Essays on Macroeconomics

Duksang Cho

Washington University in St. Louis

Follow this and additional works at: http://openscholarship.wustl.edu/art_sci_etds

Part of the Economics Commons

Recommended Citation

http://openscholarship.wustl.edu/art_sci_etds/769
Essays on Macroeconomics
by
Duksang Cho

A dissertation presented to the
Graduate School of Arts & Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2016
St. Louis, Missouri
## Table of Contents

List of Figures iv

List of Tables vi

Acknowledgments vii

Abstract ix

1 Pyramidal Business Groups and Asymmetric Financial Frictions 1
   1.1 Introduction ................................................................. 1
   1.2 A Heterogeneous Agent Model with Occupational Choices .............. 7
      1.2.1 Economic Environment .............................................. 7
      1.2.2 An Individual’s Problem ......................................... 10
   1.3 Financial Frictions and Three Types of Firms ........................... 12
      1.3.1 A Private Company ............................................... 13
      1.3.2 A Publicly Held Corporation .................................... 14
      1.3.3 A Business Group .................................................. 18
   1.4 A Matching Rule and a Stationary Equilibrium .......................... 24
      1.4.1 A Matching Rule Between Business-Group Entrepreneurs and the Others 24
      1.4.2 A Stationary Equilibrium ......................................... 26
   1.5 Remarks on the Model .................................................... 28
      1.5.1 Financial Advantage of Business Groups .......................... 28
      1.5.2 Asymmetric Financial Frictions ................................... 30
      1.5.3 External Finance Substituting for Private Finance ................. 31
   1.6 A Numerical Example of the Model ..................................... 33
      1.6.1 Setup ...................................................................... 33
      1.6.2 Observations ............................................................ 35
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>A Case of South Korea</td>
<td>49</td>
</tr>
<tr>
<td>1.7.1</td>
<td>The Prevalence of Business Groups</td>
<td>50</td>
</tr>
<tr>
<td>1.7.2</td>
<td>The Asymmetric Financial Frictions Among Firms</td>
<td>51</td>
</tr>
<tr>
<td>1.7.3</td>
<td>The Firm Size Distribution</td>
<td>52</td>
</tr>
<tr>
<td>1.7.4</td>
<td>Aggregate Variables</td>
<td>53</td>
</tr>
<tr>
<td>1.8</td>
<td>Conclusion</td>
<td>54</td>
</tr>
<tr>
<td>2</td>
<td>Peasants in the City: Consequences of Declining Labor Shares</td>
<td>56</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>56</td>
</tr>
<tr>
<td>2.2</td>
<td>A Technology Generating a Cobb-Douglas Production Function and Changing Factor Shares</td>
<td>58</td>
</tr>
<tr>
<td>2.3</td>
<td>Remarks on a Technology Substituting Capital for Labor</td>
<td>61</td>
</tr>
<tr>
<td>2.4</td>
<td>Consequences of Asymmetric Changes in Factor Shares</td>
<td>64</td>
</tr>
<tr>
<td>2.4.1</td>
<td>From Plows to Steam Engines That Substitutes Capital for Land and Labor in Agriculture</td>
<td>64</td>
</tr>
<tr>
<td>2.4.2</td>
<td>From Steam Engines to Computers That Substitutes Capital for Labor in Manufacture</td>
<td>67</td>
</tr>
<tr>
<td>2.5</td>
<td>Conclusion</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>Synchronized Durable Goods Purchases and the Business Cycle</td>
<td>72</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>72</td>
</tr>
<tr>
<td>3.2</td>
<td>A Model</td>
<td>74</td>
</tr>
<tr>
<td>3.3</td>
<td>A Stationary State</td>
<td>79</td>
</tr>
<tr>
<td>3.4</td>
<td>Conclusion and Future Research</td>
<td>80</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>82</td>
</tr>
<tr>
<td>A</td>
<td>Appendix for Chapter 1</td>
<td>86</td>
</tr>
<tr>
<td>A.1</td>
<td>Pyramidal Business Groups and Wealth Inequality</td>
<td>86</td>
</tr>
<tr>
<td>A.2</td>
<td>Auxiliary Results of the Model</td>
<td>88</td>
</tr>
<tr>
<td>A.3</td>
<td>Comparing the Model with Literature and Data</td>
<td>92</td>
</tr>
<tr>
<td>A.4</td>
<td>Proof of Proposition 1</td>
<td>99</td>
</tr>
<tr>
<td>A.5</td>
<td>Proof of Proposition 2</td>
<td>103</td>
</tr>
<tr>
<td>A.6</td>
<td>Some Algebra</td>
<td>110</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Time-line of an Individual’s Problem Within a Period . . . . . . . . . . . . . 9
1.2 Occupational Choice Given Managerial Talent $z$ and Wealth $a$ . . . . . . . . . 12
1.3 Occupational Map in an Economy With Business Groups Given $\tau = 0.5$. No population exists outside the border of orange line. . . . . . . . . . . . . . . 36
1.4 Occupational Map in an Economy With Business Groups Given $\tau = 0.1$. No population exists outside the border of orange line. . . . . . . . . . . . . . . 36
1.5 Occupational Map in an Economy Without Business Groups Given $\tau = 0.5$. 37
1.6 Occupational Map in an Economy Without Business Groups Given $\tau = 0.1$. 37
1.7 The Prevalence of Business Groups Measured by the Relative Number of Firms 38
1.8 Distributions of Capital-to-Labor Ratio . . . . . . . . . . . . . . . . . . . . . 39
1.9 Firm Size Distributions Measured by Employment . . . . . . . . . . . . . . . 40
1.10 Distributions of Managerial Talent (TFP) . . . . . . . . . . . . . . . . . . . 41
1.11 Factor Prices and Aggregate Inputs . . . . . . . . . . . . . . . . . . . . . . 42
1.12 Flotation Costs . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 44
1.13 Investment Net of Flotation Costs . . . . . . . . . . . . . . . . . . . . . . . 44
1.14 Investment Rate . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 45
1.15 Aggregate Output . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 46
1.16 External Capital Markets . . . . . . . . . . . . . . . . . . . . . . . . . . . . 48
1.17 The Distribution of Capital-to-Labor Ratio from 2010 to 2013 . . . . . . . . 51
1.18 The Distribution of Employment from 2010 to 2013 . . . . . . . . . . . . . . 53

A.1 Wealth Gini Coefficients . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 86
A.2 Upward Wealth Mobility . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 87
A.3 Downward Wealth Mobility . . . . . . . . . . . . . . . . . . . . . . . . . . . 87
A.4 Population of the Rich . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 88
A.5 Internal Capital Markets . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 89
A.6 Capital Stock ......................................................... 89
A.7 Corporate Savings ................................................... 90
A.8 Consumption Level ................................................. 91
A.9 Investment Rate ...................................................... 91
A.10 Prevalence of Business Groups ................................. 93
A.11 Underdeveloped Ext. Cap. Markets .............................. 93
A.12 The Distribution of Capital-to-Labor Ratio Within Manufacture from 2010 to 2013 ........................................... 94
A.13 The Distribution of Capital-to-Labor Ratio Within Non-Finance Service Sector from 2010 to 2013 ........................................... 95
A.14 Annual Trend of Changes in Corporate Savings for Each Country Since 2004 (Data is Collected from OECD, PWT8.1, and Masulis et al. (2011)) ........... 96
A.15 Annual Trend of Changes in Corporate Savings for Each Country Since 2004 (Data is Collected from OECD, PWT8.1, and Masulis et al. (2011)) ........... 96
A.16 Share of Household Consumption (2004, Current PPPs, Collected from PWT8.1 and Masulis et al. (2011)) ......................................................... 97
A.17 Share of Household Consumption (2004, Current PPPs, Collected from PWT8.1 and Masulis et al. (2011)) ......................................................... 97
A.18 Consumption of Fixed Capital (2004, Per Cent of GDP, Collected from OECD and Masulis et al. (2011)) ......................................................... 98
A.19 Consumption of Fixed Capital (2004, Per Cent of GDP, Collected from OECD and Masulis et al. (2011)) ......................................................... 98
A.20 Risk Sharing and Binding External Equity Finance .......... 102
A.21 Non-Negative Marginal Expected Value of Investment and Binding Private Borrowing ......................................................... 103
A.22 Non-negative Marginal Expected Value of Investment and Binding External Debt Finance of Firm 1 ......................................................... 110
List of Tables

1.1 Parameters ......................................................... 34
1.2 The Transition Probability of Managerial Talent .................. 34
1.3 Savings of the Rich ............................................. 43
1.4 Wealth of Stand-Alone Entrepreneurs in an Economy with Business Groups . 43
1.5 The Prevalence of Business Groups Measured by the Relative Number of Firms 50
Acknowledgments

First and foremost, I would like to thank my advisor, Professor Yongseok Shin. It has been an honor to be his Ph.D. student. I am grateful and indebted for his expert advice and support throughout my academic life. Without his invaluable guidance, the first chapter of this dissertation could not be written.

I would like to thank my teachers and committee members, Gaetano Antinolfi, Costas Azariadis, Rodolfo Manuelli, B. Ravikumar, Bruce Petersen, and David Wiczer. I appreciate all their contributions of time and advice to complete my dissertation.

This project would have been impossible without the financial support of Washington University in St. Louis, including the John Stuart Mill Fellowship, the Summer Research Fellowships, and the Dissertation Fellowship.

Last but not least, I thank to Han-Sung Cho and Kwang-Ja Kim for their sincere trust in my path of learning. Without their encouragement, I could have not devoted myself to accomplish this dissertation.

Duksang Cho

Washington University in St. Louis

May 2016
Dedicated to my alter ego, Heejeong Lim.
ABSTRACT OF THE DISSERTATION

Essays on Macroeconomics

by

Duksang Cho

Doctor of Philosophy in Economics

Washington University in St. Louis, 2016

Professor Yongseok Shin, Chair

My dissertation is centered on economic heterogeneity endogenously derived from market imperfections or changes in technology. By introducing specific assumptions that capture a market imperfection or a change in technology, I study how the economic realities can affect resource distributions and aggregate outcomes in an equilibrium.

Chapter 1 studies the economic impacts of business groups by focusing on their pyramidal ownership structure given capital market imperfections. An entrepreneur can alleviate financial frictions by creating a pyramidal business group in which a parent firm offers its subsidiary firm internal equity finance. This endogenous creation of pyramidal business groups can beget asymmetric financial frictions between business-group and stand-alone firms. I build a model to show that these asymmetric financial frictions can have sizable effects on resource allocation. On one hand, the financial advantage of pyramidal business groups can foster productive firms by incorporating subsidiaries. On the other hand, the asymmetrically large amount of external capital controlled by pyramidal business groups can push up the price of capital and hinder the growth of stand-alone firms. The model suggests that pyra-
midal business groups can improve the factor allocation of an economy with poor investor protection in which external capital markets are underdeveloped, but worsen the factor allocation of an economy with fine investor protection in which excessive capital is used up by unproductive business-group firms.

Chapter 2 investigates consequence of declining labor shares in manufacture. I show that a Cobb-Douglas production function can be generated with a technology that substitutes capital for labor and decreases labor shares. A simple two-sector model is used to examine consequences of declining labor shares. The model suggests that a declining labor share in manufacture can be accompanied with an increase in the labor productivity dispersion, a decrease in the labor price, and an increase in the land price.

Chapter 3 researches the possibility that changes in the number of households simultaneously purchasing durable goods can create a business cycle. Heterogeneous timings of durable goods purchases are examined as an extensive margin of aggregate consumption. I develop a model in which each household holds money to purchase durable goods and optimizes its purchase timing given adjustment costs. The model shows that a shock common to all households such as a change in the expected inflation rate or government transfers can synchronize durable goods purchases across households. I argue that altering the number of households simultaneously purchasing durable goods can generate a sizable, long-lasting business cycle without the help of sticky prices or shocks to TFP.
Chapter 1

Pyramidal Business Groups and Asymmetric Financial Frictions

1.1 Introduction

A pyramidal business group is a collection of legally independent corporations controlled by a coterie of shareholders. It is a common ownership structure for a country’s largest firms, with exceptions of some countries such as the United States or the United Kingdom.¹ The economy-wide repercussions of pyramidal business groups, however, have been unclear although they are salient economic institutions too sizable to be ignored. For instance, pyramidal business groups in South Korea not only have been acclaimed as engines of growth for the country’s fast development but also have been the subjects of controversy for their

¹La Porta, Lopez-de Silanes, and Shleifer (1999) examine 27 wealthy countries and show that most of the largest corporations in a country are business groups controlled by families or the state through pyramidal ownership schemes. La Porta, Lopez-de Silanes, Shleifer, and Vishny (2000) argue that the degree of investor protection is closely related to the corporate governance structure and that business groups are common in countries with poor investor protection. Masulis, Pham, and Zein (2011) examine 28,635 listed firms in 45 countries, including developing economies, and reaffirm that pyramidal business groups are a common ownership structure around the world. They show that the prevalence of business groups is negatively associated with the capital availability of an economy, but insignificantly associated with the degree of investor protection. They argue that business groups emerge in order to alleviate financial frictions.
economic concentration.\textsuperscript{2}

In this paper, I build a model of pyramidal business groups in a general equilibrium framework and aim to answer a question: Can pyramidal business groups affect the efficiency of an economy? I focus on a pyramidal ownership structure, which arises due to capital market imperfections and gives rise to asymmetric financial frictions between business-group and stand-alone firms.\textsuperscript{3}

Built on the span of control model developed by Lucas (1978), two assumptions are introduced. First, I assume that the investor protection of an economy is imperfect so that a firm’s ability to raise external capital is constrained.\textsuperscript{4} I assume a limited commitment problem such that an entrepreneur controlling his or her firms can divert $\tau$ fraction of the firms’ cash flow before outside investors are reimbursed. In the model, this realized diversion keeps the expected rate of return on external equity finance identical to the risk-free interest rate. Thus, an entrepreneur can earn positive profits as the private benefits of control and have an incentive to create a business group with flotation costs. Note that a common implementation of financial frictions in the literature hinges on an out-of-equilibrium path and that the diversion does not occur in an equilibrium.

Second, I allow for a business group as a private means that can alleviate financial frictions. A business group is defined as a collection of two firms connected through a pyramidal ownership structure such that a business-group entrepreneur controls a parent firm that controls a subsidiary firm. There is no limited commitment problem between the parent and the subsidiary because both firms are controlled by the common entrepreneur. Thus, the parent can offer internal finance as much as possible to the subsidiary without

\textsuperscript{2}As of 2004 in South Korea, business groups controlled by a few families hold 56\% of market capitalization in the country according to Masulis, Pham, and Zein (2011).

\textsuperscript{3}Given the fact that a business group is a dominant ownership structure of the largest corporations in a country, this study revisits a question raised by many others: if the size distribution of firms in a country affects its economic efficiency.

\textsuperscript{4}e.g. Buera, Kaboski, and Shin (2011)
financial frictions. Specifically, the financial advantage of a pyramidal business group in the model is twofold. Not only does the subsidiary use its internal equity finance offered by the parent as leverage to raise external capital, but also the parent uses its equity shares of the subsidiary as leverage to raise external capital. Thus, it is the financial advantage of a business group that makes it possible for an entrepreneur to build up a business group as a competitive ownership structure in an equilibrium.

The model shows that business groups can have non-monotonic impact on resource allocation given the degree of financial frictions. In an economy with poor investor protection, the internal capital markets of business groups substitute for underdeveloped external capital markets and foster financially constrained but productive firms. A numerical example of the model shows that the rich become business-group entrepreneurs by hiring the poor but talented as business-group managers. It also shows that an economy with business groups accumulates larger capital stock than an economy without business groups because the rich save more in order to create business groups featuring internal capital markets. This implies that business groups can be efficient private institutions at the early stage of economic development where financial frictions are rampant.

In an economy with fine investor protection, however, the asymmetric financial frictions between business-group and stand-alone firms become a source of resource misallocation. The rich but unproductive choose to create business groups despite flotation costs because they can earn ex-ante positive profits by incorporating productive subsidiaries, while their ex-post profits can be negative because the probability of launching productive subsidiaries declines with the rising managerial compensation as investor protection improves. Moreover, business-group entrepreneurs use their financial advantage to consume more and save less by raising a larger amount of external capital without increasing net capital in production. Thus, the larger demand and the lower supply of capital push up the price of capital in an equilibrium and force stand-alone entrepreneurs, most of whom are financially constrained, to
raise less external capital, produce less, and consume less. The numerical example shows that the stand-alone entrepreneurs’ wealth drops significantly and that an economy dominated by business groups features decreasing capital stock and stagnating aggregate consumption as the fraction of diversion $\tau$ goes to zero.

An interesting lesson we can learn from the model is that the relative number of business-group firms does not decrease endogenously with the improvement of investor protection. This result is consistent with Masulis, Pham, and Zein (2011) who report an insignificant association between the prevalence of family business groups and the degree of investor protection. Given the observation that the direction of effects business groups have on an economy is reversed as investor protection improves, the unvarying number of business-group firms implies that mitigating capital market imperfections may barely reduce factor misallocation or even worsen it without due consideration of pyramidal business groups, which are prevalent in many economies including developed countries.

Although I simplify the problem of business groups by focusing on the financial advantage of their internal capital markets, there is a larger pool of questions about business groups that should be examined such as monopoly, political economy, risk sharing, or intangible assets of business groups. For example, Khanna and Yafeh (2007) review the issues of business groups and conclude that their origins and effects are largely unknown. Note that the objective of this paper is to narrow down the problem and understand a trait of business groups, their internal capital markets, in a general equilibrium framework.

In the literature, the pyramidal ownership structure of a business group has been understood with two different viewpoints. First, a traditional view is that it is an expropriation device. The main argument of this view is that the pyramidal ownership structure creates discrepancy between ownership and control. Although the controlling shareholder of a business group, typically a family, owns a small portion of shares of business-group affiliates, its pyramidal scheme allows the family to take control over the business group and to
earn the private benefits of control at the expense of other shareholders. This separation of ownership from control can generate agency problems, resource misallocation, and economic entrenchment. See Morck, Wolfenzon, and Yeung (2005) for the review of this perspective.

Second, more recent studies examine pyramidal business groups as start-up breeders. They focus on the role of business groups that offer internal finance to start-up firms and help them grow larger by supplementing the inefficiency of external capital markets. Almeida and Wolfenzon (2006b) offer a theory of business groups based on the financial advantage of pyramidal business groups. In their model, the controlling shareholder of a parent firm uses the firm’s retained earnings to launch a subsidiary firm that provides cash flow to the controlling shareholder. Despite the discrepancy between ownership and control, business groups can be economically beneficial because subsidiary firms would be dismissed without the help of internal capital markets due to setup costs that cannot be raised from external capital markets given financial frictions. Bena and Ortíz-Molina (2013) use data from 38 European countries and show that business groups do play a significant role in creating new firms.

These two perspectives on pyramidal business groups are not mutually exclusive. They are rather opposite sides of the same coin, in that the first can cause the second. The opportunity to earn additional cash flow from a subsidiary firm is an incentive for the controlling shareholder of a parent firm, which offers internal finance and helps to launch its subsidiary firm.

A natural question arises. Between these two viewpoints, which aspect of business groups is dominant? Simply put, are business groups good or bad for an economy? In spite of its relevance, the answer has been unclear. This is because most researchers have focused on the internal efficiency of an individual business group. Few researchers have developed models of business groups in a general equilibrium framework.

Among them, Almeida and Wolfenzon (2006a) show that the financial advantage of busi-
ness groups can cause asymmetric financial frictions between business-group firms and stand-alone firms, which results in factor misallocation in an equilibrium. Despite its novel insight, their model is stylized so that it is hard to capture dynamic aspects of an economy allowing for forward-looking behaviors of individuals such as savings or self-financing. This can be a problem if we want to examine the economic impact of the asymmetric financial frictions because the wealth distribution of an economy is endogenously determined by the agents’ dynamic optimization, which might undo factor misallocation stemming from financial frictions (e.g. Moll (2014)).

Ševčík (2015) examines the economic impact of business groups using a heterogeneous agent model with financial frictions, in which the wealth distribution of an economy is endogenously determined. He studies to what extent internal capital markets of business groups can alleviate financial frictions and concludes that aggregate output in Canada would be reduced by 3% if its business groups were shut down. Business groups in his model, however, are partnerships rather than pyramids. This can be a problem if we want to examine the economic repercussions of pyramidal business groups that feature the separation of ownership from control. Specifically, in his model the degree of financial frictions captured by the ratio of capital to wealth is a given constant identical to all firms, while in my model the ratio is endogenously determined and business-group entrepreneurs leverage their wealth into control over capital worth vastly more through a pyramidal ownership structure.

In order to deal with these limitations, I introduce the following feature in my model. First, each individual chooses his or her consumption, savings, and occupation every period. Thus, the joint distribution of individuals’ wealth and occupation is endogenously determined. Second, an individual who chooses to be an entrepreneur also chooses his or her firms’ ownership structure. I connect corporate capital structures with corporate ownership structures given capital market imperfections. A pyramidal business group is introduced as a private means of an entrepreneur alleviating financial frictions. Thus, asymmetric financial
frictions among firms arise from the endogenous choice of firms’ ownership structure.

The rest of this paper proceeds as follows. In Section 2, I introduce an individual’s problem of occupational choices given the heterogeneity in managerial talent and wealth throughout the population. Every period, each individual chooses his or her occupation among a worker, a stand-alone entrepreneur, a business-group entrepreneur, and a manager who can be hired by a business-group entrepreneur.

In Section 3, financial frictions and three types of capital markets such as external debt, external equity, and internal equity markets are specified. These three types of capital markets are used to build up three types of firms: a private company, a publicly held corporation, and a pyramidal business group. This variety of firms’ ownership structures captures private institutions stemming from agents’ endogenous reactions against capital market imperfections, which generates asymmetric financial frictions among firms in the model.

In Section 4, a stationary equilibrium is defined by introducing a matching rule between a business-group entrepreneur and a manager. In Section 5, I remark on the model. The costs and benefits of pyramidal business groups are discussed. In Section 6, a numerical example of the model is constructed and the results of the model are presented. In Section 7, a firm-level dataset of South Korea is examined to check the model. Lastly in Section 8, I discuss the limitation of the model and propose future research.

1.2 A Heterogeneous Agent Model with Occupational Choices

1.2.1 Economic Environment

An economy consists of infinitely lived individuals. Every period, each individual is endowed with an indivisible labor force and characterized by his or her own managerial talent $z$ that
changes over periods following a Markov chain. Let’s assume that an individual consumes out of his or her own wealth $a$ such that $c \in [0, a]$ and that a utility function $u(c)$ satisfies standard conditions such that $u'(c) > 0$, $u''(c) < 0$, and $\lim_{c \to 0} u'(c) = \infty$.

Given $(z, a)$, an individual chooses his or her next period occupation $o(z, a)$ among a worker ($W$), a stand-alone entrepreneur ($SA$), or a business-group entrepreneur ($BG$). At the beginning of the next period, a worker sells his or her indivisible labor force and earns wage $w$, and an entrepreneur runs a firm and earns from the firm’s stochastic cash flow $\pi$.

An entrepreneur raises her firm’s capital $k$ given $(z, a)$. At the beginning of the next period, the entrepreneur observes a shock to the managerial talent $z'$ and hires labor $\ell$ given $k$. Then, the firm produces cash flow $\pi$ that is defined as the optimized gross output net of labor costs $w\ell$ and capital depreciation $\delta'k$ such that

$$\pi(z', \delta'|z, k) = \max_\ell z'^\alpha \ell^\theta - w\ell + (1 - \delta')k, \quad \alpha, \theta > 0, \quad \alpha + \theta < 1$$

where $\alpha + \theta < 1$ is a span of control shaping the production function into decreasing returns to scale. Suppose that the capital depreciation rate $\delta' \in (0, 1)$ is a random variable independent of $z'$.

A stand-alone entrepreneur can run either a private company or a publicly held corporation. A private company is a firm fully owned by its stand-alone entrepreneur, which raises capital from external debt markets. A publicly held corporation can be incorporated by its stand-alone entrepreneur who pays flotation costs $k^F$. It can raise capital from external equity markets as well as external debt markets.

A business group is defined as a collection of two corporations: a parent that offers

---

5An exogenous process of managerial talents can be understood as a parsimonious way of capturing the impact of financial frictions on factor allocations by abstracting away from the endogenous nature of managerial talents. In Section 6, I will specify a state space and a transition probability of managerial talent $z$.

6We can think of this timing structure, raising $k$ given $z$ and then producing cash flow $\pi$ after observing $z'$, as an entrepreneur’s investment decision taking risks.
internal equity finance and a subsidiary that receives internal equity finance. An individual of \((z_1, a_1)^7\), who chooses to be a business-group entrepreneur \(o(z_1, a_1) = BG\), runs the parent with \(z_1\) and hires a manager of \((z_2, a_2)\) who runs the subsidiary with \(z_2\). The business-group entrepreneur can choose \(z_2\), while \(a_2\) is randomly drawn with probability \(P^{BG}(z_2, a_2)\). The business-group entrepreneur earns from both firms’ cash flow at the beginning of the next period.

