Manifesting Polynomials with Small Galois Group as Rational Points

Philip Bonneville
Washington University in St. Louis

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In a 2002 paper, Nils Bruin and Noam Elkies use what appears to be a novel approach to find trinomials of the form $ax^n + bx + c$ (for $n = 7, 8$) with small Galois group over $\mathbb{Q}$. They parametrize such polynomials, up to equivalence, as (the rational points of) a copy of the projective line and then examine covers of this line that correspond to distinguishing different orderings of the $n$ roots modulo a subgroup $H$ of $S_n$. Using a correspondence between rational points on such a cover and trinomials with Galois group, a subgroup of $H$, they are able to use a technique for finding rational points on a curve to determine which trinomials of this form have a specified Galois group.

Since the basic theory of varieties over an algebraically closed field is unable to capture the arithmetic behavior at issue here, I describe how the language of scheme theory might be used to precisely state and justify this correspondence. I also examine a connection of this correspondence with work of Guralnick and Shareshian on enumerating covers of the Riemann sphere with small genus, in light of Falting’s Theorem.