Spring 2017

Washington University
Senior Honors Thesis Abstracts

Spring 2017

Two Partitioned Numerical Methods for Solving the Hodgkin-Huxley Equations

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The Hodgkin-Huxley model, a system of four-dimensional ordinary differential equations, is a fundamental mathematical model of the dynamics of a neuron's membrane voltage. Simulating neurons' activities using the Hodgkin-Huxley equations is computational expensive compared to using some other neuronal models, and there have been interests in looking for efficient numerical methods for solving the Hodgkin-Huxley equations. We aimed at looking for numerical methods that can solve the Hodgkin-Huxley equations with high stability, and studied two partitioned methods in particular. First, we adapted the Störmer/Verlet method for Hamiltonian systems to the Hodgkin-Huxley equations, resulting in a method coinciding with a modified version of the trapezoidal method. Second, we created a type of Strang splitting method. In numerical experiments, we found that the Störmer/Verlet method has higher stability than Euler's method and the Fourth Order Runge-Kutta method, two commonly used numerical methods, and that the Strang splitting method has even higher stability than the Störmer/Verlet method. Theoretically, both the Störmer/Verlet method and the Strang splitting method can be shown to have second order convergence. Furthermore, when applied to solve the van der Pol equations, a simpler nonlinear dynamical system chosen for theoretical analysis, both of these methods can be shown theoretically to preserve the size of the limit cycle to the second order because of their sympleticity.