Semi-Empirical Modeling of Two-Dimensional and Three-Dimensional Dynamic Stall

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Semi-Empirical Modeling of Two-Dimensional and
Three-Dimensional Dynamic Stall

by
Ramin Modarres

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# CONTENTS

List of Figures .................................................................................................................. v
List of Tables .................................................................................................................... ix
Nomenclature .................................................................................................................... x
Acknowledgments ............................................................................................................ xvi

1 Introduction .................................................................................................................... 1
1.1 Motivation ................................................................................................................... 1
1.2 The Phenomenon of Dynamic Stall ........................................................................... 1
   1.2.1 Dynamic Stall Events ....................................................................................... 2
   1.2.2 Stall Regimes ..................................................................................................... 4
   1.2.3 Effective Parameters on Dynamic Stall ........................................................... 5
      1.2.3.1 Geometry of the airfoil .............................................................................. 6
      1.2.3.2 Reduced Frequency .................................................................................... 6
      1.2.3.3 Amplitude and Mean Angle ....................................................................... 8
      1.2.3.4 Mach Number ............................................................................................. 8
      1.2.3.5 Plunging Oscillation ................................................................................... 9
1.3 Dynamic Stall in the Rotating Environment ............................................................. 9
   1.3.1 Double Stall Event ............................................................................................ 11
   1.3.2 Three-Dimensional Effects on Dynamic Stall .................................................. 12
1.4 Problem Statement and Approach ........................................................................... 15

2 Literature Review .......................................................................................................... 20
2.1 Modeling of Dynamic Stall ....................................................................................... 20
   2.1.1 Empirical Correlation Methods ...................................................................... 21
      2.1.1.1 Boeing-Vertol “gamma” Function Method .................................................. 21
      2.1.1.2 UTRC $\alpha, A, B$ Method ......................................................................... 22
      2.1.1.3 MIT Method ............................................................................................... 22
      2.1.1.4 Lockheed Method ....................................................................................... 23
      2.1.1.5 Time-Delay Method ................................................................................... 23
      2.1.1.6 Gangwani’s Method .................................................................................... 24
      2.1.1.7 Leishman-Beddoes Method ....................................................................... 24
      2.1.1.8 ONERA Method ......................................................................................... 27
   2.2 Development of Ahaus-Peters Unified Airloads Model ........................................... 29
      2.2.1 Finite-State Airloads Theory ......................................................................... 29
         2.2.1.1 Aerodynamic Theory ............................................................................... 29
         2.2.1.2 Glauert Expansions ............................................................................... 32
         2.2.1.3 Generalized Loads and Drag ................................................................. 34
      2.2.2 Dynamic Inflow Model .................................................................................... 35
         2.2.2.1 Peters-Karunamoorthy 2D finite state induced flow Model .................. 36
      2.2.3 Dynamic Stall Model ....................................................................................... 38
      2.2.4 Unified Model ................................................................................................ 39
### 3 Modeling the Secondary Lift Peak

3.1 Modified Stall Model ................................................................. 42

3.1.1 Determination of Parameters .................................................. 42

3.1.1.1 Determining Constants for Secondary Lift Stall .................... 45

3.1.1.2 Determining Constants for Primary Lift Stall ....................... 49

3.1.1.3 Identification of Constants for Static Hysteresis .................... 51

3.1.1.4 Final Fit of Smooth Curves ............................................... 54

3.1.2 Physical Parameters and Determining Constants ....................... 60

3.2 Validation of The modified Stall Model on VR-12 Airfoil ............... 61

### 4 Effect of Yawed Flow on Dynamic Stall

4.1 Modeling Yawed Flow .............................................................. 68

4.2 Other Three-Dimensional Studies ............................................. 74

4.3 Dynamic Stall Model ............................................................. 75

4.4 Inclusion of the Yawed Flow in Ahaus Model ............................. 78

### 5 Time-Varying Free-Stream Velocity

5.1 Previous Work ........................................................................... 83

5.2 Unsteady Free Stream with Unified Airloads Model .................... 83

5.3 Augmented Equations ............................................................... 88

5.4 Results for Lift ........................................................................ 90

5.5 Simulation of Pitching Moment ................................................ 93

### 6 Three-Dimensional Modeling of Dynamic Stall

6.1 Three-Dimensional Finite State Induced Flow Model .................. 103

6.2 Effect of Radial Coupling ......................................................... 108

6.3 Implementation of Three-Dimensional Dynamic Stall Model ....... 114

6.3.1 Simulation Condition ........................................................... 117

6.3.2 Correlation at Test Pitch Angles .......................................... 118

6.3.3 High Pitch Angle Case ......................................................... 119

6.4 Validation in Rotating Frame ................................................... 123

### 7 Future Work

7.1 Pitching Moment ...................................................................... 127

7.2 Drag ......................................................................................... 128

7.3 Higher Mach Numbers .............................................................. 128

7.4 Three-Dimensional, Rotating-Blade Validation ......................... 129

### 8 Summary and Conclusions

8.1 Summary .................................................................................. 129

8.2 Conclusions .............................................................................. 130

### References

................................................................. 132

Appendix A. Vectors and Matrices of Unified Airload Model ............. 133
Appendix B.  Wing & Wind Coordinate Systems ................................................................. 145

