining the pre-loaded target trailer weight, we conjectured that diversifying trailer weights (loaded trailer groups having different weights) and dynamically assign such trailers based upon the tractor weight information at arrival may further reduce the cost. In fact, the assignment problem is modeled by Markov decision process (MDP). We develop a algorithm, named Dynamic
Trailer Assignment (DTA), utilizing Reinforcement Learning (RL) that can be used to efficiently solve MDP problems. Specifically, we apply the non-deterministic Q-Learning, a model free RL algorithm where transition probability distribution and reward function are unknown and have probabilistic outcomes. Given the weight information of an arriving tractor, DTA dynamically determines what type of trailers should be assigned to the arriving tractor.

Using the transaction-level shipment data obtained from ABI, we evaluate the data-analytics approaches suggested above. We compare the average mismatching costs (the sum of scaleback and opportunity costs) in a monthly base. The numerical experiment reveals that ERM outperforms SAA in the static stage since the identified feature (distance) enables to better predict the uncertainty of tractor weights and customize the trailer load size accordingly. Furthermore, our experiment reveals that adding the dynamic step (DTA) after solving static Trailer Weight-load Optimization problem can significantly reduce more cost than both SAA and ERM. We identify two factors that drive the improvement: diversification and dynamics. First, diversifying trailer weight (i.e. trailer groups having different load weights) enables to take flexible actions (i.e., assigning different weights of trailers) to improve the performance. Second, dynamically assigning trailers depending on the actual weight of an arriving tractor enables to choose the optimal trailer type (i.e. weight). Furthermore, we investigate the performance bound in terms of infinite trailer inventory and perfect information on tractor weight. Surprisingly, DTA rapidly approaches such performance bounds by adding just a few more inventory (i.e. the number of trailers in each group).

The contributions of this work are summarized as follows. First, this work introduces a two-stage approach from Weight-load Optimization in the static stage to Dynamic Assignment that can be widely applied in different industries and also has a significant economic impact. Second, this paper proposes practical data-analytics methodologies to efficiently solve the problem. Especially, we propose a RL-based methodology (DTA) that dynamically incorporates the actual tractor weight information available upon its arrival, which significantly outperforms the other static methodologies (SAA and ERM). Last, this work highlights the importance of diversifying trailer weight and dynamic use of updated tractor information in the trailer shipment problem.
The remainder of this paper is organized as follows. The related literature is reviewed in Section 2. Section 3 describes the trailer shipment problem and the related logistics operations. The proposed data-analytics methodologies are described in Section 4 and 5. Using the data set obtained from ABI, the methodologies are evaluated in Section 6. Managerial insights are outlined in Section 7. This paper is concluded with the key findings and insights in Section 8.
2. Literature Review

This work is related to a container loading problem (CLP) studied in the literature. CLP is an optimization problem described as placing products into a container, typically maximizing the container space utilization. There are several practical constraints considered in CLP. Readers are referred to Bortfeldt and Wäscher (2013) for the state-of-the-art review on practical constraints considered in CLP. In the literature, the weight limit constraint is often considered (e.g., Gehring and Bortfeldt 1997, Liu and Hsiao 1997, Iori and Martello 2010, Chan et al. 2006, Egeblad et al. 2010). With the weight limit constraint considered in the methodologies, the above papers suggest various ways to deal with CLP. Gehring and Bortfeldt (1997) propose a genetic algorithm to solve CLP. Liu and Hsiao (1997) suggest a method that loads units onto a pallet to maximize the degree of stability and the utilization. Iori and Martello (2010) consider combinatorial optimization problems that combine the vehicle routing and loading. Chan et al. (2006) present a two-phase intelligent decision support system for an air-cargo pallet loading problem. Egeblad et al. (2010) propose heuristics approaches to CLP for furniture manufacturers. In addition to the gross weight restriction, several recent papers consider distribution of load weights on vehicle axles (e.g., Lim et al. 2013, Alonso et al. 2017, and Ramos et al. 2018).

In the container loading literature above, the weight limit constraint is generally assumed to be known and fixed. However, the logistics outsourcing via 3PL providers introduces a new challenge in CLP. The outsourcing practice makes the gross weight unknown and random at the time of loading containers due to the diversity of tractors owned by 3PL providers. In this case, the gross weight cannot be assumed to be a constant factor in CLP. As mentioned in the introduction, many companies are relying on 3PL providers for strategic reasons. Ton and Wheelwright (2005) point out increasing operational efficiency, cost control, reliability, and speed as the major benefits from using 3PL. In addition, Vasiliauskas and Jakubauskas (2007) recognize economies of scale, capital savings, and financial risk reduction as the key benefits as well. In the past decades, the logistics outsourcing via 3PL has attracted considerable attention in the literature. Readers are referred to Marasco (2008) and Aguezzoul (2014) for extensive reviews on the related works.
Another stream of literature that resembles our work is the inventory selection problem which optimally choose the initial inventory level for a set of products. Mahajan and Van Ryzin (2001) developed a single-period stochastic inventory model determine initial inventory levels for the assortment when faced with a type of behavior that a sequence of heterogeneous customers dynamically substitute among product variants inside a retail assortment when inventory is exhausted. Honhon et al. (2010) use a dynamic programming framework to determine the best assortment and inventory levels when a customer purchases the highest-ranked product available at the time of his visit to the store, based on his distinct preference when it comes to ordering products. Total customer demand in their model is random and consists of a fixed proportion of different types of customers. The more close related work is the joint inventory selection and online resource allocation problem, where the online matching problem considers how to match resources to demand given a fixed collection of inventory levels. Topaloglu (2013) studied to decide which sets of products to offer over the selling horizon and how many units of each product to stock so as to maximize the expected profit given that customers choose among the set of offered products according to the multinomial logit model. The most closely related work is Chen et al. (2021) which is the first to develop algorithms with provable performance guarantees for this joint problem in a multi-period and multi-product setting. We use the same ABI setting as Chen et al. (2021), however our work reflects the true dynamic nature of the problem and use reinforcement learning to allocate resource by estimating rewards from future states.

The use of data has emerged as a critical methodology both in industry and academia. As recent examples, Hamister et al. (2018) and Smyth al. (2018) provide practical issues and guidelines for the use of data analytics in supply chain management research. Readers are also referred to Choi et al. (2018) and Mišić and Perakis (2020) for the recent advances in data-driven operations management (OM) research. Even though logistics and transportation issues are extensively studied in the OM literature, data-driven applications in that area have emerged in recent years. Some representative papers are Lv et al. (2015, machine learning application to traffic flow prediction), van der Lann et
al. (2016, empirical performance analysis of the demand planning and distribution operations for humanitarian logistics), Chung et al. (2017, neural network for flight delay analysis), Jamshidi et al. (2017, data analytics for rail failure risk evaluation), Shang et al. (2017, Bayesian statistics for cargo logistics risk assessment), Wasesa et al. (2017, using trucks’ trajectory data for service rate prediction at the seaport), Xie et al. (2017, grid-based cell-structured framework to study accidents in logistics networks), Ta et al. (2018, scenario-based experiment to investigate the impact of driver disclosure and ethnicity in crowdsourced delivery systems), Kirac and Milburn (2018, evaluating the value of social data in disaster response logistics planning). Our work contributes to this stream of research by introducing practical data-analytics approaches to the trailer shipment problem.

Our work is closely related to machine learning algorithm and its application to OM issues. In recent years, machine learning techniques have received considerable attention in the literature. The applications of machine learning techniques in diverse operations areas can be found in the recent literature: waiting time prediction (Ang et al. 2015), inventory decision (Ban and Rudin 2018), and demand/sales forecast (Ferreira et al. 2015, Cui et al. 2018, Lau et al. 2018). Our work is also closely related to Q-learning that is a well-known RL algorithm. Q-learning can be applied in various real-world environment, such as online recommendation system (Chen et al. 2018), traffic light control (Abdoos et al. 2011), real-time bidding (Zhao et al. 2018). Chen et al. (2018) and Zhao et al. (2018) adopted a deep Q-learning algorithm which employs a deep neural network to approximate Q function. Abdoos et al. (2011) modeled traffic network as a multi-agent system and used Q-learning to adjust the lights according to the traffic situation. However, despite of its usefulness, there exist only a few papers that apply Q-learning algorithm, especially the non-deterministic case, in operations management fields. Our work contributes to this stream of research by suggesting diverse machine learning techniques and their application to a practical operations problem in the logistics area.
3. Problem Description: Trailer Shipment Problem

Anheuser Busch Inbev (ABI) is a world-leading beverage company, which manages more than ten production plants and operate its distribution centers in multiple locations throughout the United States. The company produces and distributes a large variety of beverage products. After being finished and packed in the plants, the products are delivered to multiple wholesalers. The company outsources the delivery service to third-party logistics (3PL) providers. The major shipping method is called *drop trailer*, which includes about 80% of the total shipments in the company. In the logistics operation, ABI loads beverages on trailers (i.e., containers) in advance, typically 4 to 48 hours before the scheduled arrival of tractors. Then, 3PL providers send tractors to the plant to pick up the loaded trailers to deliver to customers. Figure 2 illustrates a physical structure of a typical truck used in the delivery process.

