

Washington University in St. Louis

## Washington University Open Scholarship

---

Doctor of Business Administration  
Dissertations

Olin Business School

---

Spring 5-18-2018

### Skewness Risk, Jump-Diffusion Modeling and Pricing in Carry Trade

Chitsun Chen

Follow this and additional works at: <https://openscholarship.wustl.edu/dba>



Part of the [Business Administration, Management, and Operations Commons](#), and the [Finance and Financial Management Commons](#)

---

#### Recommended Citation

Chen, Chitsun, "Skewness Risk, Jump-Diffusion Modeling and Pricing in Carry Trade" (2018). *Doctor of Business Administration Dissertations*. 5.  
<https://openscholarship.wustl.edu/dba/5>

This Dissertation is brought to you for free and open access by the Olin Business School at Washington University Open Scholarship. It has been accepted for inclusion in Doctor of Business Administration Dissertations by an authorized administrator of Washington University Open Scholarship. For more information, please contact [digital@wumail.wustl.edu](mailto:digital@wumail.wustl.edu).

# Skewness Risk, Jump-Diffusion Modeling and Pricing in Carry Trade

Chitsun Chen\*

May 1, 2018

## Abstract

This paper investigates jump risk and return characteristics of currency carry trades by employing both empirical approach and analytical method. With country-level stochastic discount factor, a mathematical model is proposed to describe carry trade return dynamics that capture jump risk. Carry trade returns are modeled as jump-diffusion processes where types of jumps involve global and idiosyncratic jumps. We derive the first four moments of return process on exchange rates and use the method of moments to estimate parameters, which show the model matches excess carry trade returns in data quite well. Empirical findings show that carry trade returns exhibit an asymmetric leptokurtic feature. We also conduct asset pricing model testing and obtain results suggest that the skewness factor is a driving force to carry trade returns.

---

\*Olin Business School, Washington University in St. Louis, Email: cchen46@wustl.edu.

# 1 Introduction

Currency carry trade is very popular among large financial institutions and even small independent traders. Essentially a currency carry trade involves borrowing a currency (funding currency) with a low interest rate, and using the proceeds to finance the purchase of a currency (investment currency) with a high interest rate. Carry trade return can be decomposed into two components: a return from the interest rate differential paid between the currency pair and a return from exchange rate movements in the investment currency. When a trader enters into a carry trade, the return from interest rate differential is deterministic, but the return from exchange rate movements is stochastic. If the investing currency appreciates, the trader will have earned interest profit along with the price appreciation of the currency pair. If the depreciation in the investment currency outweighs interest profit, the carry trade will generate a negative return. Although traders may take advantage of positive interest rate spread and potential appreciation in investing currencies, carry trades do have its own inherent risks - investing currencies are subject to abrupt and sizable depreciation and may have periods of extraordinary volatility. According to the uncovered interest rate parity (UIP), the investment currency would depreciate by the amount of the positive interest rate differential between the currency pair. The fact that investment currencies tend to appreciate empirically violates the UIP, which brings about the so-called forward premium puzzle (Fama (1984)). One may think about that the strong interest rate differential increases the demand from the larger participants, which drives appreciation in the investment currency. The failure of UIP has made the carry trade profitable on average for a long time.

However, there is plenty of evidence that a carry trade strategy may produce frequent small positive excess returns but have potential extremely large losses due to depreciate sharply in investment currencies. For currency crashes, Brunnermeier and Pedersen (2009) document the features of liquidity spirals. They argue that traders provide liquidity. However, when traders reach capital constraints and close their positions, the large selling pressures can not be offset by potential buyers to maintain market equilibrium. The further drop in prices exacerbates market illiquidity and margin issues. Also, many big price changes in investment currency seem to happen in the absence of significant news or events.

Several common risk factors to account for cross-sectional variation in carry trade excess returns have been proposed in the literature, such as Lustig et al. (2011), Menkhoff et al. (2012), Ang and Xing (2016) and Rafferty (2012). For instance, Lustig et al. (2011) identify a slope factor

in exchange rates to explain the cross-sectional variation in average excess returns between high and low interest rate currencies. They form portfolios of currencies sorted on forward discounts. The forward discount is equivalent to the interest rate differential between a currency pair. They show that portfolios of currencies with higher interest rates generate higher returns. A dollar risk factor(DOL) is average excess return on all foreign currency portfolios; a slope( $HML_{FX}$ ) factor is the excess return on a portfolio for high interest rate minus low interest rate currency; and a global foreign exchange volatility risk is an aggregate market volatility.

The currency crash risk in the carry trade may be characterized by jumps in exchange rate movements. [Lee and Wang \(2017\)](#) mention that investment currencies with high negative jump beta provide higher expected returns. However, when the market experiences negative jumps, these currencies depreciate sharply. [Chernov et al. \(2013\)](#) show that on average, jumps account for 25% of total currency risk and can be as high as 40%, and also they indicate that jump risk is priced. Carr and Wu (2007) distinguish jumps up and down in FX and also allow for time-varying jump probabilities controlled by unobservable states. [Jurek and Xu \(2014\)](#) develop a model of exchange rates based on pricing kernel dynamics driven by global and country-specific shocks. [Farhi et al. \(2015\)](#) and [Farhi and Gabaix \(2016\)](#) propose a new of exchange rate, based on the hypothesis that the possibility of rare but extreme disasters is an important driver of risk premia.

This paper investigates jump risk and return characteristics of currency carry trades by employing both empirical approach and analytical method. In the empirical analysis, this paper updates some of the computations by extending the sample of [Brunnermeier et al. \(2008\)](#) from January, 1986 to December, 2016 for ten currencies by using series of 372 monthly data. Empirical results in this paper show (i) carry trade returns exhibit an asymmetric leptokurtic feature, which indicates the return distribution of carry trades is negatively skewed and has a high peak as well as fat tails; (ii) the relationship between the average 1-month carry trade return and the interest rate differential is positive, while the relationship between the average 1-month carry trade return and FX skewness of change in exchange rate movements is negative; (iii) sorted by interest rate differentials, long-short carry portfolios generate positive returns and negative skewness; (iv) high interest rate differential is a significant indicator of negative skewness of carry trade return. negative skewness.

In the analytical method, a mathematical model is proposed to describe carry trade return dynamics that capture jump risk. Carry trade returns are modeled as jump-diffusion processes where types of jumps involve global and idiosyncratic jumps. We derive the first four moments of

return process on exchange rates and use the method of moments to estimate parameters, which show the model matches excess carry trade returns in data quite well.

The rest of the paper is organized as follows. Section 2 formulates carry trade returns with various currencies and describes the data. Empirical results present the key characteristic of carry trade returns for ten currency pairs, the performance of long-short carry portfolios, and prediction of carry trade returns and FX skewness. Section 3 deals with modeling and parameter estimation of jump-diffusion carry trade dynamics, including a global jump and idiosyncratic jumps. Section 4 describes how to construct long-short trading strategies based on the third moment of return processes on exchanges and evaluate their performance. Furthermore, we test how a global currency skewness factor and a level factor explain portfolio returns of five long-short currency trading strategies. Section 5 concludes. Appendix A formulates returns on three equally weighted carry portfolios. Appendix B derives the first four moment conditions of return processes on exchange rates. Appendix C contains an extension to a model for carry trade returns with various types of jumps.

## 2 Carry Trades and Skewness Risk

### 2.1 Interest Rate Differential $r_{F,t} - r_{H,t}$

This section discusses the link of interest rate differentials between two countries to differences in spot and forward exchange rates. Later we'll use this relationship for interest rate differentials to calculate carry trade returns. In terms of exchange rates, our convention is that one unit of home currency  $H$  is the equivalent of  $e_{F/H,t}$  units of foreign currency  $F$ . Let  $e_{F/H,t}$  and  $f_t$  be the spot exchange rate and forward exchange rate at time  $t$ , respectively. Also, let  $r_{H,t}$  and  $r_{F,t}$  denote the relevant interest rates in home and foreign currencies at time  $t$ , respectively.

As an illustration, a U.S. investor has made a commitment to pay one unit of foreign currency  $F$  at time  $t + dt$ . In order to hedge the exchange rate risk, the investor may make either of the following transactions : (i) At time  $t$ , the investor deposits  $\frac{1}{f_t} \frac{1}{1+r_{H,t}dt}$  units of home currency  $H$  at interest rate  $r_{H,t}$ . Next period, at time  $t + dt$ , the investor will receive  $\frac{1}{f_t}$  units of home currency  $H$ , and exchange the proceeds into a foreign currency  $F$  at forward exchange rate  $f_t$  to receive one unit of foreign currency  $F$ . (ii) Similarly, at time  $t$ , the investor deposits  $\frac{1}{e_{F/H,t}} \frac{1}{1+r_{F,t}dt}$  units of foreign currency  $F$  at interest rate  $r_{F,t}$ . It turns out that the investor will also receive one unit of

foreign currency  $F$  at time  $t + dt$ , the same result as the operation (i).

In fact, while the transaction (i) is a strategy of an investment in the home risk-free asset and long currency forward, the transaction (ii) is an investment in the foreign risk-free asset. Obviously,  $\frac{1}{f_t} \frac{1}{1+r_{H,t}dt}$  is the present value of  $\frac{1}{f_t}$  units of home currency  $H$ ; on the other hand,  $\frac{1}{e_{F/H,t}} \frac{1}{1+r_{F,t}dt}$  is the present value of one unit of foreign currency  $F$ . As covered interest rate parity(CIP) holds, the no-arbitrage condition is satisfied with a variation on the law of one price for foreign exchange markets: equivalent assets with equal risk should offer the same price, regardless of currency denomination. This implies,

$$\frac{1}{f_t} \cdot \frac{1}{1+r_{H,t}dt} = \frac{1}{e_{F/H,t}} \cdot \frac{1}{1+r_{F,t}dt}$$

Taking the log on both sides of the equation yields

$$r_{F,t} - r_{H,t} = \log \left( \frac{f_t}{e_{F/H,t}} \right) = \log(f_t) - \log(e_{F/H,t}) \quad (1)$$

where we assume  $dt = 1$ .

## 2.2 Relationship between Exchange Rate and Stochastic Discount Factors $e_{F/H,t} = \frac{M_{H,t}}{M_{F,t}}$

In the absence of arbitrage opportunities, the exchange rate can be expressed as the ratio of two country-specific stochastic discount factors(SDF). Let  $M_{H,t}$  and  $M_{F,t}$  denote the SDFs at time  $t$  in the home currency  $H$  and foreign currency  $F$ , respectively. Then for each pair of time  $t < T$ ,

$$P_t^{(H)} = E_t \left[ \frac{M_{H,T}}{M_{H,t}} X_T^{(H)} \right]$$

$$P_t^{(F)} = E_t \left[ \frac{M_{F,T}}{M_{F,t}} X_T^{(F)} \right] = E_t \left[ \frac{M_{F,T}}{M_{F,t}} e_{F/H,T} X_T^{(H)} \right]$$

where  $P_t^{(H)}$  and  $P_t^{(F)}$  and denote prices of assets at time  $t$  in the home currency  $H$  and foreign currency( $F$ ), and  $X_T^{(H)}$  and  $X_T^{(F)}$  denote payoffs of assets at time  $T$  in the home currency  $H$  and foreign currency  $F$ , respectively. Given the fact that

$$e_{F/H,t} \times P_t^{(H)} = P_t^{(F)}$$

It follows that

$$\begin{aligned}
e_{F/H,t} \times E_t \left[ \frac{M_{H,T}}{M_{H,t}} X_T^{(H)} \right] &= E_t \left[ \frac{M_{F,T}}{M_{F,t}} e_{F/H,T} X_T^{(H)} \right] \\
E_t \left[ \frac{M_{H,T}}{M_{H,t}} X_T^{(H)} \right] &= E_t \left[ \frac{M_{F,T}}{M_{F,t}} \frac{e_{F/H,T}}{e_{F/H,t}} X_T^{(H)} \right] \\
\frac{M_{H,T}}{M_{H,t}} &= \frac{M_{F,T}}{M_{F,t}} \frac{e_{F/H,T}}{e_{F/H,t}} \tag{2}
\end{aligned}$$

### 2.3 Carry Trade Return with Two Currencies $CT_{-H/+F,t+dt}^H$

Suppose an investor forms a carry trade portfolio that takes a short position (borrowing) in the home currency  $H$  and a long position (lending) in a foreign currency  $F$ . At time  $t$ , the investor borrows a notional amount  $P_0$  units of home currency  $H$  at an interest rate  $r_H$ . The proceeds are exchanged into the foreign currency  $F$  at the exchange rate  $e_{F/H,t}$  to receive  $P_0 e_{F/H,t}$ , which are deposited at an interest rate  $r_{F,t}$ . At time  $t + dt$ , from the short position, the investor will owe  $P_0(1 + r_{H,t}dt)$  units of home currency  $H$ ; from the long position, the investor will receive  $\frac{P_0 e_{F/H,t}(1+r_{F,t}dt)}{e_{F/H,t+dt}}$  units of home currency, where  $e_{F/H,t+dt}$  is the exchange rate at time  $t + dt$ . Clearly, the gross return on this carry trade portfolio would be

$$\frac{\frac{P_0 e_{F/H,t}(1+r_{F,t})}{e_{F/H,t+dt}}}{P_0(1+r_{H,t})} = \frac{1+r_{F,t}dt}{1+r_{H,t}dt} \cdot \frac{e_{F/H,t}}{e_{F/H,t+dt}}$$

Taking the log to obtain log gross return on carry trade portfolio,

$$\begin{aligned}
\log \left( \frac{1+r_{F,t}dt}{1+r_{H,t}dt} \cdot \frac{e_{F/H,t}}{e_{F/H,t+dt}} \right) &= \log \left( \frac{1+r_{F,t}dt}{1+r_{H,t}dt} \right) + \log \left( \frac{e_{F/H,t}}{e_{F/H,t+dt}} \right) \\
&\simeq r_{F,t}dt - r_{H,t}dt - \log \left( \frac{e_{F/H,t+dt}}{e_{F/H,t}} \right)
\end{aligned}$$

Let  $dt = 1$ . The carry trade return on the portfolio of borrowing home currency  $H$  and lending foreign currency  $F$  from time  $t$  to  $t + 1$  is as follows.

$$CT_{-H/+L,t+1}^H = r_{F,t} - r_{H,t} - \log \left( \frac{e_{F/H,t+1}}{e_{F/H,t}} \right) \tag{3}$$

$$= \log \left( \frac{f_t}{e_{F/H,t}} \right) - \log \left( \frac{e_{F/H,t+1}}{e_{F/H,t}} \right) \tag{4}$$

where  $r_{F,t} - r_{H,t}$  is replaced with  $\log\left(\frac{f_t}{e_{F/H,t}}\right)$  by using covered interest rate parity. Here the superscript  $H$  of carry trade return notation  $CT_{-H/+F,t+dt}^H$  means that home currency  $H$  is a base currency; the subscript “ $-H/+F$ ” represents “taking a short position in home currency  $H$  and a long position in foreign currency  $F$ ”. This implies that  $H$  is the funding currency, and  $F$  is the investing currency.

## 2.4 Carry Trade Return with Three Currencies $CT_{-B/+L,t+dt}^H$

In the previous section, we considered carry trade portfolio containing two currencies: home currency  $H$  and a foreign currency  $F$ . This section will extend it to the general case consisting of three currencies: home currency  $H$  and two foreign currencies  $B$  and  $L$ . For instance, to take advantage of the interest rate differential between two currencies, a US investor may want to borrow in a lower interest rate currency, such as the Japanese yen, to fund investments in higher interest rate currency, such as the Australia dollar.