Appendix C.  Ellipsoidal Coordinate System ................................................................. 148

Vita ...................................................................................................................................... 150
List of Figures

Figure 1.1: Dynamic stall events for an oscillating VR-7 airfoil, $\alpha = 15^\circ + 10^\circ \sin \omega t$, $M = 0.25$, $k = 0.10$ ................................................................. 3

Figure 1.2: Dynamic stall regimes for NACA 0012 airfoil, $\alpha = \alpha_0 + 10^\circ \sin \omega t$, $k = 0.10$ .... 4

Figure 1.3: Effect of reduced frequency on the unsteady lift and pitching moment of an oscillating VR-7 airfoil, $\alpha = 10^\circ + 5^\circ \sin \omega t$, $M = 0.3$ ................................................................. 6

Figure 1.4: Effect of reduced frequency on the unsteady lift and pitching moment of an oscillating NACA 0012 airfoil, $\alpha = 10^\circ + 5^\circ \sin \omega t$, $M = 0.3$ ................................................................. 7

Figure 1.5: Effect of reduced frequency on the unsteady lift and pitching moment of an oscillating NACA 0012 airfoil in the deep stall regime, $\alpha = 15^\circ + 10^\circ \sin \omega t$, $M = 0.1$ ....... 7

Figure 1.6: Velocity distribution in forward flight ................................................................. 9

Figure 1.7: Blade angle-of-attack distribution for a helicopter in forward flight, $C_r/\sigma = 0.08$, $f/A = 0.008$, $\theta = -8^\circ$, $\mu = 0.15, 0.30, 0.45$ ................................................................. 10

Figure 1.8: Angle-of-attack distribution for a hovering rotor, calculated by free wake analysis ..... 11

Figure 1.9: Laser shadow diagram of the flow around a NACA0013 rotor blade, $\psi = 270^\circ$, $\mu = 0.4$, $r/R = 0.5$, Collective Pitch: 10°, Cyclic Pitch: $-5^\circ$ ................. 12

Figure 1.10: Lift and pitching moment data for an oscillating cantilevered wing, $k = 0.1$, $M = 0.2$ ........................................................................................................ 14

Figure 1.11: Airloads and angle-of-attack values at different blade positions for a model helicopter rotor ........................................................................................................ 15

Figure 1.12: Schematic of Ahaus-Peters unified airloads model ........................................ 17

Figure 1.13: Schematic of modified Ahaus-Peters unified airloads model ...................... 18

Figure 2.1: General airfoil coordinates system ................................................................. 30

Figure 2.2: Solid Airfoil pitching about an axis at $x = ab$ ............................................ 40

Figure 3.1: Ahaus-Peters stall model vs. the experimental lift data for NACA 0012 airfoil at $M = 0.1$ and $k = 0.02, 0.5, 0.15$ ................................................................. 44
Figure 3.2: Smooth curves and the experimental lift data for NACA 0012 airfoil at $M = 0.1$ and $k = 0.02, 0.5, 0.15$ ..........................................................46

Figure 3.3: Secondary Lift stall $\bar{\Gamma}$ versus reduced time..........................................................47

Figure 3.4: NACA 0012 fit of the smooth lift coefficient curves which includes the shift in angle of attack, $M = 0.1$ ..........................................................................................52

Figure 3.5: NACA 0012 lift coefficient data at $k = 0.02, 0.05$ and 0.15, $M = 0.1$ (simulation versus experiment) ........................................................................................................58

Figure 3.6: NACA 0012 lift coefficient data at $k = 0.004$ and 0.05, $M = 0.1$ (simulation versus experiment) ........................................................................................................59

Figure 3.7: VR-12 airfoil lift coefficient data at $k = 0.05$ and 0.1, $M = 0.2$ (simulation versus experiment) ........................................................................................................63

Figure 3.8: VR-12 airfoil lift coefficient data at $k = 0.05$ and 0.1, $M = 0.3$ (simulation versus experiment) ........................................................................................................64

Figure 3.9: VR-12 airfoil lift coefficient data at $k = 0.05$ and 0.1, $M = 0.4$ (simulation versus experiment) ........................................................................................................65

Figure 3.10: VR-12 airfoil lift coefficient data at $k = 0.05$ and 0.1, using two different set of stall parameters for the upstroke and backstroke of the lift curve, $M = 0.4$, (simulation versus experiment) ........................................................................................................67

Figure 4.1: Velocity components relative to a blade section of a helicopter in forward flight ....69

Figure 4.2: Iso-sweep angles over the rotor disk in forward flight for $\mu = 0.05$ and $\mu = 0.30$ ....70

Figure 4.3: Comparison of Gormont’s predicted yawed lift with the measured data...............71

Figure 4.4: Static lift coefficient curve for the NACA0012 Airfoil at $M = 0.4$ and $\Lambda = 0^\circ$, $\Lambda = 30^\circ$ ........................................................................................................79

Figure 4.5: Comparison of the unyawed lift data of Ref. [15] with the results obtained from our model, Leishman’s model and Barwey’s model.................................................................80

