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Essays on Consumer Shopping Behavior and Price Dispersion

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by

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Contents

List of Tables

Chapter 1

Introduction

1.1 Motivation

The interaction between consumers and firms is influenced by multiple information asymmetries. Most consumers do not know everything there is to know about all the various sellers that provide them with their needs and wants, but they certainly have the means-and occasionally the urge-to obtain some of this information. Moreover, the cost of obtaining this information differs from consumer to consumer. This is partly the result of inherent differences between individuals and partly caused by firm characteristics and strategies. Some consumers find it relatively costless to learn the pertinent facts about firms of interest. These consumers might not value the time spent on gathering information, they might obtain this information freely from their friends, or they might enjoy the information gathering process (i.e., shopping). Other consumers obtain information at a cost. Such consumers may find their time too precious to spend searching around, either because they are very wealthy and put a high premium on their time or because they are too poor and find it difficult to search.

In their struggle for consumer attention, firms also influence the cost of obtaining information about themselves. Consumers are daily bombarded with advertisements from a medley of sources that range from conventional media such as TV, radio, newspaper, and internet advertisement to movie product placements and human billboards. Some of these advertisements are informative—they reveal prices, product characteristics, or firm locations. Many are not. Much of advertising is used to convince consumers that they want to purchase the good and that they are better off purchasing it from the firm advertising. Nor are traditional advertisements the only way to grab, and more importantly keep, consumer attention. Firms can lure consumers in and hold on to them using pricing policies such as sales, price-matching or beating guarantees, or buy-now discounts; by offering better in-store or post purchase customer service; by having good refund policies and product warranties; or simply by being located closer to their target consumer.

Consumers are not the only ones who may enter the market lacking information about a particular firm. This may be equally true for that firm's rivals. This particular information asymmetry, along with a standard inability to engage in first degree price discrimination, drives many firm price and non-price policies. For instance, because firms cannot predict each other's prices, in the presence of consumer heterogeneity, they run sales of varying magnitudes even when selling the exact same good. The standard Bertrand outcome may be avoided as long as some consumers don't shop around (and enter the first firm they search, say a local store), but monopoly pricing does not prevail as long as high prices induce a proportion of consumers to search and firms understand that they can undercut each other. Thus, firms have to strike a pricing balance by determining how much they want to profit by extracting surplus from non-searching consumers and how much they want to improve their probability of capturing searching consumers with lower prices. In the face of competition, firms have to make similar decisions about advertising and other strategies. The choices that firms make in the presence of incomplete information may be very different from those they make when all firms know everything about all other firms.

This thesis explores how various firm policies and characteristics influence consumer shopping behavior in the presence of information asymmetries and in turn, how shopping behavior effects firm price and non-price policy. In the first essay (Chapter 2), I explore how price-matching guarantees can lead to shopping behavior that is profoundly different from that in a model without such guarantees. In the second essay (Chapter 3), jointly authored with Carmen Astorne-Figari, we explore asymmetries in firm pricing that stem from differences in consumer characteristics such as proximity to a particular firm. In the third essay (Chapter 4), also jointly authored with Carmen Astorne-Figari, we take a fundamentally different view of consumer reactions to advertising from those commonly used in the theory literature to explore the relationship between advertising and price, consumer welfare, and firm profitability. Work on the second and third essays contains equal theoretical contributions by both co-authors.

1.2 Essay 1: Price-Matching

A price-matching guarantee is a promise by one firm to set its price for a good to that of a rival firm upon consumer request (and in most cases, along with evidence of the rival's price). The standard theoretical view of this firm strategy, first established by Hay (1982) and then Salop (1986), is that it is a way to facilitate anti-competitive behavior. Price-matching guarantees are posited to create tacit price collusion between firms by eroding firm incentives to lower prices. The anti-competitive view received further attention in Belton (1987), Png and Hirshleifer (1987), Doyle (1988), Zhang (1995), and Hviid and Shaffer (1999), and prompted Edlin (1997) to call for an end to price-matching guarantees on antitrust grounds. Proponents of the anti-competitive view argue that price-matching guarantees make firms hesitant to lower prices because of fear of immediate retaliation by matching firms, they allow firms to engage in welfare reducing price-discrimination between "informed" and "uninformed" consumers, and they promote minimum product differentiation.

The anti-competitive view contrasts the typical consumer's impression of price-matching guarantees as signals of lower prices (Jain and Srivastava, 2000; Srivastava and Lurie, 2001). This observation has led many researchers to investigate potential pro-competitive effects of price-matching. One argument in favor of price-matching guarantees is that the price discrimination that stems from consumer heterogeneity can actually increase consumer welfare if enough consumers pay lower prices (Corts, 1996; Chen et al., 2001). An alternative argument is that when firms are also heterogeneous, firms can use price-matching to signal prices and consumers can take advantage of these signals either because the signals send them to the lower pricing firm or because they allow them to apply price-matching guarantees at higher pricing firms (Moorthy and Winter, 2006; Moorthy and Zhang, 2006).

In this essay I extend the literature on price-matching by showing that price-matching has several additional, previously unexplored effects on consumer shopping behavior. To do so, I endogenize consumer price search behavior ala Stahl (1989) and show that there are several changes in the way consumers shop when price-matching guarantees are available. Contrary to Stahl, consumers who have no cost of search may no longer purchase from the firm listing the lowest price because they can obtain this price elsewhere by using a price-matching guarantee. This behavior decreases firms' ability to capture consumers by running bigger sales, therefore encouraging them to raise prices. Price-matching also diminishes search activity in the sense that it makes consumers who do have a cost of search require higher price observations to induce them to search beyond the first firm. This contrasts the previous literature, which either does not allow "uninformed" consumers to search or posits that price-matching will somehow encourage them to search. As a result, firms have further impetus to raise prices. A third price increasing effect of price-matching guarantees is the prevalence of asymmetric equilibria where the proportion of consumers that begin their search in each firm is different. Using numerical analysis, I find that the price increasing effects of price-matching guarantees rise when more consumers make use of these guarantees and also in the amount of asymmetry that prevails in equilibrium. While this essay supports the price-increasing outcome of the standard view of price-matching, contrary to the previous literature, the underlying driver of increases in price is the change in consumer incentives regarding where to shop and whether or not to sample additional firms.

1.3 Essay 2: Asymmetric Search

Many economists have asked why price dispersion occurs in markets for homogeneous goods. A popular explanation suggests that consumers vary in the amount of information they possess about firm prices (Varian, 1980) or in the cost of obtaining that information (Stahl 1989; 1996; Janssen et al. 2005). This gives firms contrasting pricing motives: keep prices high to extract profit from consumers who remain relatively uninformed about prices in rival firms, lower them to attract consumers who are relatively informed. As a result, firms play mixed pricing strategies where they run sales of different magnitudes.

Because firms are frequently assumed to be homogeneous and the consumer search order is assumed to be random, in equilibrium, all firms choose prices from the same price distribution. However, in many real world market scenarios, different firms carrying the same products expect to encounter different types and numbers of consumers. Arbatskaya (2007) studies such a situation where consumers with different search costs all sample prices in the same order. Consumers keep searching as long as their cost is lower than the price difference between the current and subsequent firm. Armstrong et al. (2009) set up a model where all consumers prefer to search a prominent firm first, which henceforth has a different price from all remaining firms. However, in both papers, because all consumers have the same search order, the equilibrium is in pure strategies: that is, firms in a particular place in the search order always have a higher or lower price than other firms. Yet, if we compare two stores, say a supermarket and convenience store, which might expect to be visited by different proportions of different types of consumers, although one might expect the latter to have higher prices most of the time, it probably does not have higher prices all of the time. In this essay, we are able to study such a situation by supposing that some consumers can sample price freely at their local firm, but must pay for an additional sample elsewhere and that the proportions of local consumers in different firms are different.

In equilibrium, we find that the firm with more local consumers has higher prices on average and runs fewer sales, but that its sales can be just as large as those of the firm with fewer local consumers. This result generalizes Narasimhan's (1988) model where consumers are either fully informed or only informed of the price at one firm. However, we find that the highest price firms are willing to charge may be significantly lower than the consumer valuation for the good when the cost of sampling an additional firm is sufficiently small. An additional finding of note is that although average prices fall when the proportion of consumers who sample prices freely rises, the firm with more locals may run fewer sales. This turns out to have important implications for advertising policy to lower-income consumers, particularly with regard to traditional advertising using informative circulars.

1.4 Essay 3: Advertising

The theoretical study of advertising has been dominated by three views: the persuasive, the informative, and the complementary (Bagwell, 2007). The initial characterization of advertising conformed to the persuasive view, under which advertising alters consumers' valuations for advertised goods and creates product differentiation in consumers' minds, even when no meaningful differentiation exists (Braithwaite, 1928; Kaldor, 1950). As such, advertising was seen as harmful to consumers. The Chicago School took issue with the view, instead suggesting that advertisements serve to inform consumers—e.g., sellers' advertising messages inform buyers of their price and location (Stigler, 1961; Telser, 1964; Butters, 1977; Grossman and Shapiro, 1984; Robert and Stahl, 1993). Furthermore, Chicago economists argued that advertising should not change tastes, setting the basis for the complementary view, which states that advertising enters a stable utility function as a complement to the good advertised (Stigler and Becker, 1977; Becker and Murphy, 1993).

In this paper, we offer a new characterization of advertising. Following Bullmore (1999) and Ehrenberg et al. (2002), we view advertising as a form of publicity—that is, we believe that an important role of advertising is to get consumers to notice a product or brand in the first place and to retain it in memory during a purchasing situation. In the face of competitive clutter, consumers are likely to get confused about which brand a particular ad refers to (Keller, 1987; Burke and Srull 1988; Kent, 1993; Kent and Allen, 1993, 1994; Kumar and Krishnan, 2004). This gives advertising a "public good" quality which expands the market, along with its "brand building" effect. We analyze these two effects to study the relationship between advertising and economic outcomes. The equilibrium in our model is characterized by price dispersion. We find that in equilibrium, the level of market expanding advertising does not depend on price. This occurs because market expansion increases the number of consumers purchasing a product, but does not improve a firm's chances of making a sale to any particular consumer. The relationship between advertising which makes consumers more likely to seek out a particular brand and price is more complicated. Depending on the ratio of advertising cost to the maximum profit a firm can obtain from an additional consumer, all firms may engage in wasteful advertising, no firm might advertise, or firms may randomize between the two.

Chapter 2

Price-Matching in a Sequential Search Duopoly

2.1 Introduction

Currently, price-matching guarantees can be found in a large number of markets, including consumer electronics, office supplies, kitchen utensils, automobile accessories, medicine, and hotels, among many others. These guarantees typically come in the form of an offer by a firm to lower its price to that of a cheaper rival selling an identical good for consumers who can offer proof of the rival's price. The effect that these guarantees have on prices is frequently debated in the economics and marketing literature. On one hand, the guarantees may signal a firm's competitiveness. For instance, Walmart's price-matching policy states, "Our goal is always to be the low price leader in every community where we operate," suggesting the company is willing to lower its prices below those of its competitors.[1] On the other

[1] "Walmartstores.com: Price matching." *Walmartstores.com.* January 16, 2008. Retrieved September 26, 2010. ⟨http://walmartstores.com/7659.aspx⟩.

hand, price-matching guarantees are purported to foster price-increasing tacit collusion and price discrimination that allow firms to perpetually list high prices. Indeed, Walmart's policy seems to reinforce this view as well, stating, "there's no need for 'special sales'."[2]

The idea that price-matching guarantees could be anti-competitive was first raised by Hay (1982) and then Salop (1986), who suggest that these guarantees allow firms to immediately retaliate against rival price cuts without actually listing lower prices or expending resources to learn about competitor prices. This can lead to tacit collusion in a non-cooperative equilibrium by removing firms' incentives to cut prices. Subsequently, this idea was formalized in multiple settings: Bertrand oligopoly (Doyle 1988), differentiated products Stackelberg duopoly (Belton 1987), Hotelling duopoly (Zhang 1995), and differentiated products Bertrand duopoly where consumers incur hassle costs of applying price-matching guarantees (Hviid and Shaffer 1999). A parallel line of reasoning posits that price-matching guarantees allow firms to price discriminate between consumers with limited price information and those who are informed about multiple price quotes. For example, Png and Hirshleifer (1987) show that price-matching guarantees allow firms to keep list prices high to extract welfare from uninformed consumers, while attracting informed consumers by offering to price-match the rival firm when it offers a lower price.

A growing body of research argues that price-matching guarantees also have

[2]Empirically, price-matching has been argued as being consistent with both, lower prices (Moorthy and Winter 2006, Moorthy and Zhang 2006) and with higher prices (Hess and Gerstner 1991, Arbatskaya et al. 2006). More recently, laboratory market simulations have supported the latter view (see, for instance Fatas and Manez 2007), but show that price increases can be mitigated by firms' cost asymmetries (Mago and Pate 2009) or by hassle costs that consumers may face in exercising price-matching guarantees (Dugar and Sorensen 2006).

pro-competitive effects. Corts (1996) and Chen et al. (2001) use models with heterogeneous consumers to show that when firms use price-matching guarantees to price discriminate, some or all consumers may end up paying lower prices and consumer welfare can increase. Using surveys of consumers, Jain and Srivastava (2000) and Srivastava and Lurie (2001) argue that consumers perceive stores that offer price-matching guarantees to have lower prices. Moorthy and Winter (2006) and Moorthy and Zhang (2006) build on this argument by constructing models of price-matching with respectively, horizontal and vertical firm differentiation, where consumers consider their location or service preferences when choosing where to purchase and consumers who are uninformed about prices use price-matching as a signal that influences their price expectation for a particular firm. They show that when the difference in production costs between the two firms is sufficiently large and the uninformed population is sufficiently small, price-matching guarantees can be used to signal a low price and consumer welfare improves for a range of parameters.

While each of the aforementioned models has shown that price-matching can alter firm pricing behavior, as Moorthy and Winter point out, another allocative effect of price-matching is its impact on consumers' incentive to invest in information about prices (i.e., to price shop). Price-matching models of tacit collusion ignore this effect by assuming that all consumers are perfectly informed about firm pricing decisions. Consequently, in such models, either price-matching leads to a symmetric monopolistic outcome, something which is rarely observed in reality, or in order to avoid the monopoly result, the authors assume that products are somehow differentiated. In the latter case, product differentiation is interpreted as differences in firm locations since firms only price-match identical products. But this interpretation is suspect because price-matching guarantees generally require physical evidence of another firm's price, which should entail a cost to find for at least some subset of consumers. Most remaining models assume that consumers are heterogeneous with respect to the amount of price information that they possess. However, differences in price information are exogenously imposed: some consumers observe every price at no cost, while others stop at the first firm they sample with no regard to optimal shopping behavior.[3]

This paper departs from the previous literature by endogenizing the incentive to acquire price information and allowing consumers to engage in optimal price search. When the acquisition of price information is endogenous, consumers who would search both firms and make a purchase from the firm listing the lowest price in the absence of price-matching might use a price-matching guarantee to purchase from a firm listing a higher price. This contrasts recent papers like Chen et al. (2001) and Janssen and Parakhonyak (2009), where the effects of price-matching stem from an exogenously imposed increase in search activity by consumers who remain relatively uninformed in the absence of price-matching guarantees. Rather, the result is opposite: price-matching discourages such consumers from engaging in search activity instead of encouraging it. As a consequence, unlike in the papers above, consumers who use price-matching guarantees never expect them to yield lower prices. The purchasing behavior in this paper is more sensible because it

[3]Two exceptions include Lin (1988) and Janssen and Parakhonyak (2009). Lin sets up a highly stylized model where consumers can sample one or two firms without recall, but additional samples are infinitely costly. He concludes that firms use price-matching guarantees to set higher prices by encouraging consumers to engage in increasingly costly search activity. Janssen and Parakhonyak analyze an optimal search model where consumers only learn if a firm offers a price-matching guarantee after they sample it. For price-matching to have any effect, they require an exogenous proportion of costly searchers to freely learn the rival firm's price after making a purchase at the first firm.

tells us that the users of price-matching guarantees are consumers with a lower opportunity cost of using their time. In reality, price-matching can be a time consuming activity which only price conscious consumers engage in.

As in the literature which treats price-matching as a form of tacit collusion, price-matching in this paper unambiguously raises prices. However, by endogenizing consumer search, we avoid the monopoly outcome without assuming that products are differentiated. Moreover, the underlying mechanism behind price increases is very different. Firms do not directly respond to their competitors' price-matching guarantees. Rather, they are reacting to changes in consumer search behavior brought about by price-matching guarantees.

To endogenize the acquisition of price information within a model of price-match, we extend a duopoly version of Stahl's 1989 model of sequential consumer search by giving firms the option to offer price-matching guarantees before they set their prices for a homogenous good. There are two types of consumers in the market: those who face no opportunity cost of searching (referred to as shoppers) and those who do (non-shoppers). Consumers sample prices sequentially. When doing so, a consumer with a price at hand continues searching only as long as the marginal benefit of doing so is higher than the marginal cost. In equilibrium, firms randomize over lower prices to attract shoppers, who always search both firms and obtain the lower price, and over higher prices to realize greater profits from non-shoppers, who must consider how likely they are to get a low enough price to justify an additional search.

In this framework, price-matching guarantees bring about three price-increasing changes in consumer search behavior. First, because shoppers freely observe every price, in Stahl's original model, the firm with the lowest listed price captures all of them. However, when firms price-match, some shoppers can use a price-matching guarantee to obtain the lowest price at a firm listing a higher price. This option diminishes firms' incentive to lower prices because the lowest listed price no longer guarantees a firm will capture all shoppers. As the number of shoppers that make use of price-matching guarantees grows, the incentive to lower prices diminishes, leading to higher profits for firms and lower welfare for consumers. Since all consumers act rationally in this model, non-shoppers understand this price-increasing effect and anticipate higher prices in firms they have not sampled. Hence, a second price increasing effect arises from non-shoppers' willingness to pay a higher maximal price at the firm where they begin their search rather than pay the search cost to sample another firm's price.

A third consequence of price-matching guarantees is a multitude of asymmetric equilibria where otherwise homogenous firms have different pricing strategies. These equilibria fall into two categories: (i) those where the equilibrium outcome is such that both firms price-match, but firms focus on serving different consumer segments, and (ii) those where only one firm matches. When both firms offer price-matching guarantees, shoppers can obtain the lowest price wherever they make a purchase. As a result, in equilibrium more shoppers may buy from one firm than the other. In this case, the other firm plays a pricing strategy that attracts a larger proportion of non-shoppers, resulting in an equilibrium outcome where one firm serves more non-shoppers while the other expects to sell to more shoppers. In an outcome where only one firm offers price-matching guarantees, more non-shoppers frequent the firm without a guarantee and the other firm uses price-matching to capture more shoppers in equilibrium. As the disparity in the proportion of each consumer segment that firms serve grows, firm profits increase at the expense of consumers. The higher the proportion of non-shoppers a firm serves, the more profit it will lose from these "captive" consumers by lowering its price to attract shoppers, and the less inclined it is to do so. The upward shift in this firm's price distribution implies that the firm that focuses on catering to shoppers does not need to lower prices as much to expect to capture the same proportion of them and its price distribution shifts upward as well. Hence, the more asymmetry that price-matching entails, the greater the welfare loss to consumers.

The remainder of this paper is organized as follows. Section 2.2 sets up the model and equilibrium concept. Section 2.3 characterizes consumer search behavior. Section 2.4 solves for equilibrium when price-matching is imposed exogenously. Section 2.5 uses a numerical analysis to characterize the complete market equilibrium. Section 2.6 provides additional intuition and discusses potential extensions. Section 2.7 concludes. Section 2.8 is an appendix containing formal proofs.

2.2 Model and Equilibrium

With the exception of our framework for the acquisition of price information, our modeling assumptions are standard in the price-matching literature. Two firms, labeled 1 and 2, sell a homogenous good. Firms face no capacity constraints and have an identical constant cost of 0 of producing one unit of the good.[4] There is a unit mass of almost identical consumers with inelastic (unit) demand and valuation $v > 0$ for the good.

Consumers are a priori uninformed about prices, but they can learn about them through search. Following Stahl (1989), we assume that a proportion $\mu \in$

[4]The price of the good can be viewed as a price cost margin.

$(0,\ 1)$ of the consumers have 0 search cost. These consumers are viewed as having no opportunity cost of time and are henceforth referred to as shoppers.[5] The remaining $1-\mu$ consumers, called non-shoppers, pay search cost $c \in (0,\ v)$ for each firm they visit with the exception of the first. Search is sequential with costless second visits. After observing the price at the first firm for free, consumers decide whether or not to search the next one or to exit the market altogether. Consumers who have visited both firms may freely choose the cheapest price observed.[6]

In a model without price-matching, shoppers freely sample both prices and always buy from the firm with the lower listed price, but in a model where firms publically announce their intent to offer a price-matching guarantee, shoppers can obtain the low price elsewhere. We assume that when the two firms offer different prices, $\theta_S \in [0,\ 1]$ shoppers will ignore price-matching guarantees and always purchase from the firm with the lower listed price. The remaining $1-\theta_S$ will invoke a price-matching guarantee at the last firm they stopped in when one is available and necessary to obtain the lower price there and purchase from the firm with the lower listed price otherwise. In this setup, when one firm lists the lower price, while the other offers a price-matching guarantee, shoppers are indifferent between firms. Parameter θ_S allows us to subsume the full range of possible shopper behaviors.[7]

[5] Interpretations of shoppers in the literature are as individuals who read sales ads (Varian 1980), as consumers who derive enjoyment from shopping (Stahl 1989), as a coalition of consumers who freely share their search information (Stahl 1996), and as users of search engines (Janssen and Non 2008).

[6] One way to interpret the search cost is as a cost of finding out the price in a particular firm for the first time rather than as the cost of traveling there. Janssen and Parakhonyak (2010) show that when second visits are costly in a model of sequential price search, firms nevertheless use pricing strategies that are identical to the perfect recall case.

[7] θ_S may be interpreted as the proportion of shoppers who do not pay attention to price-match announcements or alternatively, as the proportion who face a positive hassle cost of using the guarantee, in which case, the shoppers prefer to purchase from the firm listing the lower price. Note, however, that both interpretations make the innocuous assumption that the remaining $1-\theta_S$ shoppers always employ a price-matching guarantee whenever they can.

Thus, we can imagine that shoppers use a price-matching guarantee not because it gives them a lower price than what they could get otherwise, but because they have an unmodeled intrinsic preference for a particular firm and the guarantee accords them the lowest price at that firm.

Firms and consumers play the following two-stage game. In the first stage, each firm simultaneously decides whether to adopt a price-matching guarantee. A firm that has adopted such a guarantee pre-commits itself to sell the good at the minimum listed price to consumers who have observed both prices and are hence able to invoke the guarantee. We view the costs of informing consumers of the decision to price-match as sunk. In the second stage, each firm's price-matching decision is known to all agents in the model. Firms then simultaneously choose prices, taking into consideration their beliefs about rival firm strategies as well as consumer search behavior. A pricing strategy consists of a price distribution F_i, where $F_i(p)$ represents the probability that firm i offers a price no higher than p. We denote the lower bound and upper bound of the support of the distribution for firm i as $\underline{p}_i$ and $\bar{p}_i$, respectively. After prices have been realized, consumers choose optimal search strategies given their beliefs about each F_i. Parameters v, c, μ, and θ_S, as well as the rationality of all agents in the model are commonly known.

The equilibrium concept used is Sequential Equilibrium. In this context, consumers who observe out of equilibrium prices believe that they are coming from a mixed strategy that puts a small weight on prices off the equilibrium path in such a way that optimal search behavior would dictate the same decision as if the unsampled firm did play its equilibrium strategy. Intuitively, we can think of consumers who observe an off-equilibrium price at the first firm they sample as treating such deviations as mistakes when forming beliefs about the remaining firm's strategy. That is, consumers believe that unsampled firms play their equilibrium strategies at all information sets. In equilibrium, because consumers are homogenous in all respects except for the cost of search, consumers who have no price information will either all prefer to begin their search at a particular firm or they will be indifferent between which firm to sample first. In this paper, we focus on equilibria where consumers who have no price information are indifferent between which firm to sample first and a positive fraction of both consumer types samples each firm first. As a result, in equilibrium, a fraction $\beta_S \in (0,\ 1)$ of shoppers and a fraction $\beta_N \in (0,\ 1)$ of non-shoppers begin their search at firm 1, while the remaining consumers begin their search at firm 2, where β_S and β_N are functions of F_i and the exogenous parameters.[8]

2.3 Consumer search behavior

It is instructive to first discuss consumer search behavior. Since we focus on equilibria where all consumers are indifferent regarding which firm to sample first (regardless of firm price-matching decisions), it suffices to consider each consumer's decision regarding whether or not to search the second firm. Consumers who have no costs of search will search both firms before making their purchase decision. Those consumers with search cost c will search the second firm only if the marginal expected benefit of continued search exceeds the cost. Thus, the optimal search rule is for a non-shopper who has freely observed the price at firm j to continue

[8] In an equilibrium where consumers who have no price information strictly prefer to begin search at a particular firm (if one exists) either β_S or β_N equals 0 or 1. It can be shown that for a small range of parameter values, there exists an equilibrium where only one firm offers a price-matching guarantee on the equilibrium path and all non-shoppers prefer to begin searching the non-matching firm first.

search if and only if the observed price is higher than a reservation price, r_i, which makes him indifferent between searching firm i and stopping. This reservation price is then defined as the solution to

$$\int_{\underline{p}_i}^{r_i} (r_i - p)\mathrm{d}F_i(p) = c \tag{2.1}$$

Note that reservation price r_i corresponds to consumers who begin their search at firm j and vice versa because consumers who begin at firm j must decide whether or not to search firm i based on the price they observed at firm j and their beliefs about firm i's pricing strategy. After integration by parts, Equation (2.1) becomes

$$\int_{\underline{p}_i}^{r_i} F_i(p)\mathrm{d}p = c \tag{2.2}$$

A price below r_i does not necessarily entail a purchase. In particular, if $r_i > v$, consumers who observe a price between r_i and v in firm j will not make a purchase, but they will also not proceed to firm i.