ABI should meet the gross weight regulation that specifies the maximum gross weight of a commercial motor vehicle limited to 80,000 pounds. From an operational perspective, it is optimal to fully utilize the trailer capacity by meeting the maximum gross weight exactly. However, it is challenging due to the fact that tractor weight is varying and unknown to ABI when it pre-loads products on trailers. Such randomness of tractor weight is due to its physical difference, drivers’ equipment/belongings on the tractor, fuel level, etc. Figure 3 plots the actual tractor weights in the arriving order at ABI’s mid-size plant in January 2014, which demonstrates the tractor weights are quite random. The high volatility of the tractor weights makes it difficult to exactly meet the total weight limit. One may wonder why ABI do not coordinate with the 3PL providers to commit trailers or share the weight information in advance. We asked ABI and they said it is simply too difficulty to execute for several reasons. First, 3PL providers typically suffer from unstable capacity (e.g., no shows of drivers and malfunctions of trucks), which makes it practically impossible to commit in advance. Second, ABI does not have the infrastructure to support this sort of coordination, which would require vast networks of communication with the hundreds of its 3PL providers. Moreover, 3PL providers often do not contract exclusively with ABI but serve many other companies as well, which is an additional complexity that makes such coordination difficult.
Hence, ABI weighs every tractor before it leaves the plant in order to assure the total weight standard (i.e., the total weight should be lower than the maximum weight limit of 80,000 pounds). If this standard is not met, an additional step is required to fix the overweight problem. It typically requires the overweighted tractor to return to the docking area for adjustment (i.e., unload some products). This action is termed scaleback, and it results in additional rework cost expressed as

\[ s(q + w - \bar{w}), \]

where \( s \) is the unit scaleback cost per weight, \( q \) is the weight of the loaded trailer, \( w \) is the weight of the tractor and \( \bar{w} \) is the maximum weight limit (80,000 pounds in our case). On the other hand, if the gross weight is less than the limit, the tractor leaves the plant for delivery. However, being underweight results in opportunity cost since the company does not fully utilize the trailer capacity. The opportunity cost can be expressed as

\[ dl(\bar{w} - q - w), \]

\[ 0 \] 100 200 300 400 500 600 700 800 900

Figure 3  Tractor Weights in Order of Arrival in January 2014
where \( d \) is the distance between the plant and the wholesaler, \( l \) is the unit logistics cost per weight. ABI determines the weight of a loaded trailer \( q \) to minimize such mismatch costs. The company’s initial approach was a simple heuristic based upon descriptive statistics on tractor weight such as the average value from historical data. Even though such a heuristics approach is simple and straightforward, it does not appropriately consider the trade-off in the above mismatching. After a consulting project with the Boeing Center for Supply Chain Innovation (BCSCI), the company is using a Newsvendor approach to setting the target weight. In what follows, we propose novel data-analytics methodologies that capture the trade-off and efficiently deal with the problem to minimize the mismatching cost.

Figure 4 illustrates ABI’s trailer shipment operation at its plants. An empty trailer is pre-loaded with products at the docking area, typically 4 to 48 hours before delivery. The pre-loaded trailer is then moved to the pick-up area and waits until it is picked up. When a tractor arrives at the plant, it goes to the pick-up area and hooks up the loaded trailer. Then, the whole truck (tractor and loaded trailer) is weighed on the scale, located at the plant exit, to make sure it is under the weight.
limit before it leaves the plant. If the tractor meets the total weight limit, it leaves the plant for delivery. However, if the tractor is overweighted, it goes back to the docking area for adjustment. After that, the tractor leaves the plant for delivery.
4. Trailer Weight-load Optimization

As a benchmark methodology, Newsvendor model is used in the analysis. SAA and ERM are non-parametric data-analytics methodologies to solving stochastic optimization problems. Those utilize the data to estimate the distribution of uncertain factors (i.e. tractor weight) in calculating the objective (i.e. mismatching cost) to be optimized. In the trailer shipment problem, those approaches are used to determine \( q \) (loaded trailer weight) that minimize the mismatching cost.

4.1. Benchmark: Newsvendor Method

The Newsvendor model (NV) is a classic method to determine inventory to minimize the expected cost. For the BCSCI consulting project, we formulated the trailer shipment problem as a Newsvendor problem, assuming the trailer weight is approximately normally distributed with mean \( \mu \) and standard deviation \( \sigma \). For each wholesaler, the overage cost \( c_o \) is \( s \) and the underage cost \( c_u = dl(\bar{w} - q - w) \) as the distance between destination and plate is known. The probability of total weight under 80,000 pounds for each shipment, also called critical ratio, is \( \frac{c_u}{c_o + c_u} \). The optimal \( q^* \) is the \( \frac{c_u}{c_o + c_u} \)th percentile of the weight distribution with a standardized value of \( z \). The optimal \( q^* \) can be expressed as

\[
(NV) \quad q = \sigma z + \mu.
\]  

4.2. Sample Average Approximation

The key information to solve the trailer shipment problem is the distribution of tractor weights. However, the true (or theoretical) distribution of such a random factor is often unknown in practice. In general, only the historical observations are available. Sample Average Approximation (SAA) is a nonparametric data-driven approach to estimate the distribution of such a random factor (i.e., tractor weight), without the need of a theoretical distribution (e.g., normal distribution). SAA utilizes the empirical distribution derived from the sample data to substitute the true (or theoretical) distribution of a random parameter. The basic idea is to use the empirical distribution to approximate the expected value function to be optimized. SAA is a popular nonparametric data-driven approach in practice (Levi et al. 2015). This methodology is also widely studied in the
literature. The application and theoretical properties of SAA in the stochastic optimization can be found in Shapiro et al. (2009). In the trailer shipment problem, the SAA approach is implemented as follows. Suppose there exists a sample data set including \( N \) observations of the actual tractor weight. The set of the tractor weights from the sample data are denoted by \( \mathbf{w}_N = \{w_1, ..., w_N\} \) where \( w_i \leq w_{i+1} \) for \( i \in \{1, ..., N-1\} \), and the corresponding delivery distances are denoted \( \{d_1, ..., d_N\} \).

Then, the expected mismatching cost is expressed as follows:

\[
(\text{SAA}) \quad \min_{q \geq 0} \mathbf{C}(q) = \frac{1}{N} \sum_{i=1}^{N} d_i (\bar{w} - q - w_i)^+ + s(q + w_i - \bar{w})^+, \tag{2}
\]

where \((x)^+ = \max\{0, x\}\). The mismatching cost is expressed as the sum of the opportunity cost and the scaleback cost. Notice that the SAA approach forms the empirical distribution by assigning the equal probability \(1/N\) to each of the observations (i.e., individual tractor weight observation in the sample data set). Based on the empirical distribution, it estimates the expected mismatching cost without knowing the true distribution of the tractor weight. Importantly, this problem is distinguished from the standard newsvendor problem. Notice that the unit opportunity cost in case of under-loaded trailers, denoted by \(d_i l\), is varying since the shipping payment is different depending on the destination and is proportional to its distance. This is a key difference from the newsvendor problem that considers the fixed unit underage and overage costs.