Figure 4.6: Comparison of the yawed lift data of Ref. [15] with the results obtained from our model, Leishman’s model and Barwey’s model.................................................................82

Figure 5.1: Philosophy of the corrections to the unified airloads model.................................93
Figure 5.2: Comparison of the lift data from the Peters et al augmented lift model and the ONERA-GBCN model, \( \alpha = 12° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined based on the \( \alpha = 12° \pm 6° \) case stall data with steady free stream) .................................................................94

Figure 5.3: Lift data from Isaacs closed form solution, Peters et al finite state model and the DSTP model, \( \alpha = 1 \) and \( \bar{U}_T = 1 + 0.8 \sin(0.2\tau) \) ........................................................................................................96

Figure 5.4: Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 9° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined based on the \( \alpha = 12° \pm 6° \) case stall data with steady free stream) .................................................................97

Figure 5.5: Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 12° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined based on the \( \alpha = 12° \pm 6° \) case stall data with steady free stream) .................................................................98

Figure 5.6: Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 15° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined based on the \( \alpha = 12° \pm 6° \) case stall data with steady free stream) .................................................................99

Figure 5.7: Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 9° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined by optimizing the entire data set of \( \alpha = 9° \pm 6° \) case) .........................................................................................................................100

Figure 5.8: Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 12° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined by optimizing the entire data set of \( \alpha = 12° \pm 6° \) case) .........................................................................................................................101

Figure 5.9: Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 15° + 6° \cos(k\tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos k\tau \). (stall parameters of the Peters et al augmented lift model are determined by optimizing the entire data set of \( \alpha = 15° \pm 6° \) case) .........................................................................................................................102

Figure 5.10: Comparison of the lift data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, \( \alpha = 9° - 13° \cos(\omega t) \) and \( M = 0.40 \) ........................................................................................................106

Figure 5.11: Comparison of the lift data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, \( \alpha = 9° - 13° \cos(\omega t) \) and \( M = 0.40 + 0.07 \cos(\omega t) \) ........................................................................................................106

vii
Figure 5.12: Comparison of the pitching moment data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$ and $M = 0.40$..................................................................................................................................................107

Figure 5.13: Comparison of the pitching moment data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$ and $M = 0.40 + 0.07 \cos(\omega t)$ ..................................................................................................................................................107

Figure 6.1: Effect of radial coupling on a NACA0013 rotor blade, $\psi = 270^\circ, \mu = 0.4, \bar{r} = 0.5$, Collective Pitch: $10^\circ$, Cyclic Pitch: $-5^\circ$..................................................................................................................................................115

Figure 6.2: Rotor blade sections ........................................................................................................................................................................116

Figure 6.3: Rotor blade shape ...........................................................................................................................................................................118

Figure 6.4: Rotor blade shape considered for the simulations..........................................................................................................................119

Figure 6.5: Lift coefficient data versus the pitch angle for segment 2, $\theta = 16.01 - 5.69 + 1.96 \cos \psi - 2.26 \sin \psi$ ..................................................................................................................................................120

Figure 6.6: Lift coefficient data versus the angle of attack for segment 2, $\theta = 16.01 - 5.69 + 1.96 \cos \psi - 2.26 \sin \psi$ ..................................................................................................................................................121

Figure 6.7: Lift coefficient data versus the pitch angle for segment 4, $\theta = 16.01 - 8.94 + 1.96 \cos \psi - 2.26 \sin \psi$ ..................................................................................................................................................122

Figure 6.8: Lift coefficient data versus the angle of attack for segment 4, $\theta = 16.01 - 8.94 + 1.96 \cos \psi - 2.26 \sin \psi$ ..................................................................................................................................................123

Figure 6.9: Lift coefficient data versus the pitch angle for segment 2, $\theta = 22.27 - 5.69 + 3.92 \cos \psi - 4.52 \sin \psi$ ..................................................................................................................................................124

Figure 6.10: Lift coefficient data versus the angle of attack for segment 2, $\theta = 22.27 - 5.69 + 3.92 \cos \psi - 4.52 \sin \psi$ ..................................................................................................................................................125

Figure 6.11: Lift coefficient data versus the pitch angle for segment 4, $\theta = 22.27 - 8.94 + 3.92 \cos \psi - 4.52 \sin \psi$ ..................................................................................................................................................126

Figure 6.12: Lift coefficient data versus the angle of attack for segment 4, $\theta = 22.27 - 8.94 + 3.92 \cos \psi - 4.52 \sin \psi$ ..................................................................................................................................................127

Figure B.1: Flow geometry in wing-axis system ..............................................................................................................................................145

Figure B.2: Flow geometry in wind-axis system ..............................................................................................................................................146

Figure C.1: Ellipsoidal coordinate system (viewed in the xz plane) ..............................................................................................................149
List of Tables

Table 3.1: Numerical values of the pulse parameters for NACA0012 airfoil, \( k = 0.02, 0.05 \) and \( k = 0.15, M = 0.1 \) .............................................................................................................48