An individual of \((z, a)\), who chooses to be a worker or a stand-alone entrepreneur \(o(z, a) \in \{W, SA\}\), can be matched to a business-group entrepreneur with probability \(P^{M}(z, a)\). If the matching is realized, the individual becomes a manager and earns managerial compensation \(w^{M}(z, a)\) at the beginning of the next period. Note that the managerial compensation \(w^{M}\) is a function of \((z, a)\) that are pinned down when the matching is realized, even though the subsidiary firm’s production will be realized with \(z’\) next period.

Figure 1.1 summarizes the timing of an individual’s problem within a period. Given \((z, a)\), firstly an individual chooses his or her occupation, secondly the matching between business-group entrepreneurs and the others are realized, and lastly output is produced with realized shocks to managerial talents \(z’\) at the beginning of the next period.

---

7I use \((z_1, a_1)\) instead of \((z, a)\) because \((z_1, a_1)\) is convenient for comparing a parent’s managerial talent \(z_1\) indexed by 1 to a subsidiary’s managerial talent \(z_2\) indexed by 2.
1.2.2 An Individual’s Problem

Every period, each individual solves the following problem given his or her managerial talent $z$ and wealth $a$ such that

$$V(z,a) = \max_{o \in \{W,SA,BG\}} \left\{ V^W(z,a), V^{SA}(z,a), V^{BG}(z,a) \right\}$$

(1.2)

given $\{r, w, w^M(z,a), P^M(z,a), P^{BG}(z_2,a_2)\}$, which respectively stand for the rate of return on capital, wage for a worker, managerial compensation, the probability of being matched with a business-group entrepreneur, and the probability of being matched with a manager featuring $(z_2,a_2)$.

$V^W(z,a)$ is the value if an individual chooses to be a worker such that

$$V^W(z,a) = \left(1 - P^M(z,a)\right) \cdot V_0^W(z,a) + P^M(z,a) \cdot \max \left\{ V_0^W(z,a), V^M(z,a) \right\},$$

$$V_0^W(z,a) = \max_{s \in [0,a]} u(a - s) + \beta E_{z'} [V(z', w + (1 + r)s) | z]$$

(1.3)

where $s$ is the risk-free asset matured in the next-period with interest rate $r$.

$V^M(z,a)$ is the value if an individual becomes a manager given $w^M(z,a)$ such that

$$V^M(z,a) = \max_{s \in [0,a]} u(a - s) + \beta E_{z'} \left[ V\left(z', w^M(z,a) + (1 + r)s \right) | z \right].$$

(1.4)

Note that both the next-period wealth for a worker $w + (1 + r)s$, and that for a manager, $w^M(z,a) + (1 + r)s$, are realized without uncertainty.

$V^{SA}(z,a)$ is the value if an individual chooses to be a stand-alone entrepreneur who runs a private company or a publicly held corporation such that

$$V^{SA}(z,a) = \left(1 - P^M(z,a)\right) \cdot V_0^{SA}(z,a) + P^M(z,a) \cdot \max \left\{ V_0^{SA}(z,a), V^M(z,a) \right\},$$

$$V_0^{SA}(z,a) = \max_{k} u\left(a - k^C\right) + \beta E_{z',\delta^C} \left[ V\left(z', a' \right) | z, k \left(k^C, k^D, k^E\right) \right]$$

(1.5)
where the firm’s capital in production \( k \) is a function of private finance \( k^C \), external debt finance \( k^D \), and external equity finance \( k^E \). The entrepreneur’s next-period wealth \( a' \) is a function of shocks to managerial talent \( z' \) and capital depreciation rate \( \delta' \) given \( \{k^C, k^D, k^E\} \).

Lastly, \( V^{BG}(z_1, a_1) \) is the value if an individual of \((z_1, a_1)\) chooses to be a business-group entrepreneur who controls a business group consisting of two corporations, a parent with \((z_1, k_1)\) and a subsidiary with \((z_2, k_2)\). The business-group entrepreneur determines both firms’ capital \( k_1 \) and \( k_2 \) by choosing \( \{k^C_i, k^D_i, k^E_i\}_{i \in \{1, 2\}} \) given \( \{z_2, w^M(z_2, a_2)\} \). \( k^C_1 \) is the private finance that the business-group entrepreneur offers to the parent, and \( k^C_2 \) is the internal equity finance that the parent offers to the subsidiary. I will specify how the business-group entrepreneur optimizes \( k_1 \) and \( k_2 \) in the following section. For now, let’s focus on that the business-group entrepreneur chooses \( z_2 \), the optimal managerial talent for the subsidiary, given \( w^M(z_2, a_2) \) and \( P^{BG}(z_2, a_2) \) such that

\[
V^{BG}(z_1, a_1) = \max_{z_2} \left[ \left( 1 - \sum_{a_2} P^{BG}(z_2, a_2) \right) \cdot V^{SA}_0(z_1, a_1) \right.
\]

\[
+ \sum_{a_2} P^{BG}(z_2, a_2) \cdot \max \left\{ V^{SA}_0(z_1, a_1), V^{BG}(z_1, a_1 | z_2, a_2) \right\} \right]
\]

\[
V^{BG}_0(z_1, a_1 | z_2, a_2) = \max_{\{k^C_i, k^D_i, k^E_i\}_{i \in \{1, 2\}}} u \left( a_1 - k^C_1 \right) + \beta \mathbb{E}_{z'_1, z'_2, \delta'_1, \delta'_2} \left[ V \left( z'_1, a'_1 \right) | z_1, z_2, k_1, k_2 \right].
\]

The business-group entrepreneur’s next-period wealth \( a'_1 \) is a function of \((z'_1, \delta'_1, z'_2, \delta'_2)\) given the firms’ capital structure \( \{k^C_i, k^D_i, k^E_i\}_{i \in \{1, 2\}} \). Note that the probability of matching with a manager \( P^{BG}(z_2, a_2) \) is endogenously determined in an equilibrium and that its sum can be less than one such that \( \sum_{a_2} P^{BG}(z_2, a_2) \leq 1 \). If the demand of \( z_2 \) is higher than the supply of \( z_2 \), some business-group entrepreneurs would fail to be matched with their targeted managers featuring \( z_2 \).

Figure 1.2 is an expository diagram of an individual’s occupational choice given his or her managerial talent \( z \) and wealth \( a \).\(^8\) First, it shows that the poor and untalented are likely

\(^8\)Figure 1.2 is not the equilibrium output of the model. It is an example constructed for clarifying the idea of an individual’s occupational choice problem.
to become workers because they are not productive enough to run firms and because they
do not have enough wealth to hire managers. Secondly, it shows that the more talented,
the more likely to become entrepreneurs. A declining line separating SA from W captures
financial frictions with which would-be entrepreneurs could become workers if they have not
enough wealth. Lastly, it shows that the rich tend to become business-group entrepreneurs
because they can pay managerial compensation and hire talented individuals as business-
group managers running subsidiary firms.

![Figure 1.2: Occupational Choice Given Managerial Talent $z$ and Wealth $a$](image)

1.3 Financial Frictions and Three Types of Firms

Suppose that an entrepreneur who controls her firm can divert $\tau$ fraction of the firm’s
cash flow. The tunneling ratio $\tau$ captures the degree of financial frictions in an economy.
Accordingly, $(1-\tau)$ captures the degree of investor protection in an economy because $(1-\tau)$ is
the residual cash flow investors can enforce on a firm if the firm does not make reimbursement.

Given financial frictions, an entrepreneur can choose her firms’ ownership structure: a
private company, a publicly held corporation, or a pyramidal business group. Specifically, an
entrepreneur can run her private company that is only allowed to access external debt markets with the help of the entrepreneur’s wealth as collateral. I assume that the external debt finance is bounded above by the firm’s lowest cash flow in order to guarantee its repayment.

For raising more external finance, an entrepreneur can pay flotation costs and incorporate a publicly held corporation that can tap into external equity markets. I assume that an entrepreneur owns all shares of her firm at the onset of its incorporation, which can be sold to outside shareholders to raise external equity finance. The extent of external equity finance her firm can raise is assumed to be proportional to the firm’s expected cash flow and the fraction of shares sold to outside shareholders.

Lastly, an entrepreneur can hire a manager with managerial compensation and build up a business group that consists of two corporations, a parent run by the entrepreneur and a subsidiary run by the manager. The business-group entrepreneur uses a pyramidal ownership structure to control both firms and makes the parent offer internal equity finance to the subsidiary without financial frictions. Similar to stand-alone corporations, both the parent and the subsidiary can sell their shares to outside shareholders and raise external equity finance.

1.3.1 A Private Company

Given her managerial talent and wealth, \((z, a)\), an entrepreneur can run a private company that is a firm fully owned by her. Due to the lack of external equity finance, a private company relies on external debt finance. The firm’s capital in production \(k\) is determined as follows. First, the entrepreneur of a private company is obliged for the company’s liability so that her wealth net of consumption \(a - c\) becomes the firm’s collateral \(k^C\) such that

\[
k^C = a - c \geq 0.
\] (1.7)
Second, given the collateral $k^C$ and the opportunity of tunneling $\tau\pi$, the firm’s capital in production $k$ is bounded above as follows.\footnote{Unlike publicly held corporations or business groups, expropriation does not occur in a private company that are fully owned by its entrepreneur.}

\[
\frac{(1 + r)k}{Debt	ext{ Repayment}} \leq \underbrace{(1 + r)k^C + (1 - \tau)\inf_{z',\delta}(\pi(z', \delta'|z, k)}_{Collateral} + \inf_{z',\delta'}[\pi(z', \delta'|z, k)]. \tag{1.8}
\]

Lastly, the entrepreneur of a private company can choose $k$ and decide how much external debt finance will be raised. I assume that the firm, or the entrepreneur, can invest in a risk-free asset by taking $k < k^C$. Thus, the entrepreneur can earn the risk-free residual cash flow from the firm such that

\[
(1 + r)(k^C - k) + \inf_{z',\delta'}[\pi(z', \delta'|z, k)]. \tag{1.9}
\]

To summarize, a stand-alone entrepreneur running a private company solves

\[
V_0^{SA}(z, a) = \max_{k^C \in [0, a], k} u(a - k^C) + \beta E_{z',\delta'}[V(z', a')|z] \tag{1.10}
\]

subject to

\[
a' = \pi(z', \delta'|z, k) + (1 + r)(k^C - k)
\]

\[
k \leq k^C + \frac{1 - \tau}{1 + r} \inf_{z',\delta'}[\pi(z', \delta'|z, k)]. \tag{1.11}
\]

### 1.3.2 A Publicly Held Corporation

An entrepreneur of $(z, a)$ can choose to incorporate her firm into a publicly held corporation with flotation costs $k^F > 0$. After its incorporation, a publicly held corporation can tap into external equity markets. The corporation’s capital in production $k$ is determined by the sum of private finance $k^C$, external debt finance $k^D$, and external equity finance $k^E$ net of
flotation costs $k^F$ such that

$$k = k^C + k^D + k^E - k^F. \quad (1.12)$$

Each type of capital is determined as follows. First, the entrepreneur can transfer a fraction of her wealth $k^C$ to her corporation. $k^C$ is determined by the entrepreneur’s wealth $a$ net of her consumption $c$ and private risk-free asset $s$. I assume that the flotation costs $k^F$ should be paid by the entrepreneur with $k^C$ before the firm’s incorporation such that\(^{10}\)

$$k^C = a - c - s \geq k^F. \quad (1.13)$$

In contrast to a private company, the entrepreneur’s wealth cannot be used as collateral for her corporation because a publicly held corporation is a legal entity that is separated from its entrepreneur. By construction, however, the wealth transfer from its entrepreneur to the publicly held corporation works as collateral, and this is why I abuse the notation of $k^C$.

Second, a publicly held corporation can use external debt finance $k^D$. Given the assumption that an entrepreneur controlling her firm can divert $\tau$ fraction of the firm’s cash flow $\pi$, the external debt finance $k^D$ is constrained in order to guarantee its repayment as follows. Note that a publicly held corporation can make an investment in a risk-free asset by taking $k^D < 0$.

$$(1 + r)k^D \leq (1 - \tau) \inf_{z', \delta'} [\pi (z', \delta' | z, k)] \quad (1.14)$$

Third, a publicly held corporation can tap into external equity markets. The corporation can raise external equity $k^E = k^E(\sigma)$ by selling its $\sigma \in [0, \bar{\sigma}_{SA}]$ fraction of shares. Suppose

\(^{10}k^F\) captures expenses such as underwriting fees, legal fees, or registration fees of issuing shares. Although in the real world flotation costs consist of fixed costs as well as costs proportional to the extent of shares issued, only the fixed costs are employed in the model with $k^F$. I exclude the proportional costs that can be paid with external financing after issuing shares because the efficiency of these back loaded costs is hardly distinguished from the degree of financial frictions $\tau$. Moreover, in the model $k^F$ is paid every periods if an entrepreneur runs a publicly held corporation successively.
that \((1 - \bar{\sigma}_{SA}) > 0\) fraction of the firm’s shares is required for an entrepreneur to take control of his or her stand-alone corporation. I assume that external capital markets are competitive and well diversified so that the publicly held corporation can raise external equity with the risk-free interest rate \(r\).

\[
\left(1 + r\right)k^E = \sigma \cdot \mathbb{E}_{z',\delta'} \left[ (1 - \tau)\pi(z', \delta'|z, k) - (1 + r)k^D \right], \quad \sigma \in [0, \bar{\sigma}_{SA}], \quad \bar{\sigma}_{SA} < 1
\]

(1.15)

As can be seen in the above equation, the firm’s cash flow \(\pi\) is sequentially distributed to the entrepreneur with tunneling \(\tau\pi\), to creditors with debt reimbursement \((1 + r)k^D\), and to shareholders with residual claims.

To summarize, a stand-alone entrepreneur running a publicly held corporation solves

\[
V_{0}^{SA}(z, a) = \max_{s \geq 0, k^C, k^D, \sigma \in [0, \bar{\sigma}_{SA}]} u\left(a - s - k^C\right) + \beta \mathbb{E}_{z',\delta'} [V(z', a')|z]
\]

subject to

\[
a' = (1 + r)s + \tau\pi(z', \delta'|z, k) + (1 - \sigma)\left\{ (1 - \tau)\pi(z', \delta'|z, k) - (1 + r)k^D \right\}
\]

\[
k = k^C + k^D + k^E - k^F
\]

\[
k^C \in \left[ k^F, a - s \right]
\]

\[
k^D \leq \frac{1 - \tau}{1 + r} \inf_{z', \delta'} \pi(z', \delta'|z, k)
\]

\[
k^E = \frac{\sigma}{1 + r} \mathbb{E}_{z',\delta'} \left[ (1 - \tau)\pi(z', \delta'|z, k) - (1 + r)k^D \right].
\]

(1.17)

**Condition 1.** The value function \(V(z, a)\) satisfies the following condition.

\[
\mathbb{E}_{z',\delta'} [V_a(z', a') \cdot \left\{ \mathbb{E}_{z',\delta'} \pi(z', \delta'|z, k) - \pi(z', \delta'|z, k) \right\} | z, k] > 0
\]

(1.18)
Condition 1 describes that the entrepreneur running a firm is risk-averse. Although the utility function of an individual is concave by construction, Condition 1 is not guaranteed in general because of the non-convexity of the individual’s choice set. The individual’s value function $V(z, a)$ might be locally convex. We need an additional structure to hold Condition 1. From now on, let’s assume that for all $(z, k)$, a minimum cash flow $\inf_{z', \delta'} \pi(z', \delta'|z, k)$ is low enough to satisfy Condition 1. Note that the marginal utility of consumption goes to infinity as consumption goes to zero by construction. Thus, a low enough minimum cash flow can make the marginal value of wealth $V_a(z', a')$ large enough to hold Condition 1.

**Proposition 1.** Given the risk-free investment opportunity for a corporation, $k^D < 0$, a stand-alone entrepreneur weakly prefers not to hold private asset such that

$$s = 0.$$ 

Given Condition 1 and the risk-free investment opportunity, a stand-alone entrepreneur of a publicly held corporation strictly prefers a full external equity finance such that

$$\sigma = \bar{\sigma}_{SA}.$$ 

**Proof.** See Appendix A.5. 

**Corollary 1.** From Proposition 1, the stand-alone entrepreneur’s choice variables degenerate into $\{k^C, k^D, \sigma\}$. Thus, we can simplify the problem of a private company and that of a publicly held corporation into the common problem of a stand-alone entrepreneur such that

$$V_0^{SA}(z, a) = \max_{k^C, k^D, \sigma \in \{0, \bar{\sigma}_{SA}\}} u\left(a - k^C\right) + \beta \mathbb{E}_{z', \delta'} [V(z', a')|z]$$ (1.19)
subject to

\[ a' = \tau \pi (z', \delta'|z, k) + (1 - \sigma) \left\{ (1 - \tau) \pi (z', \delta'|z, k) - (1 + r) k^D \right\} \]

\[ k = k^C + k^D + k^E - k^F \cdot 1_{\sigma = \bar{\sigma}_{SA}} \]

\[ k^C \in \left[ k^F \cdot 1_{\sigma = \bar{\sigma}_{SA}}, a \right] \]

\[ k^D \leq \frac{1 - \tau}{1 + r} \inf_{z', \delta'} \pi (z', \delta'|z, k) \]

\[ k^E = \frac{\bar{\sigma}_{SA} \cdot 1_{\sigma = \bar{\sigma}_{SA}}}{1 + r} \mathbb{E}_{z', \delta'} \left[ (1 - \tau) \pi (z', \delta'|z, k) - (1 + r) k^D \right] . \]

1.3.3 A Business Group

A business group is defined as a collection of two publicly held corporations, Firm 1 and Firm 2, which are controlled by a business-group entrepreneur. Let \( z_1 \) be the productivity of Firm 1 that inherits from the business-group entrepreneur and let \( z_2 \) be the productivity of Firm 2 that inherits from the manager.

Assume that a business group is connected through a pyramidal ownership structure such that Firm 2 is owned and controlled by Firm 1 that is owned and controlled by a business-group entrepreneur. More specifically, the business-group entrepreneur incorporates Firm 1 with private finance \( k^C \), keeps at least \( (1 - \bar{\sigma}_{BG}) \) shares of Firm 1, and controls Firm 1. Similarly, Firm 1 incorporates Firm 2 with internal equity finance \( k^C \), keeps at least \( (1 - \bar{\sigma}_{BG}) \) shares of Firm 2, and controls Firm 2. I assume that the manager of Firm 2 takes managerial compensation \( w^M(z_2, a_2) \), relinquishes her control rights and cash flow rights over Firm 2, and hands them over to Firm 1. As a result, the entrepreneur of a business group can control both firms and divert cash flow from both firms.

Two things are worth noting. First, the pair of managerial talent \( z_2 \) and its corresponding managerial compensation \( w^M(z_2, a_2) \) can be understood as a contract between an entrepreneur who buys \( z_2 \) and a manager who sells \( z_2 \) with the price of \( w^M(z_2, a_2) \). Thus, how to pin down \( w^M(z_2, a_2) \) can be critical in the model. Given the lack of managerial talent markets, I assume that \( w^M(z_2, a_2) \) is a certainty equivalent for an individual, who can run a
stand-alone firm or become a worker as outside options. It will be formally specified in the following section.

Second, I assume that \((1 - \bar{\sigma}_{BG})\) fraction of shares is required to acquire control rights over a business group. \(\bar{\sigma}_{BG}\) can be different from that of a stand-alone firm, \(\bar{\sigma}_{SA}\), because \((1 - \bar{\sigma}_{BG})\) needs to capture large enough block shares in order to ensure exclusive control rights over business-group firms, while \((1 - \bar{\sigma}_{SA})\) only captures stand-alone entrepreneur’s payoff structure proportional to the firm’s cash flow. Thus, I assume that \(\bar{\sigma}_{BG} \leq \bar{\sigma}_{SA}\) although the model lacks the micro foundation about how to pin down \(\bar{\sigma}_{SA}\) and \(\bar{\sigma}_{BG}\).

**Capital Structure of Firm 2**

For now, suppose that Firm 2 is run by a manager who has \(z_2\) and \(a_2\). I assume that the flotation costs \(k^F\) and the managerial compensation \(w^M = w^M(z_2, a_2)\) should be paid by Firm 1 through internal equity finance \(k^C_2\) such that

\[
k^C_2 \geq k^F + w^M.
\]

This implies that Firm 2 should be incorporated before tapping into external capital markets. Firm 2 raises external debt finance \(k^D_2\) under the following constraint given the assumption that the business-group entrepreneur, who controls Firm 1 that controls Firm 2, can expropriate cash flow from Firm 2.

\[
k^D_2 \leq \frac{1 - \tau}{1 + r} \inf_{z'_2, \delta'_2} \pi(z'_2, \delta'_2 | z_2, k_2)
\]

(1.22)
Firm 2 raises external equity finance $k_2^E$ by selling its $\sigma_2$ fraction of shares.

\[
\begin{align*}
\frac{(1 + r)k_2^E}{\text{Expected Payoff to Outside Shareholders}} &= \sigma_2 \cdot \mathbb{E}_{z_2', \delta_2'} \left[ (1 - \tau)\pi(z_2', \delta_2'|z_2, k_2) - (1 + r)k_2^D \right], \quad \sigma_2 \leq \bar{\sigma}_{BG} \\
\end{align*}
\]

From the above equations, the capital in production of Firm 2, $k_2$, is determined by the sum of internal equity finance $k_2^C$, external debt finance $k_2^D$, and external equity finance $k_2^E$ net of flotation costs $k^F$ and managerial compensation $w^M$ such that

\[
k_2 = k_2^C + k_2^D + k_2^E - k^F - w^M. \tag{1.24}
\]

**Capital Structure of Firm 1**

A business-group entrepreneur of $(z_1, a_1)$ can transfer her wealth $k_1^C$ to Firm 1. I assume that both firms’ flotation costs and Firm 2’s managerial compensation should be paid by the entrepreneur with $k_1^C$ such that

\[
k_1^C = a_1 - c - s \geq \frac{k_2^E}{\text{Flotation Costs of Firm 1}} + \frac{k^F + w^M}{\text{Gross Flotation Costs of Firm 2}}. \tag{1.25}
\]

This is not only because the timing of incorporating both Firm 1 and Firm 2 is simultaneous in the model but also because the contract between the entrepreneur and the manager should be set up before incorporating Firm 2.

Given the capital structure of Firm 2, $\{k_2^C, k_2^D, k_2^E\}$, and its cash flow, $\pi(z_2', \delta_2'|z_2, k_2)$, Firm 1 raises external debt finance $k_1^D$ under the following constraint.

\[
(1 + r)k_1^D \leq (1 - \tau)\pi_1 \quad \forall (z_1', z_2', \delta_1', \delta_2')
\]
where \( \pi_1 \) is the gross cash flow from Firm 1 defined by

\[
\begin{align*}
\pi_1 &= \pi \left( z'_1, \delta'_1 | z_1, k^*_1 = k_1 - k^C_2 \right) + (1 - \sigma_2) \left\{ (1 - \tau) \pi_2 - (1 + r) k^D_2 \right\}, \\
\pi_2 &= \pi \left( z'_2, \delta'_2 | z_2, k_2 \right).
\end{align*}
\]

\( \pi \) is Gross Output Net of Labor Costs and Capital Depreciation from Firm 1, \( \pi_2 \) is Gross Output Net of Labor Costs and Capital Depreciation from Firm 2, \( k^* = k_1 - k^C_2 \), and \( k^D = k^D_1 \).

We can rewrite the above inequality such that

\[
k^D_1 \leq \frac{1 - \tau}{1 + r} \left[ \inf_{z'_1, \delta'_1} \left\{ \pi \left( z'_1, \delta'_1 | z_1, k^*_1 \right) \right\} + (1 - \sigma_2) \left\{ (1 - \tau) \inf_{z'_2, \delta'_2} \left\{ \pi \left( z'_2, \delta'_2 | z_2, k_2 \right) \right\} - (1 + r) k^D_2 \right\} \right].
\]

Conceptually, the internal equity finance \( k^C_2 \) used by Firm 2 should be raised from Firm 1’s retained earnings (e.g. Almeida and Wolfenzon (2006b)). Given the limitation that firms are created and liquidated every period, however, I use Firm 1’s capital \( k_1 \) as the proxy for the Firm 1’s retained earnings. Thus, the internal equity finance \( k^C_2 \) is raised out of \( k_1 \), and Firm 1’s capital in production becomes \( k^*_1 = k_1 - k^C_2 > 0 \).