2.4 Firm pricing strategies

We begin by deriving the equilibria in the four possible subgames that follow firms' price-matching decisions: the subgame where neither firm price-matches, the subgame where both firms price-match, and the two subgames where only one firm matches. In Section 2.5 we will endogenize the price-matching decisions to analyze the complete equilibrium of the game.

2.4.1 Neither firm price-matches

Following Stahl (1989), Astorne-Figari and Yankelevich (2010) show that in the subgame without matching, there is a unique Sequential Equilibrium where both firms distribute prices over support $[(1-\mu)\,\bar{p}/(1+\mu),\ \bar{p}]$ with distribution $F(p) = [1+\mu-(1-\mu)\,\bar{p}/p]/(2\mu)$, where $\bar{p} = \min\{v,\ r^*\}$ and r^*, the equilibrium reservation price, is defined as

$$r^* = \begin{cases} r(\mu,\ c) = c\Big(1 - \frac{1-\mu}{2\mu}\ln\frac{1+\mu}{1-\mu}\Big)^{-1} & \text{if} \quad r(\mu,\ c) \le v \\ \infty & \text{otherwise} \end{cases} \tag{2.3}$$

In the model presented here, this equilibrium is unique up to β_S. Since there are no mass points in equilibrium, regardless of where shoppers search first, they will always purchase from the firm with the lower listed price. As a result, the equilibrium outcome is the same for all $\beta_S \in [0,\ 1]$ when firms do not offer price-matching guarantees. Symmetry tells us that $\beta_N = 1/2$.[9] Thus, in equilibrium, half the non-shoppers sample each firm first and since $\bar{p} = \min\{v,\ r^*\}$, they purchase from the first firm sampled without observing the price of the other firm. As a result, firms randomize over lower prices to attract shoppers, and over higher prices to extract greater profits from captive non-shoppers.

[9] Astorne-Figari and Yankelevich (2010) set β_N exogenously, whereas here, non-shoppers randomly choose which firm to sample first. However, from their paper, we know that when $\beta_N \neq 1/2$, the price distribution of the firm that more non-shoppers choose to sample first, first order stochastically dominates that of its rival. Since non-shoppers have correct beliefs about firm price distributions and because these distributions are bounded from above by $\min\{v,\ r_1^*,\ r_2^*\}$, this would imply that all non-shoppers would prefer to sample the firm with the dominated distribution first, a contradiction.

2.4.2 Both firms price-match

In this subsection, we focus only on equilibria where the supports of the firm equilibrium pricing distributions do not have any breaks. The appendix shows that for $\theta_S \in (0,\ 1]$, these supports can only take one of four types, two of which contain no breaks. We do not show whether equilibria with breaks exist, however, in the appendix, we outline a procedure for solving for one such equilibrium.

In any equilibrium without breaks, $\underline{p}_1 = \underline{p}_2 = \underline{p}$ and $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$ where $r_j \leq r_i$. The equilibrium is symmetric if and only if $\beta_S = \beta_N = 1/2$, in which case $r_j = r_i$. In this case, there are no mass points in equilibrium. As a result, both firms always run sales—that is, they price below $\bar{p}$ with certainty. Alternatively, the equilibrium may be asymmetric, in which case one firm has a mass point at $\bar{p}$. In equilibrium, non-shoppers who observe a price of r_j at firm i (and are hence indifferent between stopping and searching firm j) stop. Therefore, because firms never price above the smaller of the two reservation prices, non-shoppers never search in equilibrium. This means that in equilibrium, price-matching can only impact non-shoppers indirectly because they cannot use price-matching guarantees after observing only one price.

Proposition 2.1 characterizes Sequential Equilibria for the subgame where both firms offer price-matching guarantees and $\theta_S \in (0,\ 1]$. The proposition characterizes equilibria where $\beta_N \geq 1/2$. Equilibria with $\beta_N < 1/2$ can be similarly obtained and are left to the reader.

Proposition 2.1. *Suppose that firms are exogenously required to offer price-matching guarantees and $\theta_S \in (0,\ 1]$. Then there exists a set of Sequential Equilibria where both firms distribute prices over support*

$$\left[\underline{p}=\min\{v,r^{*}\}\left[\frac{(1-\mu)\,\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+(1-\mu)\,\beta_N}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}},\bar{p}=\min\{v,r^{*}\}\right]$$

and the equilibrium reservation price, $r^{*}=r_1^{*}=r_2^{*}$ *equals*

$$r^{*}=\begin{cases} r\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right) & if\quad r\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right)\leq v \\ \infty & otherwise\end{cases},$$

$$r\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right)=c\left\{1-\frac{\left[(1-\mu)\,\beta_N\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu\left(1-\beta_S\right)\left(1-\theta_S\right)}\right.$$
$$\left.\times\left\{\left[\mu\left(\beta_S\theta_S+1-\beta_S\right)+(1-\mu)\,\beta_N\right]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}-\left[(1-\mu)\,\beta_N\right]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}\right\}\right\}^{-1}.$$

Suppose that firm 1 has a mass point at $\bar{p}$*. Then it distributes prices according to*

$$F_1(p)=\left\{1+\frac{(1-\mu)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right\}\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{(1-\beta_S)\theta_S+\beta_S}{\theta_S}}\right]$$

over $\left[\underline{p},\bar{p}\right)$*, while firm 2 distributes prices according to*

$$F_2(p)=\left[1+\frac{(1-\mu)\,\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right].$$

In equilibrium, the expected prices for the two firms equal each other.

In equilibrium, β_N must be such that firm 1 is the one with the mass point at $\bar{p}$. Numerically, we can show that this requires that $\beta_N>1/2$ and that as a result, $\beta_S>1/2$. The case where firm 2 is the one with the mass point at $\bar{p}$ follows similarly and is left to the reader. When $\beta_N=\beta_S=1/2$, neither firm has a mass point and we are in support type 1 in Lemma 2.1.

In every equilibrium described in Proposition 2.1, non-shoppers randomly choose a firm to search first and make a purchase there, whereas all shoppers search both firms and obtain the lower price. However, contrary to the subgame without matching, the lower price is not always obtained at the firm with the lower listed price. For instance, if firm 1 has the lower listed price, the $1 - \beta_S$ shoppers that search firm 2 first all purchase from firm 1, but of the β_S shoppers that search firm 1 first, $\beta_S \theta_S$ purchase from firm 1 while $\beta_S (1 - \theta_S)$ obtain a price-match at firm 2. The inability to capture all shoppers at the lower listed price diminishes firms' incentive to set lower prices.

Proposition 2.1 tells us that price-matching guarantees offer no consumer benefit because a portion of consumers never uses them, while the remainder could procure the lower listed price with or without them. This is not as unreasonable as it seems at first glance. Individuals who find search costly are likely to find satisfying the non-pecuniary requirements that many firms impose on price-matching customers costly as well, so it should not be a surprise that such individuals do not use price-matching guarantees. Conversely, an individual who is willing to go through the "hassle" of applying a price-matching guarantee is likely to be an individual who is willing to shop around for price.

Because all shoppers will freely obtain the lowest price regardless of where they begin their search, they are indifferent regarding which firm to sample first in equilibrium. However, since the firm price distributions are functions of β_S in this subgame, shopper search behavior influences expected prices, which in turn determine which firm non-shoppers want to sample first. In equilibrium, for every β_S, there is a corresponding β_N that makes non-shoppers indifferent between which firm to sample first (by setting expected prices in the two firms equal to each other), resulting in a multiplicity of equilibria.

When β_N, $\beta_S > 1/2$, firm 1 will capture a higher proportion of non-shoppers while firm 2 expects to capture more shoppers. Firm 1 sets higher prices and has fewer sales than in the symmetric case because when it serves a higher proportion of non-shoppers, it loses more profit from these captive consumers whenever it lowers its price. Even though firm 2 has fewer non-shoppers than in the symmetric case, it will tend to have higher prices as well because it no longer needs to lower prices as much to have the same probability of capturing the bulk of the shoppers as it did in the symmetric case. Thus, both firms have higher prices and expected profits than in the symmetric case. Intuitively, an asymmetric equilibrium may result in the presence of price-matching because more shoppers may prefer to purchase at a particular firm, but are lexicographic, valuing a purchase at a lower price over a purchase at a preferred firm. Price-matching will allow some shoppers to purchase at a preferred firm at the lower price even when that firm does not list the lower price. If we are to compare the two firms with each other when β_N, $\beta_S > 1/2$, firm 1 runs sales less frequently than firm 2, but since expected prices are equal in equilibrium, when firm 1 does run a sale, it is expected to run a bigger one.

The next proposition says that when $\theta_S = 0$, that is, all shoppers invoke price-matching guarantees when they are available, price-matching leads to a unique monopolistic equilibrium.

Proposition 2.2. *Suppose that firms are exogenously required to offer price-matching guarantees and $\theta_S = 0$. Then there exists a unique Sequential Equilibrium where both firms set price v and $r_1^* = r_2^* = v + c$.*

When $\theta_S = 0$, lower prices do not attract additional customers. In particular, shoppers that encounter a higher price at the second firm they search will use a price-matching guarantee at the second firm rather than go back to the first firm to obtain a lower price. As a result, firms extract all consumer welfare by pricing at v.

2.4.3 Only firm 1 price-matches

In this subsection we examine equilibria that arise when firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to. The analysis when only firm 2 is exogenously required to offer price-matching guarantees follows similarly and is left to the reader.

Lemma 2.2 in the appendix narrows down the set of possible prices that firms can charge in equilibrium in the same way as Lemma 2.1. Corollary 2.2 in the appendix applies to Lemma 2.2 exactly as it did to Lemma 2.1. As before, we only examine equilibria where the firm supports do not have any breaks.

Proposition 2.3. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to. Then there exists a set of Sequential Equilibria where both firms distribute prices over support*

$$\left[\underline{p} = \min\{v, r^*\}\left\{\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]+(1-\mu)(1-\beta_N)}\right\}, \bar{p} = \min\{v, r^*\}\right]$$

and the equilibrium reservation price, $r^* = r_1^* = r_2^*$ *equals*

$$r^* = \begin{cases} r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) & if \quad r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) \leq v \\ \infty & otherwise \end{cases},$$

$$r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) = c\left\{1 - \frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]} \times \ln\left\{1+\frac{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]}{(1-\mu)(1-\beta_N)}\right\}\right\}^{-1}.$$

Firm 1 distributes prices according to

$$F_1(p) = \left\{1+\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]}\right\}\left(1-\frac{\underline{p}}{p}\right),$$

while firm 2 distributes prices according to

$$F_2(p) = \left[1 + \frac{(1-\mu)\,\beta_N}{\mu}\right]\left[1 - \left(\frac{\underline{p}}{p}\right)^{\frac{1}{\beta_S + \theta_S - \beta_S \theta_S}}\right]$$

over $\left[\underline{p}, \bar{p}\right)$ *with a mass point at* $\bar{p}$*. In equilibrium, the expected prices for the two firms equal each other.*

As in the equilibria described in Proposition 2.1, non-shoppers randomly choose a firm to search first and make a purchase there. Numerically, it can be shown that $\beta_N < 1/2$ must hold for expected prices in the two firms to equal each other and that in this case, firm 2 will be the one with the mass point at $\bar{p}$. Since $\beta_N < 1/2$, firm 1 captures a smaller proportion of non-shoppers. Knowing that it will have fewer non-shoppers with certainty, firm 1 offers a price-matching guarantee to increase the expected number of shoppers that it captures in equilibrium, while firm 2 runs fewer sales than it would if neither firm offered price-matching guarantees to exploit its larger amount of "captive" non-shoppers.

The next section completes this analysis by endogenizing firms' price-matching decisions. It will be shown that the equilibrium outcome that prevails with regard to firm matching depends on firm beliefs about the amount of asymmetry expected to occur on and off the equilibrium path. Because of the large number of equilibria that can prevail in the complete game, we limit ourselves to a numerical analysis with no randomization over price-matching.

2.5 Market equilibrium (numerical analysis)

In the first stage of the game, firms must decide whether or not to offer consumers a price-matching guarantee. For each firm, this decision depends on a comparison of the profits that it expects to obtain in each of the pricing subgames discussed in the previous section. However, this analysis is made complicated by the fact that in the subgame where both firms price-match and in the two subgames where only one firm matches, multiple equilibria may occur. For each set of price-matching decisions, these equilibria differ in the β_N, β_S pair that prevails. For example, suppose that both firms offer price-matching guarantees on the equilibrium path, half of the consumers are shoppers ($\mu = 0.5$) and half of the shoppers are willing to invoke price-matching guarantees ($\theta_S = 0.5$). Moreover, suppose that the consumer valuation for the product is high enough not to be binding (that is, $\bar{p} = r^* < v$) and search costs are normalized to one. A natural candidate for equilibrium in this case is the symmetric one where $\beta_N = \beta_S = 0.5$ and both firms choose prices from the same distribution on the equilibrium path. In this case, both firms expect profit of 1.19. An alternative outcome given the parameters above is one where $\beta_N = 0.6$ and $\beta_S = 0.8574$, in which case firm 1 expects profit of 1.22 while firm 2 expects profit of 1.38. Mathematically, multiple β_N, β_S pairs may prevail because in equilibrium, the solution for β_N and β_S is obtained using the single equation that sets the expected prices in the two firms equal to each other, making all consumers indifferent between which firm to search first. Intuitively, there is no reason to prefer one of the two outcomes above over the other because firms do not set β_N or β_S. Instead, in reality, one may think of the equilibrium outcome that prevails as the byproduct of multiple iterations of the pricing subgame for a particular industry or product category, where firm beliefs about β_N, β_S pairs evolve according to past observations of consumer behavior.

The equilibrium also depends on beliefs about β_N and β_S off the equilibrium path. Consider, for instance, the subgame where firm 1 offers a price-matching guarantee, but firm 2 does not and all the exogenous parameters are the same as in the example above. One possible equilibrium β_N, β_S pair for this subgame is $\beta_N = 0.3586$ and $\beta_S = 0.001$. In this equilibrium, firm 1 expects profit of 1.46 and firm 2 expects profit of 1.23. This subgame can be played on the equilibrium path if firms believe that in the subgame where both firms price-match, the symmetric equilibrium will prevail. However, if firms believe that when both firms price-match, the asymmetric outcome above will prevail instead, firm 2 will deviate by offering a price-matching guarantee to increase its profit from 1.23 to 1.38. For completeness, we note that the unique symmetric equilibrium in the subgame with no matching yields expected profit of 0.55 for both firms, so firm 1 will never want to deviate from offering a price-matching guarantee in this case.

The following statement summarizes the above discussion based on a numerical comparison of the equilibria that can prevail in this game.

Equilibrium. *In equilibrium, either both firms will offer price-matching guarantees or only one firm will offer a price-matching guarantee. In any equilibrium outcome where firms are given the option of offering price-matching guarantees, expected prices and firm profits are higher than when price-matching is not an option.*

For any given value of v and c, we can numerically calculate all possible equilibria. There are several factors that affect expected prices and firm profits as well as the bounds on firm price distributions in equilibrium. As in Stahl (1989), regardless of firm matching decisions, when the proportion of non-shoppers in the population increases, so do expected prices and profits. Firms are motivated to lower prices specifically to attract those consumers who observe all of them and when such consumers make up a smaller proportion of the population, there is less incentive to lower prices. Likewise, expected prices and profits grow when more shoppers accept price-matching guarantees (θ_S grows) because the firm with the lower price attracts a smaller proportion of all shoppers.

Table 2.1: Firm Profits When Both Firms Match[1,2]

		$\beta_N = 0.50$	$\beta_N = 0.55$		$\beta_N = 0.60$		$\beta_S = 0.999$	
μ	θ_S	$\pi_1 = \pi_2$	π_1	π_2	π_1	π_2	π_1	π_2
.20	.20	10.75	10.79	11.75	N/A	N/A	10.80	12.03
.50	.20	3.12	3.15	3.37	3.18	3.62	3.24	4.32
.80	.20	1.06	1.09	1.13	1.11	1.20	1.23[3]	1.71[3]
.20	.50	4.27	N/A	N/A	N/A	N/A	4.29	4.59
.50	.50	1.19	1.20	1.29	1.22	1.38	1.23	1.46
.80	.50	0.35	0.36	0.38	0.37	0.40	0.42	0.57
.20	.80	2.65	N/A	N/A	N/A	N/A	2.66	2.73
.50	.80	0.71	0.72	0.77	N/A	N/A	0.72	0.77
.80	.80	0.19	0.19	0.20	0.20	0.22	0.21	0.23

[1] μ consumers have no cost of search (shoppers), while $1-\mu$ have search cost $c = 1$ (non-shoppers). θ_S shoppers always ignore price-matching guarantees. β_N and β_S respectively represent the fraction of shoppers and non-shoppers who begin search at firm 1. Non-shoppers' valuation for the good is assumed to be strictly higher than their equilibrium reservation price (their maximum willingness to pay for the good).

[2] Equilibrium β_S varies with μ and θ_S for a given value of β_N and vice versa. N/A implies that no equilibrium exists for the given value of β_N. Rightmost column gives results for $\beta_S = 0.999$ to approximate profits with highest level of asymmetry.

[3] No equilibrium with $\beta_S = 0.999$. Results given are for $\beta_N = 0.999$ and $\beta_S = 0.80$.

Table 2.1 shows that this is the case for expected profits when both firms price-match in equilibrium while Table 2.2 does so for equilibria where only firm 2 offers a price-matching guarantee. Each entry in these tables gives the expected profit of a firm for a given value of μ and θ_S as well as a prevailing equilibrium value of β_N or β_S. Each value of β_N corresponds to a unique value of β_S which is not given in the tables and vice versa.

Tables 2.1 and 2.2 show that expected profits grow in the amount of asymmetry that prevails in equilibrium, as measured by the difference in the proportion of non-shoppers served by the two firms. Thus, as β_N, the proportion of non-shoppers who search firm 1 first, increases from 0.50 to 1, so do the profits of both firms.

Table 2.2: Firm Profits When Only Firm 2 Matches[1,2]

		$\beta_N = 0.55$		$\beta_N = 0.60$		$\beta_S = 0.999$	
μ	θ_S	π_1	π_2	π_1	π_2	π_1	π_2
.20	.20	5.78	6.29	N/A	N/A	10.75	11.99
.50	.20	0.71	0.76	0.94	1.07	3.23	4.30
.80	.20	0.16	0.16	0.18	0.19	0.45[3]	0.63[3]
.20	.50	N/A	N/A	N/A	N/A	4.29	4.58
.50	.50	0.71	0.76	0.94	1.07	1.23	1.46
.80	.50	0.16	0.16	0.18	0.19	0.42	0.57
.20	.80	N/A	N/A	N/A	N/A	2.66	2.73
.50	.80	0.71	0.76	N/A	N/A	0.72	0.77
.80	.80	0.16	0.16	0.18	0.19	0.21	0.23

[1] μ consumers have no cost of search (shoppers), while $1 - \mu$ have search cost $c = 1$ (non-shoppers). θ_S shoppers always ignore price-matching guarantees. β_N and β_S respectively represent the fraction of shoppers and non-shoppers who begin search at firm 1. Non-shoppers' valuation for the good is assumed to be strictly higher than their equilibrium reservation price (their maximum willingness to pay for the good).

[2] Equilibrium β_S varies with μ and θ_S for a given value of β_N and vice versa. N/A implies that no equilibrium exists for the given value of β_N. Rightmost column gives results for $\beta_S = 0.999$ to approximate profits with highest level of asymmetry.

[3] No equilibrium with $\beta_S = 0.999$. Results given are for $\beta_N = 0.999$ and $\beta_S = 0.80$.

Moreover, the tables show that when μ is sufficiently low or θ_S is sufficiently high, the amount of asymmetry that can occur in equilibrium is limited. Consider $\mu = \theta_S = 0.20$ in Table 2.1. If a proportion of non-shoppers only slightly higher than 0.55 searches firm 1 first, because these non-shoppers will purchase from firm 1 with certainty and since non-shoppers make up a relatively large proportion of the population, firm 1 has relatively little incentive to lower its price in an attempt to capture shoppers while firm 2 wants to lower its price to attract shoppers to compensate itself for the low proportion of non-shoppers who search it. These contrasting motivations will induce firm 1 to have a higher expected price than that of firm 2, contradicting the fact that more non-shoppers begin their search at firm 1. If θ_S increases to 0.50, even $\beta_N = 0.55$ cannot be supported in equilibrium. When fewer shoppers accept price-matching guarantees, the firm with the lower price can capture a larger proportion of them, increasing firm 2's incentive to lower its price to make up for a dearth of non-shoppers who search it.

Corresponding results for expected prices and the upper bound of firm price distributions (assuming $\bar{p} = r^* < v$) follow precisely as in Tables 2.1 and 2.2: that is expected prices and upper bounds are increasing in the proportion of non-shoppers, the usage of price-matching guarantees, and the amount of asymmetry. Results for expected prices and profits persist even if v is low enough to bind (so that $\bar{p} = v < r^* = \infty$). Thus, by varying v, we can decompose the first price increasing effect discussed in the introduction from the second—expected prices and profits increase in $1 - \mu$, $1 - \theta_S$, and in the amount of asymmetry, even if the equilibrium reservation price remains the same.

We summarize the numerical results above in the following statement.

Comparative Statics. *Given firms' equilibrium price-matching decisions, expected prices and firm profits are increasing in* $1 - \mu$, $1 - \theta_S$, *and* $|2\beta_N - 1|$. *Moreover,* r^* *is non-decreasing in* $1 - \mu$, $1 - \theta_S$, *and* $|2\beta_N - 1|$.

A casual examination of Tables 2.1 and 2.2 suggests that if firms maintain the same beliefs about β_N in both subgames (and if they believe that $1 - \beta_N$ will be played in the subgame where only Firm 1 matches), then the only possible type of equilibrium outcome is the one where both firms choose to offer price-matching guarantees. However, this outcome is inconsistent with what we observe in reality—that is, we often see only a fraction of the firms producing the same good offering price-matching guarantees. Within the context of this model, this real world observation implies that firms that choose not to price-match believe that a more symmetric equilibrium would prevail had they chosen to do so, leading to lower profits than not matching.

2.6 Discussion

2.6.1 Search

This study places the concept of price-matching into a broad literature on search. Initially, an important purpose of this literature was to address the Diamond paradox (Diamond 1974), which says that when information acquisition is costly for consumers, the prevalent pricing outcome among firms selling a homogeneous good is the monopoly price. The search literature provides explanations for why this result does not persist in the real world. Explanations outside of consumer heterogeneity (Stahl 1989) include firm cost heterogeneity (Reinganum 1979), uncertainty regarding the number of free price samples or "noisy" search (Burdett and Judd 1983), and lack of common knowledge about consumer valuation for the product on the part of firms (Kuksov 2006).