Now we consider this case in more detail. Suppose at time  $t$ , an investor creates a portfolio: borrowing foreign currency  $B$  at interest rate  $r_{B,t}$ , and lending foreign currency  $L$  at interest rate  $r_{L,t}$ . This portfolio can be replicated by a synthetic consisting of two operations: (i) borrowing home currency  $H$  at interest rate  $r_{H,t}$ , and lending foreign currency  $L$  at interest rate  $r_{L,t}$ . (ii) borrowing foreign currency  $B$  at interest rate  $r_{B,t}$ , and lending home currency  $H$  at interest rate  $r_{H,t}$ . From the expression (3), carry trade returns on the replicating synthetic are summarized as follows.

Carry trade return on the operation (i): borrowing home currency  $H$ , and lending foreign currency  $L$ ,

$$\begin{aligned} CT_{-H/+L,t+dt}^H &= r_{L,t} - r_{H,t} - \log\left(\frac{e_{L/H,t+dt}}{e_{L/H,t}}\right) \\ &= r_{L,t} - r_{H,t} - \delta e_{L/H,t+dt} \end{aligned} \tag{5}$$

where we define  $\delta e_{L/H,t+dt} \equiv \log\left(\frac{e_{L/H,t+dt}}{e_{L/H,t}}\right)$ .

Likewise, carry trade return on the operation (b): borrowing foreign currency  $B$ , and lending



home currency  $H$ ,

$$\begin{aligned}
CT_{-B/+H,t+dt}^H &= r_{H,t} - r_{B,t} - \log\left(\frac{e_{H/B,t+dt}}{e_{H/B,t}}\right) \\
&= r_{H,t} - r_{B,t} - \delta e_{H/B,t+dt} \\
&= r_{H,t} - r_{B,t} + \delta e_{B/H,t+dt}
\end{aligned} \tag{6}$$

where we define  $\delta e_{H/B,t+dt} \equiv \log\left(\frac{e_{H/B,t+dt}}{e_{H/B,t}}\right)$  and  $\delta e_{B/H,t+dt} \equiv \log\left(\frac{e_{B/H,t+dt}}{e_{B/H,t}}\right)$ .

The addition of equation (5) and (6) results in the elimination of the home currency  $H$  term. Thus this synthetic forms a replication for the original portfolio. We conclude that carry trade return on the portfolio of borrowing foreign currency  $B$  and lending foreign currency  $L$  is

$$\begin{aligned}
CT_{-B/+L,t+dt}^H &= (r_{L,t} - r_{H,t} - \delta e_{L/H,t+dt}) + (r_{H,t} - r_{B,t} + \delta e_{B/H,t+dt}) \\
&= r_{L,t} - r_{B,t} - (\delta e_{L/H,t+dt} - \delta e_{B/H,t+dt}) \\
&= r_{L,t} - r_{B,t} - \delta e_{L/B,t+dt} \\
&= r_{L,t} - r_{B,t} - \left[ \log\left(\frac{e_{L/H,t+dt}}{e_{L/H,t}}\right) - \log\left(\frac{e_{B/H,t+dt}}{e_{B/H,t}}\right) \right] \\
&= r_{L,t} - r_{B,t} + \log\left(\frac{e_{B/H,t+dt}}{e_{L/H,t+dt}}\right) - \log\left(\frac{e_{B/H,t}}{e_{L/H,t}}\right)
\end{aligned} \tag{7}$$

where we define  $\delta e_{L/B,t+dt} \equiv \delta e_{L/H,t+dt} - \delta e_{B/H,t+dt}$ . In fact,  $e_{L/B} = \frac{e_{L/H}}{e_{B/H}}$ . The exchange rate  $e_{L/B}$  indicates  $e$  units of foreign currency  $L$  per unit of foreign currency  $B$ .

## 2.5 Empirical Results

### 2.5.1 Description of the Data

The data set for analysis consists of daily spot exchange rates, 1-month spot and forward exchange rates (beginning-of-month value) for ten currencies, which include Australian dollar(AUD), Canadian dollar(CAD), European euro(EUR), Swiss franc(CHF), Norwegian krone(NOK), Japanese yen(JPY), British pound(GBP), New Zealand dollar(NZD), Danish krone(DKK), and Swedish krona(SEK).The base currency is the U.S. dollar(USD). In this study, the data sample contains information on historical currency exchange rates from January 1, 1986 through December 30, 2016 via Thomson Reuters Datastream.

With the introduction of the euro(EUR) on January 1, 1999, there was no time series data

available for the euro from January 1, 1986 to December 31, 1998. To overcome this problem posed by the incomplete data, we add the exchange rates of the Deutsche mark(DEM) to the euro data for that period from January, 1986 to December, 1998.

### 2.5.2 Jump Risk in Carry Trades (Cross-Sectional Variation)

Assume the home currency is the U.S. dollar(USD). As mentioned earlier, ten foreign currencies are AUD, CAD , CHF, DKK, EUR, GBP, JPY, NOK, NZD, and SEK, respectively. A carry trade pair is constructed by a self-financing strategy – borrowing USD and using the proceedings to investing in one of ten foreign currencies. Namely, the U.S. dollar(USD) is the funding currency, and ten foreign currencies are investment currencies.

The carry trade return on a portfolio of borrowing home currency  $H$  (USD)and investing in foreign currency  $F$  from month  $t$  and  $t + 1$ ,  $CT_{-H/+F,t+1}^H = r_{F,t} - r_{H,t} - \log\left(\frac{e_{F/H,t+1}}{e_{F/H,t}}\right)$ , has two components. The first component, interest rate differential“ $r_{F,t} - r_{H,t}$ ”, is a deterministic term at the beginning of each month, while the second component, change in the foreign exchange rate movements “ $\log\left(\frac{e_{F/H,t+1}}{e_{F/H,t}}\right)$ ”, is an unpredictable shock to exchange rates. When exchange rates become more volatile, the second stochastic term have enough of impact to move carry trade return around. In other words, the first term loses in relative importance and second stochastic term tends to dominate the variation in carry trade return.

There are several interesting features in Table 1, which reports the cross-sectional carry trade results for ten currency pairs. First, all carry trade returns for 9 currency pairs are positive, with the exception of the negative value ( $-0.0004$ ) for JPY/USD pair. Second, the interest rate differential plays a crucial role in carry trade return. There is a striking pattern that currency pairs with higher interest rate differentials earn higher average excess returns. Third, currency pairs with higher excess returns tend to be more negatively skewed. Specifically, higher excess returns on currency pairs are exposed to higher jump risk (depreciation) in investment currencies. Finally, all ten sets of carry trade returns are leptokurtic since all empirical results of excess kurtosis in Table 1 are positive. The asymmetric leptokurtic features indicate that the carry trade distribution is negatively skewed, and has a high peak and fat tails. The skewness falls from  $-0.6747$  to  $0.2655$  and the excess kurtosis from  $0.5039$  to  $3.4560$ . The leptokurtic features of carry trade returns from FX markets parallel well-known research evidences from stock markets : most equity index return series have been found to be negatively skewed and leptokurtic.

The top panel of figure 1 presents a scatterplot of the average 1-month carry trade return against the interest rate differential between January, 1986 and December, 2016 for ten currency pairs. A point in the figure represents the average 1-month carry trade return and the average interest rate differential for a given currency relative to the U.S. dollar. There is a positive relationship between the average 1-month carry trade return and the interest rate differential. The bottom panel of figure 1 shows a negative relationship between the average 1-month interest rate differential and FX skewness of change in exchange rate movements, which is measured as the skewness of daily log FX exchange rate changes,  $\log\left(\frac{e_t}{e_{t+1}}\right)$ , within a month.

The upper plot of figure 2 shows a negative relationship between the average 1-month carry trade return and FX skewness of change in exchange rate movements; likewise, a negative relationship between the average 1-month carry trade return and skewness of carry trade returns is illustrated in the lower plot of figure 2. It is apparent that carry trades with negative skewness should be rewarded with positive returns. In short, figure 1, figure 2 and Table 1 suggest that on average, currencies with positive interest rate differentials tend to have positive carry trade returns but also have negative skewness, while currencies with negative interest rate differentials tend to have negative carry trade returns but also have positive skewness. These results confirm the findings of Brunnermeier et al. (2008) who also show that carry trades are subject to crash risk.

### 2.5.3 Performance of Long-Short Carry Portfolios

As described earlier, the interest rate differentials play a key role in carry trade returns. In order to gain a better understanding of the link between interest rate differentials and returns, we construct three long-short carry portfolios where investment currencies are sorted on interest rate differentials relative to the U.S. dollar. These portfolios are formed on an equally-weighted basis. The approach to long-short portfolio construction is outlined as follows: at the beginning of each month  $t$ , carry portfolios are built by taking long positions in currencies with the highest interest rate differentials and short positions in currencies with the lowest interest rate differentials.

Now suppose the first carry portfolio consists of a long position in currency  $J$  with the highest interest rate differential  $r_{J-H,t}$  and a short position in currency  $I$  with the lowest interest rate differential  $r_{I-H,t}$  in the month  $t$ . The return on the first carry portfolio,  $R_{P1,t+1}$ , in the month

Table 1: Carry Trades for Ten Currency Pairs(Cross-Sectional Variation)

	AUD	CAD	CHF	DKK	EUR
$r_F - r_H$	0.0024	0.0006	-0.0012	0.0007	-0.0003
$\log\left(\frac{e_{t+1}}{e_t}\right)$	-0.0002	-0.0001	-0.0019	-0.0006	-0.0007
$CT_{-H/+F,t+1}^H$	0.0026	0.0007	0.0007	0.0013	0.0004
Skewness $\left(CT_{-H/+F,t+1}^H\right)$	-0.4155	-0.4248	-0.1137	-0.2245	-0.2440
Excess Kurtosis $\left(CT_{-H/+F,t+1}^H\right)$	1.5187	3.4560	1.0543	0.4939	0.5039

	GBP	JPY	NOK	NZD	SEK
$r_F - r_H$	0.0014	-0.0019	0.0018	0.0031	0.0011
$\log\left(\frac{e_{t+1}}{e_t}\right)$	0.0004	-0.0015	0.0004	-0.0009	0.0005
$CT_{-H/+F,t+1}^H$	0.0010	-0.0004	0.0014	0.0040	0.0006
Skewness $\left(CT_{-H/+F,t+1}^H\right)$	-0.6747	0.2655	-0.4505	-0.2814	-0.3177
Excess Kurtosis $\left(CT_{-H/+F,t+1}^H\right)$	2.3488	1.5973	0.9550	1.9715	1.2279

Note: The data include 10 currency pairs, January 1986 - December 2016. The table presents the means over 372 months of the  $r_{F,t} - r_{H,t}$ ,  $\log\left(\frac{e_{t+1}}{e_t}\right)$ ,  $CT_{-H/+F,t+1}^H$ , skewness and excess kurtosis.  $r_F - r_H$  is the interest rate differential between foreign currency  $F$  and home currency  $H$ (the U.S. dollar).  $\log\left(\frac{e_{t+1}}{e_t}\right)$  is the monthly change in the foreign exchange rate movements.  $CT_{-H/+F,t+1}^H$  is the carry trade return of a currency pair from borrowing in home currency  $H$  (USD) and investing in foreign currency  $F$ .

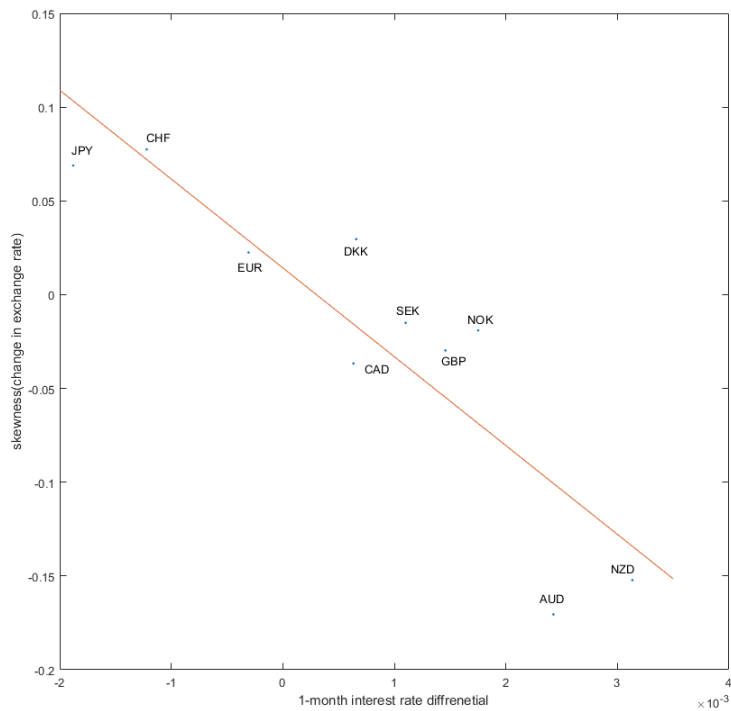
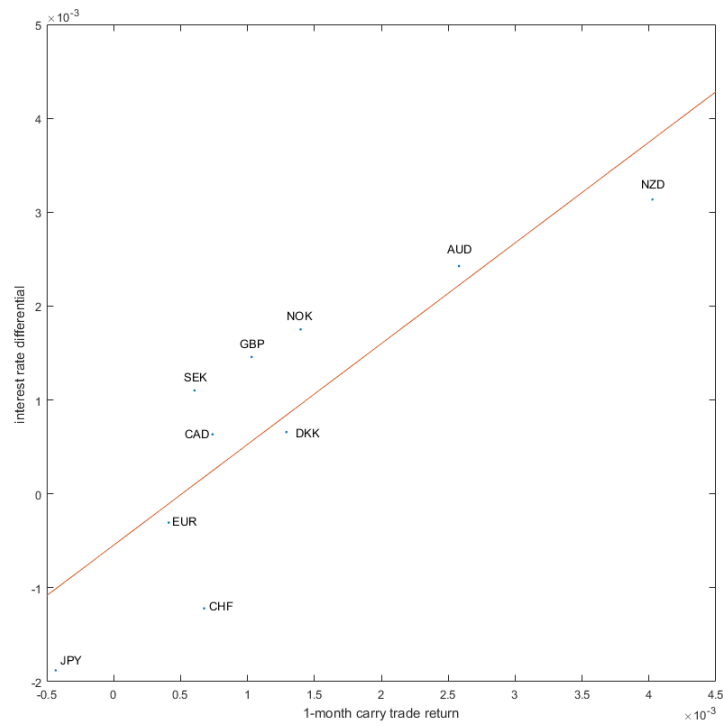


Figure 1: Top plot: Positive relationship between the average 1-month carry trade return and the average interest rate differential. Bottom plot: Negative relationship between the average 1-month interest rate differential and FX skewness of change in exchange rate movements, measured as the skewness of daily log FX exchange rate changes,  $\log\left(\frac{e_t}{e_{t+1}}\right)$ , within a month.

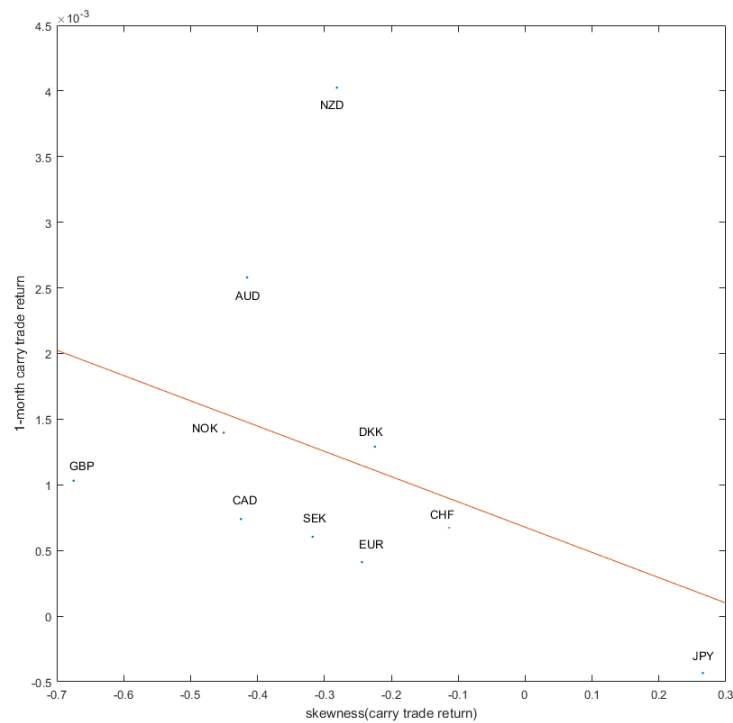
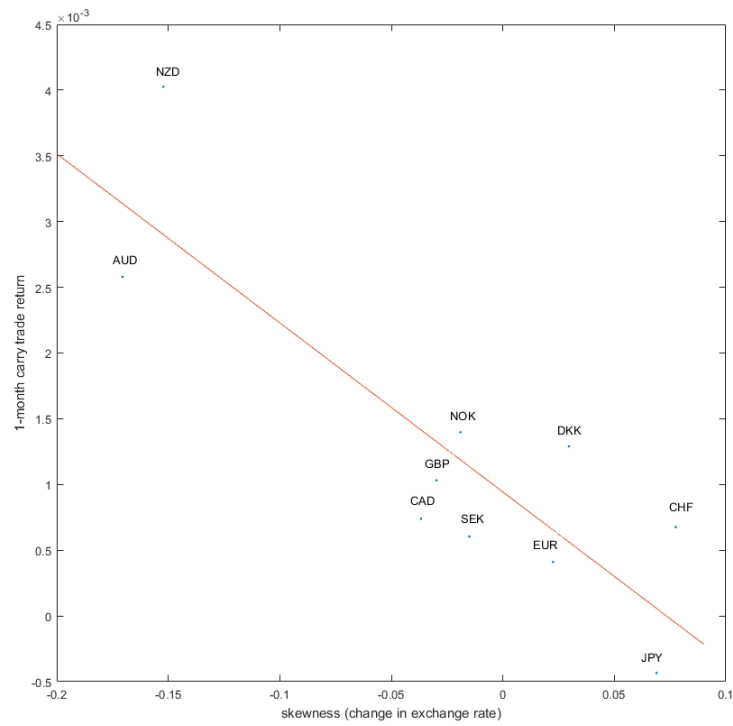


Figure 2: Top plot: Negative relationship between the average 1-month carry trade return and FX skewness of change in exchange rate movements, measured as the skewness of daily log FX exchange rate changes,  $\log\left(\frac{e_t}{e_{t+1}}\right)$ , within a month. Bottom plot: Negative relationship between the average 1-month carry trade return and skewness of carry trade returns.

$t + 1$  may be expressed as

$$R_{P1,t+1} = (r_{J-H,t} - r_{I-H,t}) - \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) - \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) \right] \quad (8)$$

where  $r_{J-H,t}$  is the interest rate differential between currency  $J$  and currency  $H$ , and  $r_{I-H,t}$  is the interest rate differential between currency  $I$  and currency  $H$  at the beginning of the month  $t$ .

Similarly, the second carry portfolio can be formed by taking a long position in two currencies  $J, J'$  with the highest interest rate differentials  $r_{J-H,t}$  and  $r_{J'-H,t}$ , as well as a short position in two currencies  $I, I'$  with the lowest interest rate differentials  $r_{I-H,t}$  and  $r_{I'-H,t}$  in the month  $t$ . The return on the second carry portfolio,  $R_{P2,t+1}$ , in the month  $t + 1$  may be expressed as

$$\begin{aligned} R_{P2,t+1} = & \frac{1}{2} \left[ (r_{J-H,t} + r_{J'-H,t}) - (r_{I-H,t} + r_{I'-H,t}) \right] - \frac{1}{2} \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) + \log \left( \frac{e_{J'/H,t+1}}{e_{J'/H,t}} \right) \right] \\ & + \frac{1}{2} \left[ \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) + \log \left( \frac{e_{I'/H,t+1}}{e_{I'/H,t}} \right) \right] \end{aligned} \quad (9)$$

In addition, the third carry portfolio can be constructed by taking a long position in three currencies  $J, J', J''$  with the highest interest rate differentials  $r_{J-H,t}$ ,  $r_{J'-H,t}$ , and  $r_{J''-H,t}$  as well as a short position in three currencies  $I, I', I''$  with the lowest interest rate differentials  $r_{I-H,t}$ ,  $r_{I'-H,t}$  and  $r_{I''-H,t}$  in the month  $t$ . The return on the third carry portfolio,  $R_{P3,t+1}$  in the month  $t + 1$  may be expressed as

$$\begin{aligned} R_{P3,t+1} = & \frac{1}{3} \left[ (r_{J-H,t} + r_{J'-H,t} + r_{J''-H,t}) - (r_{I-H,t} + r_{I'-H,t} + r_{I''-H,t}) \right] \\ & - \frac{1}{3} \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) + \log \left( \frac{e_{J'/H,t+1}}{e_{J'/H,t}} \right) + \log \left( \frac{e_{J''/H,t+1}}{e_{J''/H,t}} \right) \right] \\ & + \frac{1}{3} \left[ \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) + \log \left( \frac{e_{I'/H,t+1}}{e_{I'/H,t}} \right) + \log \left( \frac{e_{I''/H,t+1}}{e_{I''/H,t}} \right) \right] \end{aligned} \quad (10)$$

Table 2 summarizes the performance of three long-short carry portfolios. There is a monotone pattern of increasing average monthly return and standard deviation. Portfolio 3 yields the lowest mean monthly return of 0.0049 with standard deviation of 0.0257, and has negative skewness of -0.5502. By contrast, portfolio 1 generates the highest mean monthly return of 0.0072 with standard deviation of 0.0409, but has larger negative skewness of -1.1460. Furthermore, returns for three carry portfolios are leptokurtic, since all of them have positive excess kurtosis. The empirical results

for long-short portfolios listed in Table 2 also confirm the prior findings that on average, carry trade returns are positive; however, they have a high peak, fat tails and negative skewness, so they are associated with crash risk.

Table 2: Performance of Long-Short Carry Portfolios

	Portfolio 1	Portfolio 2	Portfolio 3
	(Long 1, Short 1)	(Long 2, Short 2)	(Long 3, Short 3)
Average Monthly Return	0.0072	0.0061	0.0049
Standard Deviation	0.0409	0.0306	0.0257
Skewness	-1.1460	-0.7558	-0.5502
Excess Kurtosis	3.4906	2.0292	1.7000
Annualized Sharp Ratio	0.6072	0.6962	0.6650

Note: The data include 372 months over 31 years, January 1986 - December 2016. Carry portfolios are built by taking long positions in currencies with the highest interest rate differentials and short positions in currencies with the lowest interest rate differentials at the beginning of each month. Portfolios are constructed on an equally-weighted basis.

#### 2.5.4 Forecasting Carry Trade Returns and FX Skewness

Given our findings above, in this subsection, we use panel regressions with country fixed effects to forecast carry trade returns and FX skewness based on lagged interest rate differentials and past carry trade returns. Country fixed effects are included in the regression model to control for possible unobserved heterogeneity across countries. First of all, we begin by regressing carry trade return on lagged interest rate differential as shown in equation (11) below:

$$CT_{-H/+F,t}^H = \alpha + \beta_1 (r_{F,t-h} - r_{H,t-h}) + \epsilon_t \quad (11)$$

where  $r_{F,t-h} - r_{H,t-h}$  is the  $h$ -month lagged interest rate differential. The first column of Table 3 reports regression that the coefficient on interest rate differential,  $r_{F,t-h} - r_{H,t-h}$ , is positive and statistically significant, indicating that high interest rate differentials predicts high carry trade returns over the next ten months.

Next, to see whether there exists a momentum effect in carry trade returns, we include past carry trade return as an additional observed regressor to equation (11). The primary reason for including the lagged carry trade return is to see whether there exists a momentum effect in carry



trade returns. Specifically, this country fixed effects regression model with two regressors is

$$CT_{-H/+F,t}^H = \alpha + \beta_1 (r_{F,t-h} - r_{H,t-h}) + \beta_2 CT_{-H/+F,t-h}^H + \epsilon_t \quad (12)$$

where  $CT_{-H/+F,t-h}^H$  is the  $h$ -month lagged carry trade return. As expected from previous findings, the second column of Table 3 indicates there is a strong relation between carry trade returns and lagged interest rate differentials. Monthly interest rate differentials are highly significant in explaining future carry trade returns for ten months. However, the second column shows that the coefficients on lagged carry trade returns are small and half of them are negative. This result suggests that past carry trade return is not an important determinant of future carry trade return. Although research documented the existence of a momentum effect in stock returns, we find that monthly carry trade returns for our ten currency pairs do not exhibit a momentum effect.

To forecast FX skewness based on lagged interest rate differentials, we run country fixed effects regression model of the form:

$$FXskewness_t = \alpha + \beta_1 (r_{F,t-h} - r_{H,t-h}) + \epsilon_t \quad (13)$$

where FX skewness  $_t$  corresponds to the skewness of daily log FX exchange rate changes in month  $t$ . The statistically significant coefficient estimates on interest rate differential,  $r_{F,t-h} - r_{H,t-h}$ , in the third column of Table 3 indicate that high interest rate differential is a significant indicator of negative skewness.

The fourth regression in Table 3 includes lagged carry trade return, an additional potential determinant of future carry trade returns, along with country fixed effects.

$$FXskewness_t = \alpha + \beta_1 (r_{F,t-h} - r_{H,t-h}) + \beta_2 CT_{-H/+F,t-h}^H + \epsilon_t \quad (14)$$

The estimated coefficients in the country fixed effects regression in equation (14) are negative up to the next seven month, so higher interest rate differentials are associated with higher crash risk, as measured by negative skewness. Moreover, the effect of lagged interest rate differentials  $r_{F,t-h} - r_{H,t-h}$  on FX skewness are relatively large, and the effect of past carry trade returns  $CT_{-H/+F,t-h}^H$  are relatively small. Overall, the ability to predict future carry trade returns is stronger for lagged interest rate differentials than for past carry trade returns: the predictive power of lagged interest rate differential more than decuples on past carry trade return.

Table 3: Forecasting Carry Trade Returns  $CT_{-H/+F,t}^H$  and FX Skewness  $_t$

	$CT_{-H/+F,t}^H$	$CT_{-H/+F,t}^H$	FX Skewness $_t$	FX Skewness $_t$
$r_{F,t-1} - r_{H,t-1}$	0.94 (0.15)	0.88 (0.16)	-19.75 (4.27)	-16.88 (4.04)
$r_{F,t-2} - r_{H,t-2}$	0.95 (0.18)	0.94 (0.19)	-24.56 (3.73)	-22.58 (3.35)
$r_{F,t-3} - r_{H,t-3}$	1.03 (0.15)	0.93 (0.15)	-14.66 (2.65)	-13.17 (2.53)
$r_{F,t-4} - r_{H,t-4}$	0.68 (0.10)	0.73 (0.11)	-20.95 (4.82)	-19.66 (4.83)
$r_{F,t-5} - r_{H,t-5}$	1.09 (0.12)	1.16 (0.13)	-13.12 (3.50)	-12.55 (3.46)
$r_{F,t-6} - r_{H,t-6}$	1.02 (0.18)	1.05 (0.19)	-13.51 (4.21)	-10.70 (4.14)
$r_{F,t-7} - r_{H,t-7}$	1.38 (0.20)	1.47 (0.20)	-7.78 (4.46)	-6.29 (4.33)
$r_{F,t-8} - r_{H,t-8}$	1.34 (0.19)	1.31 (0.18)	-12.43 (5.19)	-13.08 (5.16)
$CT_{-H/+F,t-1}^H$		0.04 (0.01)		-1.99 (0.21)
$CT_{-H/+F,t-2}^H$		0.01 (0.02)		-1.36 (0.27)
$CT_{-H/+F,t-3}^H$		0.07 (0.02)		-1.04 (0.25)
$CT_{-H/+F,t-4}^H$		-0.04 (0.02)		-0.92 (0.40)
$CT_{-H/+F,t-5}^H$		-0.05 (0.17)		-0.38 (0.23)
$CT_{-H/+F,t-6}^H$		-0.01 (0.01)		-1.86 (0.24)
$CT_{-H/+F,t-7}^H$		-0.06 (0.01)		-0.97 (0.26)
$CT_{-H/+F,t-8}^H$		0.02 (0.01)		0.43 (0.28)

Note: This table reports results from country fixed effects regressions of carry trade returns and FX skewness on lagged carry trade returns and lagged FX skewness for ten currency pairs over the period January 1986 - December 2016. Standard errors appear in parentheses under the estimated coefficients. Dependent variables are  $CT_{-H/+F,t}^H$  and FX Skewness  $_t$ , respectively.  $r_{F,t-h} - r_{H,t-h}$  and  $CT_{-H/+F,t-h}^H$  are the  $h$ -month lagged interest rate differential and the  $h$ -month lagged carry trade return,  $h = 1, \dots, 8$ .

### 3 Modeling and Parameter Estimation of Jump-Diffusion Carry Trade Dynamics

#### 3.1 Adding Jumps to Stochastic Discount Factor Growth

Let  $M$  be a stochastic discount factor(SDF) process. Let  $\{N_{i,t} : t \geq 0\}$  be a Poisson process with intensity  $\lambda_i$  such that the probabilities  $P(dN_{i,t}(dt) = 1) = \lambda_i dt + o(dt)$ ,  $P(dN_{i,t}(dt) = 0) = 1 - \lambda_i dt + o(dt)$  and  $P(dN_{i,t}(dt) \geq 2) = o(dt)$ . For any country  $I$ , the SDF with the jump-diffusions is as follows.

$$\frac{dM_{I,t}}{M_{I,t}} = -r_{I,t}dt - \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) (dN_{i,t} - \lambda_i dt) \quad (15)$$

where the drift term  $r_{I,t}$  of the stochastic discount factor growth is a risk free interest rate, and the volatility term  $\eta_{I,t}^T$  is the vector of market prices of risk at time  $t$  in country  $I$ . The vector  $\eta_{I,t}^T$  can be also called factor risk premium while we think of the standard diffusion  $B_t$  as factors. The jump element is a compound process, where jump sizes  $\Delta_i I$  are independent and identically distributed random variables.  $e^{\Delta_i I} - 1$  is the jump amplitude for the jumps of Poisson process  $dN_{i,t}$ .

Now we only consider the effect of jump events (i.e., ignoring the diffusion components) on the stochastic discount factor. When a jump occurs during a time interval from  $t$  to  $t + dt$ , we have  $dB_t = 0$ ,  $dt = 0$ ,  $dN_{i,t} = 1$ ,  $dt \cdot dB_t = 0$ ,  $dt \cdot dN_{i,t} = 0$ ,  $dN_{i,t} \cdot dN_{i,t} = dN_{i,t}$ . Therefore,

$$\begin{aligned} \frac{M_{I,t+dt}}{M_{I,t}} &= \frac{M_{I,t} + dM_{I,t}}{M_{I,t}} = 1 + \frac{dM_{I,t}}{M_{I,t}} \\ &= 1 - r_{I,t}dt - \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) (dN_{i,t} - \lambda_i dt) \\ &= 1 + (e^{\Delta_i I} - 1) = e^{\Delta_i I} \end{aligned}$$

We conclude that the jump effect on  $\frac{M_{I,t+dt}}{M_{I,t}}$  would be

$$\frac{M_{I,t+dt}}{M_{I,t}} = e^{\Delta_i I} \quad (16)$$

#### 3.2 Modeling Carry Trade Return with Jump-Diffusions

This section will model the carry trade return described in terms of the jump-diffusion formulations. Suppose a U.S. trader would like to construct a self-financing portfolio involving three currencies: home currency  $I$  and two foreign currencies  $B$  and  $L$ . At time  $t$ , with a notional amount of  $X_t$

units of currency  $I$ , the trader borrows  $X_t \cdot e_{B/I,t}$  units of foreign currency  $B$  with maturity  $t + dt$  at interest rate  $r_B$ . Recall the result of equation (??), which plays a role in what follows. The loan amount in the foreign currency  $B$  is

$$X_t \cdot e_{B/I,t} = X_t \cdot \frac{M_{I,t}}{M_{B,t}}$$

Using the proceeds to exchange currency  $B$  into currency  $L$  with a spot operation at an exchange rate  $e_{L/B,t}$ , the trader deposits currency  $L$  for maturity  $t + dt$  at interest rate  $r_{L,t}$ .

$$\begin{aligned} X_t \cdot e_{B/I,t} \cdot e_{L/B,t} &= X_t \cdot \frac{M_{I,t}}{M_{B,t}} \cdot \frac{M_{B,t}}{M_{L,t}} \\ &= X_t \cdot \frac{M_{I,t}}{M_{L,t}} \end{aligned}$$

At time  $t + dt$ , the trader will receive payment from the deposit in currency  $L$ :

$$X_t \cdot \frac{M_{I,t}}{M_{L,t}} \cdot (1 + r_{L,t}dt)$$

Then, converting currency  $L$  into currency  $I$  at an exchange rate  $e_{I/L,t+dt} = M_{L,t+dt}/M_{I,t+dt}$  yields

$$X_t \cdot \frac{M_{I,t}}{M_{L,t}} \cdot (1 + r_{L,t}dt) \cdot \frac{M_{L,t+dt}}{M_{I,t+dt}} \quad (17)$$

At the same time, the trader has to pay off the principal and interest in currency  $B$ ,

$$X_t \cdot \frac{M_{I,t}}{M_{B,t}} \cdot (1 + r_{B,t}dt)$$

Similarly, converting currency  $B$  into currency  $I$  at an exchange rate  $e_{I/B,t+dt} = M_{B,t+dt}/M_{I,t+dt}$  gives

$$X_t \cdot \frac{M_{I,t}}{M_{B,t}} \cdot (1 + r_{B,t}dt) \cdot \frac{M_{B,t+dt}}{M_{I,t+dt}} \quad (18)$$

From equations (17) and (18), the net payoff  $X_{t+dt}$  from the carry trade portfolio of taking a short position (borrowing) in currency  $B$  and a long position (lending) in currency  $L$  at time  $t + dt$

can be expressed as

$$\begin{aligned}
X_{t+dt} &= X_t \cdot \left[ \frac{M_{I,t}}{M_{L,t}} \cdot (1 + r_{L,t}dt) \cdot \frac{M_{L,t+dt}}{M_{I,t+dt}} - \frac{M_{I,t}}{M_{B,t}} \cdot (1 + r_{L,t}dt) \cdot \frac{M_{B,t+dt}}{M_{I,t+dt}} \right] \\
&= X_t \cdot \frac{M_{I,t}}{M_{I,t+dt}} \left[ \frac{M_{L,t+dt}}{M_{L,t}} \cdot (1 + r_{L,t}dt) - \frac{M_{B,t+dt}}{M_{B,t}} \cdot (1 + r_{B,t}dt) \right]
\end{aligned} \tag{19}$$

Given the result in equation (19), we obtain the realized excess carry trade return, denoted by  $CT_{-B/+L,t+dt}^I$ .

$$\begin{aligned}
CT_{-B/+L,t+dt}^H &= \frac{M_{I,t}}{M_{I,t+dt}} \left[ \frac{M_{L,t+dt}}{M_{L,t}} \cdot (1 + r_{L,t}dt) - \frac{M_{B,t+dt}}{M_{B,t}} \cdot (1 + r_{B,t}dt) \right] \\
&= \frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \left[ \left(1 + \frac{dM_{L,t}}{M_{L,t}}\right) (1 + r_{L,t}dt) - \left(1 + \frac{dM_{B,t}}{M_{B,t}}\right) (1 + r_{B,t}dt) \right]
\end{aligned} \tag{20}$$

To better understand the the realized excess carry trade return, it is convenient to check the jump element of the following expression, which is in the first part of equation (20):

$$\frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \cdot \left(1 + \frac{dM_{L,t}}{M_{L,t}}\right) \tag{21}$$

By substituting equation(16) into equation(21), we find that

$$\frac{M_{I,t}}{M_{I,t+dt}} \cdot \frac{M_{L,t+dt}}{M_{L,t}} = \frac{M_{L,t+dt}}{M_{L,t}} / \frac{M_{I,t+dt}}{M_{I,t}} = \frac{e^{\Delta_i L}}{e^{\Delta_i I}} = e^{\Delta_i L - \Delta_i I} \tag{22}$$

Similarly, the jump element of the following expression, which is in the second part of equation (20),

$$\frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \cdot \left(1 + \frac{dM_{B,t}}{M_{B,t}}\right)$$

can be obtained as follows.

$$\frac{M_{I,t}}{M_{I,t+dt}} \cdot \frac{M_{B,t+dt}}{M_{B,t}} = e^{\Delta_i B - \Delta_i I} \tag{23}$$

From equations (15) and (22), it follows that

$$\begin{aligned}
\frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \cdot \left(1 + \frac{dM_{L,t}}{M_{L,t}}\right) &= \frac{1}{1 - r_{I,t}dt - \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) dN_{i,t} - \sum_i (e^{\Delta_i I} - 1) \lambda_i dt} \\
&\quad \times (1 - r_{L,t}dt - \eta_{L,t}^T dB_t + \sum_i (e^{\Delta_i L} - 1) dN_{i,t} - \sum_i (e^{\Delta_i L} - 1) \lambda_i dt) \\
&= \left[1 + r_{I,t}dt + \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) \lambda_i dt + \eta_{I,t}^T \eta_{I,t} dt\right] \\
&\quad \times \left[1 - r_{L,t}dt - \eta_{L,t}^T dB_t - \sum_i (e^{\Delta_i L} - 1) \lambda_i dt\right] + \sum_i \frac{e^{\Delta_i L}}{e^{\Delta_i I}} dN_{i,t}
\end{aligned}$$

Applying Itô's Lemma to the above expression, we get

$$\begin{aligned}
\frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \cdot \left(1 + \frac{dM_{L,t}}{M_{L,t}}\right) &= 1 + r_{I,t}dt + \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) \lambda_i dt + \eta_{I,t}^T \eta_{I,t} dt \\
&\quad - r_{L,t}dt - \eta_{L,t}^T dB_t - \eta_{I,t}^T \eta_{L,t} dt - \sum_i (e^{\Delta_i L} - 1) \lambda_i dt + \sum_i \frac{e^{\Delta_i L}}{e^{\Delta_i I}} dN_{i,t}
\end{aligned}$$

Similarly, in parallel to the derivation above, we have

$$\begin{aligned}
\frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \cdot \left(1 + \frac{dM_{B,t}}{M_{B,t}}\right) &= 1 + r_{I,t}dt + \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) \lambda_i dt + \eta_{I,t}^T \eta_{I,t} dt \\
&\quad - r_{B,t}dt - \eta_{B,t}^T dB_t - \eta_{I,t}^T \eta_{B,t} dt - \sum_i (e^{\Delta_i B} - 1) \lambda_i dt + \sum_i \frac{e^{\Delta_i B}}{e^{\Delta_i I}} dN_{i,t}
\end{aligned}$$

Finally, using the above results and a little algebra, the realized excess carry trade return  $CT_{-B/+L,t+dt}^I$  can be written as

$$\begin{aligned}
CT_{-B/+L,t+dt}^I &= \frac{1}{1 + \frac{dM_{I,t}}{M_{I,t}}} \left[ \left(1 + \frac{dM_{L,t}}{M_{L,t}}\right) (1 + r_{L,t}dt) - \left(1 + \frac{dM_{B,t}}{M_{B,t}}\right) (1 + r_{B,t}dt) \right] \\
&= \sum_i [(e^{\Delta_i B} - 1) - (e^{\Delta_i L} - 1)] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt + (\eta_{B,t}^T - \eta_{L,t}^T) dB_t \\
&\quad + \sum_i [(e^{\Delta_i L - \Delta_i I} - 1) - (e^{\Delta_i B - \Delta_i I} - 1)] dN_{i,t} \tag{24}
\end{aligned}$$

In addition, the expected carry trade return  $E\left(CT_{-B/+L,t+dt}^I\right)$  can be obtained by taking

expression in equation (24),

$$\begin{aligned}
E \left( CT_{-B/+L,t+dt}^I \right) &= \sum_i [(e^{\Delta_i B} - 1) - (e^{\Delta_i L} - 1)] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt \\
&\quad + \sum_i [(e^{\Delta_i L - \Delta_i I} - 1) - (e^{\Delta_i B - \Delta_i I} - 1)] \lambda_i dt \\
&= \sum_i [(e^{\Delta_i B} - e^{\Delta_i L}) + e^{-\Delta_i I} (e^{\Delta_i L} - e^{\Delta_i B})] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt \\
&= \sum_i (1 - e^{-\Delta_i I}) (e^{\Delta_i B} - e^{\Delta_i L}) \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt \\
&= \sum_i [(e^{\Delta_i I} - 1) (e^{\Delta_i B - \Delta_i I} - e^{\Delta_i L - \Delta_i I})] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt \quad (25)
\end{aligned}$$

### 3.3 The Four Moment Conditions of $\frac{de_{J/I,t}}{e_{J/I,t}}$

The objective of this subsection is, first, to model the exchange rate dynamics with jump-diffusion processes. Second, we will present the first four moments of the returns processes of exchange rates, denoted by  $\frac{de_{J/I,t}}{e_{J/I,t}}$ , which may be appropriate for the description of statistical properties of exchange rate movements. To begin with, we introduced the relationship between exchange rate and stochastic discount factors (SDFs) in Subsection 2.2. Let  $M_{I,t}$  and  $M_{J,t}$  denote the SDFs at time  $t$  in the home currency  $I$  and the foreign currency  $J$ , respectively. Then the exchange rate  $e_{J/I,t}$  can be expressed as  $e_{J/I,t} = \frac{M_{I,t}}{M_{J,t}}$ . Furthermore, from equation (15), the SDF growths in country  $I$  and country  $J$  would be

$$\begin{aligned}
\frac{dM_{I,t}}{M_{I,t}} &= -r_{I,t} dt - \eta_{I,t}^T dB_t + \sum_i (e^{\Delta_i I} - 1) (dN_{i,t} - \lambda_i dt) \\
\frac{dM_{J,t}}{M_{J,t}} &= -r_{J,t} dt - \eta_{J,t}^T dB_t + \sum_i (e^{\Delta_i J} - 1) (dN_{i,t} - \lambda_i dt)
\end{aligned}$$

If we assume that the dynamics of exchange rate  $e_{J/I,t}$  is in a jump-diffusion nature, its stochastic

differential equation governing the evolution in this model is given by

$$\begin{aligned}
\frac{de_{J/I,t}}{e_{J/I,t}} &= \frac{d\left(\frac{M_{I,t}}{M_{J,t}}\right)}{\frac{M_{I,t}}{M_{J,t}}} \\
&= (r_{J,t} - r_{I,t}) dt + \eta_{J,t}^T (\eta_{J,t} - \eta_{I,t}) dt + (\eta_{J,t}^T - \eta_{I,t}^T) dB_t + \sum_i (e^{\Delta_i J} - e^{\Delta_i I}) \lambda_i dt \\
&\quad + \sum_i (e^{\Delta_i I - \Delta_i J} - 1) dN_{i,t}
\end{aligned} \tag{26}$$

where  $N_{i,t}$  is a standard Poisson processes which models the number of jumps, and the intensity  $\lambda_i$  is the average number of jumps per unit time. The primary reason for introducing a jump component to a pure diffusion is to capture rare or extreme events in exchange rate fluctuation.

In Appendix B, we derive the first four moments of returns processes of exchange rates  $\frac{de_{J/I,t}}{e_{J/I,t}}$  are as follows.

$$E_t \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right) = [(r_J - r_I) + \eta_J^T (\eta_J - \eta_I)] dt + \sum_i [(e^{\Delta_i I} - e^{\Delta_i J}) (e^{-\Delta_i J} - 1)] \lambda_i dt \tag{27}$$

$$E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^2 \right] = (\eta_J^T - \eta_I^T) (\eta_J - \eta_I) dt + \sum_i (e^{\Delta_i I - \Delta_i J} - 1)^2 \lambda_i dt \tag{28}$$

$$E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^3 \right] = \sum_i (e^{\Delta_i I - \Delta_i J} - 1)^3 \lambda_i dt \tag{29}$$

$$E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^4 \right] = \sum_i (e^{\Delta_i I - \Delta_i J} - 1)^4 \lambda_i dt \tag{30}$$

Note that the drift terms of returns processes of exchange rates have disappeared in the third and fourth moments in equations (29) and (30). This suggests that jumps determine the third and fourth moments of the distribution. Larger values of the intensity  $\lambda_i$  result in a large number of jumps, and lead to an increase in the third and fourth moments.

### 3.3.1 Estimating Parameters of Returns Processes $\frac{de_{J/I,t}}{e_{J/I,t}}$

For the sake of simplicity, let us assume that there are three types of jumps: a global jump  $G$  and two idiosyncratic jumps for country  $I$  and country  $J$ , respectively. When a global jump occurs, the arrival of jumps is coincident across countries; however, the size of the jump across countries may vary. The idiosyncratic jump from country  $I$  has no influence on country  $J$ , and vice versa. From



equation (24), the carry trade return of borrowing the home currency  $I$  and investing in a foreign currency  $J$  over a time interval  $[t, t + dt]$  can be written

$$\begin{aligned}
CT_{-I/+J,t+dt}^I &= \sum_i [(e^{\Delta_i I} - 1) - (e^{\Delta_i J} - 1)] \lambda_i dt + \eta_{I,t}^T (\eta_{I,t} - \eta_{J,t}) dt + (\eta_{I,t}^T - \eta_{J,t}^T) dB_t \\
&\quad + \sum_i [(e^{\Delta_i J - \Delta_i I} - 1)] dN_{i,t}, \quad i = G, I, J
\end{aligned} \tag{31}$$

Given three types of jumps,  $i = \{G, I, J\}$ , the first four moments of returns processes of exchange rates, equation (27) - equation (30), become

$$\begin{aligned}
E_t \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right) &= [(r_J - r_I) + \eta_J^T (\eta_J - \eta_I)] dt + \lambda_G [(e^{\Delta_G I} - e^{\Delta_G J}) (e^{-\Delta_G J} - 1)] dt \\
&\quad + \lambda_J (1 - e^{\Delta_J J}) (e^{-\Delta_J J} - 1) dt
\end{aligned} \tag{32}$$

$$\begin{aligned}
E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^2 \right] &= (\eta_J^T - \eta_I^T) (\eta_J - \eta_I) dt + \lambda_G (e^{\Delta_G I - \Delta_G J} - 1)^2 dt + \lambda_I (e^{\Delta_I I} - 1)^2 dt \\
&\quad + \lambda_J (e^{-\Delta_J J} - 1)^2 dt
\end{aligned} \tag{33}$$

$$E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^3 \right] = [\lambda_G (e^{\Delta_G I - \Delta_G J} - 1)^3 + \lambda_I (e^{\Delta_I I} - 1)^3 + \lambda_J (e^{-\Delta_J J} - 1)^3] dt \tag{34}$$

$$E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^4 \right] = [\lambda_G (e^{\Delta_G I - \Delta_G J} - 1)^4 + \lambda_I (e^{\Delta_I I} - 1)^4 + \lambda_J (e^{-\Delta_J J} - 1)^4] dt \tag{35}$$

To calibrate the Jump-diffusion carry trade return model with three types of jumps, we use the method of moments to estimate the model parameters. From the four moment conditions in equations (32) - (35), the parameters to be estimated are  $\eta_I, \eta_J, \lambda_G, \lambda_I, \lambda_J, \Delta_G I, \Delta_G J, \Delta_I I$ , and  $\Delta_J J$ . The subscript notation of these parameters indicates different jump sources. For example,  $\Delta_G I$  represents the jump size of country  $I$  involved with a global jump  $G$ , whereas  $\Delta_I I$  would in turn represent the jump size of country  $I$  arising from a idiosyncratic jump  $I$ .