Lastly, Firm 1 raises external equity finance \( k^E_1 \) by selling its \( \sigma_1 \) fraction of shares to outside shareholders such that

\[
(1 + r)k^E_1 = \sigma_1 \cdot \mathbb{E}_{z'_1, \delta'_1, \delta'_2} \left[ (1 - \tau) \pi_1 - (1 + r) k^D_1 \right], \quad \sigma_1 \leq \sigma_{BG}.
\]

It can be rewritten as follows.

\[
k^E_1 = \frac{\sigma_1}{1 + r} \left[ (1 - \tau) \mathbb{E}_{z'_1, \delta'_1} \left[ \pi \left( z'_1, \delta'_1 | z'_1, k^*_1 \right) \right] + \left( 1 - \tau \right) \left( 1 - \sigma_2 \right) \left\{ (1 - \tau) \mathbb{E}_{z'_2, \delta'_2} \left[ \pi \left( z'_2, \delta'_2 | z_2, k_2 \right) \right] - (1 + r) k^D_2 \right\} - (1 + r) k^D_1 \right] \]

From the above equations, the capital in production of Firm 1, \( k^*_1 \), is determined by the sum of private finance \( k^C_1 \), external debt finance \( k^D_1 \), and external equity finance \( k^E_1 \) net of...
flotation costs $k^F$ and internal equity finance $k^C_2$ such that

$$k^*_1 = k_1 - k^C_2$$

$$= k_1^C + k_1^D + k_1^E - k^F - k_2^C.$$  

(1.28)

A Business-Group Entrepreneur’s Problem

Given $(z_2, a_2)$ and $w^M = w^M(z_2, a_2)$, a business-group entrepreneur of $(z_1, a_1)$ solves

$$V_{BG}^0(z_1, a_1 \mid z_2, w^M) = \max_{s \geq 0} \left\{ k^C_i, k^D_i, k^E_i \right\}_{i \in \{1, 2\}} u \left( a_1 - s - k^C_1 \right) + \beta E_{z_1', z_2', \delta_1', \delta_2'} [V(z_1', a_1') \mid z_1, z_2] \quad (1.29)$$

subject to

$$a_1' = (1 + r)s + \tau \pi(z_1', \delta_1' \mid z_1, k^*_1) + \tau \pi(z_2', \delta_2' \mid z_2, k_2) + (1 - \sigma_1) \left\{ (1 - \tau) \pi(z_1', \delta_1' \mid z_1, k^*_1) - (1 + r)k^D_1 \right\} + (1 - \sigma_1 + \sigma_1 \tau)(1 - \sigma_2) \left\{ (1 - \tau) \pi(z_2', \delta_2' \mid z_2, k_2) - (1 + r)k^D_2 \right\}$$

Equation (21) - (28)

Condition 2. The value function $V(z_1, a_1)$ satisfies the following conditions:

$$E_{(z_1', a_1')} \left[ V_0(z_1', a_1') \cdot \left\{ E_{z_1', \delta_1'} \pi(z_1', \delta_1' \mid z_1, k^*_1) - \pi(z_1', \delta_1' \mid z_1, k^*_1) \right\} \right]_{z_1, z_2, k^*_1, k_2} > 0,$$

$$E_{(z_1', a_1')} \left[ V_0(z_1', a_1') \cdot \left\{ E_{z_2', \delta_2'} \pi(z_2', \delta_2' \mid z_2, k_2) - \pi(z_2', \delta_2' \mid z_2, k_2) \right\} \right]_{z_1, z_2, k^*_1, k_2} > 0.$$  

(1.31)

Proposition 2. Given the non-negative financial frictions, $\tau > 0$, and the risk-free investment opportunity of Firm 2 such that $k^D_2 < 0$, a business-group entrepreneur weakly prefers
no private risk-free asset and a full external debt finance of Firm 1 such that

\[ s = 0, \]
\[ k_1^{D} = \frac{1 - \tau}{1 + r} \left\{ \inf_{z_1', \delta_1'} \left[ \pi(z_1', \delta_1'|z_1, k_1^*) \right] + (1 - \sigma_2) \left\{ \left( 1 - \tau \right) \inf_{z_2', \delta_2'} \left[ \pi(z_2', \delta_2'|z_2, k_2) \right] - (1 + r)k_2^D \right\} \right\}. \]

Given Condition 2 and the risk-free investment opportunity of Firm 2, a business-group entrepreneur strictly prefers a full external equity finance of both firms such that

\[ \sigma_1 = \sigma_2 = \bar{\sigma}_{BG}. \]

**Proof.** See Appendix A.4. \( \square \)

**Corollary 2.** From Proposition 2, the business-group entrepreneur’s choice variables degenerate into \( \{k_1^C, k_2^C, k_2^D\} \). Thus, we can rewrite the business-group entrepreneur’s problem as follows.

\[
V_{BG}^0(z_1, a_1|z_2, w^M) = \max_{k_1^C, k_2^C, k_2^D} u\left(a_1 - k_1^C\right) + \beta \mathbb{E}_{z_1', z_2', \delta_1', \delta_2'} \left[ V(z_1', a'_1|z_1, z_2) \right]
\]

subject to

\[
k_1^C \in \left[ 2k_F + w^M, a \right], \quad k_2^C \in \left[ k_F + w^M, k_1 \right], \quad k_2^D \leq \frac{1 - \tau}{1 + r} \inf_{z_2', \delta_2'} \left[ \pi_2(z_2', \delta_2'|z_2, k_2) \right]
\]
\[
k_1^* = k_1^C - k_F - k_2^C + \frac{1 - \tau}{1 + r} \left\{ \bar{\sigma}_{BG} \mathbb{E}_{z_1', \delta_1'} \left[ \pi(z_1', \delta_1'|z_1, k_1^*) \right] + (1 - \bar{\sigma}_{BG}) \inf_{z_2', \delta_2'} \left[ \pi(z_2', \delta_2'|z_2, k_2) \right] - (1 - \bar{\sigma}_{BG})(1 + r)k_2^D \right\}
\]
\[
+ \frac{(1 - \tau)^2(1 - \bar{\sigma}_{BG})}{1 + r} \left\{ \bar{\sigma}_{BG} \mathbb{E}_{z_2', \delta_2'} \left[ \pi(z_2', \delta_2'|z_2, k_2) \right] + (1 - \bar{\sigma}_{BG}) \inf_{z_2', \delta_2'} \left[ \pi(z_2', \delta_2'|z_2, k_2) \right] \right\}
\]
\[
k_2 = k_2^C - k_F - w^M + (1 - \bar{\sigma}_{BG})k_2^D + \frac{1 - \tau}{1 + r} \bar{\sigma}_{BG} \mathbb{E}_{z_2', \delta_2'} \left[ \pi(z_2', \delta_2'|z_2, k_2) \right]
\]

(1.32)
\[ a'_1 = \tau \pi(z'_1, \delta'_1|z_1, k^*_1) + \tau \pi(z'_2, \delta'_2|z_2, k_2) \]

\[ + (1 - \bar{\sigma}_{BG})(1 - \tau) \left\{ \pi(z'_1, \delta'_1|z_1, k^*_1) - \inf_{z'_1, \delta'_1} \pi(z'_1, \delta'_1|z_1, k^*_1) \right\} \]

\[ - (1 - \bar{\sigma}_{BG})(1 - \tau)(1 - \bar{\sigma}_{BG}) \left\{ (1 - \tau) \inf_{z'_2, \delta'_2} \pi(z'_2, \delta'_2|z_2, k_2) - (1 + r)k^D_2 \right\} \]

\[ + (1 - \bar{\sigma}_{BG} + \bar{\sigma}_{BG}\tau)(1 - \bar{\sigma}_{BG}) \left\{ (1 - \tau)\pi(z'_2, \delta'_2|z_2, k_2) - (1 + r)k^D_2 \right\}. \]

Note that in Corollary 2, Firm 1’s capital in production \( k^*_1 \) decreases with \( k^C_2 \) but increases with the cash flow of Firm 2, \( \pi(z'_2, \delta'_2|z_2, k_2) \) in the right hand side of \( k^*_1 \). Given that \( \pi(z'_2, \delta'_2|z_2, k_2) \) increases with \( k_2 \) and that \( k_2 \) increases with \( k^C_2 \), we can see that the financial advantage of a business group derives not only from no limited commitment problem such that \( k^C_2 < k_1 \), but also from an increase in the cash flow from Firm 2 to Firm 1.

### 1.4 A Matching Rule and a Stationary Equilibrium

#### 1.4.1 A Matching Rule Between Business-Group Entrepreneurs and the Others

To complete the model, let’s consider an ad-hoc matching rule. It is designed for mitigating the gap between the model and the real world. Although the model assumes one-period matching between a business-group entrepreneur and a manager by construction, in the real world the matching between a business-group entrepreneur of \( (z, a) \) and a subsidiary Firm 2 of \( z_2 \) is stable over time.

First, let’s assume that the managerial compensation \( w^M(z_2, a_2) \) is equal to the certainty equivalent for a manager who has outside options such that

\[ V^M(z_2, a_2|w^M(z_2, a_2)) = \max \left\{ V^W_0(z_2, a_2), V^SA_0(z_2, a_2) \right\}, \quad (1.34) \]
This assumption implies that a business-group entrepreneur acquires all of gains from building a business group and that the manager of Firm 2 will have less wealth in the next period than the expected wealth a stand-alone entrepreneur would have because of the risk-averse preference.

Second, suppose that the business-group entrepreneur can choose $z_2$ but cannot choose $a_2$. A business-group entrepreneur and its manager of Firm 2 who has $a_2$ are randomly matched given $z_2$. As a result, while an individual always accepts the offer of being a manager given the managerial compensation as a certainty equivalent, a business-group entrepreneur of $(z, a)$ can turn down the opportunity of launching a subsidiary Firm 2 if the matched manager has too high $a_2$ that induces $w_M(z_2, a_2) > \bar{w}_M(z, a|z_2)$, where $\bar{w}_M(z, a|z_2)$ is the largest managerial compensation a business-group entrepreneur of $(z, a)$ can be better off such that

$$\bar{w}_M(z, a|z_2) = \sup \left\{ w_M > 0 : V_0^{BG}(z, a|z_2, w_M) \geq V_0^{SA}(z, a|\sigma \leq \bar{\sigma}_{BG}) \right\}. \quad (1.35)$$

Lastly, assume that a business-group entrepreneur, who screens out $w_M(z_2, a_2) > \bar{w}_M(z, a|z_2)$ and gives up the opportunity of launching a subsidiary Firm 2, should keep at least $(1 - \bar{\sigma}_{BG})$ shares of Firm 1. This assumption begets a business group without Firm 2, which sells only $\bar{\sigma}_{BG}$ fraction of shares, not $\bar{\sigma}_{SA}$. Although the capital structures of a business group without Firm 2 is ex-post suboptimal, it is ex-ante optimal for a business-group entrepreneur who wants to launch Firm 2 with the possibility of being matched with $w_M(z_2, a_2) \leq \bar{w}_M(z, a|z_2)$. The possibility of no subsidiary Firm 2 can be understood as an opportunity cost for a business-group entrepreneur. Given the limitation of the model defining a business group as a collection of two corporations, a business group without Firm 2 can be understood as a business group with less pyramidal layer.
A Stationary Equilibrium

Given the matching rule, a stationary equilibrium consists of a stationary joint distribution of managerial talent and wealth \( F(z,a) \); the probability of being hired as a manager \( P^M(z,a) \) and the probability of being matched with a manager \( P^{BG}(z_2,a_2) \); prices \( \{r,w,w^M(z_2,a_2)\} \); and individual policy functions such as (i) occupation \( o(z,a) \) for an individual, (ii) private risk-free asset \( s(z,a) \) for a worker or a manager, (iii) private finance \( k^C(z,a) \), external debt finance \( k^D(z,a) \), and external equity finance \( k^E(z,a) \) for a stand-alone entrepreneur, (iv) the optimal managerial talent for a subsidiary firm \( z_2(z,a) \), private finance \( k^C_1(z,a|z_2,a_2) \), internal equity finance \( k^C_2(z,a|z_2,a_2) \), and external debt finance \( k^D_2(z,a|z_2,a_2) \) for a business-group entrepreneur matched with \( w^M(z_2,a_2) \leq \bar{w}^M(z,a|z_2,a_2) \), and (v) private finance \( k^C(z,a) \) and external debt finance \( k^D(z,a) \) for a business-group entrepreneur matched with \( w^M(z_2,a_2) > \bar{w}^M(z,a|z_2,a_2) \) such that

1. Given the stationary joint distribution of managerial talent and wealth \( F(z,a) \), the probability of being hired as a manager \( P^M(z,a) \), the probability of being matched with a manager \( P^{BG}(z_2,a_2) \), and prices \( \{r,w,w^M(z_2,a_2)\} \), the individual policy functions solve the individual’s problem in Section 2.2;

2. The joint distribution of managerial talent and wealth \( F(z,a) \) is stationary such that

\[
F(z,a) = \int_{\{(\tilde{z},\tilde{a})|z' \leq \tilde{z},a' \leq \tilde{a}\}} dF(\tilde{z},\tilde{a});
\]

(1.36)

3. The probability of a worker or a stand-alone entrepreneur being hired as a manager, \( P^M(z_2,a_2) \), and the probability of a business-group entrepreneur being matched with a manager, \( P^{BG}(z_2,a_2) \), satisfy the following condition for all \( z_2 \)

\[
\int_{o(z_2,a_2) \in \{W,S,A\}} P^M(z_2,a_2) \cdot F(z_2,da_2) = \int_{o(z,a) = BG} \int_{z_2(z,a) = z_2} \int_{w^M(z_2,a_2) \leq \bar{w}^M(z,a|z_2)} P^{BG}(z_2,a_2) da_2 dF(z,a)
\]

(1.37)
4. Capital and labor markets clear such that

(capital market)

\[
\int \left\{ a - c(z, a) \right\} dF(z, a)
= \int_{o(z,a)=SA} \left\{ k(z, a) + 1_{\alpha(z,a)>0} \cdot k^F \right\} \cdot \left\{ 1 - P^M(z, a) \right\} dF(z, a)
+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \left\{ 2k^F + w^M(z_2, a_2) + k_1^*(z, a|z_2, a_2) + k_2(z, a|z_2, a_2) \right\}
\cdot P^{BG}(z_2(z, a), a_2) d\sigma dF(z, a)
+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \left\{ k^F + k_1(z, a|k_2^C = 0) \right\} \cdot \left\{ 1 - P^{BG}(z_2(z, a), a_2) \right\} d\sigma dF(z, a); (1.38)
\]

(labor market)

\[
\int_{o(z,a)=W} \left\{ 1 - P^M(z, a) \right\} dF(z, a)
= \int_{o(z,a)=SA} \int_{z'} \ell(z', k(z, a)) dG(z'|z) \cdot \left\{ 1 - P^M(z, a) \right\} dF(z, a)
+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \left\{ \int_{z'} \ell(z', k_1^*(z, a|z_2, a_2)) dG(z'|z) + \int_{z'_2} \ell(z'_2, k_2(z, a|z_2, a_2)) dG(z'_2|z_2) \right\}
\cdot P^{BG}(z_2(z, a), a_2) d\sigma dF(z, a)
+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \int_{z'} \ell(z', k_1(z, a|k_2^C = 0)) dG(z'|z) \cdot P^{BG}(z_2(z, a), a_2) d\sigma dF(z, a)
+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \int_{z'} \ell(z', k_1(z, a|k_2^C = 0)) dG(z'|z) \cdot \left\{ 1 - P^{BG}(z_2(z, a), a_2) \right\} d\sigma dF(z, a); (1.39)
\]

where \( G(z'|z) \) is a conditional cdf derived from the transition probability of managerial talents.
1.5 Remarks on the Model

1.5.1 Financial Advantage of Business Groups

In order to gauge how well internal capital markets can alleviate exogenous financial frictions in the model, let’s consider how much private wealth of an entrepreneur is required to raise a fixed amount of capital in production given the ownership structure of firms.

Suppose that a business group consists of two firms that replicate a stand-alone firm’s capital structure with identical managerial talents such that

\[ k = k_1^* = k_2, \quad z = z_1 = z_2, \quad \sigma = \bar{\sigma}_{SA} = \bar{\sigma}_{BG}. \]

Let’s compare the required level of private finance for a stand-alone firm \( k^C \) to that for a business group \( k_1^C \) in order to raise \( k = k_1^* = k_2 \). For a stand-alone firm, the feasible capital in production \( k \) is determined by the following equation.

\[
k = k^C - k^F + \frac{1 - \tau}{1 + r} \{ \bar{\sigma} E \pi(z', \delta'|z, k) + (1 - \bar{\sigma}) \inf \pi(z', \delta'|z, k) \} \quad (1.40)
\]

Similarly, the set of feasible capital in production for a business group, i.e. \( k_1^* \) for Firm 1 and \( k_2 \) for Firm 2, is determined by the following equations.

\[
k_1^* = k_1^C - k^F - k_2^C + \frac{1 - \tau}{1 + r} \{ \bar{\sigma} E \pi(z'_1, \delta'_1|z, k_1^*) + (1 - \bar{\sigma}) \inf \pi(z'_1, \delta'_1|z, k_1^*) \}
\]

\[
+ \frac{(1 - \tau)^2(1 - \bar{\sigma}) \bar{\sigma}}{1 + r} \{ E \pi(z_2', \delta_2'|z, k_2) - \inf \pi(z_2', \delta_2'|z, k_2) \}, \quad (1.41)
\]

\[
k_2 = k_2^C - k^F - \omega^M + \frac{1 - \tau}{1 + r} \{ \bar{\sigma} E \pi(z'_2, \delta'_2|z, k_2) + (1 - \bar{\sigma}) \inf \pi(z'_2, \delta'_2|z, k_2) \}
\]
By solving for the above equations with $k_1^* = k_2 = k$,

$$\begin{align*}
k_1^C &= 2k^C + w^M - \frac{(1 - \tau)^2 (1 - \bar{\sigma})}{1 + r} \left\{ E \pi (z_2', \delta_2'|z, k) - \inf \pi (z_2', \delta_2'|z, k) \right\} \\
&= 2k^C + w^M - (1 - \tau)(1 - \bar{\sigma})k^E
\end{align*}$$

(1.42)

where $k^E$ is the feasible external equity finance that a stand-alone firm with managerial talent $z$ can raise given $k^C$.

Now, we can compare the effective degree of financial frictions between business-group firms and stand-alone firms. By fixing capital in production $k = k_1^* = k_2$, let’s observe the ratio of capital in production to private finance for a stand-alone entrepreneur (SA) and for a business-group entrepreneur (BG) such that

$$\lambda_{SA} = \frac{k}{k^C}, \quad \lambda_{BG} = \frac{k_1^* + k_2}{k^C}.$$

(1.43)

Then, the financial advantage of a business group can be measured by the following ratio.

$$\left. \frac{\lambda_{BG}}{\lambda_{SA}} \right|_{k=k_1^*=k_2} = \frac{1}{1 + \frac{1}{2} \left\{ \frac{w^M}{k^E} - (1 - \tau)(1 - \bar{\sigma}) \frac{k^E}{k^C} \right\}}$$

(1.44)

The ratio depends both on the cost of building up a subsidiary firm, $w^M$, and the efficiency of external capital markets, $(1 - \tau)(1 - \bar{\sigma})k^E$. If the latter outweighs the former, the ratio becomes greater than 1. This implies that a business group raises more external finance than a stand-alone firm does given the same amount of private finance. For instance, suppose that $\frac{w^M}{k^C} = 0.4$ and $\frac{k^E}{k^C} = 20$ given $\tau = 0.3$ and $\bar{\sigma} = 0.9$. Then, the ratio becomes 2 and it means that a business group raises twice larger capital than a stand-alone firm does given the same amount of private finance.

The asymmetric financial advantage of business groups can be lessened if business groups are subject to a lower fraction of equity shares sold to outside shareholders such that $\bar{\sigma}_{BG} < \bar{\sigma}$. 

With this conditions, the above ratio can be rewritten as follows.

\[
\frac{1}{1 + \frac{1}{2} \left[ \frac{w^M}{k^E} + \left\{ 2 \left( 1 - \frac{\bar{\sigma}_{BG}}{\bar{\sigma}_{SA}} \right) - (1 - \bar{\sigma})(1 - \bar{\sigma}_{BG}) \frac{\bar{\sigma}_{BG}}{\bar{\sigma}_{SA}} \right\} \frac{k^E}{k^E} \right]}
\]

(1.45)

Given the same specification with the above but \( \bar{\sigma}_{BG} = 0.87 \) and \( \bar{\sigma}_{SA} = 0.9 \), we can observe that the ratio becomes 1.01 and the asymmetric financial advantage of business groups is almost nullified. It teaches us that the minimum equity shares \((1 - \bar{\sigma}_{BG})\), which the controlling shareholder of a business group should keep to control over the business group, can have sizable effects on mitigating the asymmetric financial advantage of business groups. However, note that this example is made up for a stark comparison and business-group entrepreneurs can choose \( z_2 \) and optimize their external financing. Thus, we can guess that \( \bar{\sigma}_{BG} \) should be much lower in order to lessen the asymmetric financial advantage of business groups in an equilibrium.

### 1.5.2 Asymmetric Financial Frictions

Given the finite amount of capital stock in an economy, the asymmetric financial advantage of business groups is in other words the asymmetric financial frictions between business-group firms and stand-alone firms, which can result in factor misallocation in a general equilibrium. Note that managerial compensation \( w^M \) is a certainty equivalent proportional to the firm’s expected cash flow net of risk premium while external equity finance \( k^E \) is proportional to the firm’s expected cash flow. This implies that as \( \tau \) decreases, \( k^E \) can grow faster than \( w^M \) and that \((1 - \tau)(1 - \bar{\sigma})k^E \) can grow much faster than \( w^M \). Thus, improvement of investor protection captured by declining \( \tau \) can increase the gap of external finance raised by business-group firms and stand-alone firms.

The asymmetric financial frictions are of concern because they can be another source of factor misallocation. In an equilibrium, alleviated financial frictions for business groups can
increase the demand of external capital and push up the price of capital. For stand-alone firms, however, the higher price of capital $r$ acts like the higher degree of financial frictions $\tau$ in that financial constraints of external finance always come with $\frac{1}{1+\tau}$ as well as $(1 - \tau)$. Thus, given the lack of internal capital markets with the higher price of capital, stand-alone firms cannot raise as much capital as they could do in an economy without business groups. As a result, an economy with business groups can give rise to the higher price of capital and lower aggregate output due to factor misallocation. Moreover, since the asymmetric financial frictions between business-group firms and stand-alone firms are intensified as the degree of financial frictions are mitigated, we can guess that costs of business groups are more likely to dominate their benefits in an equilibrium as financial frictions decrease. Last but not the least, the financial advantage of business groups increasing with investor protection $(1 - \tau)$ implies that the prevalence of business groups needs not attenuate as investor protection improves.

1.5.3 External Finance Substituting for Private Finance

As the degree of financial frictions $\tau$ decreases, the model shows that both the volume of external equity finance $k^E$ and corporate savings, or corporate lending $-k^D$, can expand without increasing capital in production $k$. Suppose that firms are financially unconstrained and that the degree of financial frictions is lessened such that

$$d\tau < 0, \quad dk = dk_1^* = dk_2 = 0.$$  \hspace{1cm} (1.46)

From Corollary 2, we can see that a business-group entrepreneur can be better off by increasing consumption $dc > 0$ and decreasing both private finance $dk_1^C < 0$ and external debt
finance $dk^D_2 < 0$ without altering the next-period wealth $da' = 0$ such that

$$dc = -dk^C_1,$$

$$
\left.\frac{da'}{dk_1^* = dk_2 = 0}\right. = (+)d\tau - \tau(1 - \tilde{\sigma}_{BG})(1 + r)dk^D_2 = 0,
\hspace{1cm} (1.47)
$$

$$dk^*_1 + dk^*_2 = (-)d\tau + dk^C_1 + \tau(1 - \tilde{\sigma}_{BG})dk^D_2 = 0.$$

Note that a decrease in private finance $dk^C_1 < 0$ without changing capital in production $dk^*_1 = dk^*_2 = 0$ means larger net external finance such that

$$d(k^D_1 + k^E_1) > 0, \quad d(k^D_2 + k^E_2) > 0. \hspace{1cm} (1.48)$$

Moreover, from Corollary 2 with $dk^*_2 = 0$, we can observe that internal equity finance $k^C_2$ increases with corporate savings $-k^D_2$ such that

$$dk^C_2 = -(1 - \tilde{\sigma}_{BG})dk^D_2 > 0. \hspace{1cm} (1.49)$$

Similarly, from Corollary 1, a stand-alone entrepreneur can be better off by increasing consumption $dc > 0$ and decreasing both private finance $dk^C < 0$ and external debt finance $dk^D < 0$ without altering the next-period wealth $da' = 0$ such that

$$dc = -dk^C,$$

$$
\left.\frac{da'}{dk=0}\right. = (+)d\tau - (1 - \sigma)(1 + r)dk^D = 0,
\hspace{1cm} (1.50)
$$

$$dk = (-)d\tau + dk^C + (1 - \tilde{\sigma}_{SA})dk^D = 0,$$

A decrease in private finance $dk^C < 0$ without changing capital in production $dk = 0$ means larger net external finance such that

$$d(k^D + k^E) > 0. \hspace{1cm} (1.51)$$

32
The above results show that the excessive amount of external equity finance can be reinvested through corporate savings for risk sharing. In case of business groups, a parent firm’s excessive external finance flows into its internal equity finance that is used by the subsidiary firm’s investment for risk sharing. Moreover, by raising more external finance, an entrepreneur can reduce wealth transferred to her firm, consume more, and save less. The declining savings ratio of the rich, most of whom are business-group entrepreneurs financially unconstrained, can result in declining capital stock of an economy. Thus, in the model, a strictly positive correlation between the price of capital and aggregate capital in production of an economy can be broken as financial frictions decrease.