The choice of Stahl (1989) for the underlying framework of this paper was made for the purpose of tractability-it proved to be a very convenient way to endogenize consumer price information acquisition. In his paper, Stahl shows that by varying the proportion of shoppers from zero to one, we can make a continuous transition from the monopoly result obtained by Diamond to the Bertrand outcome that prevails when price information is free for all consumers. Interestingly, in this paper, a similar transition can be achieved between the Diamond outcome and the Stahl price dispersed equilibrium by varying θ_S between zero and one. If we stick to the earlier suggested hassle cost interpretation of θ_S (see footnote 7), this means that if all shoppers use price-matching guarantees, the monopoly price results (Proposition 2.2). On the contrary, if all shoppers find price-matching activity costly, as in Hviid and Shaffer (1999), the guarantees are never used (though unlike in Hviid and Shaffer, the unique equilibrium is in mixed pricing strategies, not the Bertrand outcome). Throughout most of this paper, we have thus assumed that there is a positive mass of shoppers who do not find the activity of price-matching a hassle.[10]

2.6.2 Heterogeneous non-shoppers

While it is crucial that shoppers use price-matching guarantees for price-matching to have any effect in this model, as mentioned earlier, the price increasing effect stemming from a decline in non-shopper search activity is secondary. In fact, the decline is somewhat superficial in that non-shoppers do not search to begin with. An interesting extension involves heterogeneous non-shoppers who vary in the size of their search cost. Stahl (1996) shows that non-shoppers with a low enough search cost may search more than one firm in such a set up because firms can price above their reservation price as long as this still guarantees them a positive

[10]An alternative explanation suggested by Maarten Janssen and others is that all consumers face a positive hassle cost of using a price-matching guarantee, but that recall is also costly for everyone and that shoppers are heterogeneous with respect to either the size of the hassle cost or the cost of going back to a previously visited firm. Shoppers who find price-matching less costly than recalling a previously sampled lower price will use the price-matching guarantee.

mass of higher search cost non-shoppers. Since price-matching raises non-shopper reservation prices, a heterogeneous non-shopper setup may result in an observable decline in non-shopper search activity by stopping low search cost non-shoppers from searching beyond the first firm.

2.7 Conclusion

This paper explores the effects that price-matching guarantees have on firms and consumers when consumers optimally search for price. Price-matching guarantees alter the shopping behavior of both types of consumers in our model in a way that encourages firms to raise prices. When consumers who have no cost of price search use price-matching guarantees at firms that list higher prices, firms are discouraged from lowering prices in order to attract such consumers. Understanding this price increasing effect, consumers who face an opportunity cost of searching for price accept higher prices at already sampled firms because they anticipate that further search is less likely to yield a lower price. In addition, because consumers with no search costs may be able to obtain the lower price at either firm, there is a multiplicity of asymmetric equilibria where more asymmetry leads to higher expected prices and firm profits.

While the underlying mechanism driving the effects of price-matching in our model is new, this paper is not orthogonal to the previous literature. Both welfare diminishing tacit collusion and price discrimination can be found in our model. Tacit collusion occurs since firms understand that a rival's "threat" to match a lower price entails a smaller benefit from any incremental price cut. Price discrimination occurs because consumers who have no cost of price search may use a price-match to secure a lower price from the firm listing the higher price while the firm's remaining customers pay the higher listed price. However, contrary to the result in signaling models of price-match, where ex-ante asymmetries persist ex-post, we find that price-matching alone is enough to generate an asymmetric equilibrium. In the model presented, asymmetries always reduce consumer welfare, but it would be interesting to see how differences in firm production costs influence search behavior when price-matching is allowed. This is not immediately obvious from the above analysis. Ex-ante cost differences have the potential to reduce asymmetries ex-post, but it is unclear if this is a good thing because it may entail more purchases from the higher cost firm.

This study has significance for future empirical work. Recent empirical literature has focused primarily on comparisons of price observations between firms with and without price-matching guarantees (Moorthy and Winter 2006, Arbatskaya et al. 2006). However, the results in this paper suggest that such cross-sectional findings point purely to underlying cost differences between firms without telling us the effect of adopting a matching policy. If firms are homogeneous in costs, expected prices among firms that differ in the adoption of a price-matching guarantee can remain the same. Therefore, a time component is necessary for a thorough analysis; although even this may not be foolproof, as the adoption of a matching policy may follow a change in production costs. A complete analysis will look at changes in consumer shopping behavior consistent with those predicted by this paper. A survey test of our model could ask individuals who use price-matching guarantees to secure the lowest price if they would obtain that price regardless by purchasing somewhere else. An affirmative answer tells us that price-matching guarantees are in fact anti-competitive because they keep consumers away from firms with lower listed prices.

2.8 Appendix

We first define some useful notation. As with shoppers, for non-shoppers who have searched both firms, we assume that $\theta_N \in [0,\ 1]$ will ignore price-matching guarantees and always purchase from the firm with the lower listed price. The remaining $1 - \theta_N$ will invoke a price-matching guarantee at the last firm they stopped in when one is available and necessary to obtain the lower price there and purchase from the firm with the lower listed price otherwise. Let $\alpha_{S(N)} \in \left[0,\ \theta_{S(N)}\right]$ be the proportion of shoppers (non-shoppers) who buy from the first firm they searched after having observed the same price listed in both firms.[11] Let γ be the proportion of non-shoppers who do not search after freely observing a price of r_j at firm i.

Definition 2.1. *We say that firms have a mutual mass point when each firm has a mass point at the same price. We say that firms have a mutual break when each firm's equilibrium support has a break over the same price interval.*

Lemma 2.1. *Suppose that firms are exogenously required to offer price-matching guarantees and that $\theta_S \in (0,\ 1]$. In equilibrium, the supports of the firm pricing distributions can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_1 = r_2\}$.
2. *Single mass point, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm i has a mass point at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$, $r_j \leq r_i$.

[11] The restriction $\alpha_{S(N)} \leq \theta_{S(N)}$ is used for mathematical tractability. It says that when a firm undercuts a tie, it cannot lose customers.

3. *Two mass points, mutual break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has a mass point at* $r_i < \min\{v,\ r_j\}$*; mutual break over* $(r_i,\ p^u)$ *for* $p^u \in (r_i,\ \bar{p})$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$*, firm* i *has a mass point at* $\bar{p}$*.*

4. *Two mass points, single break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has a mass point at* $\bar{p}_j = r_i < \min\{v,\ r_j\}$*; firm* i *has a break over* $(r_i,\ \bar{p}_i)$ *for* $\bar{p}_i = \min\{v,\ r_j\}$ *and a mass point at* $\bar{p}_i$*.*

The following claims complete the proof of Lemma 2.1.

Claim 2.1. $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0.$

Proof. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Then, for $p_1 \in \left[\underline{p}_1, \underline{p}_2\right)$, firm 1's expected profit is

$$\begin{aligned} & p_1\{\mu[\theta_S + (1-\beta_S)(1-\theta_S)] \\ & + (1-\mu)\{\beta_N + (1-\beta_N)\{[1 - F_2(r_1)] + (1-\gamma)\Pr(p_2 = r_1)\}\}\} \end{aligned} \tag{2.4}$$

(since $\underline{p}_2 < r_2$ by definition), which is increasing in p_1, contradicting the equilibrium. If $\underline{p}_1 \leq v < \underline{p}_2$, for $p_1 \in \left[\underline{p}_1, v\right)$, firm 1's expected profit is given by Expression (2.4), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. But if $v = \underline{p}_1 < \underline{p}_2$, then $F_1(v) = 1$ (because firm 1 doesn't make any profit at prices above v) and firm 2 expects profit of $\mu\beta_S(1-\theta_S)v$ everywhere on its support. For sufficiently small $\varepsilon > 0$, firm 2 benefits by shifting its mass to $v - \varepsilon$ for expected profit of $(v-\varepsilon)\{\mu[\theta_S + \beta_S(1-\theta_S)] + (1-\mu)(1-\beta_S)\}$. Finally, if $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. By a similar argument, $\underline{p}_2 \leq \underline{p}_1$ and $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$. Since prices below zero result in negative profit, $\underline{p} \geq 0$.[12]

[12] If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$ because at $p_i = \varepsilon < \min\{r_j,\ v\}$, firm i will make money off its non-shoppers.

Suppose $v < \min\{\bar{p}_1,\ \bar{p}_2\}$. Then, for $p_i > v$, firm i expects no profit with probability $\Pr(p_j > v) > 0$ and since consumers never purchase at prices above v, firm i will only profit from consumers that accept its price-match offer after they had rejected a price no higher than v at firm j. Thus, firm i cannot lose money from such consumers by shifting mass above v down to v. However, by doing so, it now also expects to earn a positive profit with probability $\Pr(p_j > v)$, a contradiction. $\square$

Claim 2.2. *There are no mutual mass points.*

Proof. Suppose that there is a mutual mass point at p. Firm 1's expected profit at p when firm 2 charges p as well is

$$\begin{aligned} & p\{\mu[\beta_S\alpha_S + (1-\beta_S)(1-\alpha_S)] \\ & \quad + (1-\mu)\{\beta_N[\mathbb{I}_{p<r_2} + [\gamma + \alpha_N(1-\gamma)]\mathbb{I}_{p=r_2} + \alpha_N\mathbb{I}_{p>r_2}] \\ & \quad + (1-\beta_N)(1-\alpha_N)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\}\} \end{aligned} \tag{2.5}$$

where $\mathbb{I}$ is an indicator function. Suppose instead that firm 1 deviates to $p - \varepsilon$ while firm 2 maintains its price at p. Firm 1's expected profit will be

$$\begin{aligned} & (p-\varepsilon)\{\mu[\beta_S\theta_S + (1-\beta_S)] \\ & \quad + (1-\mu)\{\beta_N[\mathbb{I}_{p-\varepsilon<r_2} + [\gamma + \theta_N(1-\gamma)]\mathbb{I}_{p-\varepsilon=r_2} + \theta_N\mathbb{I}_{p-\varepsilon>r_2}] \\ & \quad + (1-\beta_N)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\}\} \end{aligned} \tag{2.6}$$

Expression (2.5) is smaller than Expression (2.6) provided that ε is sufficiently small.

Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive probability lead to a gain and with the complementary probability at worst lead to a loss of ε. Therefore, by shifting its mass point at p to $p-\varepsilon$ for sufficiently small ε firm 1 increases its expected profit, a contradiction. For the case $\alpha_S = \theta_S$, $\alpha_N = \theta_N$, and $\beta_S = \beta_N = 1$, firm 1 cannot profitably undercut the mutual mass point, but firm 2 can. □

Claim 2.3. *The only possible breaks in the equilibrium supports are:*

(i) If $\bar{p}_i < \bar{p}_j$, there is a break at $(\bar{p}_i,\ \bar{p}_j) \in S_j$.

(ii) If $r = r_i = r_j < \bar{p}_i = \bar{p}_j$, there may be a mutual break with lower bound r.

(iii) If $r_i \neq r_j$ and firm i has a mass point at r_j, there may be a mutual break with lower bound r_j.

Proof. Let S_1 and S_2 be respectively, the equilibrium supports for firms 1 and 2. Define $H = \left(p^d,\ p^u\right) \in \operatorname{int}(S_1 \cap S_2)$.

Suppose first, without loss of generality, that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at some $p_1 \in H$ is

$$\begin{aligned}
&\mu \left\{p_1 \left(\theta_S\beta_S + 1 - \beta_S\right)\left[1 - F_2\left(p_1\right)\right] + \left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2 | p_2 < p_1\right] F_2\left(p_1\right)\right\} \\
&+ (1-\mu)\left\{p_1\beta_N\left\{\mathbb{I}_{p_1<r_2} + \left\{\gamma + \theta_N\left(1-\gamma\right)\left[1-F_2\left(p_1\right)\right]\right\}\mathbb{I}_{p_1=r_2}\right.\right. \\
&\left. + \theta_N\left[1-F_2\left(p_1\right)\right]\mathbb{I}_{p_1>r_2}\right\} \\
&+ \left(1-\beta_N\right)\left\{p_1\left[1-F_2\left(r_1\right) + (1-\gamma)\Pr\left(p_2=r_1\right)\right]\mathbb{I}_{p_1<r_1} + p_1\left[1-F_2\left(r_1\right)\right]\mathbb{I}_{p_1=r_1}\right. \\
&+ \left\{p_1\left[1-F_2\left(p_1\right)\right] + r_1\left(1-\theta_N\right)(1-\gamma)\Pr\left(p_2=r_1\right)\right. \\
&\left.\left.\left. + \left(1-\theta_N\right)\mathrm{E}\left[p_2 | r_1 < p_2 < p_1\right]\left[F_2\left(p_1\right) - F_2\left(r_1\right)\right]\right\}\mathbb{I}_{p_1>r_1}\right\}\right\}
\end{aligned} \tag{2.7}$$

As firm 1 raises p_1 along H, its expected profit is increasing since $F_2\left(p_1\right)$ is constant along H (and equal to $F_2\left(r_1\right)$ if $r_1 \in H$). Thus, if $r_2 \notin H$, firm 1 could increase expected profits by shifting all its mass in H slightly below p^u (to p^u if firm 2 doesn't have a mass point there), a contradiction. If $r_2 \in H$, firm 1 can increase expected profits by shifting all mass in $\left(p^d,\ r_2\right)$ slightly below r_2, and all mass in $\left(r_2,\ p^d\right)$ either slightly below r_2 or to p^u, again contradicting the equilibrium. A similar argument applies when firm 1 has no support over H, but firm 2 does. This tells us that any breaks in $S_1 \cap S_2$ are mutual.

Now suppose that neither firm randomizes over H in equilibrium. Suppose first that $p^d \neq r_1$, $p^d \neq r_2$ and that neither firm has a mass point at p^d. Then either firm 1 has a strictly higher expected profit at p^u (or slightly below r_2 if $r_2 \in H$) than at p^d, or firm 2 has a strictly higher expected profit at p^u (or slightly below r_1 if $r_1 \in H$) than at p^d, or possibly both, if neither firm has a mass point at p^u, contradicting the equilibrium.

Suppose that firm i has a mass point at $p^d \neq r_j$. Since there are no mutual mass points, firm i could increase profits by shifting its mass point to p^u (or slightly below p^u if firm j has a mass point there, or slightly below r_j if $r_j \in H$).

If $p^d = r_j \neq r_i$ and firm i has no mass point at p^d, firm j's expected profit will be strictly higher at p^u (or slightly below p^u if firm i has a mass point there, or slightly below r_i if $r_i \in H$) than at p^d. But if firm i does have a mass point at p^d, then it is possible that profits are the same at p^d and p^u for each firm. If $\gamma \neq 1$, firm i can profitably deviate by shifting its mass point slightly below p^d. In doing so, it retains $1-\gamma$ non-shoppers who search after observing a price of r_j and have a positive probability of purchasing from firm j. However, if $\gamma = 1$, neither firm has a profitable deviation. This may also be the case if, $p^d = r_1 = r_2$.

From Claim 2.1, we know that both S_1 and S_2 have the same lower bound, $\underline{p}$, so $S_1 \Delta S_2 \in (\min\{\bar{p}_1,\ \bar{p}_2\},\ \max\{\bar{p}_1,\ \bar{p}_2\}]$. Suppose, without loss of generality, that $\bar{p}_1 > \bar{p}_2$. At $p_1 \in (\bar{p}_2,\ \bar{p}_1]$, firm 1's expected profit is

$$\begin{aligned}&\mu\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2\right]+\left(1-\mu\right)\left\{p_1\beta_N\left(\mathbb{I}_{p_1<r_2}+\gamma\mathbb{I}_{p_1=r_2}\right)\mathbb{I}_{p_1\le v}\right.\\&\left.+\left(1-\theta_N\right)\left(1-\beta_N\right)\left\{\mathrm{E}\left[p_2|r_1<p_2\right]\left[1-F_2\left(r_1\right)\right]+r_1\left(1-\gamma\right)\Pr\left(p_2=r_1\right)\right\}\right\}\end{aligned} \tag{2.8}$$

If $\bar{p}_2 < r_2$, then for $\beta_N \neq 0$, firm 1's expected profit is increasing in p_1 along $(\bar{p}_2,\ \min\{v,\ r_2\})$ and is strictly greater anywhere in $(\bar{p}_2,\ \min\{v,\ r_2\})$ than at any price above $\min\{v,\ r_2\}$. As a result, for $\varepsilon > 0$ sufficiently small, firm 1 can increase profit by shifting mass in $(\bar{p}_2,\ \bar{p}_1]$ to $\min\{v,\ r_2\} - \varepsilon$ (likewise if $\bar{p}_2 = \min\{v,\ r_2\}$). Therefore, when $\bar{p}_2 \leq r_2$, either $S_1 \Delta S_2 = \{\bar{p}_1\} = \{\min\{v,\ r_2\}\}$, or $S_1 \Delta S_2 = \varnothing$. Suppose $S_1 \Delta S_2 = \{\bar{p}_1\}$. If firm 2 has no mass point at $\bar{p}_2$, this means that firm 1's expected profit at $\bar{p}_1$ is strictly higher than its expected profit at $\bar{p}_2$, a contradiction. If firm 2 has a mass point at $\bar{p}_2 \neq r_1$, since there are no mutual mass points, firm 2 can profitably shift the mass point to slightly below $\bar{p}_1$ (or slightly below r_1 if $r_1 \in (\bar{p}_2,\ \bar{p}_1]$). However, if $\gamma = 1$ and firm 2 has a mass point at $\bar{p}_2 = r_1$, $\mathrm{E}\,\pi_1(\bar{p}_1,\ F_2(\bar{p}_1)) = \mathrm{E}\,\pi_1(\bar{p}_2,\ F_2(\bar{p}_2))$, and $F_1(r_1)$ is large enough, then neither firm has a profitable deviation. Following the proof of Claim 2.5, we will discuss why an equilibrium where $r_2 < \bar{p}_2 < \bar{p}_1$ cannot exist. A similar argument applies when $\bar{p}_2 > \bar{p}_1$. □

Corollary 2.1. *The equilibrium supports are the same except if* $\bar{p}_i = r_j < \bar{p}_j = \min\{v,\ r_i\}$.

Claim 2.4. *Firm i does not have a mass point in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof. Suppose, without loss of generality, that firm 2 has a mass point at $p \in S_1 \backslash \{\bar{p}_1\}$, and suppose that $p \neq r_1$. Firm 1's expected profit at $p - \varepsilon$ when firm 2 charges p is given by Expression (2.6), whereas its expected profit at $p + \varepsilon$ is

$$\begin{aligned} &\mu p \left(1-\theta_S\right)\left(1-\beta_S\right)+\left(1-\mu\right)\left\{\left(p+\varepsilon\right)\beta_N\left(\mathbb{I}_{p+\varepsilon<r_2}+\gamma\mathbb{I}_{p+\varepsilon=r_2}\right)\right. \\ &\left.+p\left(1-\theta_N\right)\left(1-\beta_N\right)\left[\left(1-\gamma\right)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}\right]\right\} \end{aligned} \tag{2.9}$$

Expression (2.9) is smaller than Expression (2.6) provided that ε is sufficiently small. Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 1 from lowering the price by 2ε or less is at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will with positive probability lead to a gain and with the complementary probability at worst lead to a loss of 2ε. Therefore, by shifting its mass between p and $p+\varepsilon$ to $p-\varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. By a similar argument, firm 1 cannot have a mass point at $p \in S_2 \backslash \{\bar{p}_2\}$, except possibly if $p = r_2$. $\square$

Claim 2.5. *If $\bar{p} = \bar{p}_1 = \bar{p}_2$ then either*

(i) $\bar{p} = \min\{v,\ r_1,\ r_2\}$*, the supports have no breaks, and at most one firm can have a mass point at $\bar{p}$, or*

(ii) $\bar{p} = \min\{v,\ \max\{r_1,\ r_2\}\}$*, there is a mutual break above* $\min\{r_1,\ r_2\} < \bar{p}$*, firm i has a mass point at r_j, and firm j has a mass point at $\bar{p}$.*

Proof. Suppose that $\bar{p} = \bar{p}_1 = \bar{p}_2$ and neither firm has a mass point at $\bar{p}$. From Claims 2.1 and 2.4 we know that $\underline{p} < \bar{p} \leq v$. Suppose, without loss of generality, that $\bar{p} < \min\{v,\ r_2\}$. At $\bar{p}$, firm 1's expected profit is given by Expression (2.8) (with $p_1 = \bar{p}$), which is increasing in p_1 along $(\bar{p},\ \min\{v,\ r_2\})$ when $\beta_N \neq 0$, a contradiction. Suppose instead that $\bar{p} > \min\{v,\ r_2\} = r_2$. For any $p_1 \in (r_2,\ \bar{p})$, in equilibrium, $\mathrm{E}\,\pi_1(\bar{p}) = \mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$. $\mathrm{E}\,\pi_1(\bar{p})$ is given by Expression (2.8) (with $p_1 = \bar{p}$). If $r_2 \geq r_1$, for $p_1 \in (r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ equals

$$\mu \left\{ p_1 \left(\theta_S \beta_S + 1 - \beta_S \right) \left[1 - F_2 \left(p_1 \right) \right] + \left(1 - \theta_S \right) \left(1 - \beta_S \right) \mathrm{E} \left[p_2 | p_2 < p_1 \right] F_2 \left(p_1 \right) \right\}$$
$$+ \left(1 - \mu \right) \left\{ p_1 \beta_N \theta_N \left[1 - F_2 \left(p_1 \right) \right] + \left(1 - \beta_N \right) \left\{ r_1 \left(1 - \theta_N \right) \left(1 - \gamma \right) \Pr \left(p_2 = r_1 \right) \right. \right.$$
$$\left. \left. + p_1 \left[1 - F_2 \left(p_1 \right) \right] + \left(1 - \theta_N \right) \mathrm{E} \left[p_2 | r_1 < p_2 < p_1 \right] \left[F_2 \left(p_1 \right) - F_2 \left(r_1 \right) \right] \right\} \right\} \tag{2.10}$$

Setting Expression (2.8) equal to Expression (2.10) and differentiating with respect to p_1 gives us

$$\begin{aligned} & \left[\mu \left(\theta_S \beta_S + 1 - \beta_S \right) + \left(1 - \mu \right) \left(\theta_N \beta_N + 1 - \beta_N \right) \right] \left[1 - F_2 \left(p_1 \right) \right] \\ & - \left[\mu \theta_S + \left(1 - \mu \right) \theta_N \right] p_1 {F_2}' \left(p_1 \right) = 0 \end{aligned} \tag{2.11}$$

Solving the differential equation given by Equation (2.11) using the initial value $F_2 \left(\bar{p} \right) = 1$ gives us $F_2 \left(p_1 \right) = 1$ for all $p_1 \in \left(r_2,\ \bar{p} \right]$, a contradiction. Similarly, if $r_1 \in \left(r_2,\ \bar{p} \right)$, then Expression (2.10) represents firm 1's expected profit at $\left(r_1,\ \bar{p} \right)$ and $F_2 \left(p_1 \right) = 1$ for all $p_1 \in \left(r_1,\ \bar{p} \right]$, a contradiction. If on the other hand, $r_1 \geq \bar{p}$, $\mathrm{E}\, \pi_1 \left(\bar{p} \right)$ becomes $\mu \left(1 - \theta_S \right) \left(1 - \beta_S \right) \mathrm{E} \left[p_2 \right]$ while $\mathrm{E}\, \pi_1 \left(p_1,\ F_2 \left(p_1 \right) \right)$ at $p_1 \in \left(r_2,\ \bar{p} \right)$ becomes

$$\mu \left\{ p_1 \left(\theta_S \beta_S + 1 - \beta_S \right) \left[1 - F_2 \left(p_1 \right) \right] + \left(1 - \theta_S \right) \left(1 - \beta_S \right) \mathrm{E} \left[p_2 | p_2 < p_1 \right] F_2 \left(p_1 \right) \right\}$$
$$+ \left(1 - \mu \right) \beta_N \theta_N p_1 \left[1 - F_2 \left(p_1 \right) \right] \tag{2.12}$$

Setting $\mu \left(1 - \theta_S \right) \left(1 - \beta_S \right) \mathrm{E} \left[p_2 \right]$ equal to Expression (2.12) and solving the resulting differential equation using the initial value $F_2 \left(\bar{p} \right) = 1$ again gives us $F_2 \left(p_1 \right) = 1$ for all $p_1 \in \left(r_2,\ \bar{p} \right]$, a contradiction. Hence, for $\beta_N \neq 0$, $\bar{p} = \min \left\{ v,\ r_2 \right\}$. By a similar argument, for $\beta_N \neq 1$, $\bar{p} = \min \left\{ v,\ r_1 \right\}$, so when neither firm has a mass point at $\bar{p}$, $\bar{p} = \min \left\{ v,\ r_1,\ r_2 \right\}$.