**Theorem 1.** The optimal solution to (2) is given by \( q^* = \bar{w} - w_j \), where \( w_j \in \mathbf{w}_N \) whose index is obtained by

\[
j = \inf \left\{ k \in \{1, ..., N\} : \frac{k}{N} \geq \frac{s}{d^k l + s} \right\}, \tag{3}
\]

where

\[
d^k = \frac{\sum_{i=1}^{k} d_i}{k}.
\]

**Proof.** For ease of exposition, let us define \( x = \bar{w} - q \). Then, the cost function \( \mathbf{C}(q) \) can be rewritten as a function of \( x \):

\[
\mathbf{C}(x) = \frac{1}{N} \sum_{i=1}^{N} d_i (x - w_i)^+ + s(w_i - x)^+.
\]
We solve the problem with respect to \( x \). That is, the optimal value of \( x \) is given by

\[
x^* = \arg \min_{x \in \mathbb{R}^+} C(x).
\]

Then, the optimal trailer quantity is given by \( q^* = \bar{w} - x^* \). We prove that the optimal solution of \( x \) is given by

\[
x^* = w_j \in \mathbf{w}_N \text{ such that } j = \inf \left\{ k \in \{1, \ldots, N\} : \frac{k}{N} \geq \frac{s}{d^k l + s} \right\}.
\]

First, we rewrite \( C(x) \) as

\[
C(x) = \frac{1}{N} \sum_{i=1}^{N} d_i l (x - w_i)^+ + s (w_i - x)^+.
\]

For \( w_k \leq x < w_{k+1} \), it can be shown that

\[
\frac{dC(x)}{dx} = \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l - (N - k)s \right\}.
\]

That is, \( C(x) \) is a piecewise linear function whose boundaries are defined by \( w_k \in \mathbf{w}_N \). Furthermore, for any \( w_k \in \mathbf{w}_N \)

\[
\lim_{x \to w_k^+} C(x) = \frac{1}{N} \left\{ \sum_{i=1}^{k-1} d_i l (w_k^- - w_i) + \sum_{i=k}^{N} s (w_i - w_k^-) \right\}
\]

\[
= \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l (w_k^- - w_i) - d_k l (w_k^- - w_k) + s (w_k - w_k^-) + \sum_{i=k+1}^{N} s (w_i - w_k^-) \right\}
\]

\[
= \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l (w_k^- - w_i) + \sum_{i=k+1}^{N} s (w_i - w_k^-) \right\}
\]

\[
= \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l (w_k^+ - w_i) + \sum_{i=k+1}^{N} s (w_i - w_k^+) \right\}
\]

\[
= \lim_{x \to w_k^+} C(x),
\]

which implies \( C(x) \) is continuous in \( x \).

Let us define \( \Delta C_k \doteq C(w_{k+1}) - C(w_k) \) and \( \Delta w^k \doteq w_{k+1} - w_k \). Then, from (4), we obtain

\[
\frac{\Delta C_k}{\Delta w^k} = \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l - (N - k)s \right\} \text{ for } k \in \{1, \ldots, N\}.
\]
Then, we obtain

\[
\frac{\Delta C_k}{\Delta w^k} = \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l - (N - k) s \right\} \\
< \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l + d_{k+1} l - (N - k) s + s \right\} \\
= \frac{1}{N} \left\{ \sum_{i=1}^{k+1} d_i l - (N - k - 1) s \right\} \\
= \Delta C_{k+1}^{\Delta w^{k+1}}.
\]

That is, \( C(x) \) is a convex and continuous piece wise linear function. Hence, if there exists an interior solution, it must be one of the potential tractor weights whose index \( j \) is given by

\[
j = \inf \left\{ k \in \{1, ..., N\} : \frac{1}{N} \left\{ \sum_{i=1}^{k} d_i l - (N - k) s \right\} \geq 0 \right\},
\]
equivalently,

\[
j = \inf \left\{ k \in \{1, ..., N\} : \frac{k}{N} \geq \frac{s}{d_k l + s} \right\}.
\]

\[\square\]

The optimal trailer load size \( q^* \) is obtained as a fractile solution based upon the tractor weight in the sample data set \( w_N \) (see (3)). It turns out the effect of the varying distance \( d_i \) is incorporated in the solution as the average value \( \bar{d}^k \). It is also observed that the optimal trailer load size \( q^* \) is increasing in the unit logistics cost \( l \) and is decreasing in the unit scaleback cost \( s \).

### 4.3. Empirical Risk Minimization

In practice, the tractor weight may depend on features (shipping distance, weather, oil price, shipping day, etc.) which are observable or available before decision making. These features provide additional information on the distribution of the tractor weight and it therefore helps resolve the uncertainty. This section introduces a data-analytics methodology that incorporates such influential features into the problem. The approach is based on the machine learning principle of Empirical Risk Minimization (ERM) that is used in deriving machine learning algorithms. The ERM principle derives a decision rule that minimizes the risk associated with the problem by using the training data set (such risk is called empirical risk) to obtain prediction for future outcomes (test). Readers
are referred to Sotiropoulos and Tsihrintzis (2017) for the use of the ERM principle in machine learning problems. It also can be applied in various OM problems. For example, Ban and Rudin (2018) apply the principle in an inventory problem. For details on the ERM approach in the regression learning problem, see Cherkassky and Mulier (2007).

To apply the ERM principle, we need a feature-based data set. Suppose there are \( n \) features that are related to tractor weight. Define a feature vector \( \mathbf{x} = \{x_1, \ldots, x^n\} \in \mathcal{F} \), where \( \mathcal{F} \subset \mathbb{R}^n \) is a feature space. Suppose the sample data includes \( N \) observations of tractor weight paired with the corresponding features. Then, the feature-based data set is given by \( f_N = \{(w_1, \mathbf{x}_1), \ldots, (w_N, \mathbf{x}_N)\} \), where \( w_i \) is the \( i \)-th observation of tractor weight and \( \mathbf{x}_i \in \mathcal{F} \) is a vector that includes the values of the corresponding features for the \( i \)-th tractor weight. The optimal trailer weight is given as a function of a feature vector \( \mathbf{x} \), denoted by \( q(\mathbf{x}) \). That is, the optimal trailer weight depends on the influential features. Note that the form of \( q(\mathbf{x}) \) is not restrictive. For example, a linear function of \( q(\mathbf{x}) \) can be considered such as

\[
q(\mathbf{x}) = q^0 + \sum_{i=1}^{n} q^i x^i, \quad q^i \in \mathbb{R}. \tag{5}
\]

Polynomial or logarithm transformations of features can also be considered to capture nonlinear relationships.

The basic idea of ERM is to find the function relationship \( q(\cdot) \) by computing its parameter values (e.g., \( q^i, i \in \{0, \ldots, n\} \) in (5)) which minimize the empirical risk. In the trailer shipment problem, the empirical risk is interpreted as the mismatching cost. Hence, the ERM approach is to find the optimal function of \( q(\mathbf{x}) \) that minimizes the mismatching cost calculated based on the sample data. Let \( \mathcal{Q} \) denote the collection of functions \( q : \mathcal{F} \rightarrow \mathbb{R} \). Then, the optimal trailer weight is obtained by solving the following optimization problem:

\[
\text{\textbf{(ERM)}} \quad \min_{q(\cdot) \in \mathcal{Q}} C(q(\cdot)|f_N) = \frac{1}{N} \sum_{i=1}^{N} d_i(l(\bar{w} - q(\mathbf{x}_i) - w_i) + s(q(\mathbf{x}_i) + w_i - \bar{w})^+), \tag{6}
\]

where \( C(q(\cdot)|f_N) \) is the empirical risk defined as a function of \( q(\cdot) \). The empirical risk is the expected mismatching cost calculated based on the sample tractor weight data. That is, it estimates the
mismatching cost by using the empirical distribution of the tractor weights and the corresponding feature values in the sample data set. It is challenging to obtain the analytical solution to (6). Hence, we reformulate (6) as the following optimization problem:

\[
\text{(ERM - OPT)} \quad \min_{q(\cdot) \in \mathbb{S}, u_i, v_i} \frac{1}{N} \sum_{i=1}^{N} d_i l u_i + sv_i
\]

subject to

\[
\begin{align*}
  u_i & \geq \bar{w} - q(x_i) - w_i & \forall i = 1, \ldots, N, \\
v_i & \geq q(x_i) + w_i - \bar{w} & \forall i = 1, \ldots, N, \\
u_i, v_i & \geq 0 & \forall i = 1, \ldots, N,
\end{align*}
\]

where \(u_i\) and \(v_i\) are indicator variables that represent underload and overload, respectively. If a linear function for \(q(\cdot)\) is adopted, the optimization problem (7) is formulated by a linear program.
5. Dynamic Trailer Assignment

Sample Average Approximation (SAA) and Empirical Risk Minimization (ERM) are based on deterministic assignment. These utilize no tractor weight information which can be available upon the arrival of tractors at the plant. The uncertainty of tractor weight is the major challenge to the Trailer Assignment problem. Hence, we conjectured that it can be further improved if the obtained tractor weight information can be dynamically utilized to mitigate the effect of the uncertainty.

The Dynamic Trailer Assignment Problem is aim to improve the operational by Reinforcement Learning (RL). RL is an area of machine learning, which can be used to solve MDP problems. Specifically, we use a non-deterministic Q-learning algorithm to develop our DTA algorithm that enables dynamic assignment of loaded trailers based upon the obtained weight information of arriving tractors to minimize the mismatching cost.

To utilize the updated tractor weight information, groups of loaded trailers having different weights (types) are considered. Given the different trailer groups, the manager decides which type of trailer is assigned to the arriving tractor based on the tractor weight. By doing that, the manager can utilize the updated tractor weight and differentiate the decision accordingly. Hence, the key question in this decision process is how to optimally assign the trailers (i.e., which type or weight of trailer should be assigned to the arriving tractor), utilizing the updated tractor weight information.

We formulate the trailer assignment problem as MDP, and apply the above RL framework to find the optimal policy. Since ABI operates the logistics process in a daily basis, we treat each day as one episode to be modeled in MDP, where a set of trailers need to be ship on the day. For each episode, we discretize the time step as \( t = 1, \ldots, \tau \), where exactly one tractor arrives in each time period. Then, \( \tau \) implies the number of trailers that need to be delivered on the day. The set of \( m \) types (weights) of tractors is denoted as \( T = \{1, \ldots, m\} \). For a truck type \( i \in T \), let \( w_i \) denote the truck weight. The collection of \( n \) types (weights) of loaded trailers is denoted by \( C = \{1, \ldots, n\} \), where \( n \) represent the number of trailer groups having different load size. For each trailer type \( j \in C \), we denote the trailer weight \( q_j \). In each time period, the trailer inventory
decreases as one trailer is assigned and shipped by the arriving tractor. Let $I^t_i$ denote the remaining trailer inventory for trailer type $j$ in period $t$. Notice that $\sum_{j \in C} I^t_j = \tau$. Let $I_t$ be a vector to denote the collection of the remaining inventory in time period $t$. That is, $I_t = [I^1_t, \ldots, I^n_t]$. The action in this process is to determine which type of trailer is assigned to the arriving tractor. Notice that ABI can assign a trailer whose inventory is available. Hence, ABI takes an action $j \in A(I_t)$, where $A(I_t) = \{j \in C | I^t_j > 0\}$. The reward depends on the tractor type $i$ and the trailer type $j$, which is denoted as

$$r(i, j) = -\left[d_j l(\bar{w} - q_j - w_i)^+ + s(q_j + w_i - \bar{w})^+\right].$$  \hfill (8)$$

The state is denoted as $\{I_t, i\}$ which is a combination of the inventories of different trailer types and the arriving tractor type. Given a state $\{I_t, i\}$ and an action $j \in A(I_t)$ in time period $t$, the optimal cumulative value function can be written as

$$Q_t(\{I_t, i\}, j) = r(i, j) + \gamma \sum_{k \in T} p_k \max_{a' \in A(I_t - e_j)} Q_{t+1}(\{I_t - e_j, k\}, a'),$$  \hfill (9)$$

where $Q_{\tau+1}(\cdot, \cdot) = 0$, $\gamma \in (0, 1)$ is a discount factor, $p_k$ is the arrival probability of type $k$ tractor and $e_j \in \{0, 1\}^n$ is the unit vector whose $j^{th}$ component equals 1 and others equal 0.

To solve the problem, we apply the concept of Q-learning, a non-parametric and model-free Reinforcement Learning (RL) algorithm. RL is one of the basic machine learning paradigms, which studies how a decision maker seeks to take optimal actions in a dynamic environment to achieve its goals (e.g., maximizing rewards).

This recursive definition is the basis for the Q-learning algorithm to estimate the $Q$ function. The estimate of the actual $Q$ function is denoted by $\hat{Q}(s, a)$ which is given by a numerical table where each table entry represents the estimate of $Q(s, a)$ for the pair of state and action. In the training, the agent recurrently executes an action $a$ given the current state $s$, and then earns the reward $r$ and observe the next state $s'$. At each time step of the iterative process, the agent updates $\hat{Q}(s, a)$ based upon the following rule:

$$\hat{Q}_t(s, a) \leftarrow (1 - \alpha_t)\hat{Q}_{t-1}(s, a) + \alpha_t[r + \gamma \max_{a'} \hat{Q}_{t-1}(s', a')]$$  \hfill (10)$$

This formula is known as the Q-learning update rule.
\[ \alpha_t = \frac{1}{1 + N_t(s,a)} \]  

(11)

where \( N_t(s,a) \) is the number of times this state-action pair \((s,a)\) has been visited up to and including the \( t \)th iteration. The training rule is motivated by the optimality equation (9). In each time step, the value of \( \hat{Q} \) for the current state \( s \) is updated by using the values of \( \hat{Q} \) for the new state \( s' \).

Notice that there is no need for knowing the specific forms of \( \delta(s,a) \) and \( r(s,a) \) in the training rule. Instead, it utilizes the sample data for the values of \( \delta(s,a) \) and \( r(s,a) \) to update \( \hat{Q} \).

The convergence of \( Q \) learning is presented in the following theorem that was first proved in Watkins and Dayan (1992) and later summarized in Mitchell (1997).

**Theorem 2.** The convergence of \( \hat{Q} \) to \( Q \) for nondeterministic Markov decision processes. Given bounded rewards \(|r_t| \leq R\), learning rates \( 0 \leq \alpha_t \leq 1 \), and

\[ \sum_{i=1}^{\infty} \alpha_{t^i(s,a)} = \infty, \sum_{i=1}^{\infty} \left[ \alpha_{t^i(s,a)} \right]^2 < \infty, \]  

(12)

then \( Q_t(s,a) \rightarrow Q^*(s,a) \) as \( t \rightarrow \infty \), \( \forall s,a \), with probability 1.

**Proof.** Consider \( \alpha_t = 1 \), \( \hat{Q}(s,a) \) can reduce to

\[ \hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a') \]  

(13)

We now show that the maximum error over all entries in the \( \hat{Q} \) table is reduced by at least a factor of \( \gamma \) during each such interval. Let \( \Delta_n \) be the maximum error in \( \hat{Q}_n \); that is

\[ \Delta_n \equiv \max_{s,a} \left| \hat{Q}_n(s,a) - Q(s,a) \right| \]  

(14)

Now for any table entry \( \hat{Q}_n(s,a) \) that is updated on iteration \( n + 1 \), the magnitude of the error in the revised estimate \( \hat{Q}_{n+1}(s,a) \) is
\[ |\hat{Q}_{n+1}(s,a) - Q(s,a)| = |(r + \gamma \max_{a'} \hat{Q}_n(s',a') - (r + \gamma \max_{a'} Q(s',a'))| \]
\[ = \gamma |\max_{a'} \hat{Q}_n(s',a') - \max_{a'} Q(s',a')| \]
\[ \leq \gamma \max_{a'} |\hat{Q}_n(s',a') - Q(s',a')| \]
\[ \leq \gamma \max_{s'',a'} |\hat{Q}_n(s'',a') - Q(s'',a')| \]
\[ \leq \gamma \Delta_n \]

(15)

The third line above follows from the second line because for any two functions \(f_1\) and \(f_2\) the following inequality holds:
\[ \left| \max_a f_1(a) - \max_a f_2(a) \right| \leq \max_a |f_1(a) - f_2(a)| \]

(16)

In going from the third line to the fourth line above, note we introduce a variable \(s''\) over which the maximization is performed. This is legitimate because the maximum value will be at least as great when we allow this additional variable to vary. Note that by introducing this variable we obtain an expression that matches the definition of \(\Delta_n\).