Table 3.2: Numerical values of stall, pulse and hysteresis parameters for NACA 0012 airfoil at \( M = 0.1 \) and for VR-12 airfoil at \( M = 0.2, 0.3 \) and 0.4.................................................................50

Table 3.3: Side Constraints applied on the stall parameters .................................................................51

Table 3.4: Optimization options setup in MATLAB Genetic Algorithm toolbox ...............................54

Table 4.1: Numerical values of stall, pulse and hysteresis parameters ..............................................77

Table 4.2: Error norms for Peters et al stall model as compared to error norms for the Leishmann-Beddoes stall model at \( \Lambda = 0^\circ, 30^\circ \) ........................................................................................................81

Table 5.1: Numerical values of the dynamic Stall Parameters for NACA 0012, used to correlate the experimental lift data of Ref. [88] ........................................................................................................91

Table 5.2: Numerical values of the dynamic Stall Parameters for SSC-A09 at \( M = 0.4 \) ..............104

Table 6.1: Table Method (Number of Shape Functions per Harmonic) ...........................................114

Table 8.1: Summary of airload correlations .........................................................................................131
Nomenclature

\( a \) \hspace{1cm} \text{slope of lift curve, \([\text{rad}^{-1}]\)}

\( A \) \hspace{1cm} \text{rotor disk area, } A = \pi R^2, \text{[m}^2\text{]}\)

\( [\tilde{A}_{jn}] \) \hspace{1cm} \text{transformation matrix, Eq. (4.24)}

\( b \) \hspace{1cm} \text{blade semi-chord, } b = c/2, \text{[m]}\)

\( \bar{b} \) \hspace{1cm} \text{blade section semi-chord normalized by the blade radius}\)

\( b_n \) \hspace{1cm} \text{induced flow expansion coefficients, Eq. (2.49)}

\( c \) \hspace{1cm} \text{blade chord, [m]}\)

\( C \) \hspace{1cm} \text{coefficient of change for hysteresis, Eq. (3.13)}

\( C_n^m \) \hspace{1cm} \text{arbitrary coefficient of pressure functions, Eq. (4.9), [m}^2s^2\text{]}\)

\( C_D \) \hspace{1cm} \text{section drag coefficient, } D/(\rho b U^2)\)

\( C_L \) \hspace{1cm} \text{section lift coefficient, } L/(\rho b U^2)\)

\( \Delta C_L \) \hspace{1cm} \text{lift coefficient residual}\)

\( C_M \) \hspace{1cm} \text{section pitching moment coefficient, } M/(\rho bc U^2)\)

\( \Delta C_M \) \hspace{1cm} \text{static pitching moment residual}\)

\( c_n \) \hspace{1cm} \text{generalized load}\)

\( \Delta c_n \) \hspace{1cm} \text{generalized load residual}\)

\( C_T \) \hspace{1cm} \text{thrust coefficient}\)

\( D \) \hspace{1cm} \text{drag force per unit span, [N/m]}\)

\( D_n^m \) \hspace{1cm} \text{arbitrary coefficient of pressure functions, Eq. (4.9), [m}^2s^2\text{]}\)

\( e \) \hspace{1cm} \text{lead term of the original stall model, Eq. (2.52)}\)

\( f \) \hspace{1cm} \text{equivalent drag area of rotorcraft airframe, including rotor hubs, } D/(\rho V_\infty^2)\)

\( F \) \hspace{1cm} \text{reverse flow parameter, Eq. (2.27)}\)

\( f \) \hspace{1cm} \text{factor for } \Delta C_L \text{ derivative in } \Gamma_M \text{ equation, } = f_0 + f_2(\Delta C_L)^2, \text{Eq. (4.15)}\)

\( f_L \) \hspace{1cm} \text{static correction factor to lift coefficient, } f_L = c_i\alpha/2\pi\)

\( f_s(\tau) \) \hspace{1cm} \text{normalized pulse, Eq. (3.3)}\)

\( F_c \) \hspace{1cm} \text{correction factor of the Peters, et al augmented model}\)

\( F_s(\tau) \) \hspace{1cm} \text{pulse, Eq. (3.2)}\)

\( h(x,t) \) \hspace{1cm} \text{generalized airfoil motion, [m]}\)

\( h_c \) \hspace{1cm} \text{height gradient of the normalized pulse, Eq. (3.6), [rad}^{-1}\text{]}\)
\( h_i \) initial height of the normalized pulse, Eq. (3.6)
\( h_n \) components of generalized airfoil deformation, Eq. (2.32), \([m]\)
\( h_s \) height of the normalized pulse, Eq. (3.6)
\( J_0 \) Bessel function
\( k \) reduced frequency of oscillation
\( K \) coefficient of change for pulse angle, Eq. (3.5), \([rad^{1/2}]\)
\( L \) lift force per unit span, \([N/m]\)
\( L_n \) generalized loads per unit span, Eq. (2.36), \([N/m]\)
\([\bar{L}]\) induced inflow influence coefficient matrix, Eqs. (4.22) and (4.23)
\([\bar{\bar{L}}]\) induced inflow influence coefficient matrix, Eqs. (4.25)-(4.27)
\( M \) section pitching moment per unit span, \([N\cdot m/m]\)