From Claim 2.2, we know that at most one firm can have a mass point at $\bar{p}$, say firm j. If $\gamma = 1$ or $v < r_i$, then following the argument in the paragraph above, $\bar{p} = \min \left\{ v,\ r_i \right\}$. Otherwise, firm j cannot have a mass point at $\bar{p}$ (using reasoning similar to that in the proof of Claim 2.3). Moreover, if $r_j \geq r_i$, then $\bar{p} = \min\{v,\ r_1,\ r_2\}$ and from Claim 2.3, we know that the firm supports have no breaks. Conversely, suppose $r_j < r_i$ (and therefore, $r_j < v$). Without loss of generality, let $i = 1$. From Claim 2.4, we know that firm 2 cannot have a mass point at r_2. At r_2, firm 1 expects profit of

$$\begin{aligned}&\mu\left\{r_2\left(\theta_S\beta_S + 1 - \beta_S\right)\left[1 - F_2\left(r_2\right)\right] + \right.\\ &\left.\left(1 - \theta_S\right)\left(1 - \beta_S\right)\mathrm{E}\left[p_2 | p_2 < r_2\right] F_2\left(r_2\right)\right\} + \left(1 - \mu\right)\beta_N r_2\end{aligned} \tag{2.13}$$

whereas at $p_1 \in (r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1\left(p_1,\ F_2\left(p_1\right)\right)$ is given by Expression (2.12). By definition, for $p_1 \in (r_2,\ \bar{p})$, $0 < F_2\left(r_2\right) \leq F_2\left(p_1\right)$, so for p_1 close enough to r_2, Expression (2.13) is strictly greater than Expression (2.12). Therefore, r_2 must be the lower bound for a break in S_1 and we are in Case (iii) of Claim 2.3). $\square$

Notice that Claim 2.5 rules out Case (ii) in Claim 2.3. Moreover, following the same procedure used in Claim 2.5, it is easy to show that an equilibrium where $r_j < \bar{p}_j < \bar{p}_i$ cannot exist. In particular, by setting $\mathrm{E}\,\pi_i\left(\bar{p}_i\right) = \mathrm{E}\,\pi_i\left(p_i,\ F_j\left(p_i\right)\right)$ for $p_i \in (r_j,\ \bar{p}_j]$ and solving for F_j, we see that $F_j\left(p_i\right) = 1$ for all $p_i \in (r_j,\ \bar{p}_j]$, a contradiction.

The corollary below is a technical result necessary for the existence of equilibria with support types 2, 3 or 4. It follows immediately from the proof of Lemma 2.1.

Corollary 2.2. *For equilibria with support types 2, 3 or 4 in Lemma 2.1 to exist, all non-shoppers who sample firm i first, must stop searching after observing a price of r_j unless $v < r_j$.*

Proposition 2.1. *Suppose that firms are exogenously required to offer price-matching guarantees and $\theta_S \in (0,\ 1]$. Then there exists a set of Sequential Equilibria where both firms distribute prices over support*

$$\left[\underline{p} = \min\{v, r^*\}\left[\frac{(1-\mu)\,\beta_N}{\mu\,(\beta_S\theta_S + 1 - \beta_S) + (1-\mu)\,\beta_N}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}, \bar{p} = \min\{v, r^*\}\right]$$

and the equilibrium reservation price, $r^* = r_1^* = r_2^*$ *equals*

$$r^* = \begin{cases} r\,(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) & if \quad r\,(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) \le v \\ \infty & otherwise \end{cases},$$

$$r\,(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) = c\left\{1 - \frac{[(1-\mu)\,\beta_N]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu\,(1-\beta_S)\,(1-\theta_S)}\right.$$
$$\left.\times\left\{[\mu\,(\beta_S\theta_S + 1 - \beta_S) + (1-\mu)\,\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}} - [(1-\mu)\,\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}\right\}\right\}^{-1}.$$

Suppose that firm 1 has a mass point at $\bar{p}$*. Then it distributes prices according to*

$$F_1\,(p) = \left\{1 + \frac{(1-\mu)\,(1-\beta_N)}{\mu\,[(1-\beta_S)\,\theta_S + \beta_S]}\right\}\left[1 - \left(\frac{\underline{p}}{p}\right)^{\frac{(1-\beta_S)\theta_S+\beta_S}{\theta_S}}\right]$$

over $\left[\underline{p}, \bar{p}\right)$*, while firm 2 distributes prices according to*

$$F_2\,(p) = \left[1 + \frac{(1-\mu)\,\beta_N}{\mu\,(\beta_S\theta_S + 1 - \beta_S)}\right]\left[1 - \left(\frac{\underline{p}}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right].$$

In equilibrium, the expected prices for the two firms equal each other.

Proof. Suppose that in equilibrium, firm supports have no breaks.[13] In equilibrium, a firm must be indifferent between any price in its support. Therefore, for any p_i in the support of F_j, $\mathrm{E}\,\pi_i\left(\underline{p}\right) = \mathrm{E}\,\pi_i\,(p_i,\ F_j\,(p_i))$. Then for firm 1, the profit equality condition is given by

$$\mu\,\{p_1\,(\beta_S\theta_S + 1 - \beta_S)\,[1 - F_2\,(p_1)] + (1-\theta_S)\,(1-\beta_S)\,\mathrm{E}\,[p_2|p_2 < p_1]\,F_2\,(p_1)\}$$
$$+ (1-\mu)\,\beta_N p_1 = \underline{p}\,[\mu\,(\beta_S\theta_S + 1 - \beta_S) + (1-\mu)\,\beta_N] \tag{2.14}$$

[13]See Lemma 2.1 in the appendix. Here we characterize an equilibrium for support type 2. This analysis subsumes support type 1 as a special case ($\beta_S = \beta_N = 1/2$, $r_1 = r_2$, $\Pr\,(p_i = \bar{p}) = 0$).

Differentiating Equation (2.14) with respect to p_1 and rearranging gives

$$\begin{aligned}&\mu\left\{\left(\beta_S\theta_S+1-\beta_S\right)F_2\left(p_1\right)+\theta_S p_1 F_2{}'\left(p_1\right)-\beta_S\theta_S-1+\beta_S\right\}\\&-\left(1-\mu\right)\beta_N=0\end{aligned}\tag{2.15}$$

Solving the differential equation given by Equation (2.15) using the initial value $F_2\left(\underline{p}\right)=0$ gives

$$F_2\left(p\right)=\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right]\tag{2.16}$$

We can similarly solve for $F_1\left(p\right)$ to get

$$F_1\left(p\right)=\left\{1+\frac{\left(1-\mu\right)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right\}\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{\left(1-\beta_S\right)\theta_S+\beta_S}{\theta_S}}\right]\tag{2.17}$$

Without loss of generality, suppose that firm 1 is the one with the mass point at $\bar{p}$. Setting $F_2\left(\bar{p}\right)=1$ to solve for $\underline{p}$ in terms of $\bar{p}$ gives

$$\underline{p}=\bar{p}\left[\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}\tag{2.18}$$

Substituting Equation (2.18) into Equation (2.16) gives

$$\begin{aligned}F_2\left(p\right)=&\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\\&\times\left[1-\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\left(\frac{\bar{p}}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right]\end{aligned}\tag{2.19}$$

When $r_2\leq v$, $\bar{p}=r_2$. Optimal search requires that Equation (2.2) holds. Substituting Equation (2.19) into Equation (2.2) yields

$$\begin{aligned}&\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\\&\times\int_{\underline{p}}^{r_2}\left[1-\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\left(\frac{r_2}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right]\mathrm{d}p=c\end{aligned}\tag{2.20}$$

Integrating to solve for r_2 in terms of μ, θ_S, c, β_S and β_N, we get

$$r_2(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)=c\left\{1-\frac{[(1-\mu)\beta_N]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu(1-\beta_S)(1-\theta_S)}\right.$$
$$\times\left\{[\mu(\beta_S\theta_S+1-\beta_S)+(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}\right. \quad (2.21)$$
$$\left.\left.-[(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}\right\}\right\}^{-1}$$

By assumption, non-shoppers are indifferent between which firm to sample first. Since equilibrium support type 2 in Lemma 2.1 implies that $\bar{p}=\min\{v,\ r_1,\ r_2\}$, this indifference condition is equivalent to setting the expected prices for the two firms equal to each other. By doing so, we can solve for β_S in terms of β_N and the other parameters. To obtain an expression for the expected price of each firm we proceed as in Janssen et al. (2005). For firm 2, we solve for p using Equation (2.19) to get

$$p=\bar{p}\left\{\frac{(1-\mu)\beta_N}{(1-\mu)\beta_N+\mu(\beta_S\theta_S+1-\beta_S)[1-F_2(p)]}\right\}^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}} \quad (2.22)$$

Using a change of variables with $u=F_2(p)$, we can write $\mathrm{E}_2[p]=\int_0^1 p\,\mathrm{d}u$. Substituting in p from Equation (2.22) gives

$$\mathrm{E}_2[p]=\bar{p}\int_0^1\left[\frac{(1-\mu)\beta_N}{(1-\mu)\beta_N+\mu(\beta_S\theta_S+1-\beta_S)(1-u)}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}\mathrm{d}u \quad (2.23)$$

Integrating and rearranging yields

$$\mathrm{E}_2[p]=\frac{\bar{p}[(1-\mu)\beta_N]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu(1-\beta_S)(1-\theta_S)}$$
$$\times\left\{[\mu(\beta_S\theta_S+1-\beta_S)+(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}-[(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}\right\} \quad (2.24)$$

Proceeding similarly for firm 1, we get

$$
\begin{aligned}
\mathrm{E}_1\left[p\right]=&\bar{p}\left[1-\lim_{x\to\bar{p}-}F_1\left(x\right)\right]\\
&+\frac{\bar{p}\{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]+\left(1-\mu\right)\left(1-\beta_N\right)\}^{\frac{\theta_S}{\left(1-\beta_S\right)\theta_S+\beta_S}}}{\mu\beta_S\left(1-\theta_S\right)}\\
&\times\left[\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}\\
&\times\left\{\{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]+\left(1-\mu\right)\left(1-\beta_N\right)\}^{\frac{\beta_S\left(1-\theta_S\right)}{\left(1-\beta_S\right)\theta_S+\beta_S}}\right.\\
&\left.-\left\{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]\left[1-\lim_{x\to\bar{p}-}F_1\left(x\right)\right]+\left(1-\mu\right)\left(1-\beta_N\right)\right\}^{\frac{\beta_S\left(1-\theta_S\right)}{\left(1-\beta_S\right)\theta_S+\beta_S}}\right\}
\end{aligned}
\tag{2.25}
$$

where

$$
\begin{aligned}
\lim_{x\to\bar{p}-}F_1\left(x\right)=&\left\{1+\frac{\left(1-\mu\right)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right\}\\
&\times\left\{1-\left[\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\right]^{\frac{\left(1-\beta_S\right)\theta_S+\beta_S}{\beta_S\theta_S+1-\beta_S}}\right\}
\end{aligned}
\tag{2.26}
$$

We can now implicitly solve for β_S as a function of the remaining parameters using the equation

$$
\mathrm{E}_1\left[p\right]-\mathrm{E}_2\left[p\right]=0 \tag{2.27}
$$

Equations (2.2) and (2.27) imply that $r_2=r_1$. That is,

$$
\begin{aligned}
&\int_{\underline{p}}^{r_2}p\mathrm{d}F_2(p)=\lim_{x\to r_2-}\int_{\underline{p}}^{x}p\mathrm{d}F_1(p)+\bar{p}\left[1-\lim_{x\to r_2-}F_1\left(x\right)\right]\\
\Leftrightarrow&\int_{\underline{p}}^{r_2}F_2(p)\mathrm{d}p=\int_{\underline{p}}^{r_2}F_1(p)\mathrm{d}p
\end{aligned}
\tag{2.28}
$$

The first equation sets the expected prices of the two firms equal to each other. The second follows from integration by parts.

Define r_1^* and r_2^* as the equilibrium reservation prices. If $r_2\left(\mu,\ \theta_S,\ c,\ \beta_N\right)\leq v$, r_2^* is defined by Equation (2.21). Because firms are not concerned with prices above v, if $r_2\left(\mu,\ \theta_S,\ c,\ \beta_N\right)>v$, we define r_2^* as positive infinity. According to

Equation (2.28), we can set $r_1^* = r_2^*$. $\square$

Support Type 4. Without loss of generality, suppose that β_N is such that $\bar{p}_1 = r_2 < \min\{v,\ r_1\} = \bar{p}_2$. Then a complete solution to an equilibrium with support type 4, if one exists, requires the following set of equations to hold.

$$\begin{aligned} &\mathrm{E}\,\pi_1\left(\underline{p}\right) = \mathrm{E}\,\pi_1\left(p_1, F_2\left(p_1\right)\right) \Leftrightarrow \underline{p}\left[\mu\left(\beta_S\theta_S + 1 - \beta_S\right) + \left(1-\mu\right)\beta_N\right] \\ &= \mu\left\{p_1\left(\beta_S\theta_S + 1 - \beta_S\right)\left[1 - F_2\left(p_1\right)\right]\right. \\ &+ \left.\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2|p_2 < p_1\right]F_2\left(p_1\right)\right\} + \left(1-\mu\right)\beta_N p_1 \end{aligned} \tag{2.29}$$

$$\begin{aligned} &\mathrm{E}\,\pi_1\left(\underline{p}\right) = \mathrm{E}\,\pi_1\left(r_2, F_2\left(r_2\right)\right) \Leftrightarrow \underline{p}\left[\mu\left(\beta_S\theta_S + 1 - \beta_S\right) + \left(1-\mu\right)\beta_N\right] \\ &= \mu\left\{r_2\left(\beta_S\theta_S + 1 - \beta_S\right)\Pr\left(p_2 = \bar{p}_2\right)\right. \\ &+ \left.\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2|p_2 < r_2\right]F_2\left(r_2\right)\right\} + \left(1-\mu\right)\beta_N r_2 \end{aligned} \tag{2.30}$$

$$\begin{aligned} &\mathrm{E}\,\pi_2\left(\underline{p}\right) = \mathrm{E}\,\pi_2\left(p_2, F_1\left(p_2\right)\right) \\ &\Leftrightarrow \underline{p}\left\{\mu\left[\left(1-\beta_S\right)\theta_S + \beta_S\right] + \left(1-\mu\right)\left(1-\beta_N\right)\right\} = \left(1-\mu\right)\left(1-\beta_N\right)p_2 \\ &+\mu\left\{p_2\left[\left(1-\beta_S\right)\theta_S + \beta_S\right]\left[1 - F_1\left(p_2\right)\right] + \left(1-\theta_S\right)\beta_S\,\mathrm{E}\left[p_1|p_1 < p_2\right]F_1\left(p_2\right)\right\} \end{aligned} \tag{2.31}$$

$$\begin{aligned} &\mathrm{E}\,\pi_2\left(\underline{p}\right) = \mathrm{E}\,\pi_2\left(\bar{p}_2\right) \Leftrightarrow \underline{p}\left\{\mu\left[\left(1-\beta_S\right)\theta_S + \beta_S\right] + \left(1-\mu\right)\left(1-\beta_N\right)\right\} \\ &= \mu\left(1-\theta_S\right)\beta_S\,\mathrm{E}\left[p_1\right] + \left(1-\mu\right)\left(1-\beta_N\right)\bar{p}_2 \end{aligned} \tag{2.32}$$

$$\int_{\underline{p}}^{r_2} F_1(p)\mathrm{d}p + r_1 - r_2 = c \tag{2.33}$$

$$\int_{\underline{p}}^{r_2} F_2(p)\mathrm{d}p = c \tag{2.34}$$

$$\Pr\left(p_1 = r_2\right) = 1 - \lim_{\varepsilon \to 0-} F_1\left(r_2 - \varepsilon\right) \in (0,\ 1) \tag{2.35}$$

In addition to Equations (2.29) to (2.35), the expected prices of the two firms must equal each other to make non-shoppers indifferent between which firm to search first. Moreover, the inequality, $\mathrm{E}\,\pi_1\left(r_2,\ F_2\left(r_2\right)\right) > \mathrm{E}\,\pi_1\left(\bar{p}_2-\varepsilon,\ F_2\left(\bar{p}_2-\varepsilon\right)\right)$ must hold for all $\varepsilon \in (0,\ \bar{p}_2 - r_2)$. That is, firm 1 must not wish to deviate above r_2.

We use the following procedure to look for equilibrium. First, we use Equation (2.29) and (2.31) to solve for F_2 and F_1 respectively, in terms of $\underline{p}$. Plugging F_2 into Equation (2.34) and using Equation (2.30) to solve for $\underline{p}$ we obtain r_2 in terms of $\Pr\left(p_2 = \bar{p}_2\right)$. Plugging F_1 into Equation (2.35) yields $\Pr\left(p_1 = r_2\right)$ in terms of $\Pr\left(p_2 = \bar{p}_2\right)$. Rewriting F_1 in terms of $\Pr\left(p_2 = \bar{p}_2\right)$ and plugging into Equation (2.33) yields r_1 in terms of $\Pr\left(p_2 = \bar{p}_2\right)$. Finally, using Equation (2.32) to solve for $\underline{p}$ and setting this equal to the solution obtained from Equation (2.30) we can rewrite r_1 as an alternate function of $\Pr\left(p_2 = \bar{p}_2\right)$. Setting the two expressions for r_1 equal to each other, we can now solve for $\Pr\left(p_2 = \bar{p}_2\right)$ in terms of the exogenous parameters. An equilibrium exists only if there is a solution to $\Pr\left(p_2 = \bar{p}_2\right)$ in the interval $[0,\ 1]$ such that non-shoppers are indifferent between which firm to search first and firm 1 does not wish to deviate above r_2.

Proposition 2.2. *Suppose that firms are exogenously required to offer price-matching guarantees and $\theta_S = 0$. Then there exists a unique Sequential Equilibrium where both firms set price v and $r_1^* = r_2^* = v + c$.*

Proof. We begin by showing that $\underline{p} \geq \min\{v, r_1, r_2\}$. Suppose conversely that $\underline{p} < \min\{v, r_1, r_2\}$. At any $p_1 \in \left(\underline{p},\, \min\{v, r_1, r_2\}\right)$, firm 1 captures $1 - \beta_S$ shoppers, who pay $\min\{p_1,\ p_2\}$, and the same proportion of non-shoppers (β_N, $\beta_N + (1-\gamma)(1-\beta_N)$, or 1 depending on whether firm 2 prices below, at, or above r_1), who pay p_1. Thus, firm 1's profit is increasing in p_1, a contradiction.

Claim 2.1 in the proof of Lemma 2.1 still holds. Therefore, $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq$

$\underline{p}\geq \min\{v,\ r_1,\ r_2\}$. Moreover, from Equation (2.2) we know that $\min\{r_1,\ r_2\} > \underline{p}$, so $\min\{\bar{p}_1,\ \bar{p}_2\} = v = \underline{p}$. Suppose, without loss of generality, that $\bar{p}_2 > \bar{p}_1$. At any $p_2 \in (\bar{p}_1,\ \bar{p}_2]$, firm 2 expects profit of $\mu\beta_S v$. By shifting its mass in $(\bar{p}_1,\ \bar{p}_2]$ to v, firm 2 expects an additional profit of $(1-\mu)(1-\beta_N)\,v$, a contradiction. Using a similar argument we can rule out $\bar{p}_1 > \bar{p}_2$. Thus, $v = \bar{p}_1 = \bar{p}_2 = \bar{p} = \underline{p}$. Since the unique equilibrium is symmetric and employs pure strategies, we can define $F_1(p) = F_2(p) = F(p)$ as 0 for $p < v$ and as 1 for $p \geq v$. But then, using Equation (2.2) we get $r_1^* = r_2^* = v + c$. $\square$

Lemma 2.2. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to. In equilibrium, the supports of the firm pricing distributions can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_1 = r_2\}$.

2. *Single mass point, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* i *has a mass point at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$, $r_j \leq r_i$.

3. *Two mass points, mutual break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has a mass point at* $r_i < \min\{v,\ r_j\}$*; mutual break over* $(r_i,\ p^u)$ *for* $p^u \in (r_i,\ \bar{p})$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$*, firm* i *has a mass point at* $\bar{p}$.

4. *Two mass points, single break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has a mass point at* $\bar{p}_j = r_i < \min\{v,\ r_j\}$*; firm* i *has a break over* $(r_i,\ \bar{p}_i)$ *for* $\bar{p}_i = \min\{v,\ r_j\}$ *and a mass point at* $\bar{p}_i$.

The following claims complete the proof of Lemma 2.2.

Claim 2.6. $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0$.

Proof. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Then, for $p_1 \in \left[\underline{p}_1, \underline{p}_2\right)$, firm 1's expected profit is

$$p_1 \left\{\mu + (1-\mu)\left\{\beta_N + (1-\beta_N)\left\{[1 - F_2(r_1)] + (1-\gamma)\Pr(p_2 = r_1)\right\}\right\}\right\} \tag{2.36}$$

which is increasing in p_1, contradicting the equilibrium. If $\underline{p}_1 \leq v < \underline{p}_2$, for $p_1 \in \left[\underline{p}_1, v\right)$, firm 1's expected profit is given by Expression (2.36), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. But if $v = \underline{p}_1 < \underline{p}_2$, firm 2 makes no profit on its support and for sufficiently small $\varepsilon > 0$, it benefits by shifting its mass to $v - \varepsilon$. Finally, if $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. Suppose $\underline{p}_2 < \underline{p}_1 \leq v$. Then, for $p_2 \in \left[\underline{p}_2, \underline{p}_1\right)$, firm 2's expected profit is

$$\begin{aligned} &p_2 \left\{\mu\left[\theta_S + \beta_S(1-\theta_S)\right]\right. \\ &\quad \left. + \ (1-\mu)\left\{(1-\beta_N) + \beta_N\left\{[1 - F_1(r_2)] + (1-\gamma)\Pr(p_1 = r_2)\right\}\right\}\right\} \end{aligned} \tag{2.37}$$

(since $\underline{p}_1 < r_1$ by definition) which is increasing in p_2, again contradicting the equilibrium. If $\underline{p}_2 \leq v < \underline{p}_1$, for $p_2 \in \left[\underline{p}_2, v\right)$, firm 2's expected profit is given by Expression (2.37), which is increasing in p_2, so it must be the case that $\underline{p}_2 = v$. But if $v = \underline{p}_2 < \underline{p}_1$, then $F_1(v) = 1$ (because firm 2 doesn't make any profit at prices above v) and firm 1 expects profit of $\mu(1-\beta_S)(1-\theta_S)v$ everywhere on its support. For sufficiently small $\varepsilon > 0$, firm 1 benefits by shifting its mass to $v - \varepsilon$ for expected profit of $(v-\varepsilon)\left\{\mu\left[\theta_S + (1-\beta_S)(1-\theta_S)\right] + (1-\mu)\beta_S\right\}$. Thus, $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$. Since prices below zero result in negative profit, $\underline{p} \geq 0$.[14]

The proof that $v \geq \min\{\bar{p}_1,\ \bar{p}_2\}$ follows similarly to that in Claim 2.1 and is left to the reader. $\square$

Claim 2.7. *There are no mutual mass points.*

[14]If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$ because at $p_i = \varepsilon < \min\{r_j,\ v\}$, firm i will make money off its non-shoppers.

Proof. Suppose that there is a mutual mass point at p. Firm 1's expected profit at p when firm 2 charges p as well is given by Expression (2.5). Suppose instead that firm 1 deviates to $p-\varepsilon$ while firm 2 maintains its price at p. Firm 1's expected profit will be

$$(p-\varepsilon)\left\{\mu+(1-\mu)\left\{\beta_N+(1-\beta_N)\left[(1-\gamma)\,\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}\right]\right\}\right\} \tag{2.38}$$

Expression (2.5) is smaller than Expression (2.38) provided that ε is sufficiently small.

Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive probability lead to a gain and with the complementary probability at worst lead to a loss of ε. Therefore, by shifting its mass point at p to $p-\varepsilon$ for sufficiently small ε firm 1 increases its expected profit, a contradiction. □

Claim 2.8. *The only possible breaks in the equilibrium supports are:*

(i) If $\bar{p}_i < \bar{p}_j$, there is a break at $(\bar{p}_i,\ \bar{p}_j) \in S_j$.

(ii) If $r = r_i = r_j < \bar{p}_i = \bar{p}_j$, there may be a mutual break with lower bound r.

(iii) If $r_i \neq r_j$ and firm i has a mass point at r_j, there may be a mutual break with lower bound r_j.

Proof. Let S_1 and S_2 be respectively, the equilibrium supports for firms 1 and 2. Define $H = \left(p^d,\ p^u\right) \in \operatorname{int}(S_1 \cap S_2)$.

Suppose first that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at some $p_1 \in H$ is

$$\mu \{p_1 [1 - F_2 (p_1)] + (1 - \theta_S) (1 - \beta_S) \mathrm{E} [p_2 | p_2 < p_1] F_2 (p_1)\}$$
$$+ (1 - \mu) \{p_1 \beta_N \{\mathbb{I}_{p_1 < r_2} + \{\gamma + (1 - \gamma) [1 - F_2 (p_1)]\} \mathbb{I}_{p_1 = r_2} + [1 - F_2 (p_1)] \mathbb{I}_{p_1 > r_2}\}$$
$$+ (1 - \beta_N) \{p_1 [1 - F_2 (r_1) + (1 - \gamma) \Pr (p_2 = r_1)] \mathbb{I}_{p_1 < r_1} + p_1 [1 - F_2 (r_1)] \mathbb{I}_{p_1 = r_1}$$
$$+ \{p_1 [1 - F_2 (p_1)] + r_1 (1 - \theta_N) (1 - \gamma) \Pr (p_2 = r_1)$$
$$+ (1 - \theta_N) \mathrm{E} [p_2 | r_1 < p_2 < p_1] [F_2 (p_1) - F_2 (r_1)]\} \mathbb{I}_{p_1 > r_1}\}\} \tag{2.39}$$

As firm 1 raises p_1 along H, its expected profit is increasing since $F_2 (p_1)$ is constant along H (and equal to $F_2 (r_1)$ if $r_1 \in H$). Thus, if $r_2 \notin H$, firm 1 could increase expected profits by shifting all its mass in H slightly below p^u (to p^u if firm 2 doesn't have a mass point there), a contradiction. If $r_2 \in H$, firm 1 can increase expected profits by shifting all mass in $(p^d,\ r_2)$ slightly below r_2, and all mass in $(r_2,\ p^d)$ either slightly below r_2 or to p^u, again contradicting the equilibrium.