Thus, the updated \(\hat{Q}_{n+1}(s,a)\) for any \(s,a\) is at most \(\gamma\) times the maximum error in the \(\hat{Q}_n\) table, \(\Delta_n\). The largest error in the initial table, \(\Delta_0\), is bounded because values of \(\hat{Q}_0(s,a)\) and \(Q(s,a)\) are bounded for all \(s,a\). Now after the first interval during which each \(s,a\) is visited, the largest error in the table will be at most \(\gamma \Delta_0\). After \(k\) such interval, the error will be at most \(\gamma^k \Delta_0\). Since each state is visited infinitely often, the number of such intervals is infinite, and \(\Delta_n \to 0\) as \(n \to \infty\).

Now we prove the case where \(\alpha_n = 1\). With smaller value of \(\alpha\), this term is now averaged in with the current \(\hat{Q}(s,a)\) to produce the new updated value. Notice the value of \(\alpha_n\) in Equation decreases as \(n\) increases, so that updates become smaller as training progresses. By reducing \(\alpha\) at an appropriate rate during training, we can achieve convergence to the current \(Q\) function. \(\Box\)

The revised training rule for nondeterministic cases gradually updates \(Q\) function by using the weight factor \(\alpha_t\). We also note that \(\alpha_t\) diminishes as the training progresses. It enables to achieve the convergence of \(Q\) function in the training in which the observations of \(Q\) values is varying.
The $Q$ learning algorithm is applied to the trailer assignment problem as follows. The first step for $Q$ learning is to construct $Q$ table where each row and column represent a state and an action, respectively, and the entry represents the estimated optimal cumulative reward corresponding to the pair of the state and action. At each time period, the state $\{I_t, i\}$ is observed and an action $j \in A(I_t)$ with the highest estimated value according to the $Q$ table. After taking the action, the actual reward $r(i, j)$ is then observed and the next state $\{I_t - e_j, k\}$ is attained. The final step in this period is to update the $Q$ table by by (10) in Theorem 2. This series of steps will be repeated until until the last time period (i.e., the last tractor arrival), which completes one episode. This process will be repeated for each episode to keep updating the $Q$ table. The pseudo code for this algorithm is given in the following box.

\begin{verbatim}
Initialize $Q(\{I, i\}, j) = 0, \forall I, i, j$
Repeat (For each daily episode)
    Repeat (For each time period $t$)
        Obverse current state $\{I_t, i\}$
        Take action $j \in A(I_t)$ using epsilon-greedy policy derived from $Q$
        Observe $r(i, j)$ and the next state $\{I_t - e_j, k\}$
        Update $Q$ table by (10)
    Until the last arriving tractor
\end{verbatim}

Compared with SAA or ERM which are purely exploitative, the DTA algorithm balances exploration and exploitation. Exploitation is to make decision using existing knowledge, while exploration is to gather new information by taking actions which reaches new states and potentially yields better outcomes (Poole and Mackworth 2010). Training $Q$ table is exploration which tries different actions in each scenario to see if any is better than what has been tried before. Applying $Q$ table on test data is exploitation which takes action based on what have worked best in the past.
6. Implementation to Anheuser Busch Inbev’s Trailer Shipment Problem

In this section, we implement the proposed methodologies by using the data obtained from ABI, and compare their performance. In §6.1 and §6.2, we provide the description on the obtained data, and the test scenario. §6.4-6.6 explain how each of the proposed methodologies is implemented in detail. The performance comparison result of the proposed methodologies are presented in §6.7.

6.1. Data Description

We obtained a significant amount of historical transaction-level shipping data from ABI. The data is collected from the company’s mid-size plant located in California. The length of the data is nine months, January 2014 to September 2014. The data includes the transactional information of every single shipment handled at the plant. The data size is about 700 to 1,000 shipments (containers) per month. Figure 5 illustrates the trend of the number of shipments handled at the plant in a monthly basis. The data includes the detailed information such as date, time, shipment id, plant, wholesaler id, distance, tractor weight, payment, scaleback, etc. The most important information in the data is on tractor weights. It is a key input to implement our methodologies. In the analysis, the mismatching cost is calculated based on the opportunity cost and scaleback cost. The unit opportunity cost is calculated based on the actual payments and distances of the shipments. The scaleback cost is incurred when the total weight exceeds the weight limit. In this case, the tractor

![Figure 5: Number of Monthly Shipments](image-url)
is returned to the dock station for adjustment. The estimated scaleback cost is directly given by ABI.

6.2. Test Scenario

To implement the proposed methodologies, the data is partitioned into training and test data sets. The training data set is used to derive the solution (trailer weight) based on the methodologies. Then, the obtained solution is tested over the test data set. The data set is divided in a monthly basis, and the concept of rolling horizon is used to determine the training and test data sets. That is, to test for a certain month (test data set), the model derives the solution by using the data of the previous month (training data set). For example, the January data is used as the training data set, and then the obtained solution is tested for the data in February (test data set). Similarly, the February data is used as the training data set to derive the solution, and it is then tested with the data in March as the test data set. Hence, there are eight training data sets (January to August) and eight test data sets (February to September). This concept is illustrated in Figure 6. Such a rolling horizon concept is commonly used in a dynamic environment.

6.3. Implementation of Newsvendor Method

To implement the Newsvendor Method, we group one month of training data including trailer weight and the distance from destination by warehouse. For each group, we calculate mean and standard deviation of tractor weights and the unit underage cost is the result of multiplying the distance from destination ($d$) and the unit logistics cost per weight ($l$). Overage cost is fixed. For
a group has less than 5 sample, we take the mean and standard deviation of the entire training data. For example, the distance between Antioch, CA and distribution center is 74 miles so that underage cost is $74 \times l$. There are 108 shipments in the training month with average tractor weight of 19,512 pounds and standard deviation of 381 pounds. We obtain the z value using the $\frac{74}{s+74}th$ percentile of the standard normal distribution. Then we can apply Equation (1) to gain estimated weights for tractors that deliver to warehouse in Antioch, CA. The pre-loaded trailer weight for each warehouse is the difference between 80,000 pounds and the estimated tractor weigh. Since we know the destination in advance, we can prepare pre-loaded trailers for each warehouse given the shipment information and the estimated tractor weight then calculate mismatching cost based on the actual total weight of tractor and trailer.

6.4. Implementation of SAA

For each month, we collect the tractor weights and distances in the data set. To make SAA comparable to DTA on dynamic trailer assignment problem, we sort training data by tractor weight then equally split to three groups. Within each group, SAA can be easily implemented based upon the result in Theorem 1. Besides, given the unit logistics cost $l$ and the unit scaleback cost $s$, the fractile $\frac{74}{s+74}$ can be easily calculated. Based upon the calculated fractile, we find the $k$-th tractor in the data set, which satisfies Equation (3) to calculate the optimal trailer weight for each group. The estimated trailer weights have three levels: $q_H$, $q_M$ and $q_L$. Table 2 provides the calculated load sizes for trailers using SAA. Then, using the tractor weight data in the following month, we prepare equal number of trailers in three levels given everyday’s shipment number. When one tractor arrives, we calculate the corresponding expected mismatching cost for all available trailer weights to match this tractor and select the trailer weight that would cause the lowest cost. We use this greedy heuristic approach to assign trucks to preloaded trailers.

6.5. Implementation of ERM

We identify influential features that affect the tractor weight to implement the ERM approach. This process is done by trial and error. Initially, we investigated all the available features, and then
<table>
<thead>
<tr>
<th></th>
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<th>May</th>
<th>Jun</th>
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<th>Aug</th>
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<td>60,680</td>
</tr>
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</table>

**Table 2**  Values of $q$ in SAA

**Figure 7**  Distance versus Tractor Weight Plots

28
narrowed down those based on statistical analysis. First, a scatter plot is used to identify a rough relationship, and the regression analysis is then conducted to confirm the statistical significance. Various factors such as carrier, shipping distance, weather, oil price, shipping day, and operating statistics [e.g., the recent average weight], etc. are investigated in the analysis. Eventually, carrier and the shipping distance, i.e., distance between the plant and a certain wholesaler, turn out to be a significant feature for tractor weight. To illustrate the overall relationship between distance and tractor weight, the scatter plots are presented in Figure 7. The scatter plots are drawn for each month from January to September. The horizontal axis indicates the distance in miles, and the vertical axis is for the tractor weight. A linear relationship is observed in the scatter plots. Tractor weight tends to be heavier with a longer distance. That is, there exists a positive relationship between distance and tractor weight. Intuitively, this is mainly because 3PL companies may use high-performance (i.e., heavier) tractors for longer distance, and drivers are likely to fill more gasoline as well. The labels in the scatter plots are omitted due to confidentiality. Instead, the summary statistics for distance are provided in Table 3.