1.6 A Numerical Example of the Model

1.6.1 Setup

I construct a numerical example of the model and use it to compare two economies: an economy with business groups in which an entrepreneur can choose to create a business group and an economy without business groups in which building a business group is not an option for an entrepreneur.

Table 1.1 summarizes parameters used in the numerical example. A CRRA utility function is employed such that $u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$. I choose parameters that are conventional in the literature with one exception, a time discounting factor $\beta$, which is intentionally chosen very low for the fast convergence of numerical calculation. Model specific parameters such as flotation costs and maximum equity shares sold to outside shareholders are based on the rule of thumb.\(^{11}\)

---

\(^{11}\)For example, I choose $\bar{\sigma}_{BG} = 0.7$ because it is one of the criteria Fair Trade Commission in South Korea uses to identify if a corporation is a business-group subsidiary.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discounting factor</td>
<td>$\beta = 0.85$</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma = 1.2$</td>
</tr>
<tr>
<td>Span of control</td>
<td>$\alpha + \theta = 0.8$</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha = \frac{0.8}{3}$</td>
</tr>
<tr>
<td>Labor share</td>
<td>$\theta = \frac{0.8+2}{3}$</td>
</tr>
<tr>
<td>Average capital depreciation rate</td>
<td>$E\delta' = 0.059$</td>
</tr>
<tr>
<td>Flotation costs</td>
<td>$k^F = 20$</td>
</tr>
<tr>
<td>Stand-alone firm’s equity share sold out</td>
<td>$\bar{\sigma}_{SA} = 0.9$</td>
</tr>
<tr>
<td>Business-group firm’s equity share sold out</td>
<td>$\bar{\sigma}_{BG} = 0.7$</td>
</tr>
</tbody>
</table>

Table 1.1: Parameters

The wealth space is discretized into 20 exponentially increasing grids from $a(1) = 1.0 \times 10^{-4}$ to $a(20) = 1.0 \times 10^6$. The managerial talent space is discretized into 20 exponentially increasing grids from $z(1) = 1$ to $z(20) = 4$. The transition probability of the managerial talent from $z = z(i)$ to $z' = z(j)$ is defined such that \footnote{Note that given the exponentially increasing managerial talent space, the transition probability defined in Table 2 mimics a scale-free growth process bounded below $z' = z(1)$ with negative drift, which can approximate a stationary Pareto distribution (e.g. Gabaix (1999)).}

$$\forall i \in \{1, 2, ..., 19, 20\}, j = \max \{1, \min \{20, i + k\}\} \text{ with probability } p_k, k \in \{-9, -8, ..., 8, 9\},$$

<table>
<thead>
<tr>
<th>$p_{-9}$</th>
<th>$p_{-8}$</th>
<th>$p_{-7}$</th>
<th>$p_{-6}$</th>
<th>$p_{-5}$</th>
<th>$p_{-4}$</th>
<th>$p_{-3}$</th>
<th>$p_{-2}$</th>
<th>$p_{-1}$</th>
<th>$p_0$</th>
<th>$p_{+1}$</th>
<th>$p_{+2}$</th>
<th>$p_{+3}$</th>
<th>$p_{+4}$</th>
<th>$p_{+5}$</th>
<th>$p_{+6}$</th>
<th>$p_{+7}$</th>
<th>$p_{+8}$</th>
<th>$p_{+9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.005</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Table 1.2: The Transition Probability of Managerial Talent

Lastly, I assume that the capital depreciation rate $\delta'$ is a simple random variable, which
is independent of the shocks to managerial talent such that

\[
\delta' = \begin{cases} 
\tilde{\delta} = 0.8 & \text{with probability } 0.05 \\
\tilde{\delta} = 0.02 & \text{with probability } 0.95
\end{cases}
\] (1.52)

1.6.2 Observations

**Observation 1** (Occupational Choice). *The rich choose to become business-group entrepreneurs.* The poor but talented are hired as business-group managers with positive probabilities. The northwest region of \((z, a)\), where individuals with the positive probabilities of being hired as managers reside, becomes smaller as investor protection improves. The poor, untalented become workers.

Figure 1.3 shows occupational choices of individuals given a moderate degree of financial frictions, \(\tau = 0.5\). We can see that the east where the rich reside is filled with business-group entrepreneurs and that the northwest where the poor but talented reside is filled with stand-alone entrepreneurs who can be hired as business-group managers. This occupational policy function shows that pyramidal business groups work as start-up breeders that can foster productive firms given capital market imperfections. In the southwest, a declining line separating a SA region from a W region captures that wealth is required for an individual to become a stand-alone entrepreneurs given financial frictions.

As the fraction of diversion decreases to \(\tau = 0.1\), two changes are observed in the following Figure 1.4, which depicts occupational choices of individuals in an economy with business groups given \(\tau = 0.1\). First, the rich but untalented still become business-group entrepreneurs because they expect to earn ex-ante positive profits by hiring the talented as managers. We will see that these unproductive business-group entrepreneurs can be a source of resource misallocation. If we shut down the possibility of creating pyramidal business groups, the southeast region in Figure 1.6 shows that the rich but untalented business-group
entrepreneurs would become workers in an economy without business groups given $\tau = 0.1$.

Figure 1.3: Occupational Map in an Economy With Business Groups Given $\tau = 0.5$. No population exists outside the border of orange line.

Figure 1.4: Occupational Map in an Economy With Business Groups Given $\tau = 0.1$. No population exists outside the border of orange line.
Second, Figure 1.4 shows that fewer individuals are hired as business-group managers. Note that the managerial compensation \( w^M(z_2, \omega_2) \) is likely to be increasing as financial frictions decrease because an outside option of running a stand-alone firm should be a better option with lower financial frictions. Thus, business-group entrepreneurs have to hire the more talented but still financially constrained in order to earn positive profits. The contracted upper northwest region in Figure 1.4 captures this rising cut-off value of managerial talents, which can give business-group entrepreneurs positive profits with high enough managerial talent but small enough managerial compensation.

Figure 1.5: Occupational Map in an Economy Without Business Groups Given \( \tau = 0.5 \)  

Figure 1.6: Occupational Map in an Economy Without Business Groups Given \( \tau = 0.1 \)

**Observation 2** (The Relative Number of Business-Group Firms). The prevalence of business groups shows insignificant correlation with the strength of investor protection measured by \( (1 - \tau) \). Specifically, the relative number of business-group firms out of all corporations does not decrease with \( (1 - \tau) \).

Observation 2 can be understood as a corollary of Observation 1, which states that the rich become business-group entrepreneurs regardless of the degree of financial frictions. The following Figure 1.7 shows us two interesting features about the prevalence of business
groups. First, business-group firms cannot thrive under too severe financial frictions such as \( \tau \geq 0.7 \). This is because too severe financial frictions undermine the financial advantage of a pyramidal ownership structure that leverages on external capital markets.

Second, although the total number of business-group firms is unvarying, the number of subsidiary firms decreases as financial frictions decrease. Observation 1 already shows that the number of individuals who have the positive probability of being hired as managers decreases as financial frictions decrease. We will see in the following observations that subsidiaries are more productive than parents and that this decreasing ratio of subsidiary firms can weaken the benefits of pyramidal business groups as start-up breeders.

![Figure 1.7: The Prevalence of Business Groups Measured by the Relative Number of Firms](image)

**Observation 3** (Asymmetric Financial Frictions Between Business-Group and Stand-Alone Firms). *Business-group firms have a larger ratio of capital to labor than stand-alone firms. The variance of capital to labor ratios is smaller within business-group firms than within stand-alone firms.*

Given the Cobb-Douglas production function, the ratio of capital to labor would be identical to all types of firms if an economy had no financial frictions and no shocks to
managerial talents. Thus, business-group firms’ higher mean and smaller variance of capital-to-labor ratios suggest that business-group firms have better financial conditions than stand-alone firms. Figure 1.8 shows that these asymmetric financial frictions persist and hardly vary even though investor protection improves.

Figure 1.8 also shows that public corporations achieve almost identical capital-to-labor ratios to business groups as \( \tau \) goes to zero. This implies that firms would be financially unconstrained if they could use external equity finance with fine investor protection. However, the asymmetric financial frictions between business-group and stand-alone firms do not wane because most stand-alone entrepreneur don’t pay flotation costs \( k^F \) and turn down the option of tapping into external equity markets. As can be seen in Figure 1.7, most corporations are business-group firms, and the relative number of public corporations using external equity finance decreases as \( \tau \) decreases.

![Figure 1.8: Distributions of Capital-to-Labor Ratio](image)

Figure 1.8: Distributions of Capital-to-Labor Ratio

Then, the question is if these asymmetric financial frictions have sizable effects on resource allocation. The following Observation 4 gives an answer to the question.
Observation 4 (Firm Size Distributions). *Business-group firms have the larger mean and variance of employment and also have the larger mean and variance of TFP than stand-alone firms.*

The following Figure 1.9 shows that business-group firms are larger than stand-alone firms on average. This is because business-group firms not only have better financial conditions (Figure 1.8) but also have better managerial talents on average (Figure 1.10).

Business-group firms, however, also have larger variances of employment and managerial talents. Given the persistence of asymmetric financial frictions, the large number of unproductive business-group firms can distort resource allocation in an equilibrium. Note that the distributions of business-group firms are bimodal. Small, unproductive business-group firms coexist with large, productive business-group firms. This observation complies with the occupational choice that the rich but unproductive choose to become business-group entrepreneurs regardless of the degree of financial frictions (Figure 1.4).

![Figure 1.9: Firm Size Distributions Measured by Employment](image-url)
Given that pyramidal business groups have financial advantage but also have more dispersed productivities, the effects of pyramidal business groups on resource allocation are ambiguous. Their financial advantage makes business-group firms not only to raise more capital but also to allocate more capital to low productive business-group firms. The following Observation 5 shows that the net effects of pyramidal business groups depend on the level of financial frictions, $\tau$.

**Observation 5 (Factor Prices and Aggregate Inputs).** As the strength of investor protection $(1 - \tau)$ improves in an economy with business groups, both the rate of return on capital and wage increase monotonically, while both the capital stock and labor force increase first and then decrease.

Figure 1.11 captures correlations between factor prices and aggregate inputs in the degree of financial frictions. It shows that positive correlations between factor prices and aggregate inputs are broken under the prevalence of business groups. The existence of business groups helps an economy achieve a large amount of aggregate inputs under the moderate level of
financial frictions such as $\tau \in [0.3, 0.7]$. However, a further decrease in financial frictions from $\tau = 0.2$ only pushes up factor prices and results in the smaller aggregate inputs of an economy. This non-monotonicity contrasts with strictly positive correlations between factor prices and aggregate inputs in an economy without business groups.

![Graphs showing factor prices and aggregate inputs](image)

Figure 1.11: Factor Prices and Aggregate Inputs

This negative correlation observed in Figure 1.11 derives from a decrease in savings of the rich. The following Table 1.3 captures savings of the rich\(^\text{13}\) whose wealth is top 0.14% in an economy with business groups. It shows that the rich who choose to create business groups

\(^{13}\text{I choose } a(13) = 398 \text{ as the criteria for the rich because the population of individuals whose wealth is greater than or equal to 398 hardly changes as financial frictions decrease: the population changes from 0.147% with } \tau = 0.5 \text{ to 0.136% with } \tau = 0.1%.\)
save less as financial frictions decrease. The level of their savings decreases from 0.88 to 0.53, and the share of their savings decreases from 52% to 33%. This decrease in savings can be supported by the financial advantage of business-group entrepreneurs, which allows them to substitute external finance for private finance. With the same amount of wealth, business-group entrepreneurs can consume more and save less by raising more external capital as financial frictions decrease.

<table>
<thead>
<tr>
<th>The Degree of Financial Frictions</th>
<th>( \tau = 0.1 )</th>
<th>( \tau = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>An Economy with Business Groups</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savings of the Rich (Share of Capital Stock)</td>
<td>0.53(33%)</td>
<td>0.88(52%)</td>
</tr>
<tr>
<td>Population of the Rich</td>
<td>0.14%</td>
<td>0.15%</td>
</tr>
<tr>
<td><strong>An Economy without Business Groups</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savings of the Rich (Share of Capital Stock)</td>
<td>0.93(47%)</td>
<td>0.52(35%)</td>
</tr>
<tr>
<td>Population of the Rich</td>
<td>0.21%</td>
<td>0.11%</td>
</tr>
</tbody>
</table>

Table 1.3: Savings of the Rich

<table>
<thead>
<tr>
<th>The Degree of Financial Frictions</th>
<th>( \tau = 0.1 )</th>
<th>( \tau = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wealth of SA Entrepreneurs (Share of Total Wealth)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.46(18%)</td>
<td>1.04(40%)</td>
<td></td>
</tr>
<tr>
<td><strong>Population of SA Entrepreneurs</strong></td>
<td>5.24%</td>
<td>5.78%</td>
</tr>
</tbody>
</table>

Table 1.4: Wealth of Stand-Alone Entrepreneurs in an Economy with Business Groups

It is interesting that the savings of the rich would be monotonically increasing with investor protection if we shut down the possibility of creating business groups. In an economy without business groups, savings of the rich increase from 35% to 47% as financial frictions decrease from \( \tau = 0.5 \) to \( \tau = 0.1 \). Note that the population of the rich increases,\(^{14}\) which

\(^{14}\)Given the criteria of the rich, \( a \geq a(13) = 398 \), the population of the rich in an economy without business groups increases from 0.11% with \( \tau = 0.5 \) to 0.21% with \( \tau = 0.1 \).
implies that the lower financial frictions help the talented accumulate wealth in an economy without business groups.

In an economy dominated by business groups, however, its stagnating population of the rich suggests that the poor but talented suffer from the asymmetric financial frictions and have a difficulty to accumulate their wealth. The above Table 1.4 shows this possibility. The absolute level of stand-alone entrepreneurs’ wealth decreases from 1.04 to 0.46 as financial frictions decrease from $\tau = 0.5$ to $\tau = 0.1$, and the share of their wealth also decreases from 40% to 18%. Note that the population of stand-alone entrepreneurs is barely changed as financial frictions decrease. This implies that the decrease in stand-alone entrepreneurs’ wealth derives from a decrease in their wealth on average, not from a decrease in the number of their population.\footnote{Appendix A.1 shows that pyramidal business groups can lower the wealth mobility from the bottom to the top.}

**Observation 6 (Aggregate Flotation Costs).** *An economy with business groups consumes larger flotations costs than an economy without business groups.*

![Figure 1.12: Flotation Costs](image)

![Figure 1.13: Investment Net of Flotation Costs](image)

Observation 6 teaches us creating a business group can be an efficient choice for an individual, but not for an economy. As can be seen in Figure 1.12, flotation costs of an
economy with business groups increase faster than an economy without business groups. Remember that the rich but untalented create business groups by paying flotation costs in order to launch productive subsidiaries. Thus, incorporating pyramidal business groups requires larger fixed costs. It is the problem that even though the more parent firms are incorporated, the fewer subsidiary firms are launched as financial frictions decrease.

Figure 1.13 shows the aggregate flotation costs in an economy with business groups are sizable. The aggregate investment net of flotation costs decreases as $\tau$ goes to zero. This complies with the observation that as $\tau$ goes to zero, the capital stock of an economy with business groups declines.

One might ask why the net investment declines even though financial frictions decrease and the rate of return on capital keeps rising. The following Figure 1.14 gives an explanation. It shows that the investment rate of an economy with business groups is not only larger than an economy without business groups but also increases monotonically. Thus, a decrease in financial frictions indeed increases the investment rate of an economy. The excessive flotation costs used up by business groups, however, overwhelm the increase in investment and result in the decrease in net investment used for replenishing capital depreciation in a stationary state equilibrium as $\tau$ goes to zero.

Figure 1.14: Investment Rate
Observation 7 (Aggregate Output). Let’s define aggregate output of an economy as the sum of aggregate consumption and aggregate investment. Then, the aggregate output of an economy with business groups does not monotonically increase with investor protection. When the level of investor protection is strong enough such as $(1 - \tau) \geq 0.8$, an increase in investor protection does not increase the aggregate output of an economy under the prevalence of business groups.

Pyramidal business groups make the aggregate output of an economy regress toward a moderate level over the degree of financial frictions. Figure 1.15 shows that business groups can partially nullify the impact of financial frictions on aggregate output. At the early stage of its development where financial frictions are rampant, business groups help an economy produce larger aggregate output.\textsuperscript{16} When the tunneling ratio $\tau$ goes to zero, however, Figure 1.15 shows that the aggregate output of an economy with business groups is stagnating.

\textsuperscript{16}Figure 1.15 shows that a little development of investor protection is required for business groups to help an economy produce more aggregate output. This is because the internal equity finance of business groups works as leverage for raising capital from external markets. Too large financial frictions can weaken the efficiency of the financial advantage of pyramidal business groups.
Observation 7 rebuts an argument that the economic impact of business groups would spontaneously vanish if investor protection improves. The stagnating aggregate output rather suggests that achieving good investor protection is not enough to lessen the effects of business groups on an economy and that aggregate output may not grow without restraining the prevalence of business groups. As argued in the previous remarks, business groups can be asymmetrically benefited by the improvement of investor protection in the model. The stagnating aggregate output of an economy with business groups in Figure 1.15 suggests that the asymmetric financial frictions become sizable and the benefits of business groups can be dominated by their costs when the degree of financial frictions is low enough such that $\tau \leq 0.2$.

The following Observation 8 shows how sizable the asymmetric financial frictions between business-group and stand-alone firms are and why dealing with pyramidal ownership structure is necessary for the development of external capital markets.

**Observation 8 (External Capital Markets).** Let’s define the size of external capital markets as the sum of external debt finance and external equity finance used by all firms such that

$$\begin{align*}
\text{External Capital Markets} &= \int_{(z,a)=SA} \left\{ 1 - P^M(z,a) \right\} \cdot \left\{ k^D(z,a) + k^E(z,a) \right\} dF(z,a) \\
&+ \sum_{(z,a)=BG} \sum_{i \in \{1,2\}} \mathbb{E}_{a_2} \left[ k^D_i(z,a|z_2(z,a),a_2) + k^E_i(z,a|z_2(z,a),a_2) \right] dF(z,a).
\end{align*}$$

(1.53)

Controlling for aggregate output, the external capital markets of an economy with business groups are smaller than those of an economy without business groups.

Figure 1.16 shows that the underdevelopment of external capital markets can be associated with the prevalence of business groups in an equilibrium. However, it does not mean that shutting down business groups increases the size of external capital markets. External capital markets of an economy with business groups are larger than those of an economy with-
out business groups given the moderate degree of financial frictions such that \( \tau \in [0.3, 0.6] \), while they are smaller given the low degree of financial frictions such that \( \tau \leq 0.2 \). It is a more precise interpretation of the result that business groups decrease the size of external capital used by stand-alone firms in an equilibrium. Figure 1.16 shows that more than a half of external capital is used by business groups and that external capital used by stand-alone firms is smaller than its counterparts in an economy without business groups.

This underdevelopment of external capital markets in an economy with business groups arises due to the asymmetric financial frictions between business-group firms and stand-alone firms in the model. Note that given the same degree of financial frictions, the price of capital is always higher in an economy with business groups than that without business groups. The higher price of capital impairs stand-alone firms’ external financing. Thus, stand-alone firms, which lack internal capital markets, should suffer from the tighter financial constraints and cannot but raise less external capital in an economy dominated by business groups.

To summarize, the results of this section show that the endogenous creation of pyramidal

\[17\] Note that in Figure 1.16, each point on a line is connected with two adjacent points of tunneling ratio.
business groups and the asymmetric financial frictions among firms have sizable effects on resource allocation of an economy. The following section presents a firm-level dataset of South Korea and tests the model.

1.7 A Case of South Korea

South Korea is one of many countries where the prevalence of pyramidal business groups is significant.\(^\text{18}\) I use a firm-level dataset of South Korea, the Survey of Business Structure and Activities collected by the Korean National Statistical Office from 2006 to 2013, to compare with the model. This dataset covers all firms in the country having more than fifty permanent workers and equities larger than 300 millions KRW.\(^\text{19}\) The number of all firms observed in the data is around 12000 each year.\(^\text{20}\) The dataset contains each firm’s ownership structure information if it has a parent or a subsidiary.\(^\text{21}\) Lastly, we can observe in the data since 2010 if a firm is listed in the Korean stock exchange markets, KOSPI or KOSDAQ.

I identify a business-group firm with a listed firm that has a parent or a subsidiary. Although there are many unlisted firms that have a parent or a subsidiary in the data, I only identify business-group firms with listed firms because a pyramidal business group defined in the model is a collection of listed firms, which can tap into external equity markets. Note that the financial advantage of a pyramidal ownership structure stems from the leverages on external equity finance in the model.

Stand-alone firms are defined as all firms that are not identified as business-group firms. Thus, unlisted firms that have parents or subsidiaries are also identified as stand-alone.

\(^{18}\)See Masulis, Pham, and Zein (2011).
\(^{19}\)As of March 2016, 300 millions KRW is equivalent to about 250 thousands USD.
\(^{20}\)I only use firms in non-finance sectors.
\(^{21}\)Only a firm that has more than 50% of shares of another firm is identified as a parent firm in the data. Thus, the dataset could underestimate the number of business-group firms because a de facto controlling shareholder usually holds less than 30% of block shares in South Korea.
1.7.1 The Prevalence of Business Groups

The following Table 1.5 shows the prevalence of business groups measured by the relative number of business-group firms. You can see that the prevalence of business groups is stable over the periods. The relative number of business-group firms out of all listed firms is around 0.85, which is more or less the same number observed in the numerical example of the model given $\tau \leq 0.6$. See Figure 1.7.

<table>
<thead>
<tr>
<th>Year</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business-Group Firms / All Listed Firms</td>
<td>0.85</td>
<td>0.84</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td>Business-Group Firms / All Firms</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 1.5: The Prevalence of Business Groups Measured by the Relative Number of Firms

The relative number of business-group firms out of all firms is around 0.11. This is larger than the number observed in the model such as 0.08 at $\tau = 0.01$ or 0.03 at $\tau = 0.6$. Note that the number of all firms in the data is likely to be underestimated because the dataset does not include small firms that have fewer than fifty permanent workers or equities smaller than 300 millions KRW.

---

22I implicitly assume that unlisted firms are parts of their parents or subsidiaries because unlisted firms cannot sell their shares and use them as a leverage. However, two cases are ambiguous. One is a unlisted parent that has a listed subsidiary, and another is a unlisted subsidiary that has a listed parent. The dataset only offers the existence of a parent or a subsidiary, but not the information whether they are listed or not. Although not reported in the paper, excluding these two cases shapes stand-alone firms more financially constrained and smaller such that their capital-to-labor ratios have a smaller mean and a larger variance and that their employment has a smaller mean and a larger variance.
1.7.2 The Asymmetric Financial Frictions Among Firms

Figure 1.17 shows distributions of capital-to-labor ratios for each type of firms. I use a firm’s fixed assets as its capital. A firm’s labor force is calculated by the sum of permanent and temporary workers.\(^{23}\)

![Figure 1.17: The Distribution of Capital-to-Labor Ratio from 2010 to 2013](image)

The data suggests that the asymmetric financial frictions between business-group and stand-alone firms are sizable in South Korea. You can see that business-group firms have a larger mean and a smaller variance of capital-to-labor ratios than stand-alone firms and that public corporations have more or less the same variance of capital-to-labor ratios with business-group firms, while their mean is slightly smaller than that of business-group firms. These results comply well with the numerical example of the model around \(\tau \in [0.1, 0.2]\). See Figure 1.8.

Technological differences between business-group and stand-alone firms could induce

\(^{23}\)Temporary workers consist of a large volume of employment in South Korea. In the dataset, temporary workers are more than 10% of all labor forces. However, excluding temporary workers from the total employment barely changes the results.
these asymmetric capital-to-labor ratios. For instance, business-group firms might be concentrated in capital intensive industries. Technological differences, however, cannot explain the observed patterns in South Korea. First, business groups in South Korea are so diversified that they have subsidiaries in almost all industries. We can observe the same patterns of capital-to-labor ratios within an industry such as manufacture or non-finance service sector. See Appendix A.3. Second, even if business-group firms were concentrated in an industry such as manufacture, their financial advantage should increase the price of capital in an equilibrium, which could impair the financial capacity of stand-alone firms in other industries.

1.7.3 The Firm Size Distribution

Figure 1.18 shows the firm size distributions observed in the data measured by employment. You can see that business-group firms are larger on average and more dispersed than stand-alone firms. Remember that the model teaches us that a larger variance of business-group firms can lead to factor misallocation given the asymmetric financial advantage of business-group firms, which is observed in the previous figure.