Mach number

Number of expansions in airloads theory

\([M]\) apparent mass matrix, Eq. (4.21)
\( N \) number of inflow states
\( P \) pressure, \([N/m^2]\)
\( \Delta P \) difference in pressure above and below the airfoil or disk, \([N/m^2]\)
\( p_n^m \) associated Legendre function of first kind
\( \bar{p}_n^m \) normalized Legendre function, Eq. (4.11)
\( p_e \) momentum gradient, Eq. (3.7), \([rad^{-1}]\)
\( p_i \) initial momentum, Eq. (3.7)
\( p_s \) momentum under pulse, Eq. (3.7)
\( Q \) number of blades
\( Q_n^m \) associated Legendre function of second kind
\( r \) radial position along blade, \([m]\)
\( \Delta r \) distance between the centers of two successive blade segments, \([m]\)
\( R \) rotor radius, \([m]\)
\( \bar{r} \) radial position normalized by the blade radius
\( t \) time, \([s]\)
\( \bar{t} \) nondimensional time, \( \bar{t} = \Omega t \)
\( u( \ ) \) step function
\( u_0 \) velocity component in the \( x \) direction, \([m/s]\)
\( \bar{u}_0 \) velocity of external bound vortex in negative \( x \) direction, \([m/s]\)
section resultant velocity perpendicular to the span-wise direction, \( U = \sqrt{U_T^2 + U_P^2}, \) [m/s]
\( U_c \) critical velocity, [m/s]
\( U_D \) resultant air velocity in the disk plane, \( U_D = \sqrt{U_R^2 + U_T^2}, \) [m/s]
\( U_F \) total air velocity, \( U_F = \sqrt{U_R^2 + U_P^2 + U_T^2}, \) [m/s]
\( U_p \) air velocity of blade section, perpendicular to the disk plane, [m/s]
\( U_R \) radial air velocity, [m/s]
\( U_S \) resultant air velocity perpendicular to \( U_T, U_S = \sqrt{U_R^2 + U_P^2}, \) [m/s]
\( U_T \) air velocity of blade section, tangent to the disk plane, [m/s]
\( U_{T0} \) mean value of \( U_T, \) [m/s]
\( \bar{U}_T \) \( U_T \) normalized by its mean value, \( U_{T0} \)
\( v \) total induced velocity, [m/s]
\( v_0 \) uniform velocity component in the \( y \) direction, [m/s]
\( v_1 \) velocity gradient, [m/s]
\( \bar{v} \) induced flow due to bound circulation, Eq. (2.12), [m/s]
\( V \) mass inflow parameter, Eq. (4.35), [m/s]
\( V_T \) total velocity for momentum theory, Eq. (4.29), [m/s]
\( V_\infty \) free-stream velocity, [m/s]
\( w \) total motion of flow relative to airfoil, Eq. (2.11), [m/s]
\( w_n \) components of total velocity field, Eq. (2.29), [m/s]
\( x \) Cartesian coordinate, [m]
\( X \) wake-skew parameter, Eq. (4.28)
\( y \) Cartesian coordinate, [m]
\( \alpha \) angle of attack, [rad]
\( \Delta \alpha \) shift of angle of attack in \( \Delta C_L \) look-up, Eq. (3.13), [rad]
\( \alpha_0 \) mean angle of attack, [rad]
\( \alpha_{0L} \) zero-lift angle of attack, [rad]
\( \alpha_1 \) angle-of-attack oscillation amplitude, [rad]
\( \alpha_c \) critical angle, [rad]
\( \alpha_{DS} \) dynamic stall angle, [rad]
\( \alpha_{eff} \) effective angle of attack, Eq. (3.12), [rad]
\( \alpha_{end} \) end of hysteresis portion, [rad]
\( \alpha_p \) angle at which pulse begins, [rad]
\( \alpha_{ss} \) static stall angle, [rad]
\( \alpha_j^r \) induced inflow expansion coefficients, Eq. (4.15), [m/s]
\( \dot{\alpha} \) reduced-time rate of change of angle of attack, [rad]
\( \alpha_1 \) lower critical angular rate for hysteresis, Eq. (3.13), [rad]
\( \alpha_2 \) upper critical angular rate for hysteresis, Eq. (3.13), [rad]