Carrier information provides the unique third party delivery service to which each shipment belongs. There are 27 carriers ABI used from January 2014 to September 2014. BBIG and BVHG are the two biggest carriers that are responsible for over half of shipments. The other 25 carriers individually handle less than 6% of the shipment so we count them together as one group against BBIG and BVHG. The total shipment of each carrier group in each month are provide in Table 4.

To validate the statistical significance of the relationship between distance and tractor weight among carrier groups, we conduct a linear regression analysis. The regression equation used in the analysis is

\[
Tractor \ Weight = \alpha + \beta_1 \cdot Distance + \beta_2 \cdot BVHG + \beta_3 \cdot Other,
\]  

(17)

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
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<th>Apr</th>
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<tr>
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<td>155</td>
<td>148</td>
<td>148</td>
<td>191</td>
<td>176</td>
</tr>
</tbody>
</table>

Table 3  Summary Statistics for Distance

29
where BVHG and Other are dummy variables. We conduct the regression analysis for each month. The result is given in Table 5. The analysis reveals that the values of $\beta_1$ are positive for all months, and p-values are very small for $\beta_1$, $\beta_2$ and $\beta_3$, which implies that the linear relationship is statistically significant.

Distance is adopted as a feature in the ERM approach. We apply ERM approach within each carrier group to obtain estimated tailer weights. Due to the linear relationship between distance and tractor weight, the following linear function for the solution of the trailer weight is used:

$$q(d) = q^0 + q^1d, \quad q^i \in \mathbb{R}.$$ 

That is, the trailer weight is given by a linear function of distance. Then, the ERM formulation can be rewritten as

$$(\text{ERM - LP}) \quad \min_{q^0, q^1, u_i, v_i \in \mathbb{R}} \frac{1}{N} \sum_{i=1}^{N} d_iu_i + sv_i$$

subject to

$$u_i \geq \bar{w} - q^0 - q^1d_i - w_i \quad \forall i = 1, \ldots, N,$$

$$v_i \geq q^0 + q^1d_i + w_i - \bar{w} \quad \forall i = 1, \ldots, N,$$

$$u_i, v_i \geq 0 \quad \forall i = 1, \ldots, N.$$ 

Notice that the ERM formulation is simplified to a linear program that can be efficiently solved by commercial optimization solvers. In the analysis, the optimization problem of ERM is solved

<table>
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<th></th>
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<th>Aug</th>
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Table 4 shipment of carrier groups
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<tbody>
<tr>
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<td>2077.8**</td>
<td>1890.4**</td>
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<td>1935.3**</td>
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<tr>
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<td>Aug</td>
<td>Sep</td>
<td></td>
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</tr>
<tr>
<td>Distance ($\beta_1$)</td>
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<td>1570.7**</td>
<td>1532.5**</td>
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<tr>
<td>Other ($\beta_3$)</td>
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<td>1325.9**</td>
<td>1473.6**</td>
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*Note.* **p-value < 0.01.

Table 5 Regression Coefficients for Distance

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<td>63,934</td>
<td>63,710</td>
<td>63,680</td>
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<td></td>
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<td></td>
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<td>0</td>
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Table 6 Values of $q^0$ and $q^1$ in ERM (BBIG)

by using Python 3.6 and Gurobi 9.5.0 as the optimization solver. The computed values of $q^0$ and $q^1$ for three carrier groups (BBIV, BVHG and Other) are summarized in Table 6, Table 7, and Table 8, respectively. Finally, we estimate trailer size for each shipment by applying $q^0$ and $q^1$ in the linear function given that carrier and destination information are provided by wholesalers six weeks in advance of the actual delivery. We choose three levels using 25th, 50th and 75th percentile of estimated trailer size to preload trailers. We use the same greedy heuristic approach as the SAA implementation to assign trucks to preloaded trailers that would cause the lowest mismatching cost.

6.6. Implementation of DTA

To implement the DTA algorithm, we consider three trailer groups with different load sizes. According to the load size, the trailer type is determined (i.e., $C = \{h, m, l\}$). For each trailer
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<td>61,420</td>
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<td>61,813</td>
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<td>0</td>
<td>-6.67</td>
<td>-17.78</td>
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</table>

Table 7  Values of $q^0$ and $q^1$ in ERM (BVHG)

<table>
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<th>Jul</th>
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<th>Sep</th>
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<tbody>
<tr>
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<td>60,433</td>
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<tr>
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<td>-0.23</td>
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</table>

Table 8  Values of $q^0$ and $q^1$ in ERM (Other)

For group $j \in \{h,m,l\}$, we apply the SAA methodology to obtain its loading size $q_j$. To be specific, suppose there are $N$ observations in the training data set (one-month data). We first sort the data set as $D_N = \{(w_1,d_1),\ldots,(w_N,d_N)\}$, where $w_i \leq w_{i+1}$ and $d_i \leq d_{i+1}$ for $i \in \{1,\ldots,N-1\}$. Then, we evenly divide the data into three subsets such that $D^1_N = \{(w_1,d_1),\ldots,(w_k,d_k)\}$, $D^2_N = \{(w_{k+1},d_{k+1}),\ldots,(w_{2k},d_{2k})\}$, and $D^3_N = \{(w_{2k+1},d_{2k+1}),\ldots,(w_N,d_N)\}$, where $k \equiv \lceil N/3 \rceil$. For each subset of the data, we calculate the optimal load size by using Theorem 1. The computation result is given in Table 2. To determine tractor types, we discretize the tractor weight range by a width 500. We observed that the minimum and maximum tractor weight in the data set are about 15000 and 22000, respectively. Hence, we create 14 tractor groups (types) $T = \{1,\ldots,10\}$ such that $w_i < w_{i+1}$ for $i \in \{1,\ldots,9\}$. An arriving tractor is identified as one of the tractor types. With this setup, the DTA algorithm described in §5 can be applied. However, we observed that ABI has less than 1000 tractor each month, which is not enough to train the $Q$ table. Therefore, we use bootstrap method to simulate 5,000 episodes from previous month’s data. The modified pseudo code is shown below.
Initialize $Q(\{T, i\}, j) = 0, \forall I, i, j$

Repeat (For 5,000 times)

Repeat (For each daily episode)

Simulate tractor weights from the training data

Repeat (For each time period $i$)

Obverse current state $\{I_t, i\}$

Take action $j(I_t)$ using policy derived from $Q$

Observe $r(i, j)$ and the next state $\{I_t - e_j, k\}$

Update $Q$ table by (10)

Until the last arriving tractor

The above approach is sampling 5,000 different tractor weight data sets for each episode from the training data, which significantly increases the number of episodes to better train the $Q$ table.

Table 9 demonstrate how DTA determine inventory dynamically within one day. There are 53 shipment incoming on this day. The initial inventory set include 18 low-weight trailers, 18 medium-weight ones and 17 high-weight ones. This table shows that DTA decides action based on both tractor weight and inventory level. For example, shipment 36 (16,920) has higher tractor weight than shipment 25 (16,860), while DTA selects higher trailer weight for shipment 36 than shipment 25. This decision reserves more medium-weight trailer for incoming heavier tractor, like shipment 40.