Public corporations, however, are smaller on average than business-group firms in Figure 1.18. This is a salient difference between the data (Figure 1.18) and the model (Figure 1.9). Since public corporations have slightly smaller capital-to-labor ratios on average, the significantly smaller size of public corporations implies that public corporations can be less productive than business-group firms on average, which is inconsistent with the model. This disparity might come from the assumption that managerial talent in the model follows an exogenous Markov process regardless of ownership structures. In other words, the model lacks one of the salient benefits of business groups, R&D. In the real world, business groups can use their financial advantage to improve their productivities endogenously through investment.
1.7.4 Aggregate Variables

The dataset of South Korea shows us that most listed firms are members of pyramidal business groups, that the asymmetric financial frictions between business-group and stand-alone firms are sizable, and that business-group firms are more dispersed than stand-alone firms. Thus, we can say that the model captures well the characteristics of firms with respect to ownership structures.

However, it is still ambiguous that the asymmetric financial frictions derived from pyramidal business groups have non-monotonic effects on aggregate inputs and outputs in the strength of investor protection. A future research is required to test predictions of the model. Preliminary results are presented in Appendix A.3.
1.8 Conclusion

Financial frictions can cause resource misallocation. They are understood as one of the major hindrances to economic development. Although many researchers have shown why and to what extent financial frictions affect an economy, few macroeconomic models have investigated private institutions that can arise as endogenous reactions against financial frictions. In this paper, I study the endogenous creation of pyramidal business groups and focus on the repercussions of their financial advantage given capital market imperfections.

There are three main implications of the model. First, pyramidal business groups can be efficient private institutions if external capital markets are underdeveloped due to severe financial frictions. Second, the asymmetric financial frictions between business-group and stand-alone firms can create inefficiencies that impair stand-alone firms’ external financing in an equilibrium. Third, the prevalence of business groups does not spontaneously shrink as investor protection improves.

The last implication is indeed a limitation of this paper. The unvarying number of business-group firms in the model cannot explain why the prevalence of business groups differs across developed countries. Thus, finding a rationale for the cross-country difference can be an interesting topic for future research. For instance, Kandel, Kosenko, Morck, and Yafeh (2015) argue that the U.S. pyramidal business groups have almost disappeared because the U.S. government pursued specific policy measures to regulate business groups such as the Public Utility Holding Company Act (1935) and rising inter-corporate dividend taxation (after 1935). We can use the model developed in this paper to do a counter-factual analysis that examines how effectively the regulations adopted in the U.S. can reduce the prevalence of business groups and undo factor misallocation spawned by business groups.

A stationary equilibrium employed in this paper cannot measure the dynamic effects of business groups on welfare over time. Even though an economy dominated by business
groups can feature smaller aggregate consumption and output, it does not mean that shutting down business groups immediately improves welfare because it takes time for an economy to accumulate capital stock and because in the transition periods, the economy should suffer from lower TFP. It is an interesting future research that simplifies the model, tracks down transition periods, and examines dynamic effects of changes in policies.\textsuperscript{24} A challenge is computation burden in order to deal with equilibriums over time.

Lastly, another follow-up research agenda can be the effects of pyramidal business groups on wealth inequality and socioeconomic mobility. The model developed in this paper suggests that the rich can entrench their wealth by building up pyramidal business groups, which results in a decrease in the probability of the poor accumulating wealth. Given the assumption that the inequality of entrepreneurial productivity stems from luck, business groups could be an institution that allows the rich to insure their wealth against their bad luck. This entrenchment of the rich implies that in an equilibrium, the prevalence of business groups can prevent the poor from exploiting their good luck. Thus, we can use the model to study how pyramidal business groups can change the patterns of wealth inequality and socioeconomic mobility. Preliminary results are presented in Appendix A.1.

\textsuperscript{24}e.g. Buera and Shin (2013); Buera, Moll, and Shin (2013)
Chapter 2

Peasants in the City: Consequences of Declining Labor Shares

2.1 Introduction

A recent debate on declining labor shares often focuses on the elasticity of substitution between factors of production, which is a key parameter of production functions many macroeconomic models hinge on. For instance, by estimating the elasticity greater than one Karabarbounis and Neiman (2014b) attribute the declining labor shares to a decrease in the relative price of investment goods. However, it is unclear that the elasticity of substitution is greater than one. Many studies report the opposite: Klump, McAdam, and Willman (2012) and Oberfield and Raval (2014) report that the estimates are less than one.¹

Borrowing from Houthakker (1955) and Jones (2005), I show that a Cobb-Douglas production function can be consistent with a technology that uses capital to substitute for other factors and changes factor shares. The seemingly inconsistent estimates of the elasticity of substitution of capital for labor is not the only possible determinant of the recent declining labor shares. A declining unionization or a rising offshoring might be possible causes for the declining labor shares. See Elsby et al. (2013).

¹The substitution of capital for labor is not the only possible determinant of the recent declining labor shares. A declining unionization or a rising offshoring might be possible causes for the declining labor shares. See Elsby et al. (2013).
substitution can be reconciled with the substitution technology because endogenous choices of the labor-saving technology can equip an economy with two iso-quants, one taking the technology and another not taking the technology. The envelope of these iso-quants can shape a production function with the elasticity greater than one even though each iso-quant implies the elasticity less than or equal to one.

Given this observation, I use a simple two-sector model to show that changes in factor shares can be tightly linked to changes in employment shares, factor prices, and income inequality. The industrial revolution is viewed as a technological progress that allows capital to substitute for both land and labor in agriculture.\(^2\) The model shows that an increase in the substitutability for both land and labor in agriculture can push population out from the rural area to the urban area where the employment share of manufacture is higher than the rural area.

Recent advances in information technologies are viewed as a technological progress that allows capital to substitute for labor in manufacture and decreases labor shares.\(^3\) This implies that declining labor shares in manufacture that demands less labor can intensify the agglomeration to the city where dominant industries are non-manufacture and where a labor-saving technology is still unadopted.

The model shows that the recent declining labor shares in manufacture can change factor prices. First, the price of land can increases. A substitution technology in manufacture decreases the employment share of manufacture and increases the employment share of non-manufacture industries in the city where the substitution technology is still unadopted. Given the fixed supply of land, the marginal productivity of land increases because the ratio

\(^2\)Steam engines use fossil fuel to substitute for land that was virtually the only source of energy before the revolution. Tapping into seemingly unlimited source of energy, the economy can keep growing by using a Solow production technology that is not restricted by the limited supply of land (Hansen and Prescott, 2002).

\(^3\)Koh, Santaeulalia-Llopis, and Zheng (2015) show that an increase in intellectual property capital such as software can account for the recent declining labor shares in the United States.
of labor to land increases.

Second, the price of labor can decreases. Capital is not accumulated enough to compensate for the rising labor forces in non-manufacture industries. To keep the interest rate fixed, which is determined by a time preference, the ratio of capital to labor is pushed down as the ratio of capital to land increases due to the fixed supply of land. As a result, even though aggregate output increases with a technological progress that allows capital to substitute for labor in manufacture, the marginal productivity of labor in an equilibrium can decrease.

Lastly, the model suggests that the dispersion of labor productivities can be larger as labor shares in manufacture decrease. A technology that substitutes capital for labor in manufacture increases the elasticity of labor demand with respect to labor augmenting technologies and shapes the right tale of a labor productivity distribution thicker. Thus, labor productivities are bipolarized into two distributions, a fatter right tale Pareto distribution of manufacture with fewer labor forces and a Pareto distribution of non-manufacture with larger labor forces immigrated from manufacture.

The rest of this paper proceeds as follows. Section 2 introduces a technology that substitutes capital for other factors to show that this substitution technology can change factor shares without discarding a Cobb-Douglas production function. Section 3 remarks on a technology that substitutes capital for labor. Section 4 studies consequences of declining labor shares in two different cases: a decline in agriculture and a decline in manufacture. Section 5 concludes.

2.2 A Technology Generating a Cobb-Douglas Production Function and Changing Factor Shares

Houthakker (1955) and Jones (2005) show that a Cobb-Douglas production function can be derived from stochastic factor augmenting technologies following Pareto distributions.
Suppose that output is produced by a local production technology, $\tilde{F}$, such that

$$Y = F(K, X) = \max_{i=1,\ldots,N} \tilde{F}(b_iK, a_iX).$$

(2.1)

$K$ stands for capital, and $X$ stands for the other factor such as land or labor. Assume that a pair of ideas, $(b_i, a_i)$, is drawn from independent Pareto distributions as follows.

$$\Pr[a_i \leq a] = 1 - \left( \frac{a}{\gamma_a} \right)^{-\alpha}, \quad a_i \geq \gamma_a > 0$$

(2.2)

$$\Pr[b_i \leq b] = 1 - \left( \frac{b}{\gamma_b} \right)^{-\beta}, \quad b_i \geq \gamma_b > 0$$

Given that a local production technology is Leontief, $\tilde{F}(b_iK, a_iX) = \min\{b_iK, a_iX\}$, a global production function, $F(K, X)$, converges to Cobb-Douglas as the number of ideas $N$ goes to infinity such that

$$E[Y] \approx (\gamma NK^\beta X^\alpha)^{\frac{1}{\alpha+\beta}}, \quad \gamma = \gamma_a^\alpha \gamma_b^\beta.$$  

(2.3)

Now, suppose that an economy can embody a substitution technology, $f\left(\frac{K_s}{X}\right)$, which allows capital to substitute for the other factor.

$$F\left(K, X; f\left(\frac{K_s}{X}\right)\right) = \max_{i=1,\ldots,N} \tilde{F}\left(b_i(K-K_s), a_iX \cdot f\left(\frac{K_s}{X}\right)\right) \quad \text{given} \quad K_s \in [0, K]$$

(2.4)

$f\left(\frac{K_s}{X}\right)$ captures development of a substitution technology. I assume $f\left(\frac{K_s}{X}\right)$ is a X-augmenting technology which is a function of capital per $X$. For instance, fertilizer decreases the minimum requirement of land and labor producing a unit of crops. Similarly, a computer is an exoskeleton that increases the efficiency of a unit labor force and decreases the minimum requirement of labor forces producing a unit of industrial products.
The above production function with the substitution technology can be rewritten as

\[
F\left(K, L; f\left(K_s \frac{X}{X}\right)\right) = \max_{i=1,\ldots,N} F\left(b_i(K - K_s), a_i'X\right) \quad \text{given} \quad K_s \in [0, K]
\] (2.5)

where

\[
\text{Pr}\left[a_i' \leq a\right] = 1 - \left(\frac{a}{\gamma_a f\left(K_s \frac{X}{X}\right)}\right)^{-\alpha}, \quad a_i' \geq \gamma_a f\left(K_s \frac{X}{X}\right) > 0
\] (2.6)

\[
\text{Pr}\left[b_i \leq b\right] = 1 - \left(\frac{b}{\gamma_b}\right)^{-\beta}, \quad b_i \geq \gamma_b > 0.
\]

As \(N\) goes to infinity, the production function converges to a Cobb-Douglas function.

\[
E\left[F\left(K, X; f\left(K_s \frac{X}{X}\right)\right)\right] \approx \left(\gamma_a^{\alpha} \gamma_b^{\beta} N \left\{ f\left(K_s \frac{X}{X}\right)\right\}^\alpha (K - K_s)^\beta X^\alpha\right)^{\frac{1}{\alpha + \beta}}
\] (2.7)

Lastly, let’s assume the following functional form of a substitution technology,

\[
f\left(K_s \frac{X}{X}\right) = \left(K_s / \xi X\right)^s, \quad s \in [0, 1],
\] (2.8)

where \(\xi\) captures the relative price of the substitution technology and \(s\) is the degree of substitutability. Then, the production function is optimized by choosing the level of substitution, \(K_s \frac{X}{X}\), such that

\[
E[Y] = \max_{K_s} E\left[F\left(K, X; f\left(K_s \frac{X}{X}\right)\right)\right]
\]

\[
\approx \max_{K_s} \left(\gamma_a^{\alpha} \gamma_b^{\beta} N \left\{ f\left(K_s \frac{X}{X}\right)\right\}^\alpha (K - K_s)^\beta X^\alpha\right)^{\frac{1}{\alpha + \beta}} \quad \text{as} \quad N \to \infty
\]

\[
= \max_{K_s} \left(\left(\frac{\gamma_a}{\xi^s}\right)^\alpha \gamma_b^{\beta} NK_s^{\alpha s} (K - K_s)^\beta X^{\alpha - \alpha s}\right)^{\frac{1}{\alpha + \beta}}
\]

\[
= \left(\gamma' NK^{\beta + \alpha s} X^{\alpha - \alpha s}\right)^{\frac{1}{\alpha + \beta}}, \quad \gamma' = \gamma_a^{\alpha} \gamma_b^{\beta} \left(\frac{1}{\xi}\right)\left(\frac{\alpha s}{\alpha s + \beta}\right)^{\alpha s} \left(\frac{\beta}{\alpha s + \beta}\right)^{\beta}.
\] (2.9)

Note that the production function with the substitution technology is isomorphic to the
Cobb-Douglas production function without the substitution technology but changes in the distribution of ideas: from $\beta$ to $\beta + \alpha s$ and from $\alpha$ to $\alpha - \alpha s$. Thus, the substitution technology $f$ is nothing but changes in factor shares of the production function such that

$$Y' = \max_{i=1,\ldots,N} \tilde{F}(b_i K, a_i X)$$

(2.10)

where

$$\Pr[a_i \leq a] = 1 - \left( \frac{a}{\gamma_a} \right)^{-\alpha'}, \quad \alpha' = \alpha - \alpha s, \quad \gamma'_a = \gamma_a \left( \frac{\alpha s}{\xi (\alpha s + \beta)} \right)^s$$

(2.11)

$$\Pr[b_i \leq b] = 1 - \left( \frac{b}{\gamma_b} \right)^{-\beta'}, \quad \beta' = \beta + \alpha s, \quad \gamma'_b = \frac{\gamma_b \beta}{\alpha s + \beta}.$$

converging into

$$E[Y'] \approx \left( \gamma' N K^{\beta'} X^{\alpha'} \right)^{1/\alpha' + \beta'}, \quad \gamma' = \gamma'_a \gamma'_b.$$

(2.12)

### 2.3 Remarks on a Technology Substituting Capital for Labor

Suppose that a production requires two factors, capital ($K$) and labor ($X = L$). Given the local Leontief production technology, the optimal labor demand required to produce a unit output is proportional to the inverse of the labor augmenting technology such that $L^* \sim \frac{1}{a}$. As the number of ideas $N$ grows by drawing new ideas, the expected growth rate of the labor
productivity measured by $\frac{1}{L'}$ conditional on non-negative growth is

$$E\left[\frac{1}{L'} - \frac{1}{L} \left| L' < L\right.\right] \sim E\left[\frac{a' - a}{a} \left| a' > a\right.\right]$$

$$= \int_a^\infty \frac{a' - a}{a} \cdot a^a a'^{-a-1} da'$$

$$= a a^{a-1} \left(\frac{a^{-a+1}}{a - 1} - \frac{a^{-a+1}}{a}\right)$$

$$= \frac{1}{\alpha - 1} \tag{2.13}$$

where the second equation holds under the assumption that labor augmenting technology follows a Pareto distribution.

From the above result, we can see that $\alpha$ captures the scarcity of labor augmenting technology. The higher $\alpha$ means the lower probability of finding a better labor augmenting technology $a$ and the lower growth rate of the labor productivity measured by $\frac{1}{L'}$. Note that a Cobb-Douglas production function features $\frac{\alpha}{\alpha + \beta}$ labor shares, which increases with $\alpha$. Thus, the above model tells us that the scarcer a factor is, the higher shares it takes.

Now, given the substitution technology $f \left(\frac{K_s}{L}\right)^s$, the optimal labor demand is changed such that $L^{s-1} \sim \frac{1}{a}$ and the expected growth rate of the labor productivity conditional on non-negative growth becomes

$$E\left[\frac{1/L' - 1/L}{L'} \left| L' < L\right.\right] \sim E\left[\frac{a'^{1-s} - a^{1-s}}{a^{1-s}} \left| a' > a\right.\right]$$

$$= \int_a^\infty \frac{a'^{1-s} - a^{1-s}}{a^{1-s}} \cdot a^a a'^{-a-1} da'$$

$$= a a^{-\frac{1}{1-s}} \left(\frac{a^{-a+1}}{a - \frac{1}{1-s}} - \frac{a^{-a+1}}{a}\right)$$

$$= \frac{1}{(1-s) \alpha - 1} \tag{2.14}$$

Given $s \in (0, 1)$, we can observe that the substitution technology increases the conditional
expected growth rate of the labor productivity from $\frac{1}{\alpha - 1}$ to $\frac{1}{(1-s)\alpha - 1}$. This is because the substitution technology is a scale-free replication device of labor. It amplifies the productivity of labor as a factor of $f \left( \frac{K_s}{\xi} \right)^s$ regardless of the level of $a$. Thus, the substitution technology increase the elasticity of the optimal labor demand such that $-\frac{d \log L^*}{d \log a} = \frac{1}{1-s}$.

In fact, it is a Pareto distribution of labor augmenting technology that results in the scale-free growth rate of the labor productivity.\(^4\) Note that $\alpha$ is the shape parameter of a Pareto distribution, $\Pr[a_i \leq a] = 1 - \left( \frac{a}{\gamma_{a}} \right)^{-\alpha}$. As $\alpha$ decreases, the right tale of Pareto distribution becomes fatter and the probability of drawing a better labor augmenting technology $a$ increases. Moreover, the variance of a Pareto distribution increases as $\alpha$ decreases. This implies that the shape parameter $\alpha$ of a Pareto distribution can be understood as the inverse degree of labor productivity dispersion. Thus, we can understand that a substitution technology decreasing $\alpha' = (1-s)\alpha$ not only increases the expected growth rate of labor productivity but also results in the more dispersed labor productivities across production units.

**Implications of a Technological Progress Substituting Capital for Labor**

- The labor share of income $\alpha'$ decreases with $s$, the degree of substitutability.
- The relative price of substitution technology $\xi$ determines whether substitution technology is implemented.
- Due to the endogenous choice of substitution technology, an economy can embody two different production functions, one with substitution technology ($K_s > 0$) and one without substitution technology ($K_s = 0$). Thus, the elasticity of substitution can be measured greater than 1 given two Cobb-Douglas iso-quant curves.

\(^4\)Gabaix (1999) shows that a scale-free random growth process with a positive lower bound can generate a Pareto distribution with the inverse degree of dispersion such that $\alpha = \frac{a_{\text{mean}}}{a_{\text{mean}} - a_{\text{min}}}$. 

63
• The labor share of income $\alpha'$ can decline due to a decrease in the price of substitution technology $\xi$ because the lower $\xi$ is, the more production units implement the substitution technology. This extensive margin of moving from $\alpha$ to $\alpha - \alpha s$ can decrease aggregate labor shares. With the low enough price of substitution technology, however, every production unit already adopts the substitution technology and labor share $\alpha'$ is bounded below $\alpha - \alpha s$ regardless of $\xi$.

• If we can map a wage profile into the distribution of labor-augmenting technology, $a_i$, the wage dispersion increases with the degree of substitution $s$ because the dispersion of Pareto distribution is a decreasing function of $\alpha$ such that

\[
SD(a_i) = \sqrt{\frac{1}{\alpha(\alpha - 2)}} \quad \text{where} \quad \Pr[a_i \leq a] = 1 - \left(\frac{a}{\gamma a}\right)^{-\alpha}, \alpha > 2.
\]

2.4 Consequences of Asymmetric Changes in Factor Shares

2.4.1 From Plows to Steam Engines That Substitutes Capital for Land and Labor in Agriculture

Industrial revolution changed the path of economic growth. Arguably, steam engines were the most important invention of the revolution. Steam engines allowed human to be free from cultivating land which was practically the only source of usable energy. Steam engines allowed capital to substitute for land, to be accumulated, and hence to serve for sustained economic growth. Pollan (2006) tells us a vivid story that we are in essence eating crude oil because foods we consume are cultivated by manufactured products such as chemical fertilizer, pesticide, or tractors. Hansen and Prescott (2002) use the idea of substitution for
land and build a growth model in which the path of economic growth is transformed from
the world of Malthus to that of Solow.

**Workers in Urban Area**

Let’s consider a two-sector economy consisting of agriculture in the rural area (A) and
manufacture in the urban area (M). Suppose that agricultural goods are produced with
capital $K_A$, labor $L_A$, and land $X$. In contrast, manufactured goods are produced by a
Solow production technology in which only capital $K_M$ and labor $L_M$ are required. In other
words, the land share of manufacture is assumed to be zero.

There are $L$ infinitely lived households. Each household has a unit of inelastic labor
force and sells it in the market with wage $w$. Suppose that a household selling its labor in
agriculture lives in the rural area and that a household selling its labor in manufacture lives
in the urban area. Each household consumes both agricultural goods $c_A$ and manufactured
goods $c_M$. With some conventional assumptions, the economy can be described by the
following problem.

$$
\max \sum_{t=0}^{\infty} \rho^t \left( c_{A,t}^{\theta_A} c_{M,t}^{\theta_M} \right), \quad \theta_A + \theta_M < 1, \rho < 1 \tag{2.15}
$$

subject to

\[
L \cdot c_{A,t} \leq Y_{A,t} = Z_A \left( K_{A,t}^{\beta_A} L_{A,t}^{\alpha_A} X^{\chi} \right)^{\frac{1}{\beta_A + \alpha_A + \chi}}
\]

\[
L \cdot \{ c_{M,t} + k_{t+1} - (1 - \delta)k_t \} \leq Y_{M,t} = Z_M \left( K_{M,t}^{\beta_M} L_{M,t}^{\alpha_M} \right)^{\frac{1}{\beta_M + \alpha_M}} \tag{2.16}
\]

\[
K_t = K_{A,t} + K_{M,t}
\]

\[
L = L_{A,t} + L_{M,t}
\]

In a steady state with zero capital depreciation $\delta = 0$, the following first order conditions
hold where $P_A$ is the relative price of agricultural goods.

\[
\frac{P_A Y_A}{\theta_A} = \frac{Y_M}{\theta_M}
\]

\[
w_A = \frac{\alpha_A}{\beta_A + \alpha_A + \chi} \frac{P_A Y_A}{L_A}
\]

\[
w_M = \frac{\alpha_M}{\beta_M + \alpha_M} \frac{Y_M}{L_M}
\]

(2.17)

Given the factor price equalization, the rural population is pinned down by

\[
\frac{L_A}{L} = \left(1 + \frac{\beta_A + \alpha_A + \chi \cdot \alpha_M}{\beta_M + \alpha_M} \cdot \frac{\theta_M}{\theta_A}\right)^{-1}.
\]

(2.18)

The above equation tells us several features of declining labor shares in agriculture. First, labor forces are pushed out from the rural area $\Delta L_A < 0$ as the substitution technology in agriculture increases $\Delta \beta_A > 0$. Given the fixed number of labor forces, this means that the urban agglomeration is intensified.

Labor forces have immigrated from the rural area to the urban area as an economy grows. Lucas (2004) argues that cities are better places for accumulating human capital and hence attract labor forces given the stagnating agricultural sector in the countrysides. What I argue here is the opposite side of the same coin explained by him: Labor forces are not only pulled into the urban area but also are pushed out from the rural area. A feature of the push story is that factor price equalization holds naturally. Wedges compensating for productivity gap between two regions such as externality on human capital or preference for amenity are not required. Since the rural population is not the residual of the urban population, the productivity in the countrysides does not need to be lower than that in the cities.

Second, sectoral TFPs captured by $Z_A$ and $Z_M$ are irrelevant to the urban agglomeration in the model. This is because consumption shares are constant for each sector given a specific form of preferences. If consumption share of agriculture decreases as the economy grows such
that $\Delta \theta_A < 0$, the urban agglomeration can be intensified.

Lastly, equilibrium wage does not change even though labor demand in agriculture decreases. This is because the level of capital is increased in order to keep the interest rate constant such that $1 + r = \frac{1}{\rho}$. Look at the following equations.

$$w = \frac{\alpha_M}{\beta_M} \frac{r K_M}{L_M}, \quad r = \frac{\beta_M}{\beta_M + \alpha_M} Z_M \left( \frac{L_M}{K_M} \right)^{\alpha_M \rho M + \beta_M}$$

(2.19)

$$\Delta w = 0 \quad \text{given} \quad \Delta r = \Delta \left( \frac{K_M}{L_M} \right) = 0$$

(2.20)

Given $\Delta \beta_A > 0$, labor forces are pushed out from agriculture into manufacture $\Delta L_M > 0$. In a stationary state, however, interest rate is fixed constant and hence the ratio of capital to labor in manufacture $\left( \frac{K_M}{L_M} \right)$ also should be fixed. As a result, capital in manufacture increases in order to compensate for the increase in labor such that $\Delta \frac{K_M}{L_M} = 0$. The intact wage level is possible because capital in manufacture can be accumulated. If any factor is bounded above, declining labor shares can decreases the equilibrium wage. We can see this in the next section.