\( \alpha_c \) critical angular velocity for pulse to occur, [rad]
\( \dot{\alpha}_u \) reduced-time rate of change of angle of attack at \( \alpha_{ss} \), [rad]
\( \beta_j^r \) induced inflow expansion coefficients, Eq. (4.15), [m/s]
\( \Gamma \) total bound circulation, [m²/s]
\( \Gamma_L \) blade circulation due to primary lift stall, Eq. (3.1), [m²/s]
\( \Gamma_n \) change in circulation due to primary stall of \( n^{th} \) generalized load, Eq. (2.52), [m²/s]
\( \Gamma_M \) equivalent circulation for pitching moment stall [m²/s]
\( \Gamma_s \) blade circulation due to secondary lift stall, Eq. (3.2), [m²/s]
\( \gamma \) nondimensional circulation, \( \Gamma/Ub \)
\( \gamma_b \) bound circulation per unit length, [m/s]
\( \gamma_n \) components of velocity due to bound circulation, Eq. (2.28), [m/s]
\( \gamma_w \) wake circulation per unit length, [m/s]
\( \delta \) correction factor of the DSTP model
\( \varepsilon \) distance of center of rotation forward of quarter chord/b
\( \zeta \) velocity due to the external bound vorticity, [m/s]
\( \zeta_n \) components of velocity due to the external bound vorticity, Eq. (2.31), [m/s]
\( \zeta_s \) damping ratio of secondary stall model, Eq. (3.2)
\( \eta \) damping term of original stall model, Eq. (2.52)
\( \Lambda \) ellipsoidal coordinate
rotor climb ratio, \( V_\infty \cos \chi/\Omega R \)
\( \theta \) total pitch angle, [rad]
\( \theta_0 \) mean pitch angle, [rad]
\( \theta_1 \) oscillatory pitch angle, [rad]
\( \kappa \) factor on \( F_c \) calculation
\( \Lambda \) yaw angle, Eq. (4.3), [rad]
\(\Lambda_T\) yaw angle in the wind-axis system, \([rad]\)
\(\lambda\) part of \(v\) due to shed wake, Eq. (2.13), \([m/s]\)
\(\lambda_n\) components of velocity due to shed wake, Eq. (2.30), \([m/s]\)
\(\lambda_V\) nondimensional amplitude of velocity oscillation, Eq. (4.8)
\(\mu\) rotor advance ratio, \(V_\infty \sin \chi / \Omega R\)
\(\nu\) ellipsoidal coordinate
\(\xi\) dummy variable of integration, Eqs. (2.12) and (2.13)
\(\rho\) density of air, \([kg/m^3]\)
\(\rho_n^m\) integral (0 to 1) of \(\left(P_n^m(v)\right)^2\), Eq. (4.12)
\(\tau\) reduced time, \(\tau = Ut/b\)
\(\Delta\tau\) pulse width, Eq. (3.3)
\(\tau_e\) end time of pulse, Eq. (3.3)
\(\tau_i\) start time of pulse, Eq. (3.3)
\(\tau_n\) expansion coefficients for \(\Delta P/2\), Eq. (2.24), \([m^2/s^2]\)
\(\tau_n^m\) pressure expansion coefficients, Eqs. (4.36)- (4.38), \([m^2/s^2]\)
\(\bar{\phi}_z\) average induced flow on the disk, Eq. (4.30), \([m/s]\)
\(\Phi_p\) pressure potential, Eq. (4.9), \([N/m^2]\)
\(\Phi_V\) velocity potential, Eq. (2.17), \([m^2/s]\)
\(\phi\) section inflow angle, \(\tan^{-1}(U_p / U_T)\), \([rad]\)
\(\phi_j^\tau(\bar{r})\) radial expansion shape function, Eq. (4.16)
\(\varphi\) phase shift, \([rad]\)
\(\chi\) wake skew angle, \([rad]\)
\(\psi\) azimuthal location of reference blade, \([rad]\)
\(\psi_q\) azimuth of \(q^{th}\) blade, \(\psi_q = \bar{\psi} + \frac{2\pi}{Q}(q - 1)\), \([rad]\)
\(\bar{\psi}\) ellipsoidal coordinate
\(\Omega\) frequency of input, \([rad/s]\)
\(\omega\) natural reduced frequency of original stall model, Eq. (2.52)
\(\omega_s\) natural reduced frequency of secondary stall model, Eq. (3.2)
Subscript
\( c,s \) cosine and sine part, respectively
\( n,j \) polynomial number
\( q \) blade index