Conversely, suppose that firm 1 has no support over H, but that firm 2 does. Firm 2's expected profit at some $p_2 \in H$ is

$$\begin{aligned}
&\mu p_2 [(1 - \beta_S) \theta_S + \beta_S] [1 - F_1 (p_2)] + (1 - \mu) p_2 \\
&\times \{(1 - \beta_N) \{\mathbb{I}_{p_2 < r_1} + \{\gamma + \theta_N (1 - \gamma) [1 - F_1 (p_2)]\} \mathbb{I}_{p_2 = r_1} \\
&+ \theta_N [1 - F_1 (p_2)] \mathbb{I}_{p_2 > r_1}\} \\
&+ \beta_N \{[1 - F_1 (r_2) + (1 - \gamma) \Pr (p_1 = r_2)] \mathbb{I}_{p_2 < r_2} \\
&+ [1 - F_1 (r_2)] \mathbb{I}_{p_2 = r_2} + [1 - F_1 (p_2)] \mathbb{I}_{p_2 > r_2}\}\}
\end{aligned} \tag{2.40}$$

As firm 2 raises p_2 along H, its expected profit is increasing since $F_1 (p_2)$ is constant along H (and equal to $F_1 (r_2)$ if $r_2 \in H$). Thus, if $r_1 \notin H$, firm 2 could increase expected profits by shifting all its mass in H slightly below p^u (to p^u if firm 2 doesn't have a mass point there), a contradiction. If $r_1 \in H$, firm 2 can increase expected profits by shifting all mass in $(p^d,\ r_1)$ slightly below r_1, and all mass in $(r_1,\ p^d)$ either slightly below r_1 or to p^u, again contradicting the equilibrium.

Thus, any breaks in $S_1 \cap S_2$ are mutual.

The remainder of this proof follows similarly to that in Claim 2.3 and is left to the reader. $\square$

Corollary 2.3. *The equilibrium supports are the same except if* $\bar{p}_i = r_j < \bar{p}_j = \min\{v,\ r_i\}$.

Claim 2.9. *Firm i does not have a mass point in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof. The proof to show that firm 2 does not have a mass point at $p \in S_1 \setminus \{\bar{p}_1\}$ proceeds precisely as that in Claim 2.4 and is omitted. Suppose instead that firm 1 has a mass point at $p \in S_2 \setminus \{\bar{p}_2\}$ and that $p \neq r_2$. Firm 2's expected profit at $p - \varepsilon$ when firm 1 charges p is

$$\begin{aligned}(p-\varepsilon)\{&\mu\left[(1-\beta_S)\,\theta_S+\beta_S\right]+(1-\mu)\\ &\times\{(1-\beta_N)\left[\mathbb{I}_{p-\varepsilon<r_1}+\left[\gamma+\theta_N\,(1-\gamma)\right]\mathbb{I}_{p-\varepsilon=r_1}+\theta_N\mathbb{I}_{p-\varepsilon>r_1}\right]\\ &+\beta_N\left[(1-\gamma)\,\mathbb{I}_{p=r_2}+\mathbb{I}_{p>r_2}\right]\}\}\end{aligned} \tag{2.41}$$

whereas its expected profit at $p + \varepsilon$ is

$$(1-\mu)\,(p+\varepsilon)\,(1-\beta_N)\,(\mathbb{I}_{p+\varepsilon<r_1}+\gamma\mathbb{I}_{p+\varepsilon=r_1}) \tag{2.42}$$

Expression (2.42) is smaller than Expression (2.41) provided that ε is sufficiently small. Suppose firm 1 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 2 from lowering the price by 2ε or less is at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will with positive probability lead to a gain and with the complementary probability at worst lead to a loss of 2ε. Therefore, by shifting its mass between p and $p+\varepsilon$ to $p-\varepsilon$ for sufficiently small ε, firm 2 increases its expected profit, a contradiction. $\square$

Claim 2.10. *If $\bar{p} = \bar{p}_1 = \bar{p}_2$ then either*

(i) $\bar{p} = \min\{v,\ r_1,\ r_2\}$*, the supports have no breaks, and at most one firm can have a mass point at* $\bar{p}$*, or*

(ii) $\bar{p} = \min\{v,\ max\{r_1,\ r_2\}\}$*, there is a mutual break above* $\min\{r_1,\ r_2\} < \bar{p}$*, firm i has a mass point at r_j, and firm j has a mass point at* $\bar{p}$*.*

Proof. Suppose that $\bar{p} = \bar{p}_1 = \bar{p}_2$ and neither firm has a mass point at $\bar{p}$. From Claims 2.6 and 2.9 we know that $\underline{p} < \bar{p} \leq v$. Suppose that $\bar{p} < \min\{v,\ r_2\}$. At $p_1 \in [\bar{p},\ \min\{v,\ r_2\})$, firm 1's expected profit is

$$\begin{aligned}&\mu(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2]\\&+\ (1-\mu)\{\beta_N p_1 + (1-\theta_N)(1-\beta_N)\{\mathrm{E}[p_2|r_1<p_2][1-F_2(r_1)]\\&+r_1(1-\gamma)\Pr(p_2=r_1)\}\}\end{aligned} \tag{2.43}$$

which is increasing in p_1 when $\beta_N \neq 0$, a contradiction. Suppose instead that $\bar{p} > \min\{v,\ r_2\} = r_2$. For any $p_1 \in (r_2,\ \bar{p})$, in equilibrium, $\mathrm{E}\,\pi_1(\bar{p}) = \mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$. $\mathrm{E}\,\pi_1(\bar{p})$ equals

$$\begin{aligned}&\mu(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2]\\&+\ (1-\mu)(1-\theta_N)(1-\beta_N)\{\mathrm{E}[p_2|r_1<p_2][1-F_2(r_1)]\\&+r_1(1-\gamma)\Pr(p_2=r_1)\}\end{aligned} \tag{2.44}$$

If $r_2 \geq r_1$, for $p_1 \in (r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ equals

$$\begin{aligned}&\mu\{p_1[1-F_2(p_1)] + (1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2|p_2<p_1]\,F_2(p_1)\}\\&+(1-\mu)\{p_1\beta_N[1-F_2(p_1)] + (1-\beta_N)\{r_1(1-\theta_N)(1-\gamma)\Pr(p_2=r_1)\\&+p_1[1-F_2(p_1)] + (1-\theta_N)\,\mathrm{E}[p_2|r_1<p_2<p_1][F_2(p_1)-F_2(r_1)]\}\}\end{aligned} \tag{2.45}$$

Setting Expression (2.44) equal to Expression (2.45) and differentiating with respect to p_1 gives us

$$1-F_2(p_1)-[\mu(\theta_S+\beta_S-\theta_S\beta_S)+(1-\mu)(\theta_N+\beta_N-\theta_N\beta_N)]\,p_1F_2{}'(p_1)=0 \quad (2.46)$$

Solving the differential equation given by Equation (2.46) using the initial value $F_2(\bar{p})=1$ gives us $F_2(p_1)=1$ for all $p_1\in(r_2,\ \bar{p}]$, a contradiction. Similarly, if $r_1\in(r_2,\ \bar{p})$, then Expression (2.45) represents firm 1's expected profit at $(r_1,\ \bar{p})$ and $F_2(p_1)=1$ for all $p_1\in(r_1,\ \bar{p}]$, a contradiction. If on the other hand, $r_1\geq\bar{p}$, $\mathrm{E}\,\pi_1(\bar{p})$ becomes $\mu(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2]$ while $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ at $p_1\in(r_2,\ \bar{p})$ becomes

$$\begin{aligned}&\mu\{p_1[1-F_2(p_1)]+(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2|p_2<p_1]\,F_2(p_1)\}\\&+\ (1-\mu)\,\beta_Np_1[1-F_2(p_1)]\end{aligned} \quad (2.47)$$

Setting $\mu(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2]$ equal to Expression (2.47) and solving the resulting differential equation using the initial value $F_2(\bar{p})=1$ again gives us $F_2(p_1)=1$ for all $p_1\in(r_2,\ \bar{p}]$, a contradiction. Hence, for $\beta_N\neq 0$, $\bar{p}=\min\{v,\ r_2\}$. Now suppose that $\bar{p}<\min\{v,\ r_1\}$. At $p_2\in[\bar{p},\ \min\{v,\ r_1\})$, firm 2's expected profit is $(1-\mu)(1-\beta_N)\,p_2$, which is increasing in p_2 when $\beta_N\neq 1$, a contradiction. Suppose instead, that $\bar{p}>\min\{v,\ r_1\}$. But then, at $\bar{p}$, firm 2 expects no profit, a contradiction. Thus, $\bar{p}=\min\{v,\ r_1\}$, so when neither firm has a mass point at $\bar{p}$, $\bar{p}=\min\{v,\ r_1,\ r_2\}$.

From Claim 2.7, we know that at most one firm can have a mass point at $\bar{p}$, say firm j. If $\gamma=1$ or $v<r_i$, then following the argument in the paragraph above, $\bar{p}=\min\{v,\ r_i\}$. Otherwise, firm j cannot have a mass point at $\bar{p}$ (using reasoning similar to that in the proof of Claim 2.8). Moreover, if $r_j\geq r_i$, then $\bar{p}=\min\{v,\ r_1,\ r_2\}$ and from Claim 2.8, we know that the firm supports have no breaks. Conversely, suppose $r_j<r_i$ (and therefore, $r_j<v$). First, let $i=1$. From Claim 2.9, we know that firm 2 cannot have a mass point at r_2. At r_2, firm 1

expects profit of

$$\mu \{r_2 [1 - F_2 (r_2)] + (1 - \theta_S) (1 - \beta_S) \operatorname{E} [p_2 | p_2 < r_2] F_2 (r_2)\} + (1 - \mu) \beta_N r_2 \quad (2.48)$$

whereas at $p_1 \in (r_2,\ \bar{p})$, $\operatorname{E} \pi_1 (p_1,\ F_2 (p_1))$ is given by Expression (2.47). By definition, for $p_1 \in (r_2,\ \bar{p})$, $0 < F_2 (r_2) \leq F_2 (p_1)$, so for p_1 close enough to r_2, Expression (2.48) is strictly greater than Expression (2.47). Therefore, r_2 must be the lower bound for a break in S_1 and we are in Case (iii) of Claim 2.8. Now let $j = 1$. From Claim 2.9, we know that firm 1 cannot have a mass point at r_1. At r_1, firm 2 expects profit of

$$r_1 \{\mu [(1 - \beta_S) \theta_S + \beta_S] [1 - F_1 (r_1)] + (1 - \mu) (1 - \beta_N)\} \quad (2.49)$$

whereas at $p_2 \in (r_1,\ \bar{p})$, $\operatorname{E} \pi_2 (p_2,\ F_1 (p_2))$ is given by

$$p_2 \{\mu [(1 - \beta_S) \theta_S + \beta_S] + (1 - \mu) (1 - \beta_N) \theta_N\} [1 - F_1 (p_2)] \quad (2.50)$$

By definition, for $p_2 \in (r_1,\ \bar{p})$, $0 < F_1 (r_1) \leq F_1 (p_2)$, so for p_2 close enough to r_1, Expression (2.49) is strictly greater than Expression (2.50). Therefore, r_1 must be the lower bound for a break in S_2 and we are again in Case (iii) of Claim 2.8. $\square$

Notice that Claim 2.10 rules out Case (ii) in Claim 2.8.

Proposition 2.3. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to. Then there exists a set of Sequential Equilibria where both firms distribute prices over support*

$$\left[\underline{p} = \min\{v, r^*\} \left\{ \frac{(1-\mu)(1-\beta_N)}{\mu [(1-\beta_S)\theta_S + \beta_S] + (1-\mu)(1-\beta_N)} \right\}, \bar{p} = \min\{v, r^*\}\right]$$

and the equilibrium reservation price, $r^* = r_1^* = r_2^*$ equals

$$r^{*}=\begin{cases} r\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right) & if \quad r\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right)\leq v \\ \infty & otherwise \end{cases},$$

$$r\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right)=c\left\{1-\frac{\left(1-\mu\right)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right.$$
$$\left.\times\ln\left\{1+\frac{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}{\left(1-\mu\right)\left(1-\beta_N\right)}\right\}\right\}^{-1}.$$

Firm 1 distributes prices according to

$$F_1\left(p\right)=\left\{1+\frac{\left(1-\mu\right)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right\}\left(1-\frac{\underline{p}}{p}\right),$$

while firm 2 distributes prices according to

$$F_2\left(p\right)=\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu}\right]\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{1}{\beta_S+\theta_S-\beta_S\theta_S}}\right]$$

over $\left[\underline{p},\bar{p}\right)$ *with a mass point at* $\bar{p}$*. In equilibrium, the expected prices for the two firms equal each other.*

Proof. The proof of this proposition follows very similarly to that of Proposition 2.1. As such, below we will only write down equations for the equilibrium firm distribution functions, the lower bound of firm supports, the reservation price when it is the upper bound (when it is no greater than v), and firms' expected prices.

Suppose that firm supports are represented by case 2 in Lemma 2.2. Then firm 1's distribution function is

$$F_1\left(p\right)=\left\{1+\frac{\left(1-\mu\right)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right\}\left(1-\frac{\underline{p}}{p}\right) \tag{2.51}$$

and firm 2's distribution function is

$$F_2\left(p\right)=\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu}\right]\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{1}{\beta_S+\theta_S-\beta_S\theta_S}}\right] \tag{2.52}$$

The lower bound, $\underline{p}$, is

$$\underline{p} = \bar{p}\left\{\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]+(1-\mu)(1-\beta_N)}\right\} \tag{2.53}$$

When $r_1 \leq v$, $\bar{p} = r_1$ is given by

$$\begin{aligned} &r_1(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) \\ &= c\left\{1-\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]}\ln\left\{1+\frac{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]}{(1-\mu)(1-\beta_N)}\right\}\right\}^{-1} \end{aligned} \tag{2.54}$$

Firm 1's expected price is

$$\mathrm{E}_1[p] = \frac{\bar{p}(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]}\ln\left\{1+\frac{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]}{(1-\mu)(1-\beta_N)}\right\} \tag{2.55}$$

Firm 2's expected price is

$$\begin{aligned} &\mathrm{E}_2[p] = \bar{p}\left[1-\lim_{x\to\bar{p}-}F_2(x)\right] \\ &+\bar{p}\left\{\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]+(1-\mu)(1-\beta_N)}\right\}\frac{\left[\mu+(1-\mu)\beta_N\right]^{\beta_S+\theta_S-\beta_S\theta_S}}{\mu(1+\beta_S\theta_S-\theta_S-\beta_S)} \\ &\times\left\{\left[\mu+(1-\mu)\beta_N\right]^{1+\beta_S\theta_S-\theta_S-\beta_S}\right. \\ &\left.-\left\{\mu\left[1-\lim_{x\to\bar{p}-}F_2(x)\right]+(1-\mu)\beta_N\right\}^{1+\beta_S\theta_S-\theta_S-\beta_S}\right\} \end{aligned} \tag{2.56}$$

where

$$\begin{aligned} &\lim_{x\to\bar{p}-}F_2(x) = \left[1+\frac{(1-\mu)\beta_N}{\mu}\right] \\ &\times\left\{1-\left\{\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S+\beta_S\right]+(1-\mu)(1-\beta_N)}\right\}^{\frac{1}{\beta_S+\theta_S-\beta_S\theta_S}}\right\} \end{aligned} \tag{2.57}$$

$\square$

Chapter 3

Asymmetric Sequential Search

3.1 Introduction

There is a rich literature examining how costly price search by consumers leads to price dispersion in homogeneous goods markets (Burdett and Judd 1983; Stahl 1989; Janssen et al. 2005). Because the consumer search order is usually assumed to be random, all firms expect to be visited by the same types of consumers in equal proportion and all firms have the same price distribution in equilibrium. However, there are many markets where different firms expect to encounter different types of consumers. For instance, while supermarkets serve various types of buyers, higher pricing convenience stores sell many of the same goods primarily to one type: individuals with a high opportunity cost of shopping elsewhere. Similarly, low price outlets may provide consumers with the same products sold by general retailers, but since they are located far from residential areas they mainly cater to consumers with a low opportunity cost of searching.[1] In this paper, we are

[1]For empirical evidence, see Chung and Myers 1999; Hausman and Leibtag 2007; Griffith et al. 2009.

interested in analyzing how such demand asymmetries affect prices and welfare when consumers engage in sequential price search.

In our model, two firms compete by setting prices for a homogeneous good. There are two types of consumers in the market. Fully informed consumers (referred to as shoppers) have no opportunity cost of time and observe both prices at no cost,[2] whereas non-shoppers engage in optimal sequential search. When searching sequentially, a consumer with a price at hand continues searching only as long as the marginal benefit of doing so is higher than the constant marginal cost. We adjust the commonly used assumption that the first price sample is free for all consumers by supposing that non-shoppers can only obtain the first price for free at their local firm. When one of the two firms that comprise the market has a larger local population than the other, a symmetric equilibrium no longer exists. That is, the equilibrium price distributions for the two firms differ.

We characterize the unique equilibrium of this game and provide several comparative statics. In equilibrium, firms randomize over lower prices to attract shoppers, and over higher prices to realize greater profits from local non-shoppers, who end up not searching. The equilibrium price distribution of the firm with the larger local population first order stochastically dominates that of the other firm, and as a result, the firm with the larger local population will have higher prices on average. This is because the firm with the larger local population loses more profit from non-shoppers when it lowers its price. Since both firms have the same equilibrium support, the firm with the larger local population also has an atom

[2] Interpretations of shoppers in the literature are as individuals who read sales ads (Varian 1980), as consumers who derive enjoyment from shopping (Stahl 1989), as a coalition of consumers who freely share their search information (Stahl 1996), and as users of search engines (Janssen and Non 2008).

at the upper bound of the distribution. This is interpreted to mean that it is less likely to run a sale. Moreover, the firm's propensity to run a sale decreases as the share of the total population that is local to it increases. Interestingly, both firms will continue to run sales with positive probability even when all non-shoppers are local to one firm because this firm still has an incentive to try to capture shoppers.

It turns out that the likelihood of a sale does not have to increase in the proportion of shoppers. As this proportion increases, firms tend to run better sales, but the frequency of sales can increase or decrease depending on how local non-shoppers in the two firms are affected. Both firms are clearly motivated to run better sales to attract the new shoppers in the population. However, where these new shoppers are coming from affects the frequency of sales. For instance, suppose that firm 1 has more local non-shoppers than firm 2 and that the growing number of shoppers comes primarily out of the population of non-shoppers local to firm 2. To compensate for the better sales that it runs to attract the new shoppers, firm 1 exploits its local non-shoppers by running sales less often. Nevertheless, in terms of consumer welfare, the effect of better sales always dominates.

In most models of sequential search (Stahl 1989, 1996; Janssen et al. 2005), heterogeneous consumers sample firm prices at random, so changes in the composition of consumers affect all firms in the same way. It is by biasing the sampling behavior of some of these consumers to favor a particular firm that we are able to examine the impact of demand differences on firm pricing. Arbatskaya (2007) examines an ordered search model where consumers with heterogeneous search costs *all* face the same search order. In her pure strategy equilibrium, firms at the bottom of the search order always have lower prices than those at the top. On the other hand, Armstrong et al. (2009) analyze a model where consumers with different tastes all search a prominent firm first, but find that the prominent firm has a lower price than the rest. Reinganum's (1979) model addresses the opposite question: it asks how firm cost heterogeneity affects price dispersion when all consumers are homogeneous. The papers that come closest to addressing our research question are Narasimhan (1988) and Jing and Wen (2008), which consider a market with shoppers and non-shoppers who decide where to buy according to an exogenous rule. In Narasimhan, a proportion of non-shoppers is only allowed to buy from one firm while the rest must purchase from the other firm. In Jing and Wen all non-shoppers purchase from the same firm unless the other firm underprices it by an exogenously determined amount. By endogenizing the purchasing decision, we are able to show how changes in the proportion of shoppers or the level of asymmetry affect non-shoppers' search strategies and firm pricing behavior.

The remainder of this paper is organized as follows. Section 3.2 presents the model. Section 3.3 develops the equilibrium and Section 3.4 explores the comparative statics. Section 3.5 concludes and details directions for future research. Section 3.6 consists of appendices containing formal proofs.

3.2 The model

Two firms, labeled 1 and 2, sell a homogeneous good. Firms have no capacity constraints and an identical constant cost of zero of producing one unit of the good.[3] There is a unit mass of almost identical consumers with inelastic (unit) demand and valuation $v > 0$ for the good. A proportion $\sigma \in (0, 1)$ of consumers have 0 cost of search, and will be referred to as shoppers. The remaining $1 - \sigma$

[3]The price of the good can be viewed as a price cost margin.

consumers, called non-shoppers, pay a positive search cost $c \in (0, v)$ for each firm they visit except for their local firm, which they search first. A fraction $\lambda \in [0, 1-\sigma]$ of all non-shoppers are local to firm 1, while the remaining $1-\sigma-\lambda$ are local to firm 2. Non-shoppers search sequentially. Upon observing the price at their local firm, they must decide whether to search the second one. We assume costless recall—that is, consumers who have observed both prices can freely choose to purchase the good at the lower price.

Firms and consumers play the following game. First, firms 1 and 2 simultaneously choose prices taking into consideration their beliefs about the rival firm's pricing strategy and about consumer search behavior. A pricing strategy consists of a price distribution F_i over $[\underline{p}_i, \bar{p}_i]$, where $F_i(p)$ represents the probability that firm $i = 1, 2$ offers a price no higher than p. After firms choose price distributions, prices are realized. Shoppers observe the price realizations of both firms and choose where to buy the product. Non-shoppers local to firm i only observe firm i's price realization. Given their information and beliefs about firm j's price distribution, non-shoppers choose a search strategy that specifies whether or not to search the non-local firm, whether or not to buy the product, and where to buy it (in case both firm price realizations are observed). Parameters v, c, σ, λ, as well as the rationality of all agents in the model are commonly known.

3.3 Equilibrium analysis

The equilibrium concept that we use is Sequential Equilibrium.[4] In this context, Sequential Equilibrium requires that non-shoppers who observe an off-equilibrium

[4]For an extension of the definition of sequential equilibrium to infinite action games, see Manelli (1996).

price at their local firm treat such deviations as mistakes when forming beliefs about the non-local firm's strategy. Thus, non-shoppers believe that the non-local firm plays its equilibrium strategy at all information sets.

3.3.1 Consumer behavior

The marginal benefit of searching firm j for a non-shopper local to firm i after having observed a price of p_λ at firm i is given by

$$\int_{\underline{p}_j}^{p_\lambda} (p_\lambda - p)\mathrm{d}F_j(p) \tag{3.1}$$

Expression (3.1) denotes the non-shopper's expected surplus from searching firm j. After searching firm j, the non-shopper will purchase at the lower of p_λ and the price observed at firm j. Thus, the marginal benefit of search arises from the opportunity to find a price lower than p_λ at firm j.

After integration by parts, Expression (3.1) becomes

$$\int_{\underline{p}_j}^{p_\lambda} F_j(p)\mathrm{d}p \tag{3.2}$$

A non-shopper will benefit from searching his non-local firm if and only if Expression (3.2) is no less than his cost of search c. That is,

$$\int_{\underline{p}_j}^{p_\lambda} F_j(p)\mathrm{d}p \geq c \tag{3.3}$$

The optimal strategy for a non-shopper local to firm i is to search firm j if and only if the price realization at his local firm is greater than some reservation price, denoted r_j. In subsection 3.3.3 we will show that the equilibrium reservation price is the mapping that gives the value of p_λ that makes Expression (3.3) hold with equality when such a p_λ exists and is less than or equal to v.

3.3.2 Firm pricing

Before characterizing the equilibrium of the game, we will narrow down the possible sets of prices that firms can charge.

Proposition 3.1. *In equilibrium, the firm supports can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_1 = r_2\}$.

2. *Single atom, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; *firm* i *has an atom at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\}$, $r_j \leq r_i$.

3. *Two atoms, mutual break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; *firm* j *has an atom at* $r_i < \min\{v, r_j\}$; *mutual break over* (r_i, p^u) *for* $p^u \in (r_i, \bar{p})$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\}$, *firm* i *has an atom at* $\bar{p}$.

4. *Two atoms, single break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; *firm* j *has an atom at* $\bar{p}_j = r_i < \min\{v, r_j\}$; *firm* i *has a break over* $(r_i, \bar{p}_i)$ *for* $\bar{p}_i = \min\{v, r_j\}$ *and an atom at* $\bar{p}_i$.

The proof of this proposition is contained in the appendix. Notice that in every case firms never price higher than v, so there is always complete consumer participation in the market. The completely symmetric case in Proposition 3.1 occurs if and only if $\lambda = (1 - \sigma)/2$, that is, if both firms have the same fraction of local non-shoppers. As such, it is a special case of support type 2. In support type 2, because firms never price above the smaller of the two reservation prices, non-shoppers never search in equilibrium. Support type 3 is the only one where some non-shoppers may search beyond their local firm, while support type 4 is the only case where the firm supports are not the same. However, in Proposition 3.2 we show that the last two support types cannot occur in equilibrium.

The corollary below is a technical result needed for existence of equilibrium. It follows immediately from the proof of Proposition 3.1. It states that when non-shoppers are indifferent between staying at their local firm and searching, whenever the upper bound of the firm supports is lower than v, they must stay in order for equilibrium to exist.

Corollary 3.1. *For equilibria with support types 2, 3 or 4 in Proposition 3.1 to exist, all non-shoppers must stop searching after observing a price of r_j at local firm i,*[5] *unless $v < r_j$.*

3.3.3 Equilibrium

Let $\rho_j(\sigma, \lambda, c)$ be the mapping that gives the value of p_λ that makes Expression (3.3) hold with equality. From Proposition 3.1, we know that there are no atoms at the lower bound of the support of firm price distributions. Since Expression (3.2) is increasing in p, we know that such a value of p_λ exists and that it is unique.

Going forward, we will consider only the case $\lambda \in ((1-\sigma)/2, 1-\sigma]$, so that firm 1 will always be treated as the one with more local non-shoppers. The following proposition completely describes the unique Sequential Equilibrium of this game for this case. The case $\lambda \in [0, (1-\sigma)/2)$ follows analogously and we leave it to the reader.

Proposition 3.2. *There exists a unique Sequential Equilibrium where both firms*

[5]This means $\gamma = 1$ in the proof of Proposition 3.1.

have supports $\left[\underline{p}, \bar{p}\right]$*, where* $\bar{p} = \min\{v, r_2^*\}$ *and* $\underline{p} = \frac{\lambda}{\lambda+\sigma}\bar{p}$*, and* r_1^* *and* r_2^* *are the equilibrium reservation prices for non-shoppers local to firm 2 and firm 1 respectively.* $r_1^* = \infty$*,*

$$r_2^* = \begin{cases} \rho_2(\sigma, \lambda, c) = c\left[1 - \frac{\lambda}{\sigma}\ln\left(\frac{\sigma+\lambda}{\lambda}\right)\right]^{-1} & if\ \rho_2(\sigma, \lambda, c) \leq v \\ \infty & otherwise \end{cases}$$

Firm 1 distributes prices according to

$$F_1(p) = \begin{cases} \frac{1-\lambda}{\sigma}\left[1 - \underline{p}/p\right] & \underline{p} \leq p < \bar{p} \\ 1 & p = \bar{p} \end{cases}$$

with $\Pr\left[p_1 = \bar{p}\right] = \frac{2\lambda+\sigma-1}{\sigma+\lambda}$*, and firm 2 distributes prices according to* $F_2(p) = \frac{\sigma+\lambda}{\sigma}\left[1 - \underline{p}/p\right]$*.*

Proof. We prove existence by directly computing and characterizing the equilibrium. A sketch of the proof of uniqueness is included in the appendix, where we rule out support types (3) and (4) in Proposition 3.1.

In equilibrium, a firm must be indifferent between any price in its support. Therefore, for any p_1 in the support of F_2, $\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(p_1, F_2(p_1))$. Given that the unique equilibrium has support type (2),[6]

$$\underline{p}(\sigma + \lambda) = p_1\{\sigma[1 - F_2(p_1)] + \lambda\} \tag{3.4}$$

We can solve for $F_2(p)$ to get

$$F_2(p) = \frac{\sigma + \lambda}{\sigma}\left(1 - \underline{p}/p\right) \tag{3.5}$$

Similarly, setting $\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(p_2, F_1(p_2))$, it is easy to show that

$$F_1(p) = \frac{1 - \lambda}{\sigma}\left(1 - \underline{p}/p\right) \tag{3.6}$$

[6]Support type (2) subsumes support type (1) as the special case ($\lambda = (1 - \sigma)/2$, $r_1^* = r_2^*$, $\Pr(p_i = \bar{p}) = 0$).

Since $\lambda > \frac{1-\sigma}{2}$, F_1 first order stochastically dominates F_2. Because $\bar{p}_1 = \bar{p}_2 = \bar{p}$, this implies an atom at firm 1's upper bound.

Setting $F_2(\bar{p}) = 1$, we can solve for $\underline{p}$ in terms of $\bar{p}$ and substitute it into $F_2(p)$, which becomes

$$F_2(p) = \frac{\sigma+\lambda}{\sigma}\left(1 - \frac{\lambda}{\sigma+\lambda}\frac{\bar{p}}{p}\right) \tag{3.7}$$

When $\rho_2(\sigma,\lambda,c) \leq v$, $\bar{p} = \rho_2(\sigma,\lambda,c)$. Optimal search requires that Expression (3.3) holds with equality. Substituting in Equation (3.7) yields

$$\int_{\underline{p}}^{\rho_2} \frac{\sigma+\lambda}{\sigma}\left(1 - \frac{\lambda}{\sigma+\lambda}\frac{\rho_2}{p}\right) \mathrm{d}p = c \tag{3.8}$$

Finally, integrating and solving for ρ_2, we get

$$\rho_2(\sigma,\lambda,c) = c\left[1 - \frac{\lambda}{\sigma}\ln\left(\frac{\sigma+\lambda}{\lambda}\right)\right]^{-1} \tag{3.9}$$

If $\rho_2(\sigma,\lambda,c) \leq v$, r_2^* is defined by Equation (3.9). Because firms are not concerned with prices above v, if $\rho_2(\sigma,\lambda,c) > v$, we define r_2^* as positive infinity. Since $F_1(p) < F_2(p)$, $\rho_1(\sigma,\lambda,c) > \min\{v, \rho_2(\sigma,\lambda,c)\}$, and we can define r_1^* as positive infinity. $\square$

In the equilibrium described above, all non-shoppers search their local firm and make a purchase there, whereas all shoppers purchase from the firm with the lower price. Since non-shoppers always buy from their local firm, it is more costly for firm 1 to lower its price than it is for firm 2. Firm 1 takes advantage of its location by running fewer sales and pricing higher on average.

Proposition 3.2 immediately gives rise to the following result.

Corollary 3.2. *For $\lambda \geq \frac{1-\sigma}{2}$, $\rho_2(\sigma,\lambda,c)$ is decreasing in σ and increasing in λ.*

Corollary 3.2 tells us that as long as $r_2^* \leq v$, the bounds of the firm price distributions fall in the proportion of shoppers and rise as the firm with more local non-shoppers gains market power relative to the other firm.

3.4 Comparative statics

In this section we explore how changes in λ and σ affect firm price distributions in more depth. All proofs will be contained in the appendix.

3.4.1 Changes in the proportion of locals at firm 1

Proposition 3.3. *For $\sigma \in (0,1)$,*

(i) As λ increases over $[\frac{1-\sigma}{2}, 1-\sigma]$, the probability that firm 1 runs a sale decreases. Even when $\lambda = 1-\sigma$, price dispersion persists.

(ii) Let $\frac{1-\sigma}{2} < \underline{\lambda} < \bar{\lambda} < 1-\sigma$. Then for both firms, the price distribution contitional on $\bar{\lambda}$ first order stochastically dominates the price distribution conditional on $\underline{\lambda}$.

As λ increases, the atom at $p_1 = \bar{p}$ increases in size, but equals $1-\sigma$ when $\lambda = 1-\sigma$. Hence, there is always price dispersion in equilibrium (there is no pure strategy equilibrium). This results because $\sigma \in (0,1)$ and both firms run sales to attract shoppers. The second part of Proposition 3.3 says that as firm 1 gains market power relative to firm 2, not only do the bounds on firm price distributions increase (as implied by Corollary 3.2), but also both distributions shift in a first order stochastic dominant sense. This is not completely straightforward for firm 2 because as λ increases there are two countervailing forces acting on its prices. On the one hand, when firm 2 has fewer locals, it has more incentive to lower prices to lure shoppers. On the other, because of the shift in firm 1's distribution, firm 2 no longer needs to lower prices as much to have the same probability of capturing all the shoppers as it did with a lower λ. Proposition 3.3 tells us that the latter effect dominates. The proof is included in the appendix.

3.4.2 Changes in the proportion of shoppers

When λ increases, by definition, firm one obtains a higher proportion of local non-shoppers. In contrast, when σ changes, it is not clear which firm(s) loses or gains local non-shoppers. For example, if σ increases there may be a fall in the number of local non-shoppers in firm 2 without any change in the number of non-shoppers local to firm 1. However, the fact that $\sigma + \lambda \leq 1$ imposes a constraint on how much σ can increase without affecting λ. As a result, when evaluating changes in σ, λ must be treated as a function of σ, $\varphi(\sigma)$ with derivative $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma \in [-1, 0]$. $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = 0$ means that as σ increases, only firm 2 loses local non-shoppers, $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = -1$ means that as σ increases, only firm 1 loses local non shoppers, and otherwise, both firms lose local non-shoppers as σ increases.

Proposition 3.4. *The probability that firm 1 runs a sale and both firm price distributions depend on σ and the size of $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = \lambda'$ as follows.*

(i) For λ' sufficiently close to zero, the atom increases in σ, and firm 1's price distribution increases in σ for lower prices, and decreases in σ for higher prices.

(ii) For λ' sufficiently close to -1, the atom decreases in σ, and firm 1's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ.

(iii) Firm 2's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ always.

According to Proposition 3.4, as the proportion of shoppers increases, both firms have incentives to run better sales to attract the new shoppers. That is, both firm price distributions will put more mass on lower prices. However, firm 1's atom at $\bar{p}$ may increase or decrease depending on which firm is losing more local non-shoppers. This can be interpreted respectively as a decrease or an increase in the frequency of sales run by firm 1. Suppose that that the new shoppers come primarily out of the population of non-shoppers local to firm 2. Firm 1 can compensate itself for the better sales that it runs to attract the new shoppers by pricing at $\bar{p}$ with higher probability because it still maintains its fraction of locals relative to the entire population of consumers (λ is constant). If, instead, firm 1's local non-shoppers are the ones becoming shoppers, then firm 1 will run more sales and have higher discounts because λ is decreasing.

3.4.3 Welfare implications

The welfare implications of Proposition 3.3 are straightforward. That is, expected welfare is decreasing in λ because when λ is higher there are fewer sales and those that are run tend to be worse. On the contrary, expected welfare is increasing in σ. When λ' is sufficiently close to -1, this is clear because there are more sales and those that are run tend to be better. However, when λ' is sufficiently close to zero, the welfare implications are not obvious because, while there are fewer sales, those that are run tend to be better. To show that expected welfare is still increasing in σ in this case, we define expected welfare, W, as follows:

$$W = (v - \mathbb{E}_1[p])\lambda + (v - \mathbb{E}_2[p])(1 - \lambda) \tag{3.10}$$

where $\mathbb{E}_i$ denotes the expected price under distribution F_i. Since firm 1 has a higher expected price in equilibrium, only its λ local non-shoppers are expected to purchase from it, while all other consumers are expected to buy from firm 2. It is easy to show that W increases in σ, so that the positive effect from better sales dominates the negative effect from the decline in the number of sales.

3.4.4 Limiting cases

Before concluding, we would like to explore the limiting cases when either all consumers are shoppers or all are non-shoppers. We start by looking at the case when σ approaches 1. From Corollary 3.2, we know that for sufficiently high σ, $\rho_2(\sigma, \lambda, c)$ is strictly lower than v. In addition, for any value of λ, when σ becomes large enough, $\lambda = 1 - \sigma$. Thus, the equilibrium reservation price r_2^* becomes the following expression:

$$c\left[1 + \frac{1-\sigma}{\sigma}\ln(1-\sigma)\right]^{-1} \tag{3.11}$$

As $\sigma \to 1$, Expression (3.11) approaches c and $\underline{p} \to 0$. As a consequence, Proposition 3.4 implies that as $\sigma \to 1$, the equilibrium firm price distributions collapse to a degenerate distribution at zero. Therefore, in the limit, the effects of location disappear and we are back to Stahl's (1989) result that when all consumers are shoppers, the unique Nash equilibrium is the competitive price.

The case when σ approaches 0 is not as clear cut as the previous one. The results differ depending on the value of λ. For a given value of $\lambda \in [\frac{1-\sigma}{2}, 1-\sigma)$, as $\sigma \to 0$, $\rho_2(\sigma, \lambda, c) \to \infty$, so that for sufficiently low σ, $\bar{p} = v$. Moreover, as $\sigma \to 0$, $\underline{p} \to \bar{p}$. Therefore, in the limit, when all consumers are non-shoppers, the entire distribution collapses to v. However, when $\sigma = 0$ and $\lambda = 1$, then to the contrary, the monopolistic outcome may not occur. The results of this subsection are summarized in Proposition 3.5.

Proposition 3.5.

(i) When $\sigma = 1$, the unique equilibrium is $p_1 = p_2 = 0$.

(ii) When $\sigma = 0$ and $\lambda \in [0.5, 1)$, the unique equilibrium is $p_1 = p_2 = v$, $r_1^* = r_2^* = \infty$, and consumers purchase from their local firm.

(iii) When $\sigma = 0$ and $\lambda = 1$, the set of pure strategy equilibria[7] can be characterized as follows: $p_1 \in [0, v]$. If $p_1 \in [0, v)$, $p_2 = p_1 - c$, $r_2^* = p_1 = p_2 + c < v$. If $p_1 = v$, $p_2 \in [v - c, \infty)$, and $r_2^* = \min\{v, \infty\}$.

Part (ii) of Proposition 3.5 states that Stahl's (1989) result persists even after accounting for location asymmetries. That is, when all consumers are non-shoppers and both firms have locals, the monopolistic outcome is the unique equilibrium of the game.

However, when $\lambda = 1$, there are multiple pure strategy equilibria where firm 2 underprices firm 1 by c. In such equilibria, since non-shoppers do not search, firm 1 would like to charge v. However, since $\lambda = 1$, firm 2 makes zero profit, so it can charge any price. By charging prices lower than $v - c$, firm 2 increases the marginal benefit of searching, causing r_2^* to drop below v. In order to keep its local non-shoppers from searching firm 2, firm 1 has to lower its price below v. This is contrary to what happens in Proposition 3.3 where $\sigma \in (0, 1)$ and a higher λ is associated with increasing prices.

[7] In this case there is no completely mixed strategy equilibrium. Firm 1 will always play a pure strategy in equilibrium. Even though firm 2 can play a mixed strategy in equilibrium, this does not matter to consumers because they never observe its price.

3.5 Conclusion

In this paper, we have studied the consequences of introducing a location asymmetry into a duopoly version of Stahl's (1989) seminal model of sequential consumer search. Contrary to Stahl, where all consumers can sample the first price for free at any firm, non-shoppers in our model can only obtain the first price quote for free at their local firm. When firms serve different proportions of local non-shoppers, Stahl's symmetric equilibrium can no longer exist. In this case, the price distribution of the firm with more locals (and hence, greater market power) first order stochastically dominates that of the other firm and the firm with more locals no longer runs sales all the time (as in symmetric models).

We have analyzed the following comparative statics results in this equilibrium. First, as the market power of the firm with more locals grows, it runs fewer sales and tends to offer smaller discounts in the sales it does run. Second, as the proportion of shoppers in the economy rises, this firm offers greater discounts on the sales it runs. However, it may run more or fewer sales depending on which firm is losing local non-shoppers.

A natural direction for future research is to assume that non-shoppers have a cost of recalling the first price after having searched the second firm. This has not been modeled in an asymmetric framework and we are particularly interested to learn how this feature will influence non-shopper search behavior. Another possible extension is the N firm analogue of this paper. With more than two firms reservation prices are no longer stationary and all consumers must determine an optimal sampling order for firms beyond their local one.

3.6 Appendices

3.6.1 Appendix 1: Proof of Proposition 3.1

Proposition 3.1. *In equilibrium, the firm supports can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_1 = r_2\}$.

2. *Single atom, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* i *has an atom at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\},\ r_j \leq r_i$.

3. *Two atoms, mutual break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has an atom at* $r_i < \min\{v, r_j\}$*; mutual break over* (r_i, p^u) *for* $p^u \in (r_i, \bar{p})$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\}$*, firm* i *has an atom at* $\bar{p}$.

4. *Two atoms, single break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has an atom at* $\bar{p}_j = r_i < \min\{v, r_j\}$*; firm* i *has a break over* $(r_i, \bar{p}_i)$ *for* $\bar{p}_i = \min\{v, r_j\}$ *and an atom at* $\bar{p}_i$.

The following claims complete the proof of Proposition 3.1.

Claim 3.1. $v \geq \max\{\bar{p}_1, \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0$.

Proof. Let γ be the proportion of non-shoppers who do not search after observing a price of r_j at their local firm i. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Firm 1's profit on $[\underline{p}_1, \underline{p}_2)$ equals

$$p_1\{\sigma + \lambda + (1 - \sigma - \lambda)[1 - F_2(r_1) + (1 - \gamma)\Pr(p_2 = r_1)]\} \tag{3.12}$$

which is increasing in p_1, a contradiction. Now suppose that $\underline{p}_1 \leq v < \underline{p}_2$. Then for $p_1 \in [\underline{p}_1, v)$, firm 1's profit is given by Equation (3.12), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. If $\underline{p}_1 = v < \underline{p}_2$, then firm 2 makes zero profit on its support, and would increase profits by shifting mass to v. If $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. By a similar argument, $\underline{p}_2 \geq \underline{p}_1$ and $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$.

Firms will not charge prices below zero because these yield negative profit.[8] Similarly, at prices above v firms make zero profit. As a result, all consumers in the market make a purchase. $\square$

Definition 3.1. *We say that firms have a mutual atom when each firm has an atom at the same price. We say that firms have a mutual break when each firm's equilibrium support has a break over the same price interval.*

Claim 3.2. *There are no mutual atoms.*

Proof. Let α^S be the proportion of shoppers who buy from firm 1 after having observed the same price in both firms. Let α^N be the proportion of non-shoppers who buy from their local firm after having observed the same price in both firms. Suppose that both firms have a mutual atom at p. When $p_1 = p_2 = p$, firm 1's profit is given by

$$\begin{aligned} p\{\alpha^S\sigma + \lambda[\mathbb{I}_{p<r_2} + [\gamma + \alpha^N(1-\gamma)]\mathbb{I}_{p=r_2} + \alpha^N\mathbb{I}_{p>r_2}] \\ + (1-\sigma-\lambda)(1-\alpha^N)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\} \end{aligned} \tag{3.13}$$

where $\mathbb{I}$ is an indicator function. Suppose that firm 1 sets $p_1 = p - \varepsilon$ instead of p. Then profits become

$$(p-\varepsilon)\{\sigma + \lambda + (1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\} \tag{3.14}$$

[8] If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$.

Expression (3.14) is larger than Expression (3.13) when

$$\varepsilon < \frac{p\{\sigma(1-\alpha^S)+\lambda(1-\alpha^N)[(1-\gamma)\mathbb{I}_{p=r_2}+\mathbb{I}_{p>r_2}]+(1-\sigma-\lambda)\alpha^N[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]\}}{\sigma+\lambda+(1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]}$$

Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales, so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive probability lead to a gain and with complementary probability, at worst lead to a loss of ε. Therefore, by shifting its atom at p to $p-\varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. By a similar argument, firm 2 will want to undercut a mutual atom for $\lambda \neq 1-\sigma$.

For the case $1-\alpha^S = \alpha^N = \lambda = 0$, firm 1 does not have a profitable deviation, but firm 2 does. □

Claim 3.3. *The only possible breaks in the equilibrium supports are:*

(i) if $\bar{p}_i < \bar{p}_j$ there is a break at $(\bar{p}_i, \bar{p}_j) \in S_j$

(ii) if $r = r_i = r_j < \bar{p}_i = \bar{p}_j$ there may be a mutual break with lower bound r, and

(iii) if $r_i \neq r_j$ and firm i has an atom at r_j, there may be a mutual break with lower bound r_j.

Proof. Let S_1 and S_2 denote the equilibrium supports for firms 1 and 2 respectively. Let $\hat{p} = \inf(S_1 \cap S_2)$ and $\hat{\hat{p}} = \sup(S_1 \cap S_2)$. Define $H = (p^d, p^u) \in \mathrm{int}(S_1 \cap S_2)$.

Suppose first, without loss of generality, that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at $p_1 \in H$ is

$$
\begin{aligned}
p_1\{\ & \sigma[1-F_2(p_1)] \\
& + \lambda\{\mathbb{I}_{p_1<r_2} + [\gamma + (1-\gamma)[1-F_2(p_1)]]\mathbb{I}_{p_1=r_2} + [1-F_2(p_1)]\mathbb{I}_{p_1>r_2}\} \\
& + (1-\sigma-\lambda)\{[1-F_2(r_1) + (1-\gamma)\Pr(p_2=r_1)]\mathbb{I}_{p_1<r_1} \\
& + [1-F_2(r_1)]\mathbb{I}_{p_1=r_1} + [1-F_2(p_1)]\mathbb{I}_{p_1>r_1}\}\ \}
\end{aligned}
\tag{3.15}
$$

As firm 1 raises p_1 along H, its expected profit is increasing since $F_2(p_1)$ is constant along H (and equal to $F_2(r_1)$ if $r_1 \in H$). If $r_2 \notin H$, then firm 1 could increase profits by shifting all mass in H slightly below p^u (or to p^u if firm 2 doesn't have an atom there), a contradiction. If $r_2 \in H$, then firm 1 can increase profits by shifting all mass in (p^d, r_2) to slightly below r_2, and all mass in (r_2, p^u) either to slightly below r_2 or to p^u, again contradicting the equilibrium. A similar argument applies when firm 1 has no support over H, but firm 2 does. This tells us that any breaks in $S_1 \cap S_2$ are mutual.

Now suppose that neither firm randomizes over H in equilibrium. Suppose first that $p^d \neq r_1$, $p^d \neq r_2$ and that neither firm has an atom at p^d. Then either firm 1 has a strictly higher expected profit at p^u (or slightly below r_2 if $r_2 \in H$) than at p^d, or firm 2 has a strictly higher expected profit at p^u (or slightly below r_1 if $r_1 \in H$) than at p^d, or both, if neither firm has an atom at p^u, contradicting the equilibrium.

Suppose that firm i has an atom at $p^d \neq r_j$. Since there are no mutual atoms, firm i could increase profits by shifting its atom to p^u (or slightly below p^u if firm j has an atom there, or slightly below r_j if $r_j \in H$).

If $p^d = r_j \neq r_i$ and firm i has no atom at p^d, firm j's expected profit will be strictly higher at p^u (or slightly below p^u if firm i has an atom there, or to slightly below r_i if $r_i \in H$) than at p^d. But if firm i does have an atom at p^d, then it is possible that profits are the same at p^d and p^u for each firm. If $\gamma \neq 1$, firm i can profitably deviate by shifting its atom slightly below p^d. In doing so, it retains $1-\gamma$ non-shoppers who search after observing a price r_j and have a positive probability of purchasing from firm j. However, if $\gamma = 1$, neither firm has a profitable deviation. This may also be the case if, $p^d = r_2 = r_1$.