6.7. Performance Comparison

Using the data described above, the proposed approaches are conducted to evaluate their performance. As the performance measure, we compare the average mismatching cost in the analysis. The cost is calculated for each shipment in the test data set. Then, all the costs are averaged to obtain the average average value per month, from February to September. For confidentiality, the
<table>
<thead>
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<th>Shipment</th>
<th>Tractor Weight</th>
<th>Inventory ((L, M, H))</th>
<th>Action</th>
<th>Trailer Weight</th>
<th>Total Weight</th>
<th>Cost</th>
</tr>
</thead>
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<tr>
<td>48</td>
<td>16300</td>
<td>(6.0, 0.0, 0.0)</td>
<td>L</td>
<td>60280</td>
<td>76580</td>
<td>9.23</td>
</tr>
<tr>
<td>49</td>
<td>20140</td>
<td>(5.0, 0.0, 0.0)</td>
<td>L</td>
<td>60280</td>
<td>80420</td>
<td>9.66</td>
</tr>
<tr>
<td>50</td>
<td>19100</td>
<td>(4.0, 0.0, 0.0)</td>
<td>L</td>
<td>60280</td>
<td>79380</td>
<td>18.41</td>
</tr>
<tr>
<td>51</td>
<td>20340</td>
<td>(3.0, 0.0, 0.0)</td>
<td>L</td>
<td>60280</td>
<td>80620</td>
<td>14.26</td>
</tr>
<tr>
<td>52</td>
<td>20600</td>
<td>(2.0, 0.0, 0.0)</td>
<td>L</td>
<td>60280</td>
<td>80880</td>
<td>20.24</td>
</tr>
<tr>
<td>53</td>
<td>20380</td>
<td>(1.0, 0.0, 0.0)</td>
<td>L</td>
<td>60280</td>
<td>80660</td>
<td>15.18</td>
</tr>
</tbody>
</table>

Table 9  DTA Inventory Flow
actual cost scale is not indicated. Instead, the normalized mismatching cost is presented in this section, which is given as

\[
\text{Normalized Mismatching Cost}_{i,j} (\%) = \frac{\text{Average Mismatching Cost}_{i,j}}{\max_{i,j} \{\text{Average Mismatching Cost}_{i,j}\}},
\]

where \(i\) is the index for the applied methodology (i.e., NV, SAA, ERM, DTA) and \(j\) is the index for month (i.e., February, March, April, May, June, July, August, September). The normalized cost is presented in percentage, which indicates the amount of cost relative to the maximum (i.e., worst-case) cost in all scenarios. Figure 8 compares the normalized costs of the proposed methodologies during the analysis period (February to September). The numerical values are also given in Table 10. The analysis reveals that the performance is given in the order of DTA (best performance), ERM, SAA, and NV (worst performance). NV has the worst performance as it is deterministic allocation which does not use the new updated information on the tractor weight. ERM improves SAA result by 7%. ERM diversifies the trailer weight according the delivery distance (a feature). It utilizes the distance information to better predict the tractor weight, using the relationship between distance and tractor weight.

This implies that tractor weights may have different distributions depending on the distance. For example, the distribution of tractor weights is shifted if the distance increases. In this sense, the distance determines the type of the tractor weight distribution. The SAA approach uses the distribution based on all observations without considering the distance, which is not proper for calculating the trailer load size for a certain shipment. Instead, the relevant data set that has the

<table>
<thead>
<tr>
<th></th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NV</td>
<td>82.0%</td>
<td>77.5%</td>
<td>94.5%</td>
<td>87.9%</td>
<td>100.0%</td>
<td>96.0%</td>
<td>83.8%</td>
<td>76.5%</td>
<td>87.3%</td>
</tr>
<tr>
<td>SAA</td>
<td>59.7%</td>
<td>60.9%</td>
<td>59.4%</td>
<td>52.6%</td>
<td>52.6%</td>
<td>56.9%</td>
<td>49.6%</td>
<td>50.0%</td>
<td>55.2%</td>
</tr>
<tr>
<td>ERM</td>
<td>55.3%</td>
<td>52.0%</td>
<td>51.4%</td>
<td>50.7%</td>
<td>48.4%</td>
<td>45.2%</td>
<td>42.7%</td>
<td>40.1%</td>
<td>48.2%</td>
</tr>
<tr>
<td>DTA</td>
<td>31.1%</td>
<td>31.9%</td>
<td>32.3%</td>
<td>30.2%</td>
<td>29.1%</td>
<td>32.2%</td>
<td>28.2%</td>
<td>25.6%</td>
<td>30.1%</td>
</tr>
</tbody>
</table>

Table 10 Cost Comparison
same (or similar) distances should be used in the computation to prevent the bias of the estimated distribution.

However, one drawback of SAA and ERM is that the decision maker uses greedy method in the trailer allocation process and doesn’t learn from the action using feedback of rewards or penalties. The DTA algorithm instead uses the updated feedback and dynamically allocate trailers. DTA significantly outperforms all the others, whose average normalized cost during the entire period is only 30.1% of the worst-case cost (by Newsvendor in June). Notice that we only use 3 different trailer type in this experiment, which is much smaller number, compared to Newsvendor method in which more than 40 different trailer types are calculated since it customize the trailer weight over all the different warehouses. Hence, it is easier to implement.

This result also highlight that the dynamics in DTA is a highly efficient way to reduce the mismatching cost in the trailer shipment problem, which enables to use the actual tractor weight. Besides, the DTA algorithm chooses the optimal trailer type to assign based not only on the current
Figure 9  Cost and Weight Comparison on One Day
arriving tractor weight, but also on the current trailer inventory which is more or less valuable for the future potential arrivals. Hence, the trailer type which minimizes the current cost is not always be assigned because it may save the inventory to balance the current cost with the future one if necessary. For example, a lightweight tractor arrives and a heavy trailer minimizes the current cost. In this case, a medium size trailer may be assigned if it predicts that other lighter tractors will come and heavy trailer can save more cost in the future. Figure 9 compares cost and weight of four methodologies on all shipments on a single day, using the same date of Table 9 as example. DTA outperforms other approaches on controlling the underweight cost. The avg underweight cost of DTA is at 21%, 82% and 97% underage cost of NA, ERM and SAA, respectively.
7. Managerial Insights

The result presented the previous section have shown the efficiency of DTA. We now analyze how different factors impact on DTA performance, so this solution can be effectively used in decision making at strategic and tactical level.

7.1. Impact of information on DTA performance

The perfect information case is where the company can obtain the perfect information on the tractor weights in advance. In other words, at the beginning of each day, the company knows all the weights of tractors that will arrive during the day. This would be the case where the company can coordinate with its 3PL providers to get the tractor information in advance.

Under perfect information on the arriving tractors, the trailer allocation problem becomes a deterministic optimization problem that optimally matches the loaded trailers with a fixed set of tractors. Hence, the problem can be formulated by the deterministic assignment problem:

$$\begin{align*}
\text{(PI)} & \quad \min_{z_{ij} \in \mathbb{R}} \sum_{i=1}^{N} d_i l(\bar{w} - \sum_{j} z_{ij} q^j - w_i)^+ + s(\sum_{j} z_{ij} q^j + w_i - \bar{w})^+ \\
& \quad \text{subject to} \\
& \quad \sum_{j} z_{ij} = 1 \quad \forall i = 1, \ldots, N, \\
& \quad \sum_{i} z_{ij} = n_j \quad \forall j, \\
& \quad z_{ij} \in \{0, 1\} \quad \forall i = 1, \ldots, N.
\end{align*}$$

Note that the above problem is a non-linear problem which is computationally expensive. Hence, we use indicator variables to convert the problem into the following linear mixed-integer program:

$$\begin{align*}
\text{(PI – LMIP)} & \quad \min_{z_{ij}, u_i, v_i \in \mathbb{R}} \sum_{i=1}^{N} d_i l u_i + s v_i \\
& \quad \text{subject to} \\
& \quad \sum_{j} z_{ij} = 1 \quad \forall i = 1, \ldots, N,
\end{align*}$$

(20a)
<table>
<thead>
<tr>
<th></th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTA (Base Case)</td>
<td>64.8%</td>
<td>66.4%</td>
<td>67.3%</td>
<td>62.9%</td>
<td>60.6%</td>
<td>67.0%</td>
<td>58.7%</td>
<td>53.3%</td>
<td>62.6%</td>
</tr>
<tr>
<td>Perfect Information</td>
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<td>47.1%</td>
<td>46.6%</td>
<td>48.6%</td>
<td>43.9%</td>
<td>45.9%</td>
<td>45.0%</td>
<td>40.1%</td>
<td>45.0%</td>
</tr>
<tr>
<td>Infinite Inventory</td>
<td>35.4%</td>
<td>34.8%</td>
<td>37.7%</td>
<td>37.2%</td>
<td>34.4%</td>
<td>36.0%</td>
<td>33.9%</td>
<td>31.5%</td>
<td>35.1%</td>
</tr>
</tbody>
</table>

Table 11 Performance Bounds Comparison

\[
\sum_i z_{ij} = n_j \quad \forall j, \quad (20c)
\]

\[
u_i \geq \bar{w} - \sum_j z_{ij}q^j - w_i \quad \forall i = 1, \ldots, N, \quad (20d)
\]

\[
v_i \geq \sum_j z_{ij}q^j + w_i - \bar{w} \quad \forall i = 1, \ldots, N, \quad (20e)
\]

\[
u_i, v_i \geq 0 \quad \forall i = 1, \ldots, N, \quad (20f)
\]

\[
z_{ij} \in \{0, 1\} \quad \forall i = 1, \ldots, N. \quad (20g)
\]

Notice that all the terms in the above formulation are linear which is easily solved by commercial optimization solvers. Table 11 demonstrates that the perfect information lower the cost by about 18%, compared with DTA. This shows that the perfect information can resolve some uncertainty associated with tractor weight.