Superscript
\( c,s \) spatial cosine or sine portion, respectively
\( m,r \) harmonic number

Operator
\( \cdot \) \( \partial(\cdot)/\partial t \)
\( * \) \( \partial(\cdot)/\partial \tau \)
\( \dagger \) \( \partial(\cdot)/\partial \bar{\tau} \)
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May 2016
Dedicated to my parents.
Chapter 1

Introduction

1.1. Motivation

Helicopters are generally limited in their performance by the phenomenon of dynamic stall. Unlike fixed-wing aircraft, for which stall occurs when the air speed becomes too low and when stall can cause the aircraft to crash, helicopter blades encounter stall at high forward speed and in high-g maneuvers. This rotor blade stall, however, occurs only during part of the blade rotation and does not cause the aircraft to crash. Rather, dynamic stall causes vibrations to become excessive, creating destructive fatigue damage, and causes the rotor to be unable to maintain the loads necessary for the required maneuver. It is important to be able to model the dynamic transit of any blade section in and out of stall in order to design rotor blades. It is also important to be able to model dynamic stall in real-time flight simulations so that pilots can learn how to avoid stall without creating damage to an actual aircraft. The purpose of this work is to develop a method for modeling dynamic stall that is appropriate to preliminary design and flight simulator applications.

1.2. The Phenomenon of Dynamic Stall

It is known that the airflow around an airfoil or any other flying surface produces lift and drag forces. An increase in the angle between the airflow and the airfoil chord line (angle of attack) increases the lift, but also increases the drag. In order to preserve a high lift-to-drag ratio, the flow should remain smooth and attached to the airfoil section. At a high enough angle of attack (the critical angle of attack), it becomes impossible for the air to flow smoothly over the top airfoil surface. Consequently, stall occurs; and the flow separates from the airfoil surface. In helicopter blades, the blade-element angle of attack becomes unsteady due to the pitching,
plunging, flapping, and wake inflow. Consequently, flow separation occurs in a dynamic manner; and that is why this phenomenon is called the dynamic stall.

There can be either leading-edge or trailing-edge dynamic stall. Leading-edge dynamic stall is accompanied by formation of a vortex that sheds from the leading edge region of the airfoil. The main feature that distinguishes it from static stall is a time delay in the onset of flow separation and a resultant overshoot in both the lift and the nose down pitching moment—as compared to the static case. Reference [2] gives three main reasons for this time delay in the onset of the dynamic flow separation.

The first reason is the unsteadiness of the flow that is caused by the shed vortex when it moves into the wake at the trailing edge of the airfoil. This effect reduces both the lift and the adverse pressure gradient as compared to the steady case. The second reason is the kinematics of the pitch rate of the airfoil. The positive pitch rate further decreases the effective angle of attack, the leading edge pressure, and the pressure gradient. The third reason is the existence of additional unsteady effects within the boundary layer, including the presence of flow reversal in the absence of any significant separation due to external pressure gradients. These unsteady effects also delay the onset of the stall by reducing the adverse pressure gradient. On the other hand, the reason for the larger values of the maximum lift and for the nose down pitching moment (as compared to the static case) is the time that it takes for the shed vortex from the leading edge to move down to the trailing edge and leave the surface of the airfoil. As long as this vortex is on the airfoil it builds up more lift. In this work, we conceptually treat only leading-edge stall, nevertheless, the method has been shown also to work fairly well in the presence of leading-edge stall.

1.2.1. Dynamic Stall Events

Figure 1.1 (from Ref. [3]) shows the different stages of dynamic stall for an oscillating VR-7 airfoil (a Boeing patented airfoil). At point 1, there is no separation; and the section loads can be obtained from linear, unsteady airfoil theory. One can see in the transition from point 1 to point 2 that the onset of separation has been delayed due to the reduction in the adverse pressure gradient, which is the direct result of the kinematics of: 1.) pitch rate (including camber), 2.) the influence of the shed wake, and 3.) the unsteady boundary layer response.
At Point 2, flow reversal within the boundary layer begins; and a vortex disturbance develops near the leading-edge. This disturbance spreads rearward, inducing a suction, which provides additional lift. At point 3, the pitching moment diverges to large negative values (moment stall), which results from motion of the center of pressure aft (as the vortex disturbance is swept downstream). The lift increases in this phase because it does not drop until the vortex passes into the wake. At point 4, one can see: 1.) the sudden loss of lift, 2.) a peak in pressure drag, and 3.) a maximum in nose down pitching moment after the shed vortex moves into the turbulent wake (as the flow on the upper surface progresses to full separation). At point 5, a secondary vortex is shed which produces fluctuations in the airloads. The last stage includes the reattachment process. Full flow reattachment is not obtained until the airfoil is well below its normal static stall angle because there is a time delay in the reorganization of the flow from the fully separated state until it becomes reattached. There is also an additional time delay because of the reverse kinematic effect on the leading-edge pressure gradient due to the negative pitch rate.

Figure 1.1. Dynamic stall events for an oscillating VR-7 airfoil, 
\[ \alpha = 15^\circ + 10^\circ \sin \omega t, \quad M = 0.25, \quad k = 0.10 \] [3]
1.2.2. Stall Regimes

The key parameter for determining the degree of separation is the maximum angle of attack, which for sinusoidal oscillations is $\alpha_{\text{max}} = \alpha_0 + \alpha_1$. An important aspect of dynamic stall is the large amplitude of the motion necessary to produce large values of angle of attack.

Figure 1.2 (from Ref. [6]) shows the importance of maximum angle on the stall regime. As one can see, when $\alpha_{\text{max}} = 13^\circ$ (left-hand side diagram), there is no separation visible through the cycle. For the next case ($\alpha_{\text{max}} = 14^\circ$), there is a limited separation that occurs through a small fraction of the cycle. This represents the condition in which one can obtain the maximum lift without a significant penalty in pitching moment or drag. The separation phenomenon is significant for the third case ($\alpha_{\text{max}} = 15^\circ$), and it becomes more severe as the maximum angle of attack is increased to $20^\circ$ (right-hand side diagram). It is interesting to note that, in the $C_M$ curves of Fig. 1.2, a clockwise loop is negative damping and a counter-clockwise loop is positive damping. Thus, if the clockwise part of the loop has more area than the counterclockwise, then there is the unstable condition known as stall flutter.