By Claim 3.1, we know that both S_1 and S_2 have the same lower bound $\underline{p}$, so $S_1 \Delta S_2 \in (\min\{\bar{p}_1, \bar{p}_2\}, \max\{\bar{p}_1, \bar{p}_2\}]$. Suppose, without loss of generality, that $\bar{p}_1 > \bar{p}_2$. At $p_1 \in (\bar{p}_2, \bar{p}_1]$, firm 1's expected profit is $p_1\lambda(\mathbb{I}_{p_1<r_2} + \gamma\mathbb{I}_{p_1=r_2})$. If $\bar{p}_1 > r_2$, then firm 1 can increase profits by shifting mass in $(r_2, \bar{p}_1]$ to r_2 or slightly below it if $\gamma = 0$. If $r_2 \geq \bar{p}_1$, then profits are strictly increasing in $p_1 \in (\bar{p}_2, \bar{p}_1)$, so firm 1 could increase profits by shifting mass in $(\bar{p}_2, \bar{p}_1]$ to $\min\{r_2, v\} - \varepsilon$ for $\varepsilon > 0$ sufficiently small. As a result, $S_1 \Delta S_2 = \{\min\{v, r_2\}\}$. If firm 2 has no atom at $\bar{p}_2$, firm 1's expected profit at $\bar{p}_1$ is $\bar{p}_1\lambda$, strictly higher than its expected profit of $\bar{p}_2\lambda$ at $\bar{p}_2$, a contradiction. If firm 2 has an atom at $\bar{p}_2 \neq r_1$, since there are no mutual atoms, firm 2 can profitably shift the atom to slightly below $\bar{p}_1$ (or slightly below r_1 if $r_1 \in (\bar{p}_2, \bar{p}_1]$). However, if $\gamma = 1$ and firm 2 has an atom at $\bar{p}_2 = r_1$, $\mathbb{E}\Pi_1(\bar{p}_1, F_2(\bar{p}_1)) = \mathbb{E}\Pi_1(\bar{p}_2, F_2(\bar{p}_2))$, and $F_1(r_1)$ is large enough, then neither firm has a profitable deviation. A similar argument applies when $\bar{p}_2 > \bar{p}_1$. $\square$

Corollary 3.3. *The equilibrium supports are the same except if* $\bar{p}_i = r_j < \bar{p}_j = \min\{r_i, v\}$.

Claim 3.4. *Firm* i *does not have an atom in the lower bound or the interior of firm* j*'s equilibrium support, except possibly at* r_j.

Proof. Suppose without loss of generality that firm 2 has an atom at $p \in S_1 \backslash \{\bar{p}_1\}$, and suppose that $p \neq r_1$. Firm 1's expected profit at $p - \varepsilon$ when firm 2 charges p is given by Expression (3.14), whereas its expected profit at $p + \varepsilon$ is

$$(p+\varepsilon)\lambda(\mathbb{I}_{p+\varepsilon<r_2} + \gamma\mathbb{I}_{p+\varepsilon=r_2}) \tag{3.16}$$

Expression (3.16) is smaller than Expression (3.14) for

$$\varepsilon \;<\; \frac{p\;\{\sigma+\lambda[(1-\gamma)\mathbb{I}_{p+\varepsilon=r_2}+\mathbb{I}_{p+\varepsilon>r_2}]+(1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]\}}{\sigma+\lambda[1+\mathbb{I}_{p+\varepsilon<r_2}+\gamma\mathbb{I}_{p+\varepsilon=r_2}]+(1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]}.$$

Firm 1 can increase expected profits by shifting mass from $(p_2 - \varepsilon, p_2 + \varepsilon]$ to $p_2 - \varepsilon$ for the following reason. For any price that firm 2 charges, shifting mass to $p - \varepsilon$ never reduces the number of sales for firm 1, so it loses at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will, with positive probability, lead to a gain, and with complementary probability, at worst, lead to a loss of 2ε. Therefore, by shifting its mass between p and $p+\varepsilon$ to $p-\varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. □

Claim 3.5. *If* $\bar{p}_1 = \bar{p}_2 = \bar{p}$ *then either*

(i) $\bar{p} = \min\{v, r_1, r_2\}$*, the supports have no breaks, and at most one firm can have an atom at* $\bar{p}$*, or*

(ii) $\bar{p} = \min\{v, \max\{r_1, r_2\}\}$*, there is a mutual break above* $\min\{r_1, r_2\} < \bar{p}$*, firm* i *has an atom at* r_j*, and firm* j *has an atom at* $\bar{p}$*.*

Proof. Suppose that $\bar{p}_1 = \bar{p}_2 = \bar{p}$ and neither firm has an atom at $\bar{p}$. From Claims 3.1 and 3.4 we know that $\underline{p} < \bar{p} \leq v$. Suppose $\bar{p} < \min\{v, r_2\}$. At $\bar{p}$, firm 1 expects profit of $\bar{p}\lambda$. For $\lambda \neq 0$, by raising its price to $\min\{v, r_2\} - \varepsilon$, firm 1 expects to gain $[\min\{v, r_2\} - \varepsilon - \bar{p}]\,\lambda > 0$ for sufficiently small $\varepsilon > 0$, a contradiction. Suppose instead $\bar{p} > \min\{v, r_2\}$. But then at $\bar{p}$ firm 1 expects no profit, a contradiction, so $\bar{p} = \min\{v, r_2\}$. If $\lambda = 0$, firm 1 expects no profit at $\bar{p}$ unless either $\bar{p}_1 < \bar{p}_2$ or firm 2 has an atom at $\bar{p}$, a contradiction. By a similar argument, $\bar{p} = \min\{v, r_1\}$, so $\bar{p} = \min\{v, r_1, r_2\}$.

From Claim 3.2, we know that at most one firm can have an atom at $\bar{p}$, say firm j. If $\gamma = 1$ or $v < r_i$, then following the argument in the paragraph above, $\bar{p} = \min\{v, r_i\}$. Otherwise, firm j cannot have an atom at $\bar{p}$ (using similar reasoning to that in the proof of Claim 3.3). Moreover, if $r_j \geq r_i$, then $\bar{p} = \min\{v, r_1, r_2\}$ and from Claim 3.3, we know that the firm supports have no breaks. Conversely, suppose $r_j < r_i$ (and therefore $r_j < v$). Without loss of generality, let $i = 1$. From Claim 3.4 we know that firm 2 cannot have an atom at r_2. At r_2, firm 1 expects profit of

$$r_2\{[\sigma + \lambda(1-\gamma)][1 - F_2(r_2)] + \lambda\gamma\} \tag{3.17}$$

At $p_1 \in (r_2, \bar{p})$, firm 1 expects profit of

$$p_1(\sigma + \lambda)[1 - F_2(p_1)] \tag{3.18}$$

But since $p_1 \in (r_2, \bar{p})$, by definition, $0 < F_2(r_2) \leq F_2(p_1)$, so for a small enough p_1, Expression (3.17) will be strictly greater than Expression (3.18) as long as $\gamma > 0$. Therefore, r_2 must be the lower bound for a break in S_1, so we must be in case (iii) of Claim 3.3. The second to last paragraph in the proof of Claim 3.3 implies that this equilibrium only exists for $\gamma = 1$. $\square$

Notice that Claim 3.5 rules out case (ii) in Claim 3.3.

3.6.2 Appendix B: Proof of Proposition 3.2 (uniqueness)

Proposition 3.2. *There exists a unique Sequential Equilibrium where both firms have supports* $\left[\underline{p}, \bar{p}\right]$*, where* $\bar{p} = \min\{v, r_2^*\}$ *and* $\underline{p} = \frac{\lambda}{\lambda+\sigma}\bar{p}$*, and* r_1^* *and* r_2^* *are the equilibrium reservation prices for non-shoppers local to firm 2 and firm 1 respectively.* $r_1^* = \infty$*,*

$$r_2^* = \begin{cases} \rho_2(\sigma,\lambda,c) = c\left[1 - \frac{\lambda}{\sigma}\ln\left(\frac{\sigma+\lambda}{\lambda}\right)\right]^{-1} & if\ \rho_2(\sigma,\lambda,c) \le v \\ \infty & otherwise \end{cases}$$

Firm 1 distributes prices according to

$$F_1(p) = \begin{cases} \frac{1-\lambda}{\sigma}\left[1 - \underline{p}/p\right] & \underline{p} \le p < \bar{p} \\ 1 & p = \bar{p} \end{cases}$$

with $\Pr[p_1 = \bar{p}] = \frac{2\lambda+\sigma-1}{\sigma+\lambda}$*, and firm 2 distributes prices according to* $F_2(p) = \frac{\sigma+\lambda}{\sigma}\left[1 - \underline{p}/p\right]$.

Proof. From the proof of Proposition 3.2, we know that F_1 and F_2 are the only possible equilibrium distributions when the equilibrium support set is represented by type (1) or (2) outlined in the body of the paper. It remains to show that there do not exist any probability distribution functions F_1 and F_2 that satisfy the conditions necessary for equilibrium with supports of type (3) or (4). We will sketch the proof for support type (4) when $\lambda > (1-\sigma)/2$. For a complete proof, contact the authors.

$F_1(p)$ and $F_2(p)$ are represented by Equation (3.6) and Equation (3.5) respectively over $\left[\underline{p}, \bar{p}_1 = r_2^*\right)$. Note that, $\lambda > (1-\sigma)/2 \Leftrightarrow F_1(p) < F_2(p)$, so $\bar{p}_1 = r_2^* < \bar{p}_2 = \min\{v, r_1^*\}$. A complete solution to this equilibrium requires the following set of equations to hold.

$$\begin{aligned} &\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(p_1, F_2(p_1)) \\ &\Leftrightarrow (\sigma+\lambda)\underline{p} = \{\sigma[1 - F_2(p_1)] + \lambda\}p_1 \end{aligned} \tag{3.19}$$

$$\begin{aligned} &\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(r_2^*, F_2(r_2^*)) \\ &\Leftrightarrow (\sigma+\lambda)\underline{p} = [\sigma\Pr(p_2 = \bar{p}_2) + \lambda]r_2^* \end{aligned} \tag{3.20}$$

$$\begin{aligned}&\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(p_2, F_1(p_2))\\&\Leftrightarrow (1-\lambda)\underline{p} = \{\sigma[1-F_1(p_2)] + (1-\sigma-\lambda)\}p_2\end{aligned} \tag{3.21}$$

$$\begin{aligned}&\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(\bar{p}_2)\\&\Leftrightarrow (1-\lambda)\underline{p} = (1-\sigma-\lambda)\bar{p}_2\end{aligned} \tag{3.22}$$

$$\int_{\underline{p}}^{\rho_2} F_1(p)\mathrm{d}p + (\rho_1 - \rho_2) = c \tag{3.23}$$

$$\int_{\underline{p}}^{\rho_2} F_2(p)\mathrm{d}p = c \tag{3.24}$$

$$\begin{aligned}&\mathbb{E}\Pi_1(r_2^*, F_2(r_2^*)) > \mathbb{E}\Pi_1(\bar{p}_2 - \varepsilon, F_2(\bar{p}_2 - \varepsilon))\ \forall \varepsilon \in (0, \bar{p}_2 - r_2^*)\\&\Leftrightarrow [\sigma \Pr(p_2 = \bar{p}_2) + \lambda] r_2^* \geq \bar{p}_2 \Pr(p_2 = \bar{p}_2)(\sigma + \lambda)\end{aligned} \tag{3.25}$$

$$\Pr(p_1 = \bar{p}_1^*) = 1 - \lim_{\varepsilon \to 0^-} F_1(\bar{p}_1 - \varepsilon) \in (0, 1) \tag{3.26}$$

$$\Pr(p_2 = \bar{p}_2) = 1 - \lim_{\varepsilon \to 0^-} F_2(\bar{p}_2 - \varepsilon) \in (0, 1) \tag{3.27}$$

We use the following procedure to attempt to find an equilibrium. First, we use Equation (3.19) and Equation (3.21) to solve for $F_2(p)$ and $F_1(p)$ respectively, in terms of $\underline{p}$. Plugging $F_2(p)$ into Equation (3.24) and using Equation (3.20) to solve for $\underline{p}$ we obtain ρ_2 in terms of $\Pr(p_2 = \bar{p}_2)$. Plugging $F_1(p)$ into Equation (3.26) yields $\Pr(p_1 = \bar{p}_1)$ in terms of $\Pr(p_2 = \bar{p}_2)$. Rewriting $F_1(p)$ in terms of $\Pr(p_2 = \bar{p}_2)$ and plugging into Equation (3.23) yields ρ_1 in terms of $\Pr(p_2 = \bar{p}_2)$. Finally, using Equation (3.22) to solve for $\underline{p}$ and setting this equal to the solution obtained from Equation (3.20) we can rewrite r_1 as an alternate function of $\Pr(p_2 = \bar{p}_2)$. Setting the two expressions for ρ_1 equal to each other, we can now solve for $\Pr(p_2 = \bar{p}_2)$ in terms of the exogenous parameters. We can then use this to see if Inequality (3.25) holds. The solution to $\Pr(p_2 = \bar{p}_2)$ is an implicit function. A numerical analysis shows us that there is no solution in the interval $[0,1]$ (except when $\lambda = (1-\sigma)/2$ and $\Pr(p_2 = \bar{p}_2)$=$\Pr(p_1 = \bar{p}_1) = 0$). $\square$

Corollary 3.2. *For $\lambda \geq \frac{1-\sigma}{2}$, $\rho_2(\sigma, \lambda, c)$ is decreasing in σ and increasing in λ.*

Proof.

$$\frac{\partial \rho_2}{\partial \lambda} = -\frac{c(\ln \frac{\lambda}{\sigma+\lambda} + \frac{\sigma}{\sigma+\lambda})}{\sigma(1 + \frac{\lambda}{\sigma} \ln \frac{\lambda}{\sigma+\lambda})} \;\; > 0$$

Similarly,

$$\frac{\partial \rho_2}{\partial \sigma} = -\frac{c[\lambda^{'} - \frac{\lambda(1+\lambda^{'})}{\sigma+\lambda} - (\lambda^{'} - \frac{\lambda}{\sigma}) \ln \frac{\sigma+\lambda}{\lambda}]}{\sigma(1 - \frac{\lambda}{\sigma} \ln \frac{\sigma+\lambda}{\lambda})^2} \;\; < 0$$

Where $\lambda^{'}$ is defined as in Proposition 3.4. $\square$

3.6.3 Appendix C: Proofs of Propositions 3.3, 3.4 and 3.5

Proposition 3.3. *For $\sigma \in (0,1)$,*

(i) As λ increases over $[\frac{1-\sigma}{2}, 1-\sigma]$, the probability that firm 1 runs a sale decreases. Even when $\lambda = 1-\sigma$, price dispersion persists.

(ii) Let $\frac{1-\sigma}{2} < \underline{\lambda} < \bar{\lambda} < 1-\sigma$. Then for both firms, the price distribution contitional on $\bar{\lambda}$ first order stochastically dominates the price distribution conditional on $\underline{\lambda}$.

Proof.

Part (i):

$$\frac{\partial \Pr(p_1 = \bar{p})}{\partial \lambda} = \frac{\partial \left(\frac{2\lambda+\sigma-1}{\sigma+\lambda}\right)}{\partial \lambda} = \frac{1+\sigma}{(\sigma+\lambda)^2} \;\; > \;\; 0$$

Moreover, if $\lambda = 1-\sigma$, then $Pr(p_1 = \bar{p}) = 1-\sigma > 0$.

Part (ii):

For F_1: if $v < \rho_2$,

$$\frac{\partial F_1(p)}{\partial \lambda} = -\frac{1}{\sigma}\left\{\left[1 - \frac{\lambda}{(\sigma+\lambda)}\frac{v}{p}\right] + \frac{(1-\lambda)\sigma}{(\sigma+\lambda)^2}\frac{v}{p}\right\}$$

If $v \geq \rho_2$,

$$\frac{\partial F_1(p)}{\partial \lambda} = -\frac{1}{\sigma}\left\{\left[1 - \frac{\lambda}{(\sigma+\lambda)}\frac{\rho_2}{p}\right] + \frac{1-\lambda}{\sigma+\lambda}\left[\frac{\sigma}{(\sigma+\lambda)}\frac{\rho_2}{p} + \frac{\lambda}{p}\frac{\partial \rho_2}{\partial \lambda}\right]\right\}$$

The second term inside the curly brackets in each of the two equations above is clearly positive. Given that $\bar{p}/\underline{p} = (\sigma+\lambda)/\lambda$, the first term is minimized at $\underline{p}$ and equal to zero there.

For F_2: if $v < \rho_2$,

$$\frac{\partial F_2(p)}{\partial \lambda} = -\frac{1}{\sigma}\left(1 - \frac{v}{p}\right) \leq 0$$

If $v \geq \rho_2$,

$$\frac{\partial F_2(p)}{\partial \lambda} = -\frac{1}{\sigma}\left(1 - \frac{\rho_2}{p} - \frac{\lambda}{p}\frac{\partial \rho_2}{\partial \lambda}\right) \leq 0 \qquad \square$$

Proposition 3.4. *The probability that firm 1 runs a sale and both firm price distributions depend on σ and the size of $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = \lambda'$ as follows.*

(i) For λ' sufficiently close to zero, the atom increases in σ, and firm 1's price distribution increases in σ for lower prices, and decreases in σ for higher prices.

(ii) For λ' sufficiently close to -1, the atom decreases in σ, and firm 1's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ.

(iii) Firm 2's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ always.

Proof. Let $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = \lambda'$.

$$\frac{\mathrm{d}\Pr(p_1 = \bar{p})}{\mathrm{d}\sigma} = \frac{\mathrm{d}\left(\frac{2\lambda+\sigma-1}{\sigma+\lambda}\right)}{\mathrm{d}\sigma}$$
$$= \frac{\lambda^{'}(1-\sigma) + (1-\lambda)}{(\sigma+\lambda)^2}$$

Since this is a strictly increasing function of $\lambda^{'}$ and is positive when $\lambda^{'} = 0$ but negative when $\lambda^{'} = -1$, $\mathrm{d}\Pr(p_1 = \bar{p})/\mathrm{d}\sigma$ is non-monotonic in σ.

For F_1: if $v < \rho_2$,

$$\begin{aligned}\frac{\partial F_1(p)}{\partial \sigma} &= -\frac{1}{\sigma}\left(\lambda^{'} + \frac{1-\lambda}{\sigma}\right)\left[1 - \frac{\lambda}{(\sigma+\lambda)}\frac{v}{p}\right] \\ &\quad - \frac{1-\lambda}{\sigma(\sigma+\lambda)}\frac{v}{p}\left[\lambda^{'} - \frac{\lambda(1+\lambda^{'})}{\sigma+\lambda}\right]\end{aligned}$$

If $v \geq \rho_2$,

$$\begin{aligned}\frac{\partial F_1(p)}{\partial \sigma} &= -\frac{1}{\sigma}\left(\lambda^{'} + \frac{1-\lambda}{\sigma}\right)\left[1 - \frac{\lambda}{(\sigma+\lambda)}\frac{\rho_2}{p}\right] \\ &\quad - \frac{(1-\lambda)}{\sigma(\sigma+\lambda)}\left\{\left[\lambda^{'} - \frac{\lambda(1+\lambda^{'})}{\sigma+\lambda}\right]\frac{\rho_2}{p} + \frac{\lambda}{p}\frac{\partial \rho_2}{\partial \sigma}\right\}\end{aligned}$$

When $\lambda^{'} = -1$, both of these expressions are positive. Likewise, when $\lambda^{'} = 0$, for p sufficiently low. However, when p is sufficiently high, both of these expressions become negative.

For F_2: if $v < \rho_2$,

$$\frac{\partial F_2(p)}{\partial \sigma} = \frac{\lambda^{'}\sigma - \lambda}{\sigma}\left[\frac{1}{\sigma}\left(1 - \frac{v}{p}\frac{\lambda}{\sigma+\lambda}\right) - \frac{1}{p}\frac{1}{\sigma+\lambda}\right] \;>\; 0$$

If $v \geq \rho_2$,

$$\frac{\partial F_2(p)}{\partial \sigma} = \frac{1}{\sigma}\left[\left(\lambda^{'} - \frac{\lambda}{\sigma}\right)\left(1 - \frac{\rho_2}{p}\frac{b}{\sigma+\lambda}\right) - \frac{1}{p}\left(\frac{\lambda^{'}\sigma - \lambda}{\sigma+\lambda} + \lambda\frac{\partial \rho_2}{\partial \sigma}\right)\right] \;>\; 0$$

□

Proposition 3.5.

(i) When $\sigma = 1$, the unique equilibrium is $p_1 = p_2 = 0$.

(ii) When $\sigma = 0$ and $\lambda \in [0.5, 1)$, the unique equilibrium is $p_1 = p_2 = v$, $r_1^* = r_2^* = \infty$, and consumers purchase from their local firm.

(iii) When $\sigma = 0$ and $\lambda = 1$, the set of pure strategy equilibria can be characterized as follows: $p_1 \in [0, v]$. If $p_1 \in [0, v)$, $p_2 = p_1 - c$, $r_2^* = p_1 = p_2 + c < v$. If $p_1 = v$, $p_2 \in [v - c, \infty)$, and $r_2^* = \min\{v, \infty\}$.

Proof.

Case (i) follows from the standard Bertrand argument.

Case (ii): Claim 3.1 of Proposition 3.1 still holds, so the lower bound for both firms has to be the same. However, from Equation 3.4, we know that the firm distributions must be degenerate. So it must be the case that $p_1 = p_2 = p$. Suppose $p = v$. Given consumer (correct) beliefs, $\rho_1 = \rho_2 = \rho = c + v > v$, so the reservation price is $r^* = \infty$, which implies that consumers never search. Consider any other price $p < v$. Then $\rho = c + p$ and both firms can profitably deviate to $\min\{v, p + c\}$.

Case (iii): We restrict attention to pure strategy equilibria (see footnote 5). Consider the strategies $p_1 \in [0, v)$ and $p_2 = p_1 - c$. Given consumer (correct) beliefs, $\rho_2^* = p_1 = p_2 + c < v$, so $r_2^* = \rho_2$. Following the same reasoning as in Corollary 3.1, γ must equal to 1, so consumers never search in equilibrium. Consider any $p_2 > p_1 - c$. Then $p_1 < \rho_2 = p_2 + c$ and firm 1 can profitably deviate to $p_2 + c$. Now suppose that $p_2 < p_1 - c$. But then $p_1 > \rho_2$ so all consumers leave firm 1 and never come back. Thus, firm 1 can profitably deviate to $p_2 + c$. Finally, when $p_1 = v$, firm 2 may charge any price $p_2 > p_1 - c$. Given consumer (correct) beliefs, $r_2^* = \min\{v, \infty\}$ so firm 1 has no incentive to deviate. $\square$

Chapter 4

The Bunny or the Battery: Advertising to Publicize the Brand or the Good

4.1 Introduction

According to Advertising Age's annual assessment of advertising, total spending on advertising in the United States in 2009 was approximately 250 billion, almost two percent of GDP. Much of this expenditure corresponds to firms' investment in advertising.[1] The economics literature focuses on three functions of firm advertising: altering consumer preferences (Braithwaite, 1928; Kaldor, 1950), increasing consumer information about firms and their goods (Stigler, 1961; Telser, 1964; Butters, 1977; Grossman and Shapiro, 1984; Robert and Stahl, 1993), and augmenting the utility of consuming a particular good (Stigler and Becker, 1977;

[1] *Ad Spending by Medium and Sector.* The 2011 Entertainment, Media and Advertising Market Research Handbook, 225-227.

Becker and Murphy, 1993).[2] Marketing studies of advertising suggest that another important goal of advertising is to publicize – to get noticed and remembered by consumers (Miller and Berry, 1998; Bullmore, 1999; Ehrenberg et al., 2002; Till and Baack, 2005). For example, many commercials seek to amuse at the potential expense of lost message content (Weinberger and Gulas, 1992; Weinberger et al., 1995; Krishnan and Chakravarti, 2003; Cline and Kellaris, 2007; Hansen et al., 2009). Moreover, in many advertising situations (e.g., outdoor ads, magazine ads, product placements, etc.), the exposure to an ad may be so ephemeral that it is difficult to convey much beyond publicizing a product or a brand (Taylor et al., 2006; van Meurs and Aristoff, 2009; Pieters and Wedel, 2004; Karniouchina, 2011). In this paper, we seek to establish a theoretical model of advertising as a form of publicity and to determine its effects on economic outcomes.

Since most modern advertising occurs in the midst of multi firm competition, if we are to think of advertisements as a means to reinforce a product or brand in consumer memory, it is important to consider the effects of rival ads on memory as it pertains to purchasing decisions. Experimental evidence suggests that in the face of myriad ads from different firms (Kent, 1993; Kent and Allen, 1993), consumers are likely to get confused about which brand a particular ad refers to. This occurs because there is typically a lag between consumers' exposure to advertising and their decision to purchase the advertised good (Keller, 1987) and is accentuated when consumers review similar ads with a focus on the entertainment value of each ad rather than with an intention of purchasing what is advertised (Burke and Srull, 1988) and when consumers view ads for unfamiliar brands (Kent and Allen, 1994; Kumar and Krishnan, 2004). In a particularly well known instance of brand

[2] See Bagwell (2007) for an extensive overview of advertising.

confusion, viewers of the Energizer Bunny ad mistook it for promoting Duracell batteries (Lipman, 1990). Thus, a consumer may observe an ad by Seller A and be induced to purchase from Seller B. Another instance when this occurs is when the innovator firm which introduces a new product becomes eponymous for the product itself. For instance, for years, purchasers of photocopy machines referred to them as Xeroxes regardless of the brand. Therefore, advertising can function as a public good. This quality is not necessarily bad for firms as long as brand confusion is not unidirectional and all ads still function to expand the market for the advertised good.