### 2.4.2 From Steam Engines to Computers That Substitutes Capital for Labor in Manufacture

The object of this section is to answer a question: What will occur if information technology such as computers, robots, and artificial intelligence substitutes capital for human labor forces in manufacture? Where do the labor forces go? Do any qualitative changes take place? Although general public often speculates a grim future, the answer is unclear. For instance, information technology can increase the marginal productivity of labor at least in manufacture.

To answer the question, I use a simple two-sector model again with an assumption that
the land share of income is sizable in non-manufacture industries, especially in the city. Land is assumed as a production factor offering economies of agglomeration such as searching, matching, or networking. This implies that a production function in the city can be isomorphic to that in agriculture: non-zero land share of income $\chi > 0$. Let’s abstract from agriculture and suppose that $A$ stands for non-manufacture industries or agglomeration economies in the city. Accordingly, suppose that $M$ stands for manufacture.

**Peasants in the City**

For convenience, let’s write the problem again.

$$\max \sum_{t=0}^{\infty} \rho^t \left( c_{A,t}^{\theta_A} c_{M,t}^{\theta_M} \right), \quad \theta_A + \theta_M < 1, \rho < 1 \quad (2.21)$$

subject to

$$L \cdot c_{A,t} \leq Y_{A,t} = Z_A \left( K_{A,t}^{\beta_A} L_{A,t}^{\alpha_A} X^{\chi} \right)^{\frac{1}{\beta_A + \alpha_A + \chi}}$$

$$L \cdot \left( c_{M,t} + k_{t+1} - (1-\delta)k_t \right) \leq Y_{M,t} = Z_M \left( K_{M,t}^{\beta_M} L_{M,t}^{\alpha_M} \right)^{\frac{1}{\beta_M + \alpha_M}}$$

$$K_t = K_{A,t} + K_{M,t}$$

$$L = L_{A,t} + L_{M,t} \quad (2.22)$$

Now, the question is what happens in a stationary state if the rise of a substitution technology, $\Delta \beta_M > 0$, decreases labor shares in manufacture, $\Delta \frac{\alpha_M}{\beta_M + \alpha_M} < 0$. First, factor price equalization tells us that labor forces in the city increases $\Delta L_A > 0$ as $\Delta \beta_M > 0$ such that

$$\frac{L_A}{L} = \left( 1 + \frac{\beta_A + \alpha_A + \chi}{\beta_M + \alpha_M} \cdot \frac{\alpha_M}{\alpha_A} \cdot \frac{\theta_M}{\theta_A} \right)^{-1} \quad (2.23)$$

\[\text{I assume } \delta = 0 \text{ for brevity. In the model, an increase in the depreciation rate of information technology can be implicitly captured by the large relative price of the substitution technology } \xi \text{ governing the efficiency of a substitution technology, } f \left( \frac{K_s}{L} \right)^{\xi}. \text{ Note that a production unit adopts the substitution technology } f \text{ only if } \xi \text{ is small enough to be profitable.}\]
Second, land price in the city increases. Note that a substitution technology in manufacture is endogenous in that the technology is used only if adopting it produces more than before such that \[ \Delta (P_M Y_M) > 0. \] By construction, an increase in manufactured goods means an expansion of non-manufacture industries such that \[ \Delta Y_A = \frac{\theta_A}{\theta_M} \Delta (P_M Y_M) > 0. \] Given the Cobb-Douglas production function in an economy, both the land share of income and the land supply in the city are fixed such that \[ \Delta \chi = \Delta X = 0. \] Thus, the relative price of land should increase with the rise of the substitution technology in manufacture such that

\[
\Delta \left( \frac{Y_A - rK_A - wL_A}{X} \right) = \frac{\chi}{\beta_A + \alpha_A + \chi} \frac{\Delta Y_A}{X} > 0 \quad \text{given} \quad \Delta \beta_M > 0. \tag{2.24}
\]

Lastly, the equilibrium wage decreases with declining labor shares in manufacture. As can be seen above, the production of non-manufacture industries increases such that \[ \Delta Y_A > 0 \] as \[ \Delta \beta_M > 0, \] and hence capital incomes in the city where non-manufacture industries are dominant also increase such that \[ \Delta (rK_A) = \frac{\beta_A}{\beta_A + \alpha_A + \chi} \Delta Y_A > 0. \] Given the interest rate fixed in a stationary state, this implies that capital in the city is accumulated such that \[ \Delta K_A > 0. \] Note that the land supply in the city is fixed \[ \Delta X = 0, \] that the ratio of land to capital decreases \[ \Delta \left( \frac{X}{K_A} \right) < 0, \] and that the ratio of labor to capital increases \[ \Delta \left( \frac{L_A}{K_A} \right) > 0 \] in order to keep the interest rate fixed. As a result, the equilibrium wage \( w \) decreases such that

\[
w = \frac{\alpha_A rK_A}{\beta_A L_A}, \quad r = \frac{\beta_A}{\beta_A + \alpha_A + \chi} Z_A \left( \frac{L_A}{K_A} \right) \frac{\alpha_A}{\beta_A + \alpha_A + \chi} \left( \frac{X}{K_A} \right) \frac{\chi}{\beta_A + \alpha_A + \chi} \tag{2.25}
\]

\[
\Delta w = \frac{\alpha_A}{\beta_A} r \cdot \Delta \left( \frac{K_A}{L_A} \right) < 0. \tag{2.26}
\]

It is the fixed supply of land in the city that decreases the equilibrium wage and makes land a scarcer factor when labor forces are flooded into the city. Thus, in contrast to the substitution of capital for labor and land in agriculture, the recent substitution of capital

---

\( P_M \) is the relative price of manufactured goods. I use the price of non-manufactured goods as a numeraire because no technological changes occur in non-manufacture industries.
for labor in manufacture can decrease the equilibrium wage.

Note that the equilibrium wage $w$ can increase with the productivity of non-manufacture, $Z_A$. A technology raising $Z_A$ can neutralize the declining wage derived from the declining labor shares in manufacture because an increase in $Z_A$ can accumulate capital to keep the interest rate $r$ fixed and push down the ratio of labor to capital, $\frac{L_A}{K_A}$. Given that information technology is not only a substitution technology that decreases labor shares but also a general technology that increases TFPs, it is worth studying whether advances in information technology are applied more to the substitution of capital for labor or more to the increase in TFPs.7

2.5 Conclusion

The labor share of income has significantly declined since the early 1980’s in the United States (Elsby et al., 2013), and this declining pattern has occurred in many countries (Karabarbounis and Neiman, 2014a,b). Declining labor shares are often understood as a consequence of information technology that substitutes capital for labor given a production function with an elasticity of substitution greater than one. Borrowing from Houthakker (1955) and Jones (2005), I show that labor shares can decrease without the help of an elasticity of substitution greater than one and that a Cobb-Douglas production function can be derived given a technology that substitutes capital for labor.

With this observation, I study economic consequences of declining labor shares in manufacture. A simple two-sector model is used to show that declining labor shares in manufacture

---

7Note that a substitution technology in the model changes not only the shape parameters of Pareto distributions, $\alpha$ and $\beta$, but also the minimum factor augmenting technologies, $\gamma_a$ and $\gamma_b$. In addition, The model generating a Cobb-Douglas production function tells us that the TFP of a production function increases with the minimum factor productivities such that $F(K,L) \sim \left( \gamma_a^\alpha \gamma_b^\beta \right)^{\frac{1}{\gamma_a+\gamma_b}}$. Thus, adopting a technology that substitutes capital for labor in one sector is likely to increase the TFP of the sector but not the TFPs of the other sectors.
can result in a widening dispersion of labor productivities, a declining labor price, and a rising land price. These changes are often speculated to be closely related, but underlying forces that shape these concurrent changes, if any, have been unclear (Jones, 2015). This paper proposes a possible link among these changes through the lens of a technology that substitutes capital for labor in manufacture.
Chapter 3

Synchronized Durable Goods

Purchases and the Business Cycle

3.1 Introduction

This paper examines the possibility that changes in the number of households simultaneously purchasing durable goods can create a business cycle. I build a model in which households optimize their timings of durable goods purchases given adjustment costs, which can be derived from the indivisibility and the irreversibility of durable goods.

In the model, a household can deliver its wealth from today to the future by holding money to purchase durable goods, and a household’s optimal timing of replacing durable goods depends on the rate of return on money. Thus, a shock common to all households such as a unexpected change in the inflation rate can alter the timing distribution of durable goods purchases across households and create aggregate demand fluctuation that propagates over time.

For instance, suppose that an economy is hit by a unexpected shock such that the government increases an expected inflation rate by printing money more than what it has an-
nounced. This unexpected increase in the inflation rate decreases the rate of return on money and forces households to readjust their optimal timings of durable goods purchases. Given the lower rate of return on money, some households that save too much are bunched into a group that purchases durable goods as soon as possible. As a result, the aggregate demand of durable goods can rise in the short run and stimulate the economy. However, the bunching of durable goods purchases today derives an inevitable lack of aggregate demand tomorrow. Note that the homogenized, stationary schedules of durable goods purchases across households are destroyed and that the number of households simultaneously purchasing durable goods fluctuates over time. In other words, a boom stimulated by an expansionary monetary shock can be followed up by a recession.

**Related Literature**

Grossman and Laroque (1990) show that with adjustment costs, an individual’s optimal durable goods purchase can be intermittent and follow a wait-and-purchase pattern, so called a (S,s) strategy. Their intuition is clear. A continuous replacement of durable goods is expensive due to adjustment costs, and an individual can be better off by waiting and repurchasing a lumpy amount of durable goods. Flavin and Nakagawa (2008) generalize the idea by allowing non-durable goods consumption together with durable goods purchases.

Adjustment costs of durable goods affect not only individuals’ optimizations but also aggregate consumption. Caballero (1993) shows that an individual’s (S,s) pattern can explain excessive smoothness of consumption in aggregate level. This is achieved by an ergodic consumption schedule distribution across individuals. Similarly, by aggregating individual’s (S,s) behavior with consumption commitment, Chetty and Szeidl (2016) derive a habit formation model as a reduced form. These studies exhibit the fact that an individual’s lumpy durable goods consumption can give a macroeconomic implication, which is barely captured by standard assumptions such as convex consumption sets without adjustment costs.
Lastly, the idea of synchronized durable goods purchases as a source of the business cycle has been studied by several economists. To my knowledge, De Gregorio, Guidotti, and Vegh (1998) are the first to present a model of synchronized durable goods purchases. They show that an inflation stabilization policy can have significant effects on the business cycle through the bunching of durable goods purchases. Leahy and Zeira (2005) use an overlapping generation model to show that even allowing for non-durable goods consumption, the bunching of durable goods purchases still have sizable effects on aggregate demand.

3.2 A Model

Suppose that an economy consists of $H$ number of infinitely-lived households. Each household supplies its labor $n(t)$ and earns wage $w(t)$ such that $w(t)n(t)$. It derives utility from consuming the flow of non-durable goods $c(t)$ and holding the stock of durable goods $s(t)$. Durable goods a household holds depreciate over time with the rate of $\delta$ such that $ds(t) = -\delta s(t)dt$. Suppose that the stock of durable goods $s(t)$ is indivisible and irreversible in the sense that a household has to sell its existing stock of durable goods and purchase a new one in order to adjust the existing stock of durable goods.

Let’s assume that the economy has an aggregate production technology $F$ such that

$$F(N, S) = \bar{w}N + (1 - \kappa)S, \quad \kappa \in (0, 1),$$

(3.1)

where $N$ and $S$ are the aggregate supply of labor and the aggregate supply of second-hand durable goods households sell, respectively. $\bar{w}$ is the average labor productivity of households such that $\bar{w} = \frac{\sum_h w_h n_h}{N}$ where $N = \sum_h n_h$. Given adjustment costs captured by $\kappa \in (0, 1)$, the optimal durable goods purchase must be infrequent or intermittent over time because any continuous readjustment of durable goods can be dominated by a wait-and-consume
strategy due to the positive adjustment costs.

Then, we can write a household’s problem as follows. $T$ is the optimal timing of replacing durable goods, and $m$ and $s$ is the real money balance and the stock of durable goods a household holds at $t = 0$, respectively.

$$V(m, s) = \max_{c(t), n(t), T, m', s'} E_0 \left[ \int_0^T e^{-\rho t} \left\{ u(c(t), e^{-\delta t} s) - \chi \frac{n(t)^{1+1/\nu}}{1+1/\nu} \right\} dt + e^{-\rho T} V(m', s') \right]$$

subject to

$$m' + s' \leq E_0 \left[ (1 - \kappa) e^{-\delta T} s + \int_0^T e^{-\mu(t)(T-t)} \left\{ w(t)n(t) + \tau(t) - c(t) \right\} dt + e^{-\int_0^T \mu(t) dt} m \right]$$

$$m' \geq m,$$

where $\mu(t)$, $\tau(t)$, and $m$ are the inflation rate, the real money transfer from the government to a household, and the borrowing limit for a household, respectively. Note that the borrowing limit $m$ only works when a household purchases its durable goods. Thus, as long as a household meets the borrowing condition at $t = T$, it can consume non-durable goods without any frictions.

Suppose that the utility function $u(c, s)$ satisfies the following standard conditions,

$$u_1 > 0, \lim_{c \to 0} u_1(c, s) = \infty, u_2 > 0, \lim_{s \to 0} u_2(c, s) = \infty, u_{11} < 0, u_{22} < 0, \text{ and } u_{12} \geq 0.$$

Then, the following market clearing condition characterizes an equilibrium.

$$\sum_{h=1}^H c_h(t) + \sum_{h=1}^H s'_h \cdot 1_{T_h(t) = 0} = \sum_{h=1}^H w_h(t)n_h(t) + (1 - \kappa) \sum_{h=1}^H s_h(t) \cdot 1_{T_h(t) = 0}$$

The LHS of the equation is the demand of non-durable goods consumption and durable
goods purchases, and the RHS is the supply of labor forces and second-hand durable goods net of adjustment costs. Note that only a fraction of households with $T_h(t) = 0$ contributes both the demand and the supply of durable goods, while all households consume non-durable goods and supply labor forces.

**Proposition 3.** Suppose that the inflation rate $\mu(t)$ satisfies the following condition.

$$\mu(t) > -\rho \quad \forall t \in [0, T]$$

Then, the labor supply $n(t)$ increases and the non-durable consumption $c(t)$ decreases over time such that

$$\frac{dn(t)}{dt} > 0 \quad \text{and} \quad \frac{dc(t)}{dt} < 0.$$ 

**Proof.** Let’s rewrite the above problem as a Lagrangian such that

$$V(m, s) = E_0 \left[ \int_0^T e^{-\rho t} \left\{ u(c(t), e^{-\delta t} s) - \frac{\chi n(t)^{1+1/\nu}}{1+1/\nu} \right\} dt + e^{-\rho T} V(m', s') \right]$$

$$+ \lambda_T E_0 \left[ (1 - \kappa) e^{-\delta T} s + \int_0^T e^{-\mu(t)(T-t)} \{ w(t)n(t) + \tau(t) - c(t) \} \ dt + e^{-\int_0^T \mu(t) dt} m - m' - s' \right]$$

$$+ \lambda_m (m' - m). \quad (3.5)$$

Then, the envelope condition with respect to $m$ gives

$$V_1(m, s) = E_0 \left[ e^{-\int_0^T \mu(t) dt} \right] \lambda_T, \quad (3.6)$$

and the first order conditions with respect to $n(t)$ and $c(t)$ are

$$\chi n(t)^{1/\nu} = w(t)e^{(\mu(t)+\rho)t-\mu(t)T} \lambda_T,$$

$$u_1(c(t), e^{-\delta t} s) = e^{(\mu(t)+\rho)t-\mu(t)T} \lambda_T. \quad (3.7)$$
By substituting the envelope condition into the first order conditions,

\[
    n(t) = E_0 \left[ \exp \left\{ \nu(\mu(t) + \rho)t - \nu \mu(t)T + \nu \int_0^T \mu(x)dx \right\} \right] \left\{ \frac{w(t)}{\chi} V_1(m, s) \right\}^\nu,
\]

\[
    u_1(c(t), e^{-\delta t} s) = E_0 \left[ \exp \left\{ (\mu(t) + \rho)t - \mu(t)T + \int_0^T \mu(x)dx \right\} \right] V_1(m, s).
\]

(3.8)

Then, it is immediate that \( \frac{dn(t)}{dt} > 0 \) and \( \frac{dc(t)}{dt} < 0 \) given \( \mu(t) > -\rho \) for all \( t \in [0, T] \). \( \square \)

Proposition 3 tells us that changes in the timing distribution of durable goods purchases across households can affect not only the aggregate demand of non-durable goods consumption but also the aggregate supply of labor forces. Specifically, it shows that an increase in the number of households that are about to purchase durable goods can increase the aggregate supply of labor forces and help the model achieve an equilibrium by clearing goods market featuring the bunching of households that purchase durable goods simultaneously.

**Proposition 4.** A household’s non-durable consumption \( c(t) \) is proportional to its stock of durable goods \( s(t) \).

**Proof.** From the first order condition with respect to \( c(t) \) at \( t = 0 \),

\[
    u_{11}(c, s)dc + u_{12}(c, s)ds = 0.
\]

(3.9)

Then, given the conditions for the utility function \( u(c, s) \), it is immediate to show that

\[
    \frac{dc}{ds} = -\frac{u_{12}(c, s)}{u_{11}(c, s)} > 0.
\]

(3.10)

\( \square \)

Proposition 4 is nothing but the complementarity of a household’s utility function between non-durable goods and durable goods. Note that this complementarity, together with
intermittent durable goods purchases, can give rise to a lagged response of the aggregate non-
durable consumption $\sum_{h=1}^{H} c_h(t)$ to the aggregate durable goods purchases $\sum_{h=1}^{H} \{ s_h' \cdot 1_{T_h(t)=0} \}$.

Non-durable goods consumption can be peaked right after the peak of durable goods pur-
chases because households that have just purchased durable goods would consume more
non-durable goods with the replenished stock of durable goods.

**Proposition 5.** Suppose that $\mu(t) > -\rho$ for all $t \in [0,T]$. Then, the borrowing limit
condition is binding such that $m' = \overline{m}$ at $t = T$ when a household purchases durable goods.

**Proof.** The first order condition with respect to the optimal durable goods purchase $s'$ guarantees that the budget constraint is binding such that

$$\lambda_T = e^{-\rho T} V_2(m', s') > 0.$$ 

The first order condition with respect to the optimal money balance $m'$ at $t = T$ is

$$\lambda_T - \lambda_m = e^{-\rho T} V_1(m', s').$$

By substituting into the above equation a smooth pasting condition such that

$$V_1(m', s') = \lim_{(m, s) \rightarrow (m', s')} V_1(m, s) = e^{-\int_0^T \mu(t) dt} \lambda_T,$$

it is immediate to show that

$$\lambda_m = \lambda_T \left\{ 1 - \exp \left( -\rho T - \int_0^T \mu(t) dt \right) \right\} > 0 \quad \text{given} \quad \mu(t) > -\rho \quad \forall t \in [0, T].$$

With Proposition 5, we can manipulate $\overline{m}$ to examine the effects of financial shocks on
aggregate consumption. For example, a decrease in $\overline{m}$ can increase aggregate durable goods

purchases by increasing the number of households simultaneously purchasing durable goods in the short run. The government can continue to lower $m$ and keep an elevated level of aggregate consumption by forcing more households to purchase durable goods earlier. Once it touches the limit and there is no room for lowering $m$, however, we can guess that the bunching of households simultaneously purchasing durable goods cannot be sustained and that a recession derived from the lack of demand should be followed as a reaction of the expedited, foregone durable goods purchases.

3.3 A Stationary State

**Definition 1** (Stationary State). *Given the constant inflation rate $\mu(t) = \mu$ and the corresponding real money transfer $\tau(t) = \tau$, an economy is in a stationary state if its aggregate demand is constant over time such that*

$$
\frac{d}{dt} \left\{ \sum_{h=1}^{H} c_h(t) + \sum_{h=1}^{H} s_h(t) \cdot 1_{T_h(t)=0} \right\} = 0 \quad \forall t \geq 0.
$$

Suppose that an economy is in a stationary state with a constant labor productivity such that $w(t) = w$ for all $t \geq 0$. From Proposition 1, we can rewrite the baseline model as follows.

$$
V(m, s(0)) = \frac{1}{1 - e^{-\rho T}} \cdot \max_{T,n(0)} \int_{T}^{T} e^{-\rho t} \left\{ u(c(t), e^{-\delta t} s(0)) - \chi \frac{n(t)^{1+1/\nu}}{1 + 1/\nu} \right\} dt \quad (3.11)
$$

subject to

$$
s(0) \leq \frac{1}{1 - (1 - \kappa) e^{-\delta T}} \left\{ \int_{0}^{T} e^{-\mu(T-t)} \left\{ wn(t) + \tau - c(t) \right\} dt + (e^{-\mu T} - 1) m \right\} \quad (3.12)
$$

$$
m \leq 0 \quad \text{given}
$$
where
\[
c(t) = u_1^{-1} \left( \frac{\chi}{w} n(t)^{1/\nu} \right) s(t) = e^{-\delta t} s(0) \]
\[
n(t) = e^{(\mu + \rho) t} n(0) \]

Given the inflation rate \( \mu \), the corresponding real money transfer, \( \tau = \tau(\mu) \), is pinned down by the following market clearing condition.

\[
\left\{ 1 - (1 - \kappa) e^{-\delta T} \right\} s(0) + \int_0^T c(t) dt = \int_0^T wn(t) dt
\]

A stationary state can be used as a benchmark economy. It can be understood as a long-run equilibrium given parameters. Starting from a stationary state, thus, we can introduce a shock common to all households and study the effects of the shock on aggregate consumption.

### 3.4 Conclusion and Future Research

I develop a model in which each household optimizes its flow of non-durable goods consumption and its timing of durable goods purchase given adjustment costs. A novel feature of the model is that a household’s labor supply can be pinned down by its optimized timing of durable goods purchase. Thus, goods market can clear with the endogenous, elastic aggregate labor supply that are determined by the timing distribution of durable goods purchases across households.

Given the model developed in this paper, a numerical example should be constructed in order to solve for an equilibrium, which will be the next step of this paper. Then, we can use the model to study to what extent the extensive margin of aggregate consumption derived from the heterogeneous timings of durable goods purchases can affect the business cycle. For instance, the impact of a government policy that expedites durable goods purchases can be
examined. Some historical observations such as Hausman (2016) might be useful to test the model.
References


Appendix A

Appendix for Chapter 1

A.1 Pyramidal Business Groups and Wealth Inequality

The wealth Gini coefficient of an economy with business groups rises first and then declines as the strength of investor protection \((1 - \tau)\) improves. See the following Figure A.1.

![Figure A.1: Wealth Gini Coefficients](image)

This non-monotonic change in wealth inequality under the prevalence of business groups...
is not a trivial result because by construction, the model intends to feature a rising wealth inequality with investor protection. Given capital market imperfections, the less financial frictions allows entrepreneurs to use the more capital in production and to accumulate the more wealth. Wealth inequality in the model would follow the underlying managerial-talent inequality generated by a Markov process if no financial frictions existed. Thus, the hump-shaped Gini coefficients observed in Figure A.1 suggests that business groups hinder the talented from accumulating wealth as $\tau$ goes to zero.

Figure A.2 shows that business groups can lower a upward wealth mobility. It captures the probability of individuals moving from $a = a(6)$ to $a' \geq a(13)$ after ten periods. An individual of $a(6)$ living in an economy with business groups has a lower probability of being rich than that in an economy without business groups.

In contrast, Figure A.3 shows that pyramidal business groups can lower a downward wealth mobility. However, this effect of lowering the downward wealth mobility decreases as investor protection improves. Figure A.3 depicts the probability of individuals moving from $a = a(6)$ to $a' \leq a(12)$ after ten periods. Given the observation that the prevalence of business groups increases abruptly at $\tau = 0.6$ and persists as $\tau$ decreases, this rising downward mobility from $\tau = 0.5$ teaches us that creating a business group as an insurance
for the rich becomes less efficient as $\tau$ decreases. Remember that the probability of launching a subsidiary firm is decreasing as $\tau$ decreases, which can be seen in Figure 1.7.

With the low upward wealth mobility and the U-shaped downward wealth mobility, business groups result in the non-monotonic population of the rich with respect to $\tau$. Figure A.4 captures the population of the rich increases first and then decreases as investor protection $(1 - \tau)$ improves. Note that the population of the rich follows the similar pattern with the capital stock (Figure 1.11) and with the net aggregate investment (Figure 1.13) in an economy with business groups. This implies that a lower upward wealth mobility can be a symptom of inefficiencies pyramidal business groups induce in an equilibrium.

![Figure A.4: Population of the Rich](image)

### A.2 Auxiliary Results of the Model

**Observation 9** (Internal Capital Markets). *Let’s define the size of internal capital markets as the sum of internal equity finance used by all business groups such that*

$$
\text{Internal Capital Markets} = \int_{o(z,a)=BG} \mathbb{E}_{a_2} \left[ k(z,a)z_2(z,a,a_2) \right] dF(z,a). \tag{A.1}
$$
Then, internal capital markets of business groups are larger than external capital markets if the degree of financial frictions is moderate ($\tau \in [0.4, 0.6]$) or low enough ($\tau = 0.01$).