We formulate the objective function as the sum of the squared differences between the four

moment conditions by the model and those calculated from the data.

$$\begin{aligned}
f &= f(\eta_I, \eta_J, \lambda_G, \lambda_I, \lambda_J, \Delta_G I, \Delta_G J, \Delta_I I, \Delta_J J) \\
&= w_1 \times \left\| E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{model} - E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right\|^2 \\
&\quad + w_2 \times \left\| E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{model}^2 - E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^2 \right\|^2 \\
&\quad + w_3 \times \left\| E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{model}^3 - E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^3 \right\|^2 \\
&\quad + w_4 \times \left\| E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{model}^4 - E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^4 \right\|^2
\end{aligned} \tag{36}$$

where we assume  $w_1 = 1$  and let

$$\begin{aligned}
w_2 \times \left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^2 \right]^2 &= w_1 \times \left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right]^2, & w_2 &= \frac{\left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right]^2}{\left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^2 \right]^2} \\
w_3 \times \left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^3 \right]^2 &= w_1 \times \left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right]^2, & w_3 &= \frac{\left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right]^2}{\left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^3 \right]^2} \\
w_4 \times \left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^4 \right]^2 &= w_1 \times \left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right]^2, & w_4 &= \frac{\left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data} \right]^2}{\left[ E \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)_{data}^4 \right]^2}
\end{aligned} \tag{37}$$

Note that we have included weights for the the squared differences in moment conditions. These do not affect estimates of parameters and they have the advantage of reducing unstable issues in the optimization process. The parameters are estimated as the solution by solving the following nonlinear optimization problem, which minimizes the square of the difference between the moment conditions from the model and the moments from the data.

$$\begin{aligned}
&\underset{\substack{\eta_I, \eta_J, \lambda_G \\ \lambda_I, \lambda_J, \Delta_G I \\ \Delta_G J, \Delta_I I, \Delta_J J}}{\text{minimize}} & f(\eta_I, \eta_J, \lambda_G, \lambda_I, \lambda_J, \Delta_G I, \Delta_G J, \Delta_I I, \Delta_J J) \\
&\text{subject to} & \eta_I, \eta_J > 0 \\
& & \lambda_G, \lambda_I, \lambda_J > 0
\end{aligned} \tag{38}$$

where the nonlinear optimization problem is constrained with positive values for market price of risk  $\eta$  and the jump intensity  $\lambda$ .

Table 4 reports estimates of the parameters for return processes of jump-diffusion exchange rates along with CT(carry trade) return and FX skewness. The base currency is the USD; the funding currency  $I$  is also USD, and the investing currency  $J$  is AUD, CAD, JPY, NZD, NOK, CHF, GBP, EUR, SEK, and DKK respectively. The last two columns of the table 4 give the carry trade return and FX skewness, which are obtained by substituting those estimated parameters into equation (31) and equation (26). The results were desirable as they successfully demonstrated that positive carry trade returns are associated with negative FX skewness. JPY is the only investing currency that earned negative excess return associated with positive FX skewness.

Table 4: Parameter Estimates for the Returns Processes of Exchange Rate: 9 parameters  
(Funding Currency: I = USD; Base Currency: USD)

Investing Currency	Price of Risk		Jump Intensity			Jump Size				CT	FX
	$\eta_I$	$\eta_J$	$\lambda_G$	$\lambda_I$	$\lambda_J$	$\Delta_G I$	$\Delta_G J$	$\Delta_I I$	$\Delta_J J$	Return	Skewness
AUD	0.1367	0.1659	0.2439	0.0530	0.0788	0.6800	0.5088	-0.3228	0.0711	0.0310	-0.1738
CAD	0.1303	0.1417	0.1298	0.0573	0.1054	0.5606	0.4255	-0.1328	0.1328	0.0089	-0.1589
JPY	0.0646	0.1291	0.0392	0.0148	0.1095	0.6072	0.3623	-0.1162	0.2608	-0.0052	0.0392
NZD	0.1784	0.2028	0.3731	0.0439	0.0467	0.7266	0.5783	-0.3814	0.0641	0.0483	-0.1612
NOK	0.1045	0.1371	0.1620	0.0774	0.1015	0.5879	0.3996	-0.2466	0.0788	0.0168	-0.1757
CHF	0.0979	0.1411	0.0848	0.1125	0.1055	0.5914	0.3832	-0.2653	0.1219	0.0081	-0.0831
GBP	0.0811	0.1170	0.1336	0.0737	0.1000	0.5610	0.3708	-0.2238	0.0738	0.0124	-0.2697
EUR	0.0672	0.1153	0.0554	0.1200	0.1151	0.5984	0.3634	-0.2064	0.1288	0.0049	-0.1078
SEK	0.0571	0.1097	0.0679	0.1231	0.1122	0.5753	0.3249	-0.1920	0.1169	0.0073	-0.1568
DKK	0.0844	0.1228	0.1636	0.0756	0.0996	0.5766	0.4069	-0.2633	0.0729	0.0155	-0.1088

Note: The table reports estimates of 9 parameters for the returns processes of exchange rates. The base currency is the USD; the funding currency  $I$  is also USD, and the investing currency  $J$  is AUD, CAD, JPY, NZD, NOK, CHF, GBP, EUR, SEK, and DKK respectively. The estimated parameters,  $\eta_I, \eta_J, \lambda_G, \lambda_I, \lambda_J, \Delta_G I, \Delta_G J, \Delta_I I,$  and  $\Delta_J J$ , are obtained by minimizing the squared differences between the four moment conditions by the model and those calculated from the data. CT Return stands for Carry Trade Return per annum, and CT Skewness for Skewness of Exchange Rate Movements.

### 3.3.2 Estimating Parameters for the Returns Processes of $\frac{de_{B/I,t}}{e_{B/I,t}}$ and $\frac{de_{L/I,t}}{e_{L/I,t}}$

Now consider the case that the base currency  $I$  is USD; the funding currency  $B$  is also JPY, and the investing currency  $L$  is AUD, CAD, NZD, NOK, CHF, GBP, EUR, SEK, and DKK respectively. With four types of jumps: a global jump  $G$  and three idiosyncratic jumps for country  $I, B$  and  $L$ ,

the carry trade return of borrowing currency  $B$  and investing in currency  $L$  over a time interval  $[t, t + dt]$  is

$$\begin{aligned}
CT_{-B/+L,t+dt}^I &= \sum_i [(e^{\Delta_i B} - 1) - (e^{\Delta_i L} - 1)] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt + (\eta_{B,t}^T - \eta_{L,t}^T) dB_t \\
&\quad + \sum_i [(e^{\Delta_i L - \Delta_i I} - 1) - (e^{\Delta_i B - \Delta_i I} - 1)] dN_{i,t}, \quad i = G, I, B, L
\end{aligned} \tag{39}$$

The first four moments of returns processes of exchange rates  $\frac{de_{B/I,t}}{e_{B/I,t}}$  become

$$\begin{aligned}
E_t \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right) &= [(r_B - r_I) + \eta_B^T (\eta_B - \eta_I)] dt + \lambda_G [(e^{\Delta_{GI}} - e^{\Delta_{GB}}) (e^{-\Delta_{GB}} - 1)] dt \\
&\quad + \lambda_B (1 - e^{\Delta_{BB}}) (e^{-\Delta_{BB}} - 1) dt
\end{aligned} \tag{40}$$

$$\begin{aligned}
E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \right] &= (\eta_B^T - \eta_I^T) (\eta_B - \eta_I) dt + \lambda_G (e^{\Delta_{GI} - \Delta_{GB}} - 1)^2 dt + \lambda_I (e^{\Delta_{II}} - 1)^2 dt \\
&\quad + \lambda_B (e^{-\Delta_{BB}} - 1)^2 dt
\end{aligned} \tag{41}$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^3 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GB}} - 1)^3 + \lambda_I (e^{\Delta_{II}} - 1)^3 + \lambda_B (e^{-\Delta_{BB}} - 1)^3] dt \tag{42}$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^4 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GB}} - 1)^4 + \lambda_I (e^{\Delta_{II}} - 1)^4 + \lambda_B (e^{-\Delta_{BB}} - 1)^4] dt \tag{43}$$

Similarly, the first four moments of returns processes of exchange rates  $\frac{de_{L/I,t}}{e_{L/I,t}}$  are

$$\begin{aligned}
E_t \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right) &= [(r_L - r_I) + \eta_L^T (\eta_L - \eta_I)] dt + \lambda_G [(e^{\Delta_{GI}} - e^{\Delta_{GL}}) (e^{-\Delta_{GL}} - 1)] dt \\
&\quad + \lambda_L (1 - e^{\Delta_{LL}}) (e^{-\Delta_{LL}} - 1) dt
\end{aligned} \tag{44}$$

$$\begin{aligned}
E_t \left[ \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right] &= (\eta_L^T - \eta_I^T) (\eta_L - \eta_I) dt + \lambda_G (e^{\Delta_{GI} - \Delta_{GL}} - 1)^2 dt + \lambda_I (e^{\Delta_{II}} - 1)^2 dt \\
&\quad + \lambda_L (e^{-\Delta_{LL}} - 1)^2 dt
\end{aligned} \tag{45}$$

$$E_t \left[ \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^3 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GL}} - 1)^3 + \lambda_I (e^{\Delta_{II}} - 1)^3 + \lambda_L (e^{-\Delta_{LL}} - 1)^3] dt \tag{46}$$

$$E_t \left[ \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^4 \right] = \left[ \lambda_G (e^{\Delta_{GI} - \Delta_{GL}} - 1)^4 + \lambda_I (e^{\Delta_I} - 1)^4 + \lambda_L (e^{-\Delta_{LL}} - 1)^4 \right] dt \quad (47)$$

From the four moment conditions in equations (40) - (47), the 13 parameters to be estimated are  $\eta_I, \eta_B, \eta_L, \lambda_G, \lambda_I, \lambda_B, \lambda_L, \Delta_{GI}, \Delta_{GB}, \Delta_{GL}, \Delta_I, \Delta_B, \Delta_L$ . We estimate these 13 parameters by solving the following nonlinear optimization problem, which minimizes the squared deviation of 8 moment conditions (equations (40)- (47)) from the moments calculated by the data.

$$\begin{aligned} & \underset{\substack{\eta_I, \eta_B, \eta_L, \lambda_G, \lambda_I \\ \lambda_B, \lambda_L, \Delta_{GI}, \Delta_{GB} \\ \Delta_{GL}, \Delta_I, \Delta_B, \Delta_L}}{\text{minimize}} & \sum_{i=1}^4 w_i \times \left\| E \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^i_{\text{model}} - E \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^i_{\text{data}} \right\|^2 \\ & + \sum_{j=1}^4 w_j \times \left\| E \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^j_{\text{model}} - E \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^j_{\text{data}} \right\|^2 & (48) \\ & \text{subject to} & \eta_I, \eta_B, \eta_L > 0 \\ & & \lambda_G, \lambda_I, \lambda_B, \lambda_L > 0 \end{aligned}$$

where weights  $w_i$  and  $w_j$  are similar to those in the expression (36). The 13 estimated Parameters for the Returns Processes of  $\frac{de_{B/I,t}}{e_{B/I,t}}$  and  $\frac{de_{L/I,t}}{e_{L/I,t}}$  are listed in Table 5. By substituting the estimated parameters into equations (39) - (47), we obtain the reconstructed carry trade returns and moments and compare to their values computed from the data, as shown in Table 6. We find that the model matches the four moments and carry trade returns in data quite well.

## 4 Trading Strategies and Asset Pricing Tests

### 4.1 Trading Strategies

In subsection 2.5.3, we demonstrated the performance of long-short carry portfolios, which were built by taking long positions in currencies with the highest interest rate differentials and short positions in currencies with the lowest interest rate differentials. Carry trade returns can be characterized by skewness, which arises from sudden but large movements in exchange rates. Now, to exhibit the skewness effect on the carry trade return, we'll construct long-short trading strategies based on the third moment of returns on exchange rates.

Recall that for currency  $I$  and  $J$ , the jump effect on exchange rate return,  $\frac{de_{J/I,t}}{e_{J/I,t}}$ , over a period

Table 5: Parameter Estimates for the Returns Processes of  $\frac{de_{B/I,t}}{e_{B/I,t}}$  and  $\frac{de_{L/I,t}}{e_{L/I,t}}$ : 13 parameters  
(Funding Currency: B = JPY; Base Currency: I = USD)

	Price of Risk			Jump Intensity				Jump Size					
	$\eta_I$	$\eta_B$	$\eta_L$	$\lambda_I$	$\lambda_B$	$\lambda_L$	$\lambda_G$	$\Delta_I I$	$\Delta_B B$	$\Delta_L L$	$\Delta_G I$	$\Delta_G B$	$\Delta_G L$
AUD	0.410	0.468	0.395	0.020	0.003	0.175	0.539	-0.466	0.510	-0.001	0.354	0.250	0.222
CAD	0.422	0.431	0.400	0.116	0.001	0.132	0.591	0.138	-0.573	0.141	-0.053	0.086	-0.048
NZD	1.056	1.068	1.017	0.494	0.008	0.185	0.029	-0.148	0.339	0.001	0.495	0.217	0.185
NOK	0.391	0.430	0.384	0.026	0.023	0.164	0.437	-0.384	0.339	-0.004	0.265	0.146	0.127
CHF	0.366	0.412	0.377	0.033	0.001	0.097	0.567	-0.417	0.495	0.142	0.224	0.118	0.110
GBP	0.416	0.431	0.395	0.018	0.071	0.200	0.388	-0.395	0.262	0.001	0.130	0.002	-0.007
EUR	0.369	0.411	0.391	0.035	0.007	0.183	0.504	-0.365	0.473	0.004	0.236	0.123	0.115
SEK	0.276	0.327	0.301	0.040	0.010	0.199	0.294	-0.339	0.423	0.003	0.299	0.166	0.134
DKK	0.405	0.431	0.385	0.153	0.018	0.184	0.151	-0.210	0.362	0.009	0.243	0.070	0.066

Note: The table reports estimates of 13 parameters for the returns processes of exchange rates  $e_{B/I,t}$  and  $e_{L/I,t}$ . The base currency is the *USD*; the funding currency *B* is *JPY*, and the investing currency *L* is *AUD, CAD, NZD, NOK, CHF, GBP, EUR, SEK, and DKK* respectively. The estimated parameters,  $\eta_I, \eta_B, \eta_L, \lambda_I, \lambda_B, \lambda_L, \lambda_G, \Delta_I I, \Delta_B B, \Delta_L L, \Delta_G I, \Delta_G B, \Delta_G L$ , are obtained by minimizing the squared differences between the four moment conditions by the model and those calculated from the data.

of length  $dt$  is given by

$$\frac{de_{J/I,t}}{e_{J/I,t}} = e^{\Delta_i I - \Delta_i J} - 1$$

and the third moment of  $\frac{de_{J/I,t}}{e_{J/I,t}}$ ,

$$E_t \left[ \left( \frac{de_{J/I,t}}{e_{J/I,t}} \right)^3 \right] = \sum_i (e^{\Delta_i I - \Delta_i J} - 1)^3 \lambda_i dt$$

In addition, the expected carry trade return,  $E \left( CT_{-B/+L,t+dt}^I \right)$ , of taking a long position in currency *L* financed by a short position in currency *B* can be rewritten,

$$\begin{aligned} E \left( CT_{-B/+L,t+dt}^I \right) &= \sum_i [(e^{\Delta_i I} - 1) (e^{\Delta_i B - \Delta_i I} - e^{\Delta_i L - \Delta_i I})] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt \\ &= \sum_i (e^{\Delta_i I} - 1) [(e^{\Delta_i B - \Delta_i I} - 1) - (e^{\Delta_i L - \Delta_i I} - 1)] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt \end{aligned}$$

The above expression shows that a large  $(e^{\Delta_i B - \Delta_i I} - 1)$  and small  $(e^{\Delta_i L - \Delta_i I} - 1)$  may lead to a higher return, implying that we should take a short position in currency *B* with small third moment and a long position in currency *L* with large third moment of returns on exchange rates.

Table 6: Reconstructed Moments and Carry Trade Returns

	Moments(Funding Currency:JPY)				Moments(Investing Currencies)				CT
	1st	2nd	3rd	4th	1st	2nd	3rd	4th	Return
JPY/AUD:data	-0.01148	0.01313	-0.00051	0.00054	0.00415	0.01377	0.00045	0.00061	0.03628
JPY/AUD:recon.	-0.01148	0.01313	-0.00051	0.00054	0.00415	0.01377	0.00045	0.00061	0.03573
JPY/CAD:data	-0.01148	0.01313	-0.00051	0.00054	0.00112	0.00536	0.00008	0.00010	0.01407
JPY/CAD:recon.	-0.01051	0.00938	-0.00056	0.00047	0.00114	0.00588	0.00008	0.00010	0.01343
JPY/CHF:data	-0.01148	0.01313	-0.00051	0.00054	-0.01613	0.01391	-0.00053	0.00059	0.01332
JPY/CHF:recon.	-0.01148	0.01313	-0.00051	0.00054	-0.01613	0.01391	-0.00053	0.00059	0.01342
JPY/DKK:data	-0.01148	0.01313	-0.00051	0.00054	-0.00206	0.01161	0.00006	0.00041	0.02069
JPY/DKK:recon.	-0.01148	0.01313	-0.00051	0.00054	-0.00206	0.01161	0.00006	0.00041	0.01876
JPY/EUR:data	-0.01148	0.01313	-0.00051	0.00054	-0.00264	0.01205	0.00005	0.00044	0.00988
JPY/EUR:recon.	-0.01148	0.01313	-0.00051	0.00054	-0.00264	0.01313	0.00005	0.00044	0.00823
JPY/GBP:data	-0.01148	0.01313	-0.00051	0.00054	0.00968	0.01062	0.00060	0.00038	0.01758
JPY/GBP:recon.	-0.01148	0.01313	-0.00051	0.00054	0.00968	0.01062	0.00060	0.00038	0.01505
JPY/NOK:data	-0.01148	0.01313	-0.00051	0.00054	0.00963	0.01230	0.00059	0.00048	0.02196
JPY/NOK:recon.	-0.01148	0.01313	-0.00051	0.00054	0.00963	0.01230	0.00059	0.00048	0.02041
JPY/NZD:data	-0.01148	0.01313	-0.00051	0.00054	-0.00393	0.01472	0.00011	0.00069	0.05351
JPY/NZD:recon.	-0.01148	0.01313	-0.00051	0.00054	-0.00393	0.01472	0.00011	0.00069	0.05366
JPY/SEK:data	-0.01148	0.01313	-0.00051	0.00054	0.01309	0.01338	0.00076	0.00058	0.01245
JPY/SEK:recon.	-0.01148	0.01313	-0.00051	0.00054	0.01309	0.01338	0.00076	0.00058	0.01011

Note: The table reports the reconstructed moments and carry trade returns that are obtained by substituting the estimated parameters from Table 5 into equations (39) - (47), which are then compared to their values computed from the data. “recon.” stands for reconstructed. CT Return stands for Carry Trade Return.

The long-short trading strategies are constructed as follows. At each month, ten currencies are sorted on the basis of the mean third moment of returns on exchange rates over a T-month rolling window. Then these sorted currencies, B1, B2, B3, B4, B5, L5, L4, L3, L2, L1, are allocated into five trading strategies (portfolios). The first strategy consists of a short position in currency B1 with the smallest third moment of returns on exchange rates, and a long position in currency L1 with the largest third moment of returns on exchange rates. The second strategy is constructed by a short position in currency B2 with the second smallest third moment, and a long position in currency L2 with the second largest third moment of returns on exchange rates, and so on up to the fifth strategy, which takes a short position in currency B5 and a long position in currency L5.

Table 7 reports the performance of five long-short currency trading strategies. Figure 3 plots 1-month portfolio returns for five long-short strategies with various rolling windows. Comparing the 1-month returns of five long-short trading strategies, the first strategy which borrows currency B1 and lends currency L1 consistently yields higher returns than other strategies, regardless of rolling window selection. Namely, the trading strategy that borrows currency with the largest third moment of return on exchange rate and lends currency with the smallest third moment outperforms other strategies. on the other hand, the patterns in 1-month returns are not pronounced for the third, fourth and fifth strategies.

Table 8 reports the frequency of currencies which are selected as B1 and L1 for the first long-short trading strategy with various rolling window. As is seen in the top panel, the high-frequency selection for a short position includes currencies such as JPY and CHF. In the bottom panel, the high-frequency selection for a long position includes NZD, AUD and SEK. From Table 1, JPY and CHF had low interest rate differentials relative to the U.S. dollar of  $-0.0019$  and  $-0.0012$ , and NZD and AUD had high interest rate differentials of  $0.0031$  and  $0.0024$ . Moreover, from Table 2, the first carry portfolio that borrows in a currency with the lowest interest rate differential and lends a currency with the highest interest rate differential generates the highest mean monthly return of  $0.0072$ . Consistent with our prior results in Table 1 and 2, the results in Table 7 and 8 also provide evidence that currencies with smaller third moment of returns on exchange rates tend to have low interest rate differentials, while currencies with larger third moment of returns on exchange rates tend to have high interest rate differentials.



Table 7: Performance of Five Long-Short Currency Trading Strategies

Rolling Window		1 year			2 years		
Strategies	Return	Standard Dev.	Skewness	Return	Standard Dev.	Skewness	
Strategy 1 (-B1/+L1)	<b>0.0033</b>	0.0379	-0.2501	<b>0.0045</b>	0.0362	-0.2185	
Strategy 2 (-B2/+L2)	-0.0005	0.0341	-0.0340	0.0037	0.0348	-0.2117	
Strategy 3 (-B3/+L3)	-0.0006	0.0478	-8.1606	0.0019	0.0301	0.0577	
Strategy 4 (-B4/+L4)	-0.0018	0.0310	-0.5832	0.0022	0.0476	8.0313	
Strategy 5 (-B5/+L5)	-0.0001	0.0255	-0.1479	0.0016	0.0247	0.2576	
Rolling Window		3 years			4 years		
Strategies	Return	Standard Dev.	Skewness	Return	Standard Dev.	Skewness	
Strategy 1 (-B1/+L1)	<b>0.0041</b>	0.0361	-0.3704	<b>0.0052</b>	0.0355	-0.2387	
Strategy 2 (-B2/+L2)	0.0021	0.0353	-0.0898	0.0010	0.0531	6.3308	
Strategy 3 (-B3/+L3)	0.0005	0.0296	-0.3915	0.0018	0.0257	-0.1154	
Strategy 4 (-B4/+L4)	0.0032	0.0481	9.5746	-0.0007	0.0297	-0.1769	
Strategy 5 (-B5/+L5)	-0.0016	0.0245	-0.7591	0.0014	0.0258	0.0358	
Rolling Window		5 years			6 years		
Strategies	Return	Standard Dev.	Skewness	Return	Standard Dev.	Skewness	
Strategy 1 (-B1/+L1)	<b>0.0073</b>	0.0360	-0.1436	<b>0.0061</b>	0.0379	-0.4322	
Strategy 2 (-B2/+L2)	0.0019	0.0525	6.9131	0.0026	0.0355	-0.5309	
Strategy 3 (-B3/+L3)	0.0013	0.0297	-0.1564	-0.0016	0.0294	-0.2969	
Strategy 4 (-B4/+L4)	-0.0022	0.0296	-0.9370	-0.0003	0.0497	8.8605	
Strategy 5 (-B5/+L5)	0.0016	0.0244	0.8442	0.0014	0.0197	1.6221	
Rolling Window		7 years			8 years		
Strategies	Return	Standard Dev.	Skewness	Return	Standard Dev.	Skewness	
Strategy 1 (-B1/+L1)	<b>0.0066</b>	0.0366	-0.4290	<b>0.0053</b>	0.0370	-0.4523	
Strategy 2 (-B2/+L2)	0.0023	0.0354	-0.3898	0.0036	0.0338	-0.8724	
Strategy 3 (-B3/+L3)	0.0000	0.0306	-0.3465	0.0004	0.0278	-0.3563	
Strategy 4 (-B4/+L4)	-0.0002	0.0284	0.2759	0.0008	0.0301	0.1902	
Strategy 5 (-B5/+L5)	0.0013	0.0442	12.7079	0.0025	0.0440	13.3472	
Rolling Window		9 years			10 years		
Strategies	Return	Standard Dev.	Skewness	Return	Standard Dev.	Skewness	
Strategy 1 (-B1/+L1)	<b>0.0061</b>	0.0385	-0.4337	<b>0.0057</b>	0.0375	-0.7797	
Strategy 2 (-B2/+L2)	0.0049	0.0303	-6.9131	0.0034	0.0336	-0.8747	
Strategy 3 (-B3/+L3)	-0.0003	0.0269	-0.5280	0.0004	0.0282	-0.0463	
Strategy 4 (-B4/+L4)	0.0016	0.0308	0.4800	0.0008	0.0287	-0.4776	
Strategy 5 (-B5/+L5)	0.0007	0.0452	12.9881	0.0027	0.0466	12.2800	

Note: The table reports the performance of five long-short trading strategies. Ten currencies are sorted monthly into five trading strategies based on the mean third moment of returns on exchange rates over a specific rolling window. Strategy 1(-B1/+L1) indicates the first strategy taking a short position in currency B with the smallest third moment of returns on exchange rates, and a long position in currency L with the largest third moment of returns on exchange rates, and so on up to Strategy 5(-B5/+L5). Long-short strategies are constructed assuming a holding period of 1 month.

Table 8: Frequencies of Currencies Selected as B1 and L1 for The First Long-Short Strategy

Currencies Selected as B1 (Short Position)										
Currncies	Rolling Window									
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
AUD	19	14	12	5	2	0	0	0	0	0
CAD	43	54	46	28	26	7	3	6	8	13
JPY	96	108	111	126	153	89	201	206	210	202
NZD	24	12	4	4	4	2	2	4	0	0
NOK	32	25	8	11	3	7	12	12	11	0
CHF	64	79	94	99	76	66	59	47	36	35
GBP	28	17	20	12	15	13	3	0	0	0
EUR	1	0	0	0	0	0	0	0	0	0
SEK	47	38	42	36	34	17	9	2	0	3
DKK	7	2	0	4	0	0	0	0	0	0

Currencies Selected as L1(Long Position)										
Currncies	Rolling Window									
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
AUD	53	42	53	67	69	60	63	89	82	73
CAD	28	17	12	3	0	0	0	0	0	0
JPY	53	39	26	16	17	15	5	1	0	0
NZD	74	117	104	100	97	11	117	102	99	96
NOK	16	17	29	15	12	8	0	0	0	0
CHF	26	24	34	34	34	24	12	0	0	0
GBP	28	25	10	10	11	13	14	6	3	2
EUR	24	28	21	26	12	5	3	0	0	0
SEK	44	28	34	44	53	65	74	79	81	82
DKK	15	12	14	10	8	0	1	0	0	0

Note: The table reports the frequency of currencies which are selected as B1 and L1 for the first long-short trading strategy with various rolling window. At each month, ten currencies are sorted on the basis of the third moment of returns on exchange rates. Currency B1 has the smallest third moment of , while currency L1 has the largest third moment. Data: January 1986 - December 2016.

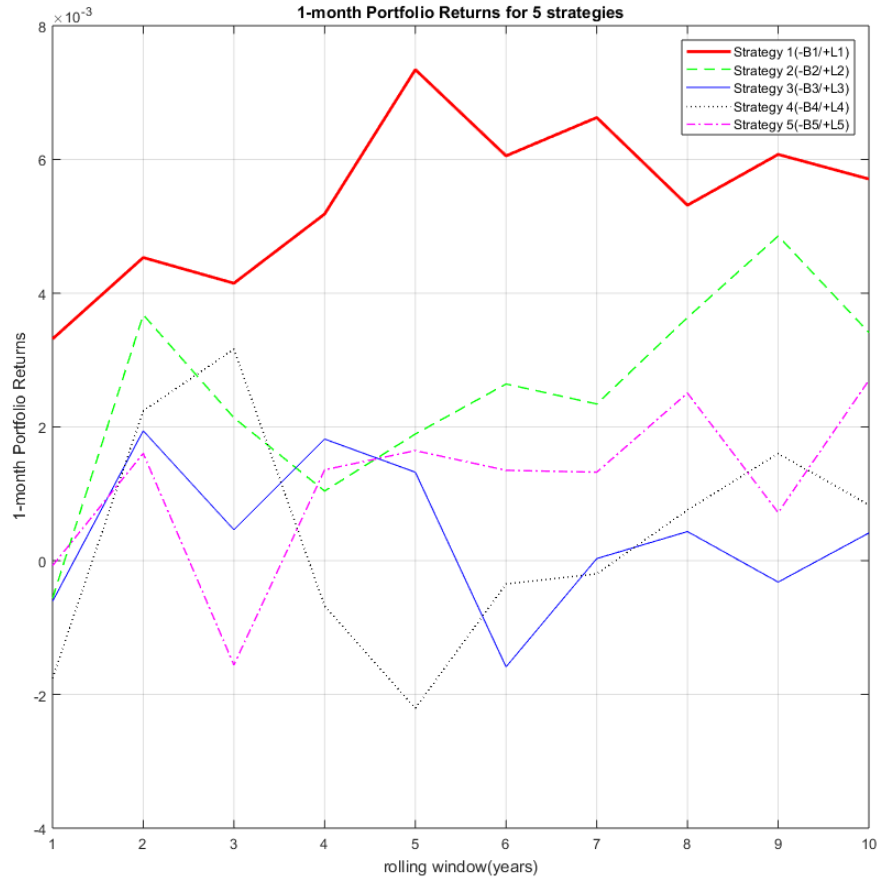


Figure 3: The figure displays 1-month portfolio returns for five long-short strategies with various rolling windows. Ten currencies are sorted monthly into five trading strategies based on the mean third moment of returns on exchange rates over a specific rolling window. Strategy 1(-B1/+L1) indicates the first strategy taking a short position in currency B with the smallest third moment of returns on exchange rates, and a long position in currency L with the largest third moment of returns on exchange rates, and so on up to Strategy 5(-B5/+L5). Long-short strategies are constructed assuming a holding period of 1 month.

## 4.2 Testing Asset Pricing Models

In this subsection, we use the Fama-MacBeth two-stage regression to test how risk factors explain portfolio returns of five long-short currency trading strategies, as mentioned in the previous subsection 4.1. The goal is to find the risk premia from exposure to these factors. The first risk factor is a global currency skewness risk factor, which was proposed by Rafferty (2012). The global currency skewness risk factor can be thought as an aggregate skewness of individual currencies. Let  $FXskew_{F,t}$  represent the skewness of daily log FX exchange rate changes for currency  $F$  in month  $t$ . The global currency skewness risk factor,  $Gskew_t$ , in month  $t$  is defined as

$$Gskew_t = \frac{1}{10} \sum_F^{10} sign(r_F - r_{USD}) FXskew_{F,t} \quad (49)$$

where  $r_F$  is the interest rate in the country  $F$ , and  $r_{USD}$  is the interest rate in the USA. The 10 foreign currencies include AUD, CAD, JPY, NZD, NOK, CHF, GBP, EUR, SEK, and DKK.

The second risk factor is a market factor, which is similar to a dollar risk factor(DOL). The market factor, denoted by Market, is defined as the average returns of five long-short currency trading strategies.

$$Market_t = \frac{1}{5} \sum_{i=1}^5 R_{i,t} \quad (50)$$

We have 5 portfolio returns over  $T = 313$  months. For the 1-factor model, in the first stage the portfolio factor betas are obtained by calculating 5 time series regressions of each currency portfolio return on the risk factor Gskew.

$$R_{i,t} = \alpha_i + \beta_{i,Gskew} Gskew_t + \epsilon_{i,t} \quad i = 1, \dots, 5; t = 1, \dots, 313 \quad (51)$$

where  $R_{i,t}$  is the currency portfolio return from strategy  $i$  in month  $t$ ,  $Gskew_t$  is the global currency skewness risk factor in month  $t$ ,  $\beta_{i,Gskew}$  is the factor exposure that describes how portfolio returns are exposed to this global currency skewness risk factor Gskew, and month  $t$  goes from 1 to 313. The second stage is to compute  $T = 313$  cross-sectional regression of portfolio returns on the estimated betas ( $\hat{\beta}_{i,Gskew}, i = 1, \dots, 5$ ) from the first stage.

$$R_{i,t} = \gamma_{t,0} + \gamma_{t,Gskew} \hat{\beta}_{i,Gskew} + \epsilon_{i,t} \quad i = 1, \dots, 5; t = 1, \dots, 313 \quad (52)$$

where  $R_{i,t}$  is the currency portfolio return from strategy  $i$  in month  $t$ ,  $\gamma$  are regression coefficients that are used to calculate the risk premium for the global currency skewness risk factor, and in each regression  $i$  goes from 1 to 5.

Similarly, for a 2-factor model, there are 5 currency portfolio returns and 2 risk factors. In the first stage the factor exposures,  $\beta_{i,Gskew}$  and  $\beta_{i,Market}$ , are obtained by calculating 5 regressions, each one on 2 factors: a global currency skewness risk factor,  $Gskew$ , and a market risk factor,  $Market$ .

$$R_{i,t} = \alpha_i + \beta_{i,Gskew}Gskew_t + \beta_{i,Market}Market_t + \epsilon_{i,t} \quad i = 1, \dots, 5; t = 1, \dots, 313 \quad (53)$$

The second stage is to run  $T = 313$  cross-regressions of the currency portfolio returns on the estimated betas,  $(\hat{\beta}_{i,Gskew}, i = 1, \dots, 5)$  and  $(\hat{\beta}_{i,Market}, i = 1, \dots, 5)$ , from the first stage to get the risk premium  $\gamma_{Gskew}$  and  $\gamma_{Market}$  for each factor.

$$R_{i,t} = \gamma_{t,0} + \gamma_{t,Gskew}\hat{\beta}_{i,Gskew} + \gamma_{t,Market}\hat{\beta}_{i,Market} + \epsilon_{i,t} \quad i = 1, \dots, 5; t = 1, \dots, 313 \quad (54)$$

Table 9 reports the main results of testing factor models for returns on five long-short trading strategies. For the 1-factor model, the global currency skewness risk captures 28.3% of the cross-sectional excess returns in our five long-short currency trading strategies. This result confirms the existing finding that currency trading returns are driven by a skewness factor. For the 2-factor model, the factor risk premium  $\gamma_{Market}$  is not significantly different from zero and that cross sectional differences in portfolio returns explained by the relationship (54) does not improve significantly when we add the market risk factor as the second risk factor.

## 5 Conclusion

In this paper, we investigate jump risk and return characteristics of currency carry trades by employing both data-driven approach and analytical method. First of all, using monthly data from January 2006 to December 2016, we construct carry trades for ten currency pairs, where USD is the funding currency, and AUD, CAD, CHF, DKK EUR, GBP, JPY NOK, NZD, SEK is the investing currency, respectively. The results of cross-sectional variation suggest that on average, currencies with positive interest rate differentials tend to have positive carry trade returns but

Table 9: Estimates of Factor Models with Five Currency Portfolios

Factor Model	Risk Factor	$\gamma_{Gskew}$	$\gamma_{Market}$
1-factor Model	Gskew	0.283 (2.418)	
2-factor Model	Gskew Market	0.255 (2.303)	0.004 (0.491)

Note: This table reports the Fama-MacBeth estimates of the price premiums using 5 currency portfolio returns and 2 risk factors: a global currency skewness risk factor and a market risk factor. The 5 currency portfolios are formed by strategies sorted on the 3rd Moment of Exchange Rate Movements. The t statistics are in parenthesis

also have negative skewness, while currencies with negative interest rate differentials tend to have negative carry trade returns but also have positive skewness.

To gain a better understanding of the link of interest rate differentials and carry trade returns, three equally weighted carry trade portfolios are built by taking long positions in currencies with the highest interest rate differentials and short positions in currencies with the lowest interest rate differentials. we find that excess returns for these long-short carry portfolios are leptokurtic. In order to forecast carry trade returns and FX skewness based on lagged interest rate differentials and past carry trade returns, we use panel regressions with country fixed effects. The evidences support that high interest rate differentials predict high carry trade returns over the next ten months, and high interest rate differential is a significant indicator of negative skewness. However, empirical results suggest that past carry trade return is not an important determinant of future carry trade return. Although research documented the existence of a momentum effect in stock returns, we find that monthly carry trade returns for our ten currency pairs do not exhibit a momentum effect.

Starting with the stochastic discount factor growth, a mathematical model is proposed to describe carry trade return dynamics that capture jump risk. Carry trade returns are modeled as jump-diffusion processes where types of jumps involve global and idiosyncratic jumps. We derive the first four moments of return process on exchange rates and use the method of moments to estimate parameters, which show the model matches excess carry trade returns in data quite well. Empirical results indicate that carry trade returns exhibit an asymmetric leptokurtic feature.

We investigate the relationship between currency portfolio returns and the third moment of exchange rate changes by studying the performance of five long-short currency trading strategies.

To implement this, we form five long-short strategies based on the third moment of returns on exchange rates by taking a short position in currencies with smallest third moment and a long position in currencies with largest third moments of returns on exchange rates. The trading strategy that borrows currency with the largest third moment of return on exchange rate and lends currency with the smallest third moment outperforms other strategies. It is interesting to see that currencies with smaller third moment of returns on exchange rates tend to have low interest rate differentials, while currencies with larger third moment of returns on exchange rates tend to have high interest rate differentials.

Finally, we conduct asset pricing experiments to test how global currency skewness risk factor and market factor explain portfolio returns of five long-short currency trading strategies. The risk premium for the global currency skewness risk factor is positive and statistically significant, whereas the risk premium for the market factor is not significantly different from zero. This result confirms that the skewness factor is a driving force to carry trade returns.

## References

- Ang, Joseph Chen, Andrew, and Yuhang Xing, 2016, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.
- Brunnermeier, Markus K, Stefan Nagel, and Lasse H. Pedersen, 2008, Carry Trades and Currency Crashes, in Kenneth Rogoff, Michael Woodford, and Daron Acemoglu, eds., *NBER Macroeconomics Annual 2008*, volume 23, 313–347.
- Brunnermeier, Markus K, and Lasse H. Pedersen, 2009, Market Liquidity and Funding Liquidity, volume 22, 2201–2238.
- Chernov, Mikhail, Jeremy Graveline, and Irina Zviadadze, 2013, Crash Risk in Currency Returns, Working paper, London School of Economics.
- Fama, Eugene F., 1984, Forward and Spot Exchange Rates, *Journal of Monetary Economics* 14, 319–338.
- Farhi, Emmanuel, Samuel Paul Fraiberger, Xavier Gabaix, Romain Ranciere, and Adrien Verdelhan, 2015, Crash Risk in Currency Markets, Harvard working paper.
- Farhi, Emmanuel, and Xavier Gabaix, 2016, Rare Disasters and Exchange Rates, *Quarterly Journal of Economics* 131, 1–52.
- Jurek, Jakub W., and Zhikai Xu, 2014, Option-implied currency risk premia, Working paper, Princeton University.
- Lee, Suzanne S., and Minh Wang, 2017, The Impact of Jumps on Carry Trade Returns, *Journal of Financial Economics* .
- Lustig, Hanno, Nick Roussanov, and Adrien Verdelhan, 2011, Common Risk Factors in Currency Returns, *Review of Financial Studies* 24, 3731–3777.
- Menkhoff, Lukas, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf, 2012, Carry Trades and Global Foreign Exchange Volatility, *Journal of Finance* 67, 681–718.
- Rafferty, Barry, 2012, Currency Returns, Skewness and Crash Risk, Working paper, Duke University.



# Appendices

## A Formulating Returns on Three Equally Weighted Carry Portfolios

In subsection 2.4, three equally weighted carry portfolios are constructed by taking a short position in currencies with lowest interest rate differentials, and a long position in currencies with highest interest rate differentials. The first carry portfolio, which consists of a short position in currency  $I$  with the lowest interest rate differential and a long position in currency  $J$  with the longest interest rate differential in the month  $t$ , can be replicated by two portfolios: (i) portfolio that involves a short position in currency  $I$  and a long position in currency  $H$ ; (ii) portfolio that involves a short position in currency  $H$  and a long position in currency  $J$ . Then the return on the first carry portfolio,  $R_{P1,t+1}$ , in the month  $t + 1$  can be written as

$$R_{P1,t+1} = CT_{-I/+H,t+1}^H + CT_{-H/+J,t+1}^H$$

$CT_{-I/+H,t+1}^H$  with a minus sign would mean reversing the cash flow, and hence  $CT_{-I/+H,t+1}^H = -CT_{-H/+I,t+1}^H$ . The above expression gives

$$\begin{aligned} R_{P1,t+1} &= CT_{-H/+J,t+1}^H - CT_{-H/+I,t+1}^H \\ &= \left[ r_{J,t} - r_{H,t} - \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) \right] - \left[ r_{I,t} - r_{H,t} - \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) \right] \\ &= (r_{J-H,t} - r_{I-H,t}) - \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) - \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) \right] \end{aligned}$$

where we define  $r_{J-H,t} \equiv r_{J,t} - r_{H,t}$  and  $r_{I-H,t} \equiv r_{I,t} - r_{H,t}$ . By covered interest rate parity,  $R_{P1,t+1}$  may be computed by

$$R_{P1,t+1} = \left[ \log \left( \frac{f_{J/H,t}}{e_{J/H,t}} \right) - \log \left( \frac{f_{I/H,t}}{e_{I/H,t}} \right) \right] - \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) - \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) \right]$$

where  $f_{J/H,t}$  is forward exchange rate of currency  $J$  relative to currency  $H$  at time  $t$ .

Similarly, the second carry portfolio are built by taking a long position in two currencies  $J, J'$  with the highest interest rate differentials  $r_{J-H,t}$  and  $r_{J'-H,t}$ , as well as a short position in two currencies  $I, I'$  with the lowest interest rate differentials  $r_{I-H,t}$  and  $r_{I'-H,t}$  in the month  $t$ . The

return on the second carry portfolio , $R_{P2,t+1}$ , in the month  $t + 1$  may be expressed as

$$\begin{aligned}
R_{P2,t+1} &= \frac{1}{2} \left[ r_{J-H,t} - \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) \right] + \frac{1}{2} \left[ r_{J'-H,t} - \log \left( \frac{e_{J'/H,t+1}}{e_{J'/H,t}} \right) \right] \\
&\quad - \left\{ \frac{1}{2} \left[ r_{I-H,t} - \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) \right] + \frac{1}{2} \left[ r_{I'-H,t} - \log \left( \frac{e_{I'/H,t+1}}{e_{I'/H,t}} \right) \right] \right\} \\
&= \frac{1}{2} \left[ \left( r_{J-H,t} + i_{J'-H,t} \right) - \left( r_{I-H,t} + r_{I'-H,t} \right) \right] - \frac{1}{2} \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) + \log \left( \frac{e_{J'/H,t+1}}{e_{J'/H,t}} \right) \right] \\
&\quad + \frac{1}{2} \left[ \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) + \log \left( \frac{e_{I'/H,t+1}}{e_{I'/H,t}} \right) \right]
\end{aligned}$$

Finally, the third carry portfolio can be constructed by taking a long position in three currencies  $J, J', J''$  with the highest interest rate differentials  $r_{J-H,t}$ ,  $r_{J'-H,t}$ , and  $r_{J''-H,t}$  as well as a short position in three currencies  $I, I', I''$  with the lowest interest rate differentials  $r_{I-H,t}$ ,  $r_{I'-H,t}$  and  $r_{I''-H,t}$  in the month  $t$ . The return on the third carry portfolio,  $R_{P3,t+1}$  in the month  $t + 1$  may be expressed as

$$\begin{aligned}
R_{P3,t+1} &= \frac{1}{3} \left[ r_{J-H,t} - \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) \right] + \frac{1}{3} \left[ r_{J'-H,t} - \log \left( \frac{e_{J'/H,t+1}}{e_{J'/H,t}} \right) \right] \\
&\quad + \frac{1}{3} \left[ r_{J''-H,t} - \log \left( \frac{e_{J''/H,t+1}}{e_{J''/H,t}} \right) \right] - \frac{1}{3} \left[ r_{I-H,t} - \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) \right] \\
&\quad - \frac{1}{3} \left[ r_{I'-H,t} - \log \left( \frac{e_{I'/H,t+1}}{e_{I'/H,t}} \right) \right] - \frac{1}{3} \left[ r_{I''-H,t} - \log \left( \frac{e_{I''/H,t+1}}{e_{I''/H,t}} \right) \right] \\
&= \frac{1}{3} \left[ \left( r_{J-H,t} + r_{J'-H,t} + i_{J''-H,t} \right) - \left( r_{I-H,t} + r_{I'-H,t} + r_{I''-H,t} \right) \right] \\
&\quad - \frac{1}{3} \left[ \log \left( \frac{e_{J/H,t+1}}{e_{J/H,t}} \right) + \log \left( \frac{e_{J'/H,t+1}}{e_{J'/H,t}} \right) + \log \left( \frac{e_{J''/H,t+1}}{e_{J''/H,t}} \right) \right] \\
&\quad + \frac{1}{3} \left[ \log \left( \frac{e_{I/H,t+1}}{e_{I/H,t}} \right) + \log \left( \frac{e_{I'/H,t+1}}{e_{I'/H,t}} \right) + \log \left( \frac{e_{I''/H,t+1}}{e_{I''/H,t}} \right) \right]
\end{aligned}$$

## B Deriving The First Four Moments of $\frac{de_{F/H,t}}{e_{F/H,t}}$

This section will present the first four moments of  $\frac{de_{F/H,t}}{e_{F/H,t}}$ , the net return on exchange rate  $e_{F/H}$ . From equation (??), we begin with

$$e_{F/H,t} = \frac{M_{H,t}}{M_{F,t}}$$

Let's define  $de_{F/H,t} \equiv e_{F/H,t+dt} - e_{F/H,t}$ . To investigate the impact of jump events on  $\frac{de_{F/H,t}}{e_{F/H,t}}$ , now we ignore its diffusion part. As jumps occur over the interval  $[t, t + dt]$ , we know  $M_{H,t+dt} = M_{H,t}e^{\Delta_i H}$  and  $M_{F,t+dt} = M_{F,t}e^{\Delta_i F}$ . The change in exchange rate movement can be expressed as

$$\begin{aligned} de_{F/H,t} &\equiv e_{F/H,t+dt} - e_{F/H,t} = \frac{M_{H,t+dt}}{M_{F,t+dt}} - \frac{M_{H,t}}{M_{F,t}} \\ &= \frac{M_{H,t}e^{\Delta_i H}}{M_{F,t}e^{\Delta_i F}} - \frac{M_{H,t}}{M_{F,t}} = \frac{M_{H,t}}{M_{F,t}} \left( \frac{e^{\Delta_i H}}{e^{\Delta_i F}} - 1 \right) \end{aligned}$$

The jump effect on exchange rate return,  $\frac{de_{F/H,t}}{e_{F/H,t}}$ , over a period of length  $dt$  is given by

$$\frac{de_{F/H,t}}{e_{F/H,t}} = \frac{\frac{M_{H,t}}{M_{F,t}} \left( \frac{e^{\Delta_i H}}{e^{\Delta_i F}} - 1 \right)}{\frac{M_{H,t}}{M_{F,t}}} = \frac{e^{\Delta_i H}}{e^{\Delta_i F}} - 1 = e^{\Delta_i H - \Delta_i F} - 1$$

The following is the application of Itô's formula to compute  $\frac{de_{F/H,t}}{e_{F/H,t}}$  by using the rule for ratios. That is, let  $X$  and  $Y$  be Itô's processes, and  $Z = \frac{X}{Y}$ , then  $\frac{dZ}{Z}$  can be written as

$$\frac{dZ}{Z} = \frac{dX}{X} - \frac{dY}{Y} + \left( \frac{dY}{Y} \right)^2 - \frac{dX}{X} \frac{dY}{Y}$$

From equation (15), we get the SDFs with the jump-diffusions for home currency  $I$  and foreign currency  $F$

$$\begin{aligned} \frac{dM_{H,t}}{M_{H,t}} &= -r_{H,t}dt - \eta_{H,t}^T dB_t + \sum_i (e^{\Delta_i H} - 1) (dN_{i,t} - \lambda_i dt) \\ \frac{dM_{F,t}}{M_{F,t}} &= -r_{F,t}dt - \eta_{F,t}^T dB_t + \sum_i (e^{\Delta_i F} - 1) (dN_{i,t} - \lambda_i dt) \end{aligned}$$

If  $Z = e_{F/H,t}$ ,  $X = M_{H,t}$  and  $Y = M_{F,t}$ , then  $\frac{de_{F/H,t}}{e_{F/H,t}}$  will be

$$\begin{aligned}
\frac{de_{F/H,t}}{e_{F/H,t}} &= \frac{d\left(\frac{M_{H,t}}{M_{F,t}}\right)}{\frac{M_{H,t}}{M_{F,t}}} \\
&= \frac{dM_{H,t}}{M_{H,t}} - \frac{dM_{F,t}}{M_{F,t}} + \left(\frac{dM_{F,t}}{M_{F,t}}\right)^2 - \frac{dM_{H,t}}{M_{H,t}} \frac{dM_{F,t}}{M_{F,t}} + \sum_i (e^{\Delta_i H - \Delta_i F} - 1) dN_{i,t} \\
&= -r_{H,t} dt - \eta_{H,t}^T dB_t - \sum_i (e^{\Delta_i H} - 1) \lambda_i dt + r_{F,t} dt + \eta_{F,t}^T dB_t + \sum_i (e^{\Delta_i F} - 1) \lambda_i dt \\
&\quad + \eta_{F,t}^T \eta_{F,t} dt - \eta_{F,t}^T \eta_{H,t} dt + \sum_i (e^{\Delta_i H - \Delta_i F} - 1) dN_{i,t} \\
&= (r_{F,t} - r_{H,t}) dt + \eta_{F,t}^T (\eta_{F,t} - \eta_{H,t}) dt + (\eta_{F,t}^T - \eta_{H,t}^T) dB_t + \sum_i (e^{\Delta_i F} - e^{\Delta_i H}) \lambda_i dt \\
&\quad + \sum_i (e^{\Delta_i H - \Delta_i F} - 1) dN_{i,t} \tag{55}
\end{aligned}$$

Taking expectation of  $\frac{de_{F/H,t}}{e_{F/H,t}}$  in equation (55) gives us

$$\begin{aligned}
E_t \left( \frac{de_{F/H,t}}{e_{F/H,t}} \right) &= (r_{F,t} - r_{H,t}) dt + \eta_{F,t}^T (\eta_{F,t} - \eta_{H,t}) dt + \sum_i (e^{\Delta_i F} - e^{\Delta_i H}) \lambda_i dt \\
&\quad + \sum_i (e^{\Delta_i H - \Delta_i F} - 1) \lambda_i dt \\
&= [(r_F - r_H) + \eta_F^T (\eta_F - \eta_H)] dt + \sum_i [(e^{\Delta_i F} - e^{\Delta_i H} + e^{\Delta_i H - \Delta_i F} - 1) \lambda_i dt] \\
&= [(r_F - r_H) + \eta_F^T (\eta_F - \eta_H)] dt + \sum_i [e^{\Delta_i H} (e^{-\Delta_i F} - 1) + e^{\Delta_i F} (1 - e^{-\Delta_i F})] \lambda_i dt \\
&= [(r_F - r_H) + \eta_F^T (\eta_F - \eta_H)] dt + \sum_i [e^{\Delta_i H} (e^{-\Delta_i F} - 1) - e^{\Delta_i F} (e^{-\Delta_i F} - 1)] \lambda_i dt \\
&= [(r_F - r_H) + \eta_F^T (\eta_F - \eta_H)] dt + \sum_i [(e^{\Delta_i H} - e^{\Delta_i F}) (e^{-\Delta_i F} - 1)] \lambda_i dt \tag{56}
\end{aligned}$$

Thus, the first moment of  $\frac{de_{F/H,t}}{e_{F/H,t}}$  is

$$\frac{1}{dt} E_t \left( \frac{de_{F/H,t}}{e_{F/H,t}} \right) = [(r_F - r_H) + \eta_F^T (\eta_F - \eta_H)] + \sum_i [(e^{\Delta_i H} - e^{\Delta_i F}) (e^{-\Delta_i F} - 1)] \lambda_i \tag{57}$$

The jump component of  $E_t \left[ \left( \frac{de_{F/H,t}}{e_{F/H,t}} \right)^2 \right]$  can be written as:

$$\begin{aligned}
\frac{1}{dt} E_t \left[ \left( \frac{e_{F/H,t+1} - e_{F/H,t}}{e_{F/H,t}} \right)^2 \right] &= \frac{1}{dt} E_t \left[ \left( \frac{e_{F/H,t+1}}{e_{F/H,t}} - 1 \right)^2 \right] \\
&= \frac{1}{dt} E_t [f] \quad \text{let } f = \left( \frac{e_{F/H,t+1}}{e_{F/H,t}} - 1 \right)^2 \\
&= \frac{1}{dt} \sum_i [(\lambda_i dt) f_{N_i=1} + (1 - \lambda_i dt) f_{N_i=0}] \\
&= \frac{1}{dt} \sum_i \left[ (\lambda_i dt) \left( \frac{e_{N_i=1}}{e_{N_i=0}} - 1 \right)^2 + (1 - \lambda_i dt) \left( \frac{e_{N_i=0}}{e_{N_i=0}} - 1 \right)^2 \right] \\
&= \frac{1}{dt} \sum_i \left[ (\lambda_i dt) \left( \frac{M_{H,t} e^{\Delta_i H}}{M_{F,t} e^{\Delta_i F}} - 1 \right)^2 + (1 - \lambda_i dt) \left( \frac{M_{H,t}}{M_{F,t}} - 1 \right)^2 \right] \\
&= \sum_i \lambda_i \left( \frac{e^{\Delta_i H}}{e^{\Delta_i F}} - 1 \right)^2 = \sum_i \lambda_i (e^{\Delta_i H - \Delta_i F} - 1)^2 \quad (58)
\end{aligned}$$

From equations (55) and (58), we obtain the second moment of  $\frac{de_{F/H,t}}{e_{F/H,t}}$  by using Itô Lemma

$$\frac{1}{dt} E_t \left[ \left( \frac{de_{F/H,t}}{e_{F/H,t}} \right)^2 \right] = (\eta_F^T - \eta_H^T) (\eta_F - \eta_H) + \sum_i (e^{\Delta_i H - \Delta_i F} - 1)^2 \lambda_i \quad (59)$$

The third and fourth moments of  $\frac{de_{F/H,t}}{e_{F/H,t}}$  are similarly computed. Obviously, the diffusion parameters disappear and only the jump parameters, such as jump sizes  $\Delta_i$  and jump intensity  $\lambda_i$ , show in the third and fourth moments.

$$\frac{1}{dt} E_t \left[ \left( \frac{de_{F/H,t}}{e_{F/H,t}} \right)^3 \right] = \sum_i (e^{\Delta_i H - \Delta_i F} - 1)^3 \lambda_i \quad (60)$$

$$\frac{1}{dt} E_t \left[ \left( \frac{de_{F/H,t}}{e_{F/H,t}} \right)^4 \right] = \sum_i (e^{\Delta_i H - \Delta_i F} - 1)^4 \lambda_i \quad (61)$$

## C Model Extension with Different Types of Jumps

For the sake of simplicity, let us assume that there are three types of jumps: a global jump  $G$  and two idiosyncratic jumps for country  $I$  and country  $J$ , respectively. When a global jump occurs, the arrival of jumps is coincident across countries; however, the size of the jump across countries may

vary. The idiosyncratic jump from country  $I$  has no influence on country  $J$ , and vice versa. From equation (24), the carry trade return of borrowing the home currency  $I$  and investing in a foreign currency  $J$  over a time interval  $[t, t + dt]$  can be written

Now consider the case that the base currency  $I$  is USD; the funding currency  $B$  is also JPY, and the investing currency  $L$  is AUD, CAD, NZD, NOK, CHF, GBP, EUR, SEK, and DKK respectively. Suppose there are 1 global jump, 3 idiosyncratic jumps, and 3 bi-currency jumps. The bi-currency jump only affect two currencies. When a bi-currency jump occurs, the arrival of jumps is coincident across two countries; however, the size of the jump across two countries may vary. The carry trade return of borrowing the home currency  $I$  and investing in a foreign currency  $J$  over a time interval  $[t, t + dt]$  can be written

$$CT_{-B/+L,t+dt}^I = \sum_i [(e^{\Delta_i B} - 1) - (e^{\Delta_i L} - 1)] \lambda_i dt + \eta_{I,t}^T (\eta_{B,t} - \eta_{L,t}) dt + (\eta_{B,t}^T - \eta_{L,t}^T) dB_t \\ + \sum_i [(e^{\Delta_i L - \Delta_i I} - 1) - (e^{\Delta_i B - \Delta_i I} - 1)] dN_{i,t}, \quad i = G, I, B, L, BI, LI, BL \quad (62)$$

The first four moments of returns processes of exchange rates  $\frac{de_{B/I,t}}{e_{B/I,t}}$  become

$$E_t \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right) = [(r_B - r_I) + \eta_B^T (\eta_B - \eta_I)] dt + \lambda_G [(e^{\Delta_{GI}} - e^{\Delta_{GB}}) (e^{-\Delta_{GB}} - 1)] dt \\ + \lambda_B (1 - e^{\Delta_{BB}}) (e^{-\Delta_{BB}} - 1) dt \quad (63)$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \right] = (\eta_B^T - \eta_I^T) (\eta_B - \eta_I) dt + \lambda_G (e^{\Delta_{GI} - \Delta_{GB}} - 1)^2 dt + \lambda_I (e^{\Delta_{II}} - 1)^2 dt \\ + \lambda_B (e^{-\Delta_{BB}} - 1)^2 dt \quad (64)$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^3 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GB}} - 1)^3 + \lambda_I (e^{\Delta_{II}} - 1)^3 + \lambda_B (e^{-\Delta_{BB}} - 1)^3] dt \quad (65)$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^4 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GB}} - 1)^4 + \lambda_I (e^{\Delta_{II}} - 1)^4 + \lambda_B (e^{-\Delta_{BB}} - 1)^4] dt \quad (66)$$

Similarly, the first four moments of returns processes of exchange rates  $\frac{de_{L/I,t}}{e_{L/I,t}}$  are

$$E_t \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right) = [(r_L - r_I) + \eta_L^T (\eta_L - \eta_I)] dt + \lambda_G [(e^{\Delta_{GI}} - e^{\Delta_{GL}}) (e^{-\Delta_{GL}} - 1)] dt + \lambda_L (1 - e^{\Delta_{LL}}) (e^{-\Delta_{LL}} - 1) dt \quad (67)$$

$$E_t \left[ \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right] = (\eta_L^T - \eta_I^T) (\eta_L - \eta_I) dt + \lambda_G (e^{\Delta_{GI} - \Delta_{GL}} - 1)^2 dt + \lambda_I (e^{\Delta_{II}} - 1)^2 dt + \lambda_L (e^{-\Delta_{LL}} - 1)^2 dt \quad (68)$$

$$E_t \left[ \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^3 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GL}} - 1)^3 + \lambda_I (e^{\Delta_{II}} - 1)^3 + \lambda_L (e^{-\Delta_{LL}} - 1)^3] dt \quad (69)$$

$$E_t \left[ \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^4 \right] = [\lambda_G (e^{\Delta_{GI} - \Delta_{GL}} - 1)^4 + \lambda_I (e^{\Delta_{II}} - 1)^4 + \lambda_L (e^{-\Delta_{LL}} - 1)^4] dt \quad (70)$$

Moreover, the cross-moments of  $\frac{de_{B/I,t}}{e_{B/I,t}}$  and  $\frac{de_{L/I,t}}{e_{L/I,t}}$  are

$$E_t \left( \frac{de_{B/I,t}}{e_{B/I,t}} \times \frac{de_{L/I,t}}{e_{L/I,t}} \right) = \left[ (\eta_B - \eta_I) (\eta_L - \eta_I) + \sum_i (e^{\Delta_{iI} - \Delta_{iB}} - 1) (e^{\Delta_{iI} - \Delta_{iL}} - 1) \lambda_i \right] dt \quad (71)$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right) \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right] = \sum_i (e^{\Delta_{iI} - \Delta_{iB}} - 1) (e^{\Delta_{iI} - \Delta_{iL}} - 1)^2 \lambda_i dt \quad (72)$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right) \right] = \sum_i (e^{\Delta_{iI} - \Delta_{iB}} - 1)^2 (e^{\Delta_{iI} - \Delta_{iL}} - 1) \lambda_i dt \quad (73)$$

$$E_t \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right] = \sum_i (e^{\Delta_{iI} - \Delta_{iB}} - 1)^2 (e^{\Delta_{iI} - \Delta_{iL}} - 1)^2 \lambda_i dt \quad (74)$$

From the twelve moment conditions in equations (63) - (74), the 22 parameters to be estimated are  $\eta_I, \eta_B, \eta_L, \lambda_G, \lambda_I, \lambda_B, \lambda_L, \lambda_{BI}, \lambda_{LI}, \lambda_{BL}, \Delta_{GI}, \Delta_{GB}, \Delta_{GL}, \Delta_{II}, \Delta_{IB}, \Delta_{LL}, \Delta_{BII}, \Delta_{BIB}, \Delta_{LII}, \Delta_{LIL}, \Delta_{BLB}, \Delta_{BLL}$ . We estimate these 22 parameters by solving the following nonlinear optimization problem, which minimizes the squared deviation of 12 moment conditions (equations (63)- (74))

from the moments calculated by the data.

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^4 w_i \times \left\| E \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^i_{model} - E \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^i_{data} \right\|^2 \\
& + \sum_{j=1}^4 w_j \times \left\| E \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^j_{model} - E \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^j_{data} \right\|^2 \\
& + w_9 \times \left\| E \left( \frac{de_{B/I,t}}{e_{B/I,t}} \times \frac{de_{L/I,t}}{e_{L/I,t}} \right)_{model} - E \left( \frac{de_{B/I,t}}{e_{B/I,t}} \times \frac{de_{L/I,t}}{e_{L/I,t}} \right)_{data} \right\|^2 \\
& + w_{10} \times \left\| E \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right) \right]_{model} - E \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right) \right]_{data} \right\|^2 \\
& + w_{11} \times \left\| E \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right) \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right]_{model} - E \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right) \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right]_{data} \right\|^2 \\
& + w_{12} \times \left\| E \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right]_{model} - E \left[ \left( \frac{de_{B/I,t}}{e_{B/I,t}} \right)^2 \times \left( \frac{de_{L/I,t}}{e_{L/I,t}} \right)^2 \right]_{data} \right\|^2 \\
\text{subject to} \quad & \eta_I, \eta_B, \eta_L > 0 \\
& \lambda_G, \lambda_I, \lambda_B, \lambda_L, \lambda_{BI}, \lambda_{LI}, \lambda_{BL} > 0
\end{aligned} \tag{75}$$