However, firms can develop advertising strategies that are better "branded" (Unnava and Burnkrant, 1991; Law, 2002; Till and Baack, 2005). To understand the effects of successful brand advertising, we first consider how consumers in the market decide which brand to buy. Building on research in psychology, experimental studies of consumers posit that once the consumer decides to buy the product, he considers his list of viable brand alternatives, referred to as his consideration set (Miller and Berry, 1998; and Romaniuk and Sharp, 2004). After evoking a certain number of brands in his consideration set, he evaluates other attributes such as price, and then makes his decision of which brand to buy. Bullmore (1999) succinctly summarizes this decision process: "Most of us have clusters of brands which we find perfectly satisfactory. We will allocate share of choice within this repertoire according to chance, promotions, advertising, availability, price, impulse or recommendation. Brands may move in or out of that repertoire, but only infrequently." For example, a consumer shopping for toothpaste at the supermarket has an idea of which brands he is willing to buy even before stepping into the toothpaste aisle. He then compares the prices of the first couple of brands that come into his mind and buys the cheapest. Alternatively, a consumer who is considering buying a car chooses to visit a subset of local car dealerships. After looking at the different makes, he purchases the one with the best gas mileage. As such, firms find it crucial to be on top of consumers' minds. By promoting the brand, firms attempt to achieve a prominent position in the consumer consideration set.

We have identified two qualities of advertising publicity: a public good quality and a branding quality. We define the public good quality as *ad breadth.* Ads with greater breadth capture consumers' attention in such a way that they are more likely to enter the market, but they do not assure a firm that convinces an additional consumer to enter the market that the consumer will make a purchase from that firm. To paraphrase, in our paper, an ad with greater breadth directly enhances promotion of a good, but only indirectly benefits the individual brand. On the other hand, the branding quality, which we refer to as *brand salience*, increases the probability that when a consumer goes to make a purchase, he will evoke that brand out of his consideration set.

This paper sets up two theoretical models of advertising as a form of publicity, one to investigate the effects of ad breadth and another to analyze the effects of brand salience on economic outcomes. We find that in equilibrium, the level of ad breadth does not depend on price. Intuitively, this occurs because while greater breadth increases the number of consumers purchasing a product, it does not improve a firm's chances of making a sale to any particular consumer. Therefore, firms continue to compete on price with the same intensity regardless of the number of consumers.

The relationship between brand salience and price is substantially more complicated. If salient advertising is either sufficiently expensive or sufficiently cheap, all firms either forego or engage in advertising, respectively. In both cases, the equilibrium price distributions are identical and both firms have an equal probability of being evoked. This means that profit is strictly lower in the low cost case, making advertising wasteful. The effect of salient advertising is lost because both firms advertise with equal intensity. Intermediate levels of advertising cost lead to a mixed strategy equilibrium in advertising. Expected prices are higher in this equilibrium because, unlike in the pure strategy cases, where advertising is ineffective, firms can successfully use advertising to get consumers to pay attention to their brand, dampening the effectiveness of having a lower price.

Price dispersion is a consequence of our setup because firms know that some consumers who have evoked them will also evoke rival brands, and they wish to remain competitive over such consumers. As in Burdett and Judd's (1983) noisy sequential search framework, price dispersion occurs because every consumer has a positive probability of evoking one or more firms. However, in this paper, the distribution is endogenously determined by advertising.[3] A consumer who enters the market for a good is aware of all potential competitors in that market, but he only compares the prices of brands he has evoked.[4] In fact, as we discuss later, our model is robust to various forms of consumer heterogeneity as long as there exists a positive mass of consumers with a finite number of evocations.

The remainder of this paper is organized as follows. Section 4.2 sets up the

[3]This model differs from models of informative advertising with price dispersion (e.g., Butters, 1977; Robert and Stahl, 1993) where consumers cannot purchase from a particular firm without either obtaining a price ad from that firm or having paid a cost to search that firm.

[4]This contrasts both Chioveanu's (2008) model of persuasive advertising, where all consumers choose the lowest priced good unless they are "convinced" by advertising to become loyal to a certain brand and the framework of Haan and Moraga-Gonzlez (2009) where consumers are more likely to search firms with more salient advertising, but where search is required before a firm is considered.

model of breadth and analyzes its equilibrium and comparative statics. Section 4.3 does the same for salience. Section 4.4 discusses and Section 4.5 concludes. Section 4.6 contains supplementary formal proofs.

4.2 Model of Breadth

There are $n \geq 2$ identical firms that compete to sell a homogeneous good. Each firm pays a constant marginal cost of c to produce the good. Firm i makes a twofold decision: it sets its price, p_i and determines its level of ad breadth, $a_i^b \in \mathbb{R}_+$. For the remainder of the section, we treat the words breadth and advertising synonymously. The cost of advertising is given by $A(a_i^b)$, which is increasing and strictly convex. We assume there is no cost of no advertising: $A(0) = 0$.

On the other side of the market, there is a mass of identical consumers with valuation $v > c$ for the good. Consumers cannot purchase the product without remembering an ad from at least one firm. We can think of ads in this model as reminding consumers that they want the product being advertised, or, in the extreme case, as informing them that it exists. A consumer who remembers an ad enters the market and evokes one or two firms from his consideration set (which, for simplicity, is the set of all firms). Note that ads are remembered while firms are evoked. If only one firm is evoked, the consumer purchases from that firm. If two different firms are evoked, the consumer chooses the firm offering the lowest price. If both firms have the same price, each of these firms has an equal probability of being chosen.

The number of consumers who remember at least one ad is given by $r(\sum_{i=1}^{n} a_i^b)$, which is increasing and strictly concave in a_i^b for all i. There is no market for the good when no firm advertises: $r(0) = 0$. The probability that a consumer evokes firm i from his consideration set is σ_i. Consumers make two evocations with replacement (thus, the probability that a consumer evokes the same firm i twice is σ_i^2). In this framework, advertising increases the number of consumers in the market, $r(\sum_{i=1}^{n} a_i^b)$, but does not help any individual firm distinguish itself from its rivals.

Firms and consumers play the following game. First, the n firms in the market simultaneously choose prices and the level of advertising. Then, consumers in the market observe the prices of the firms they evoke and make a purchase decision. A pricing strategy for firm i is a price distribution F_i, where $F_i(p)$ denotes the probability that firm i offers a price smaller than or equal to p. An advertising strategy for firm i is a correspondence a_i^b defined over the support of F_i, where $a_i^b(p)$ denotes the level(s) of ad breadth at price p.

4.2.1 Equilibrium Analysis

We restrict attention to the symmetric equilibrium, where $\sigma_i = \frac{1}{n}$, $a_i^b = a^b$ and prices are distributed according to $F_i(p_i) = F(p)$ over some interval $[\underline{p}, \bar{p}]$ for all i. At each price $p \in [\underline{p}, \bar{p}]$, the optimal level of advertising is unique. This follows directly by strict concavity of r and strict convexity of A. Therefore, in equilibrium, a^b is a surjective function of p. Before we derive the equilibrium of this game, we make two restrictions on the equilibrium support.

Claim 4.1. *There are no atoms in the equilibrium price distribution.*

Claim 4.2. *The upper bound of the equilibrium firm price distribution is v.*

Claim 4.1 follows by a standard atom undercutting argument. Claim 4.2 follows because firms make no profits at prices above v, but they always have an incentive to raise prices to v for consumers who have evoked them and no other firms. Both proofs are in the appendix.

Since there are no atoms in equilibrium, the profit equation at each price p in the support of F is

$$\begin{aligned} \mathbb{E}\Pi_i\left(p, a^b(p)\right) = &\int_{\underline{p}}^{v} \dots \int_{\underline{p}}^{v} r\left(a^b(p) + (n-1)a^b(x_j)\right) \prod_{j \neq i} \mathrm{d}F(x_i) \\ &* \left\{ \left(\tfrac{1}{n}\right)^2 + \tfrac{2}{n}\left(1 - \tfrac{1}{n}\right)[1 - F(p)] \right\} (p - c) - A\left(a^b(p)\right) \end{aligned} \tag{4.1}$$

From the first order condition for maximization with respect to a^b, we have

$$\begin{aligned} &\left\{ \left(\tfrac{1}{n}\right)^2 + \tfrac{2}{n}\left(1 - \tfrac{1}{n}\right)[1 - F(p)] \right\} (p - c) \\ &= A'(a^b(p))[\int_{\underline{p}}^{v} \dots \int_{\underline{p}}^{v} r'\left(a^b(p) + (n-1)a_j^b(x_j)\right) \prod_{j \neq i} \mathrm{d}F(x_i)]^{-1} \end{aligned} \tag{4.2}$$

Substituting (4.2) into (4.1), we get

$$\begin{aligned} \mathbb{E}\Pi_i = &\int_{\underline{p}}^{v} \dots \int_{\underline{p}}^{v} r\left(a^b(p) + (n-1)a_j^b(x_j)\right) \prod_{j \neq i} \mathrm{d}F(x_i) \\ &* \frac{A'(a^b(p))}{\int_{\underline{p}}^{v} \dots \int_{\underline{p}}^{v} r'\left(a^b(p) + (n-1)a_j^b(x_i)\right) \prod_{j \neq i} \mathrm{d}F(x_i)} - A\left(a^b(p)\right) \end{aligned} \tag{4.3}$$

Equation (4.3) tells us that, in equilibrium, breadth does not depend on price. Therefore we can solve for the equilibrium price distribution by setting expected profit at v equal to expected profit for an arbitrary p in the support, keeping a^b constant. This gives us

$$F(p) = 1 - \frac{v - p}{2(n-1)(p-c)} \tag{4.4}$$

The lower bound of the equilibrium price distribution is

$$\underline{p} = \frac{v - c}{2n - 1} + c \tag{4.5}$$

Setting the right hand side of Expression (4.3) to Expression (4.1) evaluated at $\underline{p}$, we obtain the optimal level of breadth

$$a^{b^*} = \Psi^{-1}\left(\frac{v-c}{n^2}\right) \tag{4.6}$$

where $\Psi\left(a^{b^*}\right) = \frac{A'(a^{b^*})}{r'\left(na^{b^*}\right)}$.

Claim 4.3 summarizes the above derivation of equilibrium.

Claim 4.3. *There exists a unique symmetric Nash Equilibrium where both firms distribute prices according to* $F(p) = \frac{(2n-1)(p-c)-(v-c)}{2(n-1)(p-c)}$ *over support* $[\frac{v-c}{2n-1}+c, v]$ *and the equilibrium level of breadth* $a^{b^*} = \Psi^{-1}\left(\frac{v-c}{n^2}\right)$ *is independent of price.*

It is now easy to see that breadth increases in consumers' valuation for the good and decreases in the number of firms and in firms' cost of production.

Distributions with higher consumer valuation or marginal cost of production first order stochastically dominate those with lower consumer valuation and marginal cost of production. On the contrary, equilibrium price distributions with a larger number of firms are first order stochastically dominated by those with a smaller number of firms. Likewise, higher consumer valuation and marginal cost of production increase the lower bound of the firm price distribution, and a larger number of firms decreases it.

4.3 Model of Salience

There are 2 identical firms that compete to sell a homogeneous good. Each firm pays a constant marginal cost of c to produce the good. Firm i makes a twofold decision: it sets price, p_i, and determines whether to engage in salient advertising ($a_i^s = 1$) or not ($a_i^s = 0$). If it chooses to engage in salient advertising, it incurs a cost of $A > 0$.

On the other side of the market, there is a unit mass of identical consumers with valuation $v > c$ for the good. Unlike in the previous model, any consumers can readily purchase the product at the outset of the game from either firm that he evokes – the two firms comprise consumers' consideration sets. Instead, ads influence the probability that an individual consumer evokes a particular firm from his consideration set. Thus, in this model, ads can make a brand more salient to a consumer.

The probability that a consumer evokes firm i from his consideration set is given by $\sigma_i\left(a_i^s, a_j^s\right)$. In particular, if $a_1^s = a_2^s$, then $\sigma_1 = \sigma_2 = \frac{1}{2}$. If $a_i^s > a_j^s$, then $\sigma_i = \alpha$ and $\sigma_j = 1 - \alpha$, $\alpha \in \left(\frac{1}{2}, 1\right)$. As in the previous model, consumers make two evocations with replacement. If only one firm is evoked, the consumer purchases from that firm. If two different firms are evoked, the consumer chooses the firm offering the lower price. If both firms have the same price, each of them has an equal probability of being chosen. In this framework, salient advertising raises the probability that a firm i will be evoked, but does not expand the market.

Firms and consumers play a game similar to that in the previous model. First, the 2 firms in the market simultaneously choose prices and decide whether or not to engage in salient advertising. Then, consumers in the market observe the prices of the firms they evoke and make a purchase decision. A pricing strategy for firm i is a price distribution F_i. A pure advertising strategy is a mapping from the support of F_i, to the binary variable $a_i^s \in \{0, 1\}$. A mixed advertising strategy can then be denoted by a probability, β that $a_i^s = 1$, where $\beta(p)$ is the probability that a firm advertises at price p.

4.3.1 Equilibrium Analysis

We restrict attention to the symmetric equilibrium and dispense with subscripts on F. The characterization of equilibrium depends on the cost to maximum benefit ratio of advertising, $A/(v-c)$. Firms always play mixed pricing strategies. However, when the cost to maximum benefit ratio of advertising is sufficiently high or sufficiently low, both firms either forego advertising or always advertise, respectively. On the other hand, intermediate values of this ratio can lead to mixed strategy equilibria in advertising where firms are either always indifferent between advertising and not advertising, or they are indifferent for low(high) prices in the support of F, but prefer to advertise(not advertise) for high(low) prices. Therefore, we divide the equilibrium analysis into five cases in which the cost to maximum benefit ratio of advertising is: high, low, medium, medium-high, and medium-low.

As in the previous section, the equilibrium price distribution has no atoms, and its upper bound is the consumer valuation v. The proofs follow similarly. As a result, expected profit when firm i does not invest in salient advertising is:

$$\begin{aligned}\mathbb{E}\Pi_i(p,0) = &\left\{\tfrac{1}{4}+\tfrac{1}{2}[1-F(p)]\right\}(1-\mathbb{E}\beta)(p-c)\\ &+\left\{(1-\alpha)^2+2\alpha(1-\alpha)[1-F(p)]\right\}\mathbb{E}\beta(p-c)\end{aligned} \tag{4.7}$$

where $\mathbb{E}\beta = \int_{\underline{p}}^{v} \beta(x)\mathrm{d}F(x)$. Likewise, expected profit when it invests in salience is:

$$\begin{aligned}\mathbb{E}\Pi_i(p,1) = &\left\{\alpha^2+2\alpha(1-\alpha)[1-F(p)]\right\}(1-\mathbb{E}\beta)(p-c)\\ &+\left\{\tfrac{1}{4}+\tfrac{1}{2}[1-F(p)]\right\}\mathbb{E}\beta(p-c)-A\end{aligned} \tag{4.8}$$

Case 1: High cost to maximum benefit ratio of advertising

We say that the cost to maximum benefit ratio of advertising is high whenever $A/(v-c) \geq \alpha^2 - \frac{1}{4}$. In the unique equilibrium of this case, firms do not engage in salient advertising. Prices are distributed according to:

$$F(p) = 1 - \frac{v - p}{2(p - c)} \tag{4.9}$$

Equilibrium price distributions with higher consumer valuation and marginal cost of production first order stochastically dominate those with lower consumer valuation and marginal production cost, respectively.

The lower bound of the equilibrium price distribution is increasing in both consumer valuation and marginal cost of production. It is given by

$$\underline{p} = \frac{v + 2c}{3} \tag{4.10}$$

Expected firm profit is positive and equal to

$$\mathbb{E}\Pi = \frac{v - c}{4} \tag{4.11}$$

Case 2: Low cost to maximum benefit ratio of advertising

We say that the cost to maximum benefit ratio of advertising is low whenever $A/(v-c) \leq \frac{1}{3}(\alpha^2 - \frac{1}{4})$. In the unique equilibrium of this case, both firms engage in salient advertising. The equilibrium firm price distribution and lower bound are the same as the those in Equation (4.9) and Equation (4.10). However, firm expected profit is strictly smaller than that in Equation (4.11) since firms have to incur the cost A.

In this case, firms end up in a prisoner's dilemma where advertising is effectively wasted. It is not worthwhile for firm i to deviate to no advertisement because when doing so there is an overly large probability that consumers will only evoke the rival firm, but when advertising, it finds itself competing on price just as heavily as if neither firm was to advertise.

Case 3: Medium cost to maximum benefit ratio of advertising

In this case, $A/(v-c) \in [\frac{1}{4} - (1-\alpha)^2, \alpha - \frac{1}{2})$. The unique equilibrium involves firms engaging in salient advertising with positive probability at every price in the support of F. This means that the expected probability that a firm is advertising, $\mathbb{E}\beta$, is a number between zero and one. Setting Equation (4.7) equal to Equation (4.8) at v, we can solve for $\mathbb{E}\beta$ to get

$$\mathbb{E}\beta = \frac{\left(\alpha - \frac{1}{2}\right)\left(\alpha + \frac{1}{2}\right) - \frac{A}{v-c}}{2\left(\alpha - \frac{1}{2}\right)^2} \tag{4.12}$$

Since $\mathbb{E}\beta$ is an expectation over price, it should be the same for every price in the support of F. Therefore, to find the equilibrium firm price distribution we set Equation (4.7) equal to Equation (4.8) at an arbitrary price in the support of F after substituting $\mathbb{E}\beta$ from Equation (4.12). This yields

$$F(p) = 1 - \frac{\frac{A}{p-c} - \frac{A}{v-c}}{2\left(\alpha - \frac{1}{2} - \frac{A}{v-c}\right)} \tag{4.13}$$

The lower bound of the support is

$$\underline{p} = \frac{A}{2\left(\alpha - \frac{1}{2}\right) - \frac{A}{v-c}} + c \tag{4.14}$$

Observe that the lower bound of the distribution in this case is higher than the lower bound of the equilibrium price distribution in the high and low cost to maximum benefit ratio of advertising cases. In fact, the entire distribution in this case first order stochastically dominates the two former cases. Unlike in the case with a low cost to maximum benefit ratio of advertising, firms can effectively compete on advertising and might prefer to do so in place of setting a low price. However, competition in advertising is relatively unprofitable. Expected firm profit is

$$\mathbb{E}\Pi = (v-c)\left[\frac{1}{4}\left(1 - \mathbb{E}\beta\right) + (1-\alpha)^2\,\mathbb{E}\beta\right] \tag{4.15}$$

Since $(1-\alpha)^2 < \frac{1}{4}$, it can be readily seen that expected profit in Equation (4.15) is smaller than when the cost to maximum benefit ratio of advertising is high. Moreover, expected profit here may be larger or smaller than that in the low cost to maximum benefit ratio of advertising depending on the size of A in each of these cases. The problem for firms is that while advertising is effective relative to the low cost to maximum benefit ratio case, it is also much more expensive. By engaging in expensive advertising with a high enough probability, firms do not fully recoup the cost of advertising with the higher average prices that it entails.

Equilibrium price distributions with higher marginal cost of production and higher cost of advertising first order stochastically dominate those with lower costs. Similarly, the lower bound is increasing in both costs. Surprisingly, these results do not hold for higher consumer valuation. To the contrary, the lower bound as well as average price fall in consumer valuation. A higher consumer valuation lowers the cost to maximum benefit ratio of advertising, thereby raising the probability that a firm will advertise in equilibrium ($\mathbb{E}\beta$). Knowing that rival firms are more likely to advertise in equilibrium, firms find that the marginal benefit of advertising declines and opt to intensify their price competition.

Case 4: Medium-high cost to maximum benefit ratio of advertising

In this case, $A/(v-c) \in [\alpha - \frac{1}{2}, \alpha^2 - \frac{1}{4}]$. We conjecture that the unique equilibrium has no advertising at low prices. At high prices, firms randomize between advertising and not advertising. The expected probability of advertising on the support is given by Equation (4.12).

Case 5: Medium-low cost to maximum benefit ratio of advertising

In this case, $A/(v-c) \in [\frac{1}{3}(\alpha^2 - \frac{1}{4}), \frac{1}{4} - (1-\alpha)^2]$. We conjecture that the unique equilibrium has advertising at high prices. At low prices, firms randomize between advertising and not advertising. The expected probability of advertising on the support is given by Equation (4.12).

4.4 Discussion

In both of our models, all consumers can evoke at most two firms. Increasing the number of evocations does not alter our findings as long as the number of evocations is finite. This happens because there is always a positive probability that only one firm gets evoked. Therefore, there is always an incentive to raise prices. Because the probability of a single firm getting evoked goes down and because there is a chance of more than two firms being evoked, price competition will increase with the number of evocations. We should expect that price distributions with fewer evocations first order stochastically dominate those with more.

As mentioned in the introduction, the models above are robust to various forms of consumer heterogeneity. First, consider what happens if we introduce consumers who are loyal to a particular firm. In our model, this is equivalent to having consumers who only evoke that particular firm. As long as there is a sufficient number of consumers who evoke other firms, firms will still have an incentive to lower prices and price dispersion will persist. Alternatively, we can introduce consumers who conduct a thorough search of all brands. We can think of these consumers as individuals who evoke every firm in the market to get the lowest price. As in our original framework, firms will still have incentives to charge high prices as long as there is a sufficient number of consumers who evoke only a finite number of firms. The Bertrand outcome will not occur because there is a positive probability that only one firm gets evoked by some individuals.

4.5 Conclusion

In this paper, we have analyzed advertising as a form of publicity. Publicity has two effects for firms: it increases the size of the market and it makes brands more salient to consumers. We separately analyze these two effects. Advertising expands the market by informing or reminding consumers that they need a certain product. When a particular firm's advertising for this product is not salient, it can send consumers of the product to a rival firm. In contrast, a firm that engages in salient advertising can increase its chances of coming to a consumer's mind in a buying situation.

This paper remains a work in progress. Although the market expanding effects of advertising have been fully characterized, we have not yet analyzed equilibrium when the ratio of cost to maximum benefit of advertising is medium-low or medium-high. In addition, we intend to show that our binary model of salient advertising is sufficient in the sense that it tells us everything that a model of salient advertising where advertising is treated continuously might tell us. Moreover, we intend to expand this model to the n firm case. Finally, we intend to examine various asymmetries that can occur. These include asymmetries in the cost of production and the cost of advertising, as well as in consumer preferences regarding different brands. In particular, consumers evoking a preferred brand, A, and a second brand, B, may require a substantially lower price for B to induce them to purchase it. This can result in substantial changes in advertising by both firms.

4.6 Appendix

Claim 4.1. *There are no atoms in the equilibrium price distribution.*

Proof. Suppose the equilibrium price distributions have an atom at price $\hat{p}$. Let $\hat{a}^b(\hat{p})$ be the level of advertising at $\hat{p}$. Firm i's expected profit at $\hat{p}$ is

$$\begin{aligned}&\mathbb{E}\Pi_i\left(\hat{p}, \hat{a}^b(\hat{p})\right) = \int_{\underline{p}}^{v} \dots \int_{\underline{p}}^{v} r\left(\hat{a}^b(\hat{p}) + (n-1)a_j^b(x_j)\right) \prod_{j \neq i} \mathrm{d}F(x_i)\\ &* \left\{\left(\tfrac{1}{n}\right)^2 + \tfrac{2}{n}\left(1 - \tfrac{1}{n}\right)\{[1 - F(\hat{p})] + \mathbb{P}[p = \hat{p}]\}\right\}(\hat{p} - c) - A\left(\hat{a}^b(\hat{p})\right)\end{aligned} \tag{4.16}$$

Suppose that firm i shifts mass from $\hat{p}$ to $\hat{p}-\varepsilon$, without changing breadth. Expected profit becomes

$$\begin{aligned}&\mathbb{E}\Pi_i\left(\hat{p} - \varepsilon, \hat{a}^b(\hat{p})\right) = \int_{\underline{p}}^{v} \dots \int_{\underline{p}}^{v} r\left(\hat{a}^b(\hat{p}) + (n-1)a_j^b(x_j)\right) \prod_{j \neq i} \mathrm{d}F(x_i)\\ &* \left\{\left(\tfrac{1}{n}\right)^2 + \tfrac{2}{n}\left(1 - \tfrac{1}{n}\right)[1 - F(\hat{p} - \varepsilon)]\right\}(\hat{p} - \varepsilon - c) - A\left(\hat{a}^b(\hat{p})\right)\end{aligned} \tag{4.17}$$

For ε small enough, profit at $\hat{p}-\varepsilon$ is strictly higher than profit at $\hat{p}$, a contradiction. □

Claim 4.2. *The upper bound of the equilibrium firm price distribution is v.*

Proof. A firm pricing at $\bar{p} > v$ makes no profit. Suppose that the upper bound is $\bar{p} < v$. At $\bar{p}$, firm i will only sell if it is evoked both times, since it will be underpriced for sure. Thus, it can increase its profit by raising its price to v, a contradiction. □

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