![Figure 1.2. Dynamic stall regimes for NACA 0012 airfoil, $\alpha = \alpha_0 + 10^\circ \sin \omega t$, $k = 0.10$, solid lines denote increasing $\alpha$, dashed lines decreasing $\alpha$ [6]](image-url)
One can divide the dynamic stall seen in Fig. 1.2 into two different categories: light stall and deep stall. The tendency toward negative damping is strongest during light stall. For this type of stall, the vertical extent of the boundary layer tends to be on the order of the airfoil thickness. Therefore, either the zonal method or the thin-layer Navier Stokes method with straightforward turbulence modeling is used to predict the engineering quantities of interest. The parameters that can affect the quantitative behavior of light stall are airfoil geometry, reduced frequency, maximum incidence, Mach number, type of motion, and three-dimensional effects; but the qualitative behavior of stall is sensitive to: 1.) boundary-layer separation 2.) change in separation with maximum angle of attack, 3.) reduced frequency, and 4.) free-stream Mach number.

In deep stall, where there are large angles of attack, lift, moment, and drag coefficients can exceed their static values because of the shed vortices. In addition, the size of the hysteresis loop is larger than for light stall. The qualitative behavior of deep stall is less sensitive to the airfoil geometry, airfoil motion, Reynolds number and Mach number than is light stall. Moreover, the quantitative airloads are mainly dependent on the time history of the angle of attack for the portion of the cycle for which the angle of attack exceeds the static angle of attack. In the work here, we do not need to differentiate between these two regions, as the model treats both in a unified manner.

1.2.3. Effective Parameters on Dynamic Stall

Dynamic stall has been widely studied over the years, mostly with two-dimensional oscillating airfoils in wind tunnels. These wind tunnel tests are done with airfoils that have imbedded pressure taps that are used to integrate the pressures to find lift, moment and drag. In addition, smoke is often used to visualize the boundary-layer separation associated with stall. Because of the importance of this problem, the extent of dynamic stall data is rather extensive. It includes many different types of airfoils, the addition of leading-edge droop and trailing-edge flaps, and a wide range of pitch angles and reduced frequencies. Unfortunately, there are fewer tests available that are at full-scale Reynolds numbers and Mach numbers. However, there are good results that are meaningful; and much can be gained from a study of them. The results of these experimental studies show that the dynamic stall is highly dependent on the following parameters.
1.2.3.1. Geometry of the airfoil

The effect of airfoil shape on the problem of dynamic stall is still not fully understood. However, the results of Refs. [4], [5] and [6] show that the geometry of the leading edge is an important stall parameter, especially in the light stall regime. Airfoils with sharp nose radius produce high adverse pressure gradients near the leading edge, which results in separation of the laminar boundary layer at low angles of attack (leading edge stall). In contrast, blunt nose airfoils or airfoils with large leading-edge camber display separation that starts from the trailing edge and proceeds toward the leading edge (trailing edge stall). However, in the deep stall regime—where vortex shedding becomes fully developed—the difference between the two cases is not significant.

1.2.3.2. Reduced Frequency

Another important effective parameter on dynamic stall is the reduced frequency. Its effect depends on the stall type (leading edge or trailing edge stall) and the stall regime (light or deep stall). Figures 1.3 and 1.4 (from Ref. [7]) show the effect of the reduced frequency on stall for the VR-7 and NACA 0012 airfoils, exhibiting trailing-edge and the leading-edge stall respectively. Both airfoils were tested under identical conditions. As one can see, they exhibit different behaviors.

![Figure 1.3](image)

**Figure 1.3.** Effect of reduced frequency on the unsteady lift and pitching moment of an oscillating VR-7 airfoil at $\alpha = 10^\circ + 5^\circ \sin \omega t$ and $M = 0.3$ [7]
Figure 1.4. Effect of reduced frequency on the unsteady lift and pitching moment of an oscillating NACA 0012 airfoil at $\alpha = 10^\circ + 5^\circ \sin \omega t$ and $M = 0.3$ [7]

Figure 1.5 (from Ref. [8]) shows the effect of the reduced frequency on NACA 0012 airfoil in the deep stall regime. As one can see, vortex shedding evolves for $k \geq 0.05$; and its strength becomes independent of reduced frequency for $k \geq 0.15$.

Figure 1.5. Effect of reduced frequency on the unsteady lift and pitching moment of an oscillating NACA 0012 airfoil in the deep stall regime at $\alpha = 15^\circ + 10^\circ \sin \omega t$ and $M = 0.1$ [8]
1.2.3.3. Amplitude and Mean Angle

As mentioned in Section 1.2.2., although the maximum angle of attack ($\alpha_{\text{max}} = \alpha_0 + \alpha_1$) is the primary parameter that defines the degree of separation, variation in the mean angle ($\alpha_0$) or the oscillating amplitude ($\alpha_1$) can also cause different stall behavior. Experiments in Ref. [6] show that, at the deep stall regime, if the time histories of the angles of attack, the static stall angles, maximum angles of attack and reattachment angles match closely, stall behavior will be similar. As a result, the aerodynamic coefficients—at least for pitching motions—will agree closely.

1.2.3.4. Mach Number

It is known that even at relatively low free-stream Mach numbers, local transonic or supersonic flow can occur near the leading edge of the airfoil. The experiments reported in Ref. [6] showed no evidence that local supersonic flow resulted in the occurrence of shock waves on the Vertol VR-7 airfoil. However, the experiments reported in Refