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A criterion for the solvability of a μ-synthesis problem

Abstract

We give a solvability criterion for the following μ-synthesis problem. Let μ be the structured singular value for the diagonal matrices with entries in $\mathbb{C}$.

Problem. Given distinct points $\lambda_1, \dots, \lambda_n$ in the open unit disc $\mathbb{D}$ and target 2×2 complex matrices $W_1, \dots, W_n$ such that $\mu(W_j) \leq 1$ for all $j = 1, \dots, n$, find a holomorphic 2×2 matrix function F on $\mathbb{D}$ such that $F(\lambda_j) = W_j$ for each j, and $\mu(F(\lambda)) \leq 1$ for all $\lambda \in \mathbb{D}$.

By [1, Theorem 9.2], this problem is equivalent to the following interpolation problem: does there exist a holomorphic function x from the disc to the tetrablock $\overline{\mathbb{E}}$ such that $x(\lambda_j) = (w_{11}^j, w_{22}^j, \det W_j)$ for each j? The tetrablock is the domain in $\mathbb{C}^3$ defined by

$$\overline{\mathbb{E}} := \{(x_1, x_2, x_3) \in \mathbb{C}^3 : 1 - x_1 z - x_2 w + x_3 zw \neq 0 \text{ for all } z, w \in \mathbb{D}\}.$$

In this talk we show such an x exists if and only if, for distinct $z_1, z_2, z_3 \in \mathbb{D}$, there are positive $3n$-square matrices $[N_{il,jk}]$, of rank 1, and $[M_{il,jk}]$ such that

$$\left[1 - \overline{\frac{z_l x_{3i} - x_{1i}}{x_{2i} z_l - 1}} \frac{z_k x_{3j} - x_{1j}}{x_{2j} z_k - 1}\right] \geq [(1 - \overline{z_l} z_k) N_{il,jk}] + [(1 - \overline{\lambda_i} \lambda_j) M_{il,jk}],$$

where $(x_{1j}, x_{2j}, x_{3j}) = (w_{11}^j, w_{22}^j, \det W_j)$ for each j.

The talk is based on a joint work with Z. A. Lykova and N. J. Young.

[1] A. A. Abouhajar, M. C. White and N. J. Young, A Schwarz lemma for a domain related to μ-synthesis, *J. Geom. Anal. 17*, (2007), pp. 717-750

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