An economy with business groups features sizable internal capital markets of business groups growing up early, lasting long. Figure A.5 shows that the internal capital markets are even bigger and matured earlier than external capital markets given the moderate degree of financial frictions. As can be seen in Figure A.6, controlling for aggregate output, the early development of internal capital markets makes the capital in production of an economy with business groups larger than that of an economy without business groups given the high degree of financial frictions such that $\tau \geq 0.5$. The faster growth of internal capital markets suggests that business groups can be good substitutes for underdeveloped external capital markets in the early stage of an economy where financial frictions are rampant.

Another salient observation is the longevity of internal capital markets of business groups. Figure A.5 shows that internal capital markets do not monotonically wane as financial frictions decrease. The size of internal capital markets decreases when the degree of financial frictions decreases from $\tau = 0.6$ to $\tau = 0.1$. However, it increases again when the degree of financial frictions decreases from $\tau = 0.1$ to $\tau = 0.01$. With low enough financial frictions, 

\[ \tau \in \{0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\} \]
business groups are likely to be financially unconstrained and accumulate corporate savings. The rebounding internal capital markets in Figure A.5 captures these excessive corporate savings of business groups.²

Observation 10 (Corporate Savings). Let’s define corporate savings as the sum of assets that are not used in production but re-invested in external capital markets such that

\[
\text{Corporate Savings} = \int_{o(z,a)=SA} \{1 - P^M(z, a)\} \cdot \{-k^D(z, a) \cdot \mathbb{1}_{k^F(z,a)>0}\} dF(z, a)
\]
\[+ \int_{o(z,a)=BG} \sum_{i \in \{1, 2\}} \mathbb{E}_{a_{2}} \left[-k^D(z, a|z_2, a_2) \big| P^{BG}(z_2(z, a), a_2)\right] dF(z, a). \tag{A.2}
\]

Then, given the level of aggregate output, the corporate savings of an economy with business groups are larger than those of an economy without business groups, and the gap of corporate savings between these two economies is enlarged as the degree of financial frictions decreases.

![Figure A.7: Corporate Savings](image)

Given the asymmetric financial advantage of business groups, an economy with business groups

²See Appendix A.2 for the more detailed description of corporate savings. I define corporate savings as the assets corporations hold, which are not used in production but re-invested in external capital markets for risk sharing.
groups features excessive corporate savings. Figure A.7 shows that corporate savings grow much faster in an economy with business groups than those in an economy without business groups do. It also shows that most corporate savings are piled up by business groups.

Corporate savings per se is good for an entrepreneur because the firm’s minimum cash flow increases with corporate savings in the model. However, the asymmetrically large corporate savings of business groups can induce factor misallocation in an equilibrium. With smaller financial frictions \((d\tau < 0)\), a business-group entrepreneur can raise more external equity finance \((dk_i^E > 0)\), which is used not only to reduce private finance \((dk_i^C < 0)\) for more consumption \((dc > 0)\) but also to increase corporate savings \((d(-k_D^i) > 0)\) for risk sharing, without increasing capital in production \((dk_i = 0)\). Thus, the excessive corporate savings of business groups imply that business-group entrepreneurs can save less \((dk_i^C < 0)\) by taking control of larger external capital \((d(k_i^D + k_i^E) > 0)\). Note that in an economy with business groups, capital stock (Figure A.6) is shrinking while corporate savings of business groups (Figure A.7) are soaring up as the tunneling ratio \(\tau\) goes to zero.

**Observation 11 (Aggregate Consumption and Investment Rate).** *Controlling for aggregate output, aggregate consumption in an economy with business groups is smaller than that in an economy without business groups, and accordingly the investment rate of an economy with business groups is higher than that of an economy without business groups.*
Figure A.8 shows that business groups can lower the aggregate consumption and make it stagnate even though the degree of financial frictions decreases.

The smaller aggregate consumption in an economy with business groups derives from a decrease in consumption of stand-alone entrepreneurs. Although the stand-alone entrepreneurs’ population barely changes, their consumption decreases from 0.22 to 0.11 as financial frictions decrease from $\tau = 0.5$ to $\tau = 0.1$. As the share of aggregate consumption, stand-alone entrepreneurs’ consumption accounts for 24% given $\tau = 0.5$ but only for 12% given $\tau = 0.1$. This contrasts with the rising consumption share of business-group entrepreneurs: from 13% to 17%.

Figure A.9 shows that the investment rate of an economy with business groups is significantly higher than that of an economy without business groups with $\tau < 0.4$. Since a stationary equilibrium is employed in the model, the lower consumption level makes a pair with the higher investment rate. Note that both the lower consumption level and higher investment rate of an economy with business groups become more salient as the degree of financial frictions decreases. The declining capital stock of an economy with business groups in Figure A.6 implies that the rising investment rate depicted in Figure A.9 does not increase capital stock but is used as flotation costs for propping up pyramidal ownership structures which can be seen in Figure 1.12.

A.3 Comparing the Model with Literature and Data

Replicating Masulis, Pham, and Zein (2011)

Masulis, Pham, and Zein (2011) report that the prevalence of family business groups in a country has strong negative association with the availability of external capital. They also report a positive but insignificant association between the prevalence of family business groups and the degree of investor protection. Their observations are replicated by the nu-
merical example of the model. Let’s define the prevalence of business groups as the ratio of the number of business-group firms to the number of firms incorporated, which is the definition Masulis, Pham, and Zein (2011) use in their paper.

First, Figure A.10 shows that business groups initially thrive when severe financial frictions are mitigated and that they are partially disciplined but do not vanish as moderate financial frictions attenuate. Thus, the degree of investor protection, which is the inverse of financial frictions, can have a positive but insignificant association with the prevalence of business groups.

Second, Figure A.11 shows that the prevalence of business groups moves in opposite direction with the difference in the size of external capital markets between two economies with and without business groups. This negative correlation implies that the prevalence of business groups is significantly associated with the availability of capital in an economy. Admittedly, the size of external capital markets in the model is not the exact counterpart of capital availability for stand-alone firms Masulis, Pham, and Zein (2011) use in their paper because the former captures an equilibrium output but the latter captures the supply of capital.

This negative correlation between business groups and external capital markets can be
explained by two competing channels. First, business groups can deter the development of external capital markets. Second, business groups can arise because of the underdevelopment of external capital markets.

The model developed in this paper suggests that the second channel is dominant with severe financial frictions and that the first channel is dominant with low enough financial frictions. Given severe financial frictions, an economy with business groups has larger capital stock with larger external capital markets than an economy without business groups. In contrast, given the low degree of financial frictions, an economy with business groups has smaller capital stock and smaller external capital markets than an economy without business groups. Since the prevalence of pyramidal business groups does not change in the degree of financial frictions, these reversals in capital markets imply that the direction of causality for the negative correlation can be reversed as the degree of financial frictions decreases.

Capital-to-Labor Ratios Within an Industry, South Korea

Figure A.12: The Distribution of Capital-to-Labor Ratio Within Manufacture from 2010 to 2013
Faster Growth of Corporate Savings, Lower Household Consumption, and Higher Consumption of Fixed Capital

I examine three characteristics of an economy with business groups that the model predicts: faster growth of corporate savings, lower household consumption, and higher consumption of fixed capital.

First, the model shows that corporate savings in an economy with business groups grow faster than those in an economy without business groups do when financial frictions attenuate. Figure A.14 and Figure A.15 depict annual trends of changes in corporate savings for 23 countries from 2004 to 2014. The Annual trend of corporate savings for a country is estimated with changes in corporate-savings ratios, which are non-financial corporate gross savings divided by gross national disposable income collected from OECD.
I assume that the degree of financial frictions has been decreased since 2004 as the global capital markets have been expanded. I also assume that each country has its own prevalence of business groups because of country specific environment such as government regulations. Given the assumptions, the model predicts that corporate savings in a country with the higher prevalence of business groups grow faster than those in a country with the lower prevalence of business groups do.

Figure A.14, in which the prevalence of business groups is measured by the relative number of family and non-family business-group firms, shows that countries with more than 20% of the prevalence tend to have corporate savings growing faster. The average growth rate of corporate savings in countries with above 20% of business-group firms is 0.15%, and that in countries with below 20% of business-group firms is almost 0%. Figure A.15, in which the prevalence of business groups is measured by the relative number of family-group firms only, shows that there is no strict association between the prevalence of family groups and the growth rates of corporate savings. The average growth rate of corporate savings in countries with above 10% of family-group firms is 0.12%, and that in countries with below
10% of family-group firms is 0.076%.

Second, the model shows that the consumption level of an economy with business groups is significantly lower than that of an economy without business groups unless financial frictions are too severe. Figure A.16 and Figure A.17 show that the share of household consumption has negative association with the prevalence of business-group firms within the group of countries above $30,000 real GDP per capita. Note that the negative correlation disappears if real GDP per capita is less than $30,000.

![Figure A.16: Share of Household Consumption (2004, Current PPPs, Collected from PWT8.1 and Masulis et al. (2011))](image1)

![Figure A.17: Share of Household Consumption (2004, Current PPPs, Collected from PWT8.1 and Masulis et al. (2011))](image2)

Because the model predicts significant lower aggregate consumption if an economy dominated by business groups produces higher aggregate output, I divide countries into two groups, one with real GDP per capita greater than or equal to $30,000 and the other with real GDP per capita less than $30,000.\(^3\) The data of real GDP per capita and the share of household consumption is collected from Penn World Table 8.1 for 44 countries. I use the year of 2004 data points because Masulis, Pham, and Zein (2011) collect the prevalence of business groups as of 2004.

Lastly, the model predicts that the consumption of fixed capital is significantly higher in

---

\(^3\)Out of 44 countries in the sample, the number of countries with above $30,000 real GDP per capital is 17, and the number of countries with below $30,000 real GDP per capital is 27.
an economy with business groups than that in an economy without business groups unless financial frictions are too severe. Figure A.18 and Figure A.19 show that the consumption of fixed capital is positively associated with the prevalence of business groups and that the association is stronger within the group of countries above $30,000 real GDP per capita.

Note that the model employs a stationary equilibrium and so that investment in the model is equivalent to the consumption of fixed capital or capital depreciation. Given that the model predicts significantly higher investment rates of an economy with business groups if it produces higher aggregate output, countries are divided into two groups, one with real GDP per capita greater than or equal to $30,000 and the other with real GDP per capita less than $30,000. Consumption of fixed capital as a share of GDP for 23 countries in 2004 is collected from OECD and real GDP per capita in 2004 is collected from Penn World Table 8.1.

$30,000 real GDP per capita is the median of the sample.
A.4 Proof of Proposition 1

Let’s define \( \phi \in [\underline{\phi}, 1] \) and \( \nu \leq 1 \) such that

\[
  k^C = \phi a, \quad k^D = \frac{1 - \tau}{1 + r} \nu \inf_{z', \delta'} [\pi(z', \delta'|z, k)]. \tag{A.3}
\]

Then, a stand-alone entrepreneur running a publicly held corporation solves the following problem.

\[
  \mathcal{L}(z, a) = u((1 - \phi)a - s) + \beta E_{z', \delta'}[V(z', a')|z] + \lambda_s s + \lambda_\phi (\phi - \phi) + \lambda_\nu (1 - \nu) + \lambda_a (\bar{\sigma} - \sigma) \tag{A.4}
\]

where

\[
  a' = (1 + r)s + \tau \pi(z', \delta'|z, k) + (1 - \sigma)(1 - \tau) \left\{ \pi(z', \delta'|z, k) - \nu \inf_{z', \delta'} [\pi(z', \delta'|z, k)] \right\}
\]

\[
  k = \phi a - k^F + \frac{1 - \tau}{1 + r} \left\{ \sigma E_{z', \delta'}[\pi(z', \delta'|z, k)] + (1 - \sigma) \nu \inf_{z', \delta'} [\pi(z', \delta'|z, k)] \right\} \tag{A.5}
\]

\[
  \phi = \frac{k^F}{a}.
\]

To simplify notations, let’s suppress arguments of functions and operators unless there is ambiguity. The corresponding Kuhn-Tucker conditions are as follows. For the optimal private saving, \( s \),

\[
  \lambda_s s = 0,
\]

\[
  \lambda_s = u'( (1 - \phi)a - s ) - (1 + r)\beta E_v a \geq 0. \tag{A.6}
\]
For the optimal private finance, $k^C = \phi a$,

$$\lambda_\phi (\phi - \phi) = 0,$$

$$\lambda_\phi = a \left[ u'((1 - \phi)a - s) - (1 + r)\beta E V_a \right] - a\beta E [V_a \cdot \{-(1 + r) + AB\}] = \lambda_a \geq 0$$

(A.7)

where

$$A \equiv \left[ 1 - \frac{1 - \tau}{1 + r} \left\{ \sigma E_{z',\delta'} \left[ \frac{d}{dk} \pi(z',\delta'|z,k) \right] + (1 - \sigma) \nu \inf_{z',\delta'} \left[ \frac{d}{dk} \pi(z',\delta'|z,k) \right] \right\} \right]^{-1}$$

$$B \equiv \tau \frac{d}{dk} \pi(z',\delta'|z,k) + (1 - \sigma)(1 - \tau) \left\{ \frac{d}{dk} \pi(z',\delta'|z,k) - \nu \inf_{z',\delta'} \left[ \frac{d}{dk} \pi(z',\delta'|z,k) \right] \right\}.$$  

(A.8)

For the optimal external debt finance, $k^D = \frac{1 - \tau}{1 + r} \nu \inf \pi$,

$$\lambda_\nu (1 - \nu) = 0,$$

$$\lambda_\nu = (1 - \sigma) \frac{1 - \tau}{1 + r} \inf \pi \beta E [V_a \cdot \{-(1 + r) + AB\}]$$

$$= (1 - \sigma) \frac{1 - \tau}{1 + r} \inf \pi A \cdot \left\{ \frac{\beta E [V_a \cdot \{E\pi' - (1 + r)\}]}{\text{Marginal Value of Expected Return}} - \frac{(1 - \sigma + \sigma \tau) \beta E [V_a \cdot \{E\pi' - \pi'\}]}{\text{Marginal Cost of Risk}} \right\}$$

$$\geq 0$$

(A.9)

where

$$\pi' \equiv \frac{d}{dk} \pi(z',\delta'|z,k).$$
Lastly, given $\sigma > 0$, the optimal external equity finance, $k^E = \frac{\sigma}{1+r} \left\{ (1-\tau)E\pi - (1+r)k^D \right\}$, satisfies the following conditions.

$$\lambda_\sigma (\bar{\sigma} - \sigma) = 0,$$

$$\lambda_\sigma = \frac{(1-\tau)\beta E [V_\alpha \cdot \{E\pi - \pi\}]}{\beta E \left[ V_\alpha \cdot \frac{d\pi'}{d\sigma} \right]_{d(k,\sigma)=0}} > 0$$

Marginal Value of Risk Sharing

Given the Fixed Amount of Capital $k$

$$\frac{1 - \tau}{1+r} (E\pi - \nu \inf \pi) A \cdot \beta E [V_\alpha \cdot \{E\pi' - (1+r) - (1 - \sigma + \sigma \tau)(E\pi' - \pi')\}]$$

$$= \frac{d\pi}{d\sigma} \cdot \beta E \left[ V_\alpha \cdot \{E\pi - \pi\} \right]_{d(k,\sigma)=0} > 0$$

$$\geq 0$$

**Proof.** From the Kuhn-Tucker condition for $\lambda_\phi$,

$$\lambda_s = \frac{1}{a} \left\{ \lambda_\phi + |J| \lambda_\nu \right\} \text{ where } |J| = \left| \frac{d\nu}{d\phi} \right|_{d(k,\nu)=0} \frac{E\pi - \nu \inf \pi}{(1-r) \inf \pi} > 0.$$

Given the assumption that a firm is allowed to invest in a risk-free asset, the external debt finance $k^D$ is only bounded above such that $\lambda_\nu \geq 0$. If $\lambda_\nu > 0$, $\lambda_s > 0$ and the optimal private saving is bounded below such that $s = 0$. If $\lambda_\nu = 0$, $\lambda_s = \frac{\lambda_\phi}{a}$ and the optimal private saving and the optimal private finance are undetermined because the marginal costs of them are aligned such that $1_{\lambda_s > 0} = 1_{\lambda_\phi > 0}$. Thus, the zero private saving, $s = 0$, is weakly preferred and the optimization can be achieved by choosing $\{\phi, \nu, \sigma\}$ with $s = 0$.

Given Condition 1 and $\lambda_\nu \geq 0$, the marginal value of external equity finance is always greater than zero such that

$$\lambda_\sigma = (1-\tau)\beta E [V_\alpha \cdot \{E\pi - \pi\}] + \left| \frac{d\nu}{d\sigma} \right|_{d(k,\sigma)=0} \cdot \lambda_\nu$$

$$> 0.$$
Thus, given $\sigma > 0$, the optimal external equity finance is bounded above such that $\sigma = \bar{\sigma}$.

Figure A.20 shows that the optimal external equity finance is binding. Given the entrepreneur’s managerial talent and wealth, $(z, a)$, there is a downward sloping curve on which the marginal expected value of investment is zero such that $\lambda_{\nu}(\sigma, k(\sigma, \nu), a'(\sigma, \nu)|s, \phi) = 0$. From the Proposition 1, the marginal value of external equity finance is always positive on the curve such that $\lambda_{\sigma}|_{\lambda_{\nu}=0} = \beta E \left[ V_a \cdot \frac{da'}{d\sigma} \bigg|_{dk=0} \right] = (1 - \tau) \beta E \left[ V_a \cdot \{ E\pi - \pi \} \right] > 0$ because of the positive marginal benefit of risk sharing through external equity finance. The entrepreneur, thus, sells her firm’s shares as many as possible until the constraint for the external equity finance is binding such that $\sigma = \bar{\sigma}_{SA}$.

![Figure A.20: Risk Sharing and Binding External Equity Finance](image)

Second, Figure A.21 shows how the optimal private saving becomes zero. The risk-free investment opportunity keeps the marginal opportunity cost of private saving greater than or equal to that of private finance such that $a\lambda_s \geq \lambda_{\phi} \geq 0$. Given $a\lambda_s \geq \lambda_{\phi}$, the indifference curve $V = V(\phi, s)$ cuts from below the line of constant marginal opportunity cost of private saving, $\lambda_s = \bar{\lambda}_s(c, a')$, which is achieved by $dc(s, \phi) = da'(s, \nu(s, \phi), k(\phi, \nu)) = dk(\phi, \nu(s, \phi)) = 0$. Thus, the indifference curve is pushed down until the borrowing constraint of an entrepreneur is binding such that $s = 0$. 

102
A.5 Proof of Proposition 2

Let's define \( \phi \in \left[ \phi, 1 \right] \), \( \nu_1 \leq 1 \), and \( \nu_2 \leq 1 \) such that

\[
k^C_1 = \phi a,
\]

\[
k^D_1 = \frac{1 - \nu_1}{1 + r} \left( 1 - \sigma_2 \right) \inf_{z_1', \delta_1'} \left\{ \pi(z_1', \delta_1' | z_1, k^*_1) \right\} + (1 - \sigma_2) \left\{ (1 - \tau) \inf_{z_2', \delta_2'} \left[ \pi(z_2', \delta_2' | z_2, k^*_2) \right] - (1 + r) k^D_2 \right\},
\]

\[
k^D_2 = \frac{1 - \tau}{1 + r} \nu_2 \inf_{z_2', \delta_2'} \pi((z_2', \delta_2' | z_2, k^*_2)).
\]

(A.11)

Then, given \((z_2, w^M)\), a business-group entrepreneur with \((z_1, a)\) solves the following problem.

\[
\mathcal{L}(z_1, a | z_2, w^M) = u \left( (1 - \phi)a - s \right) + \beta \mathbb{E}_{z_1', z_2', \delta_1', \delta_2'} \left[ V(z_1', a') | z_1 \right]
\]

\[+ \lambda_s s + \lambda_\phi (\phi - \phi) + \lambda_{k^C_2} \left( k^C_2 - k^F - w^M \right)
\]

\[+ \lambda_{\nu_1} (1 - \nu_1) + \lambda_{\nu_2} (1 - \nu_2) + \lambda_{\sigma_1} (\bar{\sigma} - \sigma_1) + \lambda_{\sigma_2} (\bar{\sigma} - \sigma_2)
\]

(A.12)
where
\[
a' = (1 + r)s + \tau \pi(z'_1, \delta'_1|z_1, k^*_1) + (1 - \sigma_1)(1 - \tau) \pi(z'_1, \delta'_1|z_1, k^*_1)
- (1 - \sigma_1)(1 - \tau) \nu_1 \left\{ \inf_{z'_2, \delta'_2} \left[ \pi(z'_1, \delta'_1|z_1, k^*_1) \right] + (1 - \sigma_2)(1 - \tau) \inf_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] \right\} 
+ \tau \pi(z'_2, \delta'_2|z_2, k_2) + (1 - \sigma_1 + \sigma_1 \tau)(1 - \sigma_2)(1 - \tau) \left\{ \pi(z'_2, \delta'_2|z_2, k_2) - \nu_2 \inf_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] \right\}
\]
\[
k^*_1 = \phi a - k^F - k^C + \frac{1 - \tau}{1 + r} \left\{ \sigma_1 E_{z'_1, \delta'_1} \left[ \pi(z'_1, \delta'_1|z_1, k^*_1) \right] + (1 - \sigma_1) \nu_1 \inf_{z'_1, \delta'_1} \left[ \pi(z'_1, \delta'_1|z_1, k^*_1) \right] \right\} 
+ \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} \sigma_1 \left\{ E_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] - \nu_2 \inf_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] \right\} 
+ \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} (1 - \sigma_2) \nu_1 (1 - \nu_2) \inf_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] 
\]
\[
k_2 = k^C_2 - k^F - w^M + \frac{1 - \tau}{1 + r} \left\{ \sigma_2 E_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] + (1 - \sigma_2) \nu_2 \inf_{z'_2, \delta'_2} \left[ \pi(z'_2, \delta'_2|z_2, k_2) \right] \right\}.
\]
(A.13)

The corresponding Kuhn-Tucker conditions are as follows. Let’s suppress arguments of functions and operators for simplicity unless there is ambiguity. For the optimal private saving, \(s\),
\[
\lambda_a s = 0
\]
\[
\lambda_s = u' ((1 - \phi) a - s) - (1 + r) \beta EV_a \geq 0.
\]
(A.14)

For the optimal private finance of Firm 1, \(k^C_1 = \phi a\),
\[
\lambda_\phi (\phi - \phi) = 0
\]
\[
\lambda_\phi = a \left\{ u' ((1 - \phi) a - s) - (1 + r) \beta EV_a \right\}_{\lambda_s} - a \beta E \left\{ V_a \cdot \{(1 + r) + A_1 B_1\} \right\}
\]
\[
\geq 0
\]
(A.15)
where

\[
A_1 \equiv \left[ 1 - \frac{1 - \sigma_1}{1 + r} \left\{ \sigma_1 E_{\pi_1^\prime} \hat{\pi}_1^\prime + (1 - \sigma_1) \nu_1 \inf_{z_1^\prime, \delta_1^\prime} \pi_1^\prime \right\} \right]^{-1}
\]

\[
B_1 \equiv \tau \hat{\pi}_1^\prime + (1 - \sigma_1)(1 - \tau)(\hat{\pi}_1^\prime - \nu_1 \inf \pi_1^\prime)
\]

\[
\pi_1^\prime \equiv \frac{d}{dk_1^\ast} \pi_1(z_1^\prime, \delta_1^\prime | z_1, k_1^\ast).
\]

For the optimal external debt finance of Firm 1, \( k_1^D = \frac{1 - \tau}{1 + r} \nu_1 \left[ \inf \pi_1 + (1 - \sigma_2)(1 - \nu_2)(1 - \tau) \inf \pi_2 \right] \),

\[
\lambda_{\nu_1}(1 - \nu_1) = 0
\]

\[
\lambda_{\nu_2} = (1 - \sigma_1) \left[ \frac{1 - \tau}{1 + r} (\inf \pi_1 + (1 - \sigma_2)(1 - \tau)(1 - \nu_2) \inf \pi_2) \cdot \beta E [V_a \cdot \{-(1 + r) + A_1 B_1}\} \right] = (1 - \sigma_1) \left[ \frac{1 - \tau}{1 + r} \{\inf \pi_1 + (1 - \sigma_2)(1 - \tau)(1 - \nu_2) \inf \pi_2 \} A_1 \right] = \frac{d k_1^\ast}{d \pi_1^\prime}
\]

\[
\begin{cases} 
\beta E [V_a \cdot \{\hat{E}_{\pi_1^\prime} - (1 + r)\}] - (1 - \sigma_1 + \sigma_1 \tau) \beta E [V_a \cdot \{\hat{E}_{\pi_1^\prime} - \pi_1^\prime\}] \\
\text{Marginal Value of Expected Return} \quad \text{Marginal Cost of Risk}
\end{cases}
\]

\[
\geq 0.
\]

(A.17)

For the optimal external equity finance of Firm 1,

\[
k_1^E = \frac{\sigma_1}{1 + r} \left[ (1 - \tau) E \pi_1 + (1 - \tau)(1 - \sigma_2) \left\{ (1 - \tau) E \pi_2 - (1 + r) k_2^D \right\} - (1 + r) k_1^D \right],
\]
\[ \lambda_{\sigma_1}(\bar{\sigma} - \sigma_1) = 0 \]
\[
\lambda_{\sigma_1} = (1 - \tau)\beta \mathbb{E} \left[ V_0 \cdot \{ E_{\pi_1} - \pi_1 \} \right] + (1 - \tau)^2(1 - \sigma_2)\beta \mathbb{E} \left[ V_0 \cdot \{ E_{\pi_2} - \pi_2 \} \right]
\]
\[= \beta \mathbb{E} \left[ V_0 \cdot \frac{d\sigma}{d\sigma_1} \right]_{dk_1^*(\sigma_1, \nu_1) = 0} > 0 \]
\[= \frac{ak_1^*}{\pi_1} \]
\[\geq 0. \quad (A.18)\]

For the optimal internal equity finance from Firm 1 to Firm 2, \( k_2^C \),
\[
\lambda_{k_2^C}(k_2^C - k^F - \omega^M) = 0
\]
\[
\lambda_{k_2^C} = \beta \mathbb{E} \left[ V_0 \cdot \{ A_1B_1 - A_{12}A_2A_1B_1 - A_2B_2 \} \right] \quad (A.19)
\]
\[\geq 0 \]

where
\[
A_{12} \equiv \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} \left\{ \sigma_1 \left( E_{x_2, \delta_2} \pi_{2'} - \nu_2 \inf_{x_2, \delta_2} \pi_{2'} \right) + (1 - \sigma_1)\nu_1(1 - \nu_2) \inf_{x_2, \delta_2} \pi_{2'} \right\}
\]
\[
A_2 \equiv \left[ 1 - \frac{1 - \tau}{1 + r} \left\{ \sigma_2 E_{x_2, \delta_2} \pi_{2'} + (1 - \sigma_2)\nu_2 \inf_{x_2, \delta_2} \pi_{2'} \right\} \right]^{-1}
\]
\[
B_2 \equiv \tau \pi_{2'} + (1 - \sigma_1 + \sigma_1\tau)(1 - \sigma_2)(1 - \tau)(\pi_{2'} - \nu_2 \inf_{x_2, \delta_2} \pi_{2'})
\]
\[- (1 - \tau)^2(1 - \sigma_1)(1 - \sigma_2)\nu_1(1 - \nu_2) \inf_{x_2, \delta_2} \pi_{2'}. \quad (A.20)\]
For the optimal external debt finance of Firm 2, \( k_2^D = \frac{1-\tau}{1+r} \nu_2 \inf \pi_2 \),

\[
\lambda_{\nu_2} (1-\nu_2) = 0 \\
\lambda_{\nu_2} = \frac{1-\tau}{1+r} (1-\sigma_2) \inf \pi_2 \{ \tau + (1-\tau)(1-\sigma_1)(1-\nu_1) \} A_1 \\
\quad \quad = \frac{\partial \pi_1^*}{\partial \nu_2} \bigg|_{\nu_2=0} \rightarrow 0 \; \text{as} \; \tau \rightarrow 0 \; \text{with} \; \nu_1 = 1 \\
\quad \quad \cdot \beta \mathbb{E} \left[ V_a \cdot \left\{ (\mathbb{E}\pi_1' - (1+r)) - (1-\sigma_1 \tau)(\mathbb{E}\pi_1' - \pi_1') \right\} \right] \\
\quad \quad = \lambda_{k_2^D} \\
\geq 0.
\]

For the optimal external equity finance of Firm 2, \( k_2^E = \frac{\sigma_2}{1+r} \left\{ (1-\tau)\mathbb{E}\pi_2 - (1+r)k_2^D \right\} \),

\[
\lambda_{\sigma_2} (\bar{\sigma} - \sigma_2) = 0 \\
\lambda_{\sigma_2} = \beta \mathbb{E} \left[ V_a \cdot \frac{1-\tau}{1+r} (\mathbb{E}\pi_2 - \nu_2 \inf \pi_2) \right. \\
\quad \quad \cdot \left\{ (-1 + A_1 B_1 + A_2 B_2 + \{ \tau + (1-\tau)(1-\sigma_1)(1-\nu_1) \}) \{ -(1+r) + A_1 B_1 \} \right\} \\
\quad \quad = |J| \lambda_{\nu_2} \; \text{where} \; |J| = \frac{\partial \pi_2}{\partial \nu_2} \bigg|_{\nu_2=0} = \frac{\mathbb{E}\pi_2 - \nu_2 \inf \pi_2}{\tau = \sigma_2 \inf \pi_2} \\
\quad \quad + \frac{(1-\tau)^2}{1+r} (1-\sigma_1) \nu_1 (\mathbb{E}\pi_2 - \inf \pi_2) A_1 \cdot \beta \mathbb{E} \left[ V_a \cdot \left\{ (\mathbb{E}\pi_1' - (1+r)) - (1-\sigma_1 \tau)(\mathbb{E}\pi_1' - \pi_1') \right\} \right] \\
\quad \quad = \frac{\partial \pi_1^*}{\partial \sigma_2} \bigg|_{\sigma_2=0} \\
\quad \quad + \beta \mathbb{E} \left[ V_a \cdot (1-\tau)(1-\sigma_1 + \sigma_1 \tau) \{ \mathbb{E}\pi_2 - \pi_2 \} \right] \\
\quad \quad = \beta \mathbb{E} \left[ V_a \frac{d\pi_1'}{d\sigma_2} \bigg|_{\sigma_1=\nu_2=0} \right] \\
\quad \quad \text{Marginal Value of Risk Sharing Through Firm 2 Given Capital} \; (k_1^*, k_2) \\
\geq 0.
\]
Proof. From the Kuhn-Tucker condition for $\lambda_{\phi}$,

$$
\lambda_s = \frac{1}{a}\{\lambda_{\phi} + |J| \lambda_{\nu_1}\} \text{ where } |J| = \left| \frac{d\nu_1}{d\phi} \right|_{dk_1^* = 0} > 0.
$$

Given the assumption that firms are allowed to invest in a risk-free asset, the external debt finance of Firm 1 is only bounded above such that $\lambda_{\nu_1} \geq 0$. If $\lambda_{\nu_1} > 0$, $\lambda_s > 0$ and the optimal private saving is bounded below such that $s = 0$. If $\lambda_{\nu_1} = 0$, $\mathbb{1}_{\lambda_s} = \mathbb{1}_{\lambda_{\phi}}$ and the optimal private saving and the optimal private finance are undetermined unless they are binding together. Thus, the zero private saving is weakly preferred and the optimization can be achieved with $s = 0$.

From the Kuhn-Tucker condition for $\lambda_{\nu_2}$,

$$
\lambda_{\nu_1} = C \cdot \lambda_{k_2^C} + D \cdot \lambda_{\nu_2}, \quad C, D > 0 \text{ given } \tau > 0.
$$

Since firms are allowed to invest in a risk-free asset, the external debt finance of Firm 2 is only bounded above such that $\lambda_{\nu_2} \geq 0$. If $\lambda_{\nu_2} > 0$, $\lambda_{\nu_1} > 0$ and the optimal external debt finance of Firm 1 is bounded above such that $\nu_1 = 1$. If $\lambda_{\nu_2} = 0$, $\mathbb{1}_{\lambda_{\nu_1}} = \mathbb{1}_{\lambda_{k_2^C}}$ and the optimal external debt finance of Firm 1 and the optimal internal equity finance are undetermined unless they are binding together. Thus, the full external debt finance of Firm 1 is weakly preferred and the optimization can be achieved with $\nu_1 = 1$.

Given Condition 2 and $\lambda_{\nu_1}, \lambda_{\nu_2} \geq 0$, the marginal values of external equity finance of Firm 1 and Firm 2 are always greater than zero such that,

$$
\lambda_{\sigma_1} \geq \beta \mathbb{E} \left[ V_a \cdot \left. \frac{da'}{d\sigma_1} \right|_{dk_1^* = 0} \right] > 0, \\
\lambda_{\sigma_2} \geq \beta \mathbb{E} \left[ V_a \cdot \left. \frac{da'}{d\sigma_2} \right|_{dk_1^* = dk_2^* = 0} \right] > 0.
$$

108
Thus, the optimal external equity finance is binding such that $(\sigma_1, \sigma_2) = (\bar{\sigma}, \bar{\sigma})$.

The intuition of Proposition 2 is similar to that of Proposition 1. Given the non-negative value of investment, the risk sharing motive makes an entrepreneur to sell both her shares of Firm 1 and Firm 1’s shares of Firm 2 as many as possible. Thus, the constraints for the external equity finance of Firm 1 and Firm 2 are binding.

Moreover, the risk-free investment opportunity of firms makes an entrepreneur to take advantage of external debt finance of Firm 1 and carry it over into Firm 2. It is entrepreneur’s relegated saving in the sense that the risk-free cash flow of Firm 2 is diverted out to the entrepreneur due to financial frictions. Note that financial frictions are required to link $\lambda_{\nu_1}$ and $\lambda_{\nu_2}$. If $\tau = 0$, the Kuhn-Tucker conditions are collapsed into $\lambda_{\nu_2} = \lambda_{k^C_2} = 0$ regardless of $\lambda_{\nu_1}$ and the full external debt finance of Firm 1 is not guaranteed anymore.

The following Figure A.22 shows that the borrowing constraint for Firm 1 is binding. The risk-free investment opportunity of Firm 2 keeps the marginal value of external debt finance of Firm 1 is greater than or equal to the marginal opportunity cost of internal equity finance such that $\lambda_{\nu_1} \geq C\lambda_{k^C_2} \geq 0$. Given $\lambda_{\nu_1} \geq C\lambda_{k^C_2}$, the indifference curve $V = V(\nu_1, k^C_2)$ cuts from above the curve of constant marginal value of external debt finance of Firm 1, $\lambda_{\nu_1} = \lambda_{\nu_1}(k^*_1, a')$, which is achieved by $dk^*_1(\nu_1, k^C_2, \nu_2(\nu_1, k^C_2), k^*_2) = dk^*_2(k^C_2, \nu_2(\nu_1, k^C_2)) = da'(\nu_1, \nu_2(\nu_1, k^C_2), k^*_1, k^*_2) = 0$. Thus, the indifference curve is pushed up until the borrowing constraint of Firm 1 is binding such that $\nu_1 = 1$. 

109
A.6 Some Algebra

The following algebra is omitted in the above entrepreneur’s problem for brevity.

A Stand-Alone Entrepreneur’s Problem

From $\lambda_\nu \geq 0$, $\lambda_\nu \geq C \lambda_{k_2}$,

$$-(1 + r) + AB = A \left[ -(1 + r)A^{-1} + B \right]$$

$$= A \left[ -(1 + r) + (1 - \tau) \{ \sigma \mathbb{E}\pi' + (1 - \sigma)\nu \text{inf} \pi' \} + (1 - \sigma + \sigma\tau)\pi' - (1 - \sigma)(1 - \tau)\nu \text{inf} \pi' \right]$$

$$= A \left[ \mathbb{E}\pi' - (1 + r) - (1 - \sigma + \sigma\tau)(\mathbb{E}\pi' - \pi') \right]$$

$$= A \left[ \frac{\nu_1}{k_1^*} \lambda_{k_2} \right]$$

$$\lambda_\nu = \frac{\lambda_\nu}{k_1^*} \lambda_{k_2} \geq 0$$

$$\frac{d\nu_1}{d(k_2^*/k_1)} \bigg|_{\lambda_\nu = \lambda_{k_1^*}} = \frac{(k_1^*)^2}{C k_1^*}$$

$$V = \overline{V}(k_2^*, \nu_1), \nabla V = \left( -\frac{(k_1^*)^2}{k_1^*} \lambda_{k_2}, \lambda_{k_1^*} \right)$$

Figure A.22: Non-negative Marginal Expected Value of Investment and Binding External Debt Finance of Firm 1
where

\[
A \equiv \left[ 1 - \frac{1 - \tau}{1 + r} \{ \sigma E \pi' + (1 - \sigma) \nu \inf \pi' \} \right]^{-1} \tag{A.24}
\]

\[
B \equiv \tau \pi' + (1 - \sigma)(1 - \tau) \{ \pi' - \nu \inf \pi' \}.
\]

From \(\lambda_\sigma \geq 0\),

\[
A^{-1} dk(\nu, \sigma) = \frac{1 - \tau}{1 + r} (1 - \sigma) \inf \pi \nu d\nu + \frac{1 - \tau}{1 + r} (E \pi - \nu \inf \pi) d\sigma
\]

\[
\left. \frac{d\nu}{d\sigma} \right|_{dk(\nu, \sigma) = 0} = - \frac{E \pi - \nu \inf \pi}{(1 - \sigma) \inf \pi}
\]

and

\[
\left. \frac{da'(\nu, \sigma)}{d\sigma} \right|_{dk = 0} = -(1 - \sigma)(1 - \tau) \inf \pi \nu d\nu - (1 - \tau)(\pi - \nu \inf \pi) d\sigma
\]

\[
\left. \frac{da'(\nu, \sigma)}{d\sigma} \right|_{dk = 0} = -(1 - \sigma)(1 - \tau) \inf \pi \cdot \left. \frac{d\nu}{d\sigma} \right|_{dk = 0} - (1 - \tau)(\pi - \nu \inf \pi)
\]

\[
= (1 - \tau)(E \pi - \nu \inf \pi) - (1 - \tau)(\pi - \nu \inf \pi)
\]

\[
= (1 - \tau)(E \pi - \pi).
\]

In the proof of Proposition 1,

\[
A^{-1} dk(\phi, \nu) = a \phi + \frac{1 - \tau}{1 + r} (1 - \sigma) \inf \pi \nu d\nu
\]

\[
|J| = \left| \left. \frac{d\nu}{d\phi} \right|_{dk(\phi, \nu) = 0} \right.
\]

\[
= \left. \frac{a}{1 - \frac{\tau}{1 + r} \sigma \inf \pi} \right.
\]

The line of constant marginal opportunity cost of private saving,

\[
\lambda_s = \lambda_s(c(s, \phi), a'(s, \nu(s, \phi), k(\phi, \nu))|\sigma),
\]
is derived by solving for the following system of equations

\[
dc(s, \phi) = -ds - ad\phi = 0
\]
\[
A^{-1}dk(\phi, \nu) = ad\phi + \frac{1 - \tau}{1 + r}(1 - \sigma) \inf \pi d\nu = 0
\]  \hspace{1cm} (A.28)
\[
da'(s, \nu)|_{dk=0} = (1 + r)ds - (1 - \sigma)(1 - \tau) \inf \pi d\nu = 0
\]

such that

\[
ad\phi = -ds = -\frac{1 - \tau}{1 + r}(1 - \sigma) \inf \pi d\nu.
\]  \hspace{1cm} (A.29)

Note that \( da' = 0 \) is redundant with \( dc = dk = 0 \).

**A Business-Group Entrepreneur’s Problem**

From \( \lambda_{\sigma_1} \geq 0 \),

\[
A^{-1}k^*_1(\nu_1, \sigma_1)|_{dk_2=0} = \left\{ \frac{1 - \tau}{1 + r}(1 - \sigma_1) \inf \pi_1 + \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r}(1 - \sigma_1)(1 - \nu_2) \inf \pi_2 \right\} d\nu_1
\]
\[
+ \left\{ \frac{1 - \tau}{1 + r}(E\pi_1 - \nu_1 \inf \pi_1) + \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r}(E\pi_2 - \nu_2 \inf \pi_2) - \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r}\nu_1(1 - \nu_2) \inf \pi_2 \right\} d\sigma_1
\]
\[
da'(\nu_1, \sigma_1)|_{dk^*_1=dk_2=0} = \left\{ -(1 - \sigma_1)(1 - \tau) \inf \pi_1 - (1 - \tau)^2(1 - \sigma_1)(1 - \sigma_2)(1 - \nu_2) \inf \pi_2 \right\} d\nu_1
\]
\[
+ \left\{ -(1 - \nu_1 \inf \pi_1) + (1 - \tau)^2(1 - \sigma_2)\nu_1(1 - \nu_2) \inf \pi_2 - (1 - \tau)^2(1 - \sigma_2)(\pi_2 - \nu_2 \inf \pi_2) \right\} d\sigma_1.
\]  \hspace{1cm} (A.30)

Adding to the bottom equation the upper equation multiplied by \((1+r)\) with taking \( dk^*_1 = 0 \),

\[
da'|_{dk^*_1=dk_2=0} = \left\{ (1 - \tau)(E\pi_1 - \pi_1) + (1 - \tau)^2(1 - \sigma_2)(E\pi_2 - \pi_2) \right\} d\sigma_1
\]
\[
\frac{da'}{d\sigma_1}|_{dk^*_1=dk_2=0} = \left\{ (1 - \tau)(E\pi_1 - \pi_1) + (1 - \tau)^2(1 - \sigma_2)(E\pi_2 - \pi_2) \right\}.
\]  \hspace{1cm} (A.31)
From $\lambda_{\nu_2}$,

$$A_2^{-1}dk_2(k_c^2, \nu_2) = dk_2^C + \frac{1 - \tau}{1 + r} (1 - \sigma_2) \inf \pi_2 d\nu_2$$

and

$$A_1^{-1}dk_1^*(k_c^2, \nu_2)\bigg|_{dk_2=0} = -dk_2^C + \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} \inf \pi_2 \{-\sigma_1 - (1 - \sigma_1)\nu_1\} d\nu_2$$

$$\frac{dk_1^*(k_c^2, \nu_2)}{d\nu_2}\bigg|_{dk_2=0} = -A_1 \frac{dk_2^C}{d\nu_2}\bigg|_{dk_2=0} - \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} \inf \pi_2 \{\sigma_1 + (1 - \sigma_1)\nu_1\} A_1$$

$$= \frac{1 - \tau}{1 + r}(1 - \sigma_2) \inf \pi_2 \{\tau + (1 - \tau)(1 - \sigma_1)(1 - \nu_1)\} A_1.$$  \hfill (A.33)

From $\lambda_{\sigma_2}$,

$$A_1^{-1}dk_1^*(\nu_2, \sigma_2)\bigg|_{dk_2=0} = -\frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} \{\sigma_1 + (1 - \sigma_1)\nu_1\} \inf \pi_2 d\nu_2$$

$$- \frac{(1 - \tau)^2}{1 + r} \{\sigma_1(E\pi_2 - \nu_2 \inf \pi_2) + (1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2\} d\sigma_2$$

$$A_2^{-1}dk_2(\nu_2, \sigma_2) = \frac{1 - \tau}{1 + r}(1 - \sigma_2) \inf \pi_2 d\nu_2 + \frac{1 - \tau}{1 + r} (E\pi_2 - \nu_2 \inf \pi_2) d\sigma_2.$$  \hfill (A.34)

Adding to the upper equation the bottom equation multiplied by $(1 - \tau)\{\sigma_1 + (1 - \sigma_1)\nu_1\}$ with taking $dk_2 = 0$,

$$A_1^{-1}dk_1^*(\nu_2, \sigma_2)\bigg|_{dk_2=0} = \frac{(1 - \tau)^2}{1 + r} \left[(E\pi_2 - \nu_2 \inf \pi_2) \{-\sigma_1 + \sigma_1 + (1 - \sigma_1)\nu_1\} - (1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2\right] d\sigma_2$$

$$= \frac{(1 - \tau)^2(1 - \sigma_1)\nu_1}{1 + r} (E\pi_2 - \inf \pi_2) d\sigma_2$$  \hfill (A.35)

$$\frac{dk_1^*(\nu_2, \sigma_2)}{d\sigma_2}\bigg|_{dk_2=0} = \frac{(1 - \tau)^2(1 - \sigma_1)\nu_1}{1 + r} (E\pi_2 - \inf \pi_2) A_1.$$
By adding up the following two equations with taking $dk_1^* = dk_2 = 0$,

\[
dk_1^*(k_2^C, \nu_2, \sigma_2) = -dk_2^C - \frac{(1 - \tau)^2}{1 + r} \{\sigma_1(E\pi_2 - \nu_2 \inf \pi_2) + (1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2\} d\sigma_2
\]
\[+ \frac{(1 - \tau)^2(1 - \sigma_2)}{1 + r} \{-\sigma_1 \inf \pi_2 - (1 - \sigma_1)\nu_1 \inf \pi_2\} d\nu_2
\]
\[
dk_2^*(k_2^C, \nu_2, \sigma_2) = dk_2^C + \frac{1 - \tau}{1 + r} (E\pi_2 - \nu_2 \inf \pi_2) d\sigma_2 + \frac{1 - \tau}{1 + r} (1 - \sigma_2) \inf \pi_2 d\nu_2,
\]

(A.36)

we can derive

\[
\frac{d\nu_2}{d\sigma_2} \bigg|_{dk_1^* = dk_2 = 0} = \frac{-\sigma_1(1 + \sigma_1\tau)(E\pi_2 - \nu_2 \inf \pi_2) + (1 - \tau)(1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2}{(1 - \sigma_2) \inf \pi_2 \{\tau + (1 - \tau)(1 - \sigma_1)(1 - \nu_1)\}}.
\]

(A.37)

Then, by substituting for $\frac{d\nu_2}{d\sigma_2} \bigg|_{dk_1^* = dk_2 = 0}$,

\[
\frac{da'(\nu_2, \sigma_2)}{d\sigma_2} \bigg|_{dk_1^* = dk_2 = 0} = \left\{(1 - \tau)^2(1 - \sigma_1)\nu_1 - (1 - \sigma_1 + \sigma_1\tau)(1 - \tau)\right\} (1 - \sigma_2) \inf \pi_2 \cdot \frac{d\nu_2}{d\sigma_2} \bigg|_{dk_1^* = dk_2 = 0}
\]
\[+ (1 - \tau)^2(1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2 - (1 - \sigma_1 + \sigma_1\tau)(1 - \tau)(\pi_2 - \nu_2 \inf \pi_2)
\]
\[= -(1 - \tau)^2(1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2 + (1 - \sigma_1 + \sigma_1\tau)(1 - \tau)(E\pi_2 - \nu_2 \inf \pi_2)
\]
\[+ (1 - \tau)^2(1 - \sigma_1)\nu_1(1 - \nu_2) \inf \pi_2 - (1 - \sigma_1 + \sigma_1\tau)(1 - \tau)(\pi_2 - \nu_2 \inf \pi_2)
\]
\[= (1 - \sigma_1 + \sigma_1\tau)(1 - \tau)(E\pi_2 - \pi_2).
\]

(A.38)

Lastly, the curve of constant marginal value of external debt finance of Firm 1,

\[
\lambda_{\nu_1} = \sum_{\nu_1} (k_1^*(\nu_1, k_2^C, \nu_2, k_2^C), k_2(k_2^C, \nu_2), a'(\nu_1, k_2^C, k_1^*, k_2(k_2^C, \nu_2))),
\]
is derived by solving for the following system of equations with taking $dk^*_1 = dk_2 = da' = 0$

$$A_1^{-1} dk^*_1(\nu_1, k_C^2, \nu_2) = -dk_C^2 + \frac{1 - \tau}{1 + r} (1 - \sigma_1) \{ \inf \pi_1 + (1 - \sigma_1)(1 - \tau)(1 - \nu_2) \inf \pi_2 \} d\nu_1$$
$$+ \frac{(1 - \tau)^2 (1 - \sigma_2)}{1 + r} \inf \pi_2 \{ -\sigma_1 - (1 - \sigma_1) \nu_1 \} d\nu_2$$
$$A_2^{-1} dk_2(k_C^2, \nu_2) = dk_C^2 + \frac{1 - \tau}{1 + r} (1 - \tau) \inf \pi_2 d\nu_2$$
$$da'(\nu_1, \nu_2) = -(1 - \sigma_1)(1 - \tau) \{ \inf \pi_1 + (1 - \sigma_2)(1 - \nu_2)(1 - \tau) \inf \pi_2 \} d\nu_1$$
$$- (1 - \sigma_2)(1 - \tau) \inf \pi_2 \{(1 - \sigma_1 + \sigma_1 \tau) - (1 - \sigma_1)(1 - \tau) \nu_1 \} d\nu_2$$

such that
$$dk_C^2 = \frac{\frac{1 - \tau}{1 + r} (1 - \sigma_1) \{ \inf \pi_1 + (1 - \sigma_2)(1 - \nu_2)(1 - \tau) \inf \pi_2 \}}{\tau + (1 - \sigma_1)(1 - \nu_1)(1 - \tau)} d\nu_1$$
$$= -\frac{1 - \tau}{1 + r} (1 - \sigma_2) \inf \pi_2 d\nu_2.$$ 

Note that $da' = 0$ is redundant with $dk^*_1 = dk_2 = 0$. 

115