An extension of the Beurling-Chen-Hadwin-Shen theorem for noncommutative Hardy spaces associated with finite von Neumann algebras

Abstract

In 2015, Yanni Chen, Don Hadwin and Junhao Shen proved a noncommutative version of Beurling’s theorem for a continuous unitarily invariant norm $\alpha$ on a tracial von Neumann algebra $(M, \tau)$ such that $\alpha$ is one dominating with respect to $\tau$. The role of $H^\infty$ is played by a maximal subdiagonal algebra $A$. In the talk, we first will show that if $\alpha$ is a continuous normalized unitarily invariant norm on $(M, \tau)$, then there exists a faithful normal tracial state $\rho$ on $M$ and a constant $c > 0$ such that $\alpha$ is a $c$ times one norm-dominating norm on $(M, \rho)$. Moreover, $\rho(x) = \tau(xg)$, where $x \in M$, $g$ is positive in $L^1(Z, \tau)$, where $Z$ is the center of $M$. Here $c$ and $\rho$ are not unique. However, if there is a $c$ and $\rho$ so that the Fuglede-Kadison determinant of $g$ is positive, then Beurling-Chen-Hadwin-Shen theorem holds for $L^{(\alpha)}(M, \tau)$. The key ingredients in the proof of our result include a factorization theorem and a density theorem for $L^{(\alpha)}(M, \rho)$.