### 7.2. Impact of inventory level on DTA performance

In the infinite inventory case, there exists infinite loaded trailers for each trailer type. That is, \(I^n \to \infty, n \in C\). Notice that the problem still has the dynamics since the tractor weight is uncertain. Hence, the trailer assignment problem under infinite inventory can be treat as a MDP problem. The following theory characterizes the optimal policy in the infinite inventory case.

**Theorem 3.** If there exists infinite trailer inventory (i.e., \(I^n \to \infty, n \in C\)), the optimal assignment of trailer is obtained by \(j^* = \arg \max_{j \in C} [r(i, j)]\).
Proof. The Bellman equation for the trailer assignment problem can be written as

\[ V^*_t(I_t, i) = \max_{j \in A(I_t)} \left[ r(i, j) + \gamma \sum_{k \in \mathcal{T}} p_k V^*_{t+1}(I_t - e_j, k) \right], \]  

(21)

where \( p_k \) is the arrival probability of type \( k \) tractor and \( e_j \in \{0, 1\}^n \) is the unit vector whose \( j^{th} \) component equals 1 and others equal 0. Let us use \( \tilde{I}_t \) to denote the inventory vector \( I_t \) such that \( I^n_t \to \infty, n \in \mathcal{C} \). Then, \( \lim_{I^n_t \to \infty} I_t - e_j = \lim_{I^n_t \to \infty} \tilde{I}_t = \tilde{I}_t \). Hence, the above Bellman equation can be rewritten as

\[
\lim_{I^n_t \to \infty} V^*_t(I_t, i) = \lim_{I^n_t \to \infty} \max_{j \in A(I_t)} \left[ r(i, j) + \gamma \sum_{k \in \mathcal{T}} p_k V^*_{t+1}(\tilde{I}_t - e_j, k) \right],
\]

(22)

\[
\iff V^*_t(\tilde{I}_t, i) = \max_{j \in A(\tilde{I}_t)} \left[ r(i, j) + \gamma \sum_{k \in \mathcal{T}} p_k V^*_{t+1}(\tilde{I}_t, k) \right]
\]

(23)

Notice that the last term \( \gamma \sum_{k \in \mathcal{T}} p_k V^*_{t+1}(\tilde{I}_t, k) \) is independent of the decision variable \( j \). Hence,

\[
\lim_{I^n_t \to \infty} \arg \max_{j \in A(I_t)} \left[ r(i, j) + \gamma \sum_{k \in \mathcal{T}} p_k V^*_{t+1}(I_t - e_j, k) \right] = \arg \max_{j \in \mathcal{C}} [r(i, j)].
\]

(23)

□

Theorem 3 implies that the myopic policy that maximizes the current-period reward is optimal when there exists infinite trailer inventory. That is, only considering the current-period problem.
is enough for optimally assigning trailers to minimize the cost. This is because the current-period
decision does not affect the inventory scarcity in the following periods. In other words, there exists
available inventory for all trailer types in any period regardless of the previous assignment decision.

The two cases, perfect information and infinite inventory can be adopted as the performance
bounds for DTA. Table 11 demonstrates the cost comparison of the performance bounds. Compared
with DTA, the perfect information and infinite inventory cases lower the cost by about 18% and
28%, respectively. Even though perfect information can eliminate some of the uncertainty associated
with tractor weight, keeping infinite inventory is more efficient to reduce the cost. However, it
is not practical to keep high inventory for several reasons. Hence, we investigate how adding a
few inventory affects the performance. We test the performance improvement by adding 2 and 5
more trailers for each trailer type and compare the results. Figure 10 illustrates the comparison
results. Surprisingly, adding just a few inventory significantly reduces the cost. It shows that the
cost exponentially decreases and fast approaches to the infinite inventory case as adding just a
few inventory. Especially, when adding 5 more inventory, the performance is even better than
the perfect information case. This result provides interesting managerial insights on the trailer
assignment problem. Coordination with 3PL providers to resolve the tractor uncertainty may not
be an efficient way to manage the trailer shipment problem. Instead, adding just a few more trailer
inventory and dynamically utilizing it in the assignment problem may be more beneficial to reduce
the cost.

7.3. Impact of multiple target weights on DTA performance

In addition to weight information and inventory level, we also studied how the use of multiple
target weights impacts on DTA performance. The computation time is a big issue in choosing how
many target weights. Adding one more target weights means that the vector \( \mathcal{I}_t \), the collection of
the remaining inventory in time period \( t \), now has one more element for each state and hence the
\( Q \) table, \( \hat{Q}(s,a) \), must be updated for all newly added pairs of state and action. We compared the
cost of using 2 or 4 target weights to the base case of 3 for each month. Figure 11 shows that using
3 target weights outperforms using 2 targets in 5 of 8 months. Using 4 targets performs using 3 targets in 6 of 8 months doubles the processing time. This numerical experiment demonstrates that increasing the target weights does not always improve the efficiency of DTA performance on cost.
8. Conclusion

This paper introduces the trailer shipment problem faced by ABI. The logistics outsourcing practice via 3PL providers causes a major challenge to this problem. Since the weight of tractors owned by 3PL providers is random and unknown to ABI when it pre-loads trailers, it is challenging for ABI to meet the gross weight limit regulation. We propose practical data-analytics methodologies that efficiently solve the trailer shipment problem using the historical data.

Sample Average Approximation (SAA) uses the empirical distribution using the sample data to approximate the expected mismatching cost to be minimized. Empirical Risk Minimization (ERM) is a feature-based approach, which incorporates key features that are related to the uncertain tractor weight into the solution process. ERM provides the optimal trailer weight as a function of these features. The distance between the brewery and wholesaler is identified as such a key feature in this methodology. Dynamic Trailer Allocation (DTA) utilizes the tractor weight information that is available upon arrival to dynamically assign different weights of trailers by considering the value of the current inventory on the future arrivals.

Using the transaction-level data obtained from ABI, we evaluate the suggested methodologies by comparing the average mismatching cost. The numerical experiment reveals that ERM outperforms SAA. Utilizing the distance information as a feature, it enables to better predict the uncertainty tractor weight and to customize the trailer load size accordingly. However ERM is deterministic allocation which does not use the available information on the tractor weight. On the other hand, DTA prepares only a few trailer load sizes (three types in our experiment) and dynamically allocate the trailers based upon the incoming tractor and the current trailer inventory. Surprisingly, DTA considerably outperforms both SAA and ERM.

We also investigate two performance bounds for DAT: perfect information and infinite inventory cases. The infinite inventory case performs better than the perfect information one. More interestingly, adding just a few inventory significantly reduces the cost, even better than the perfect information case. Considering the fact that coordination with the 3PL provides is practically infeasible, this result highlights that keeping a few more trailer inventory and dynamically utilizing it
may be a more practical and efficient way to manage the trailer shipment problem rather than obtaining the tractor information via coordination with 3PL providers.

The key contributions of this work are summarized as follows. This work introduces the trailer shipment problem, which can be widely applied in different industries and makes a significant economic impact. This paper proposes practical data-analytics approaches that efficiently solve the problem. Especially, we propose a RL-based methodology (DTA) that dynamically incorporates the actual tractor weight information into the trailer assignment decision, which significantly outperforms the other static methodologies (SAA and ERM). This work also highlights the importance of keeping a few more trailer inventory and dynamically utilizing it to further improve the associated cost in the trailer shipment problem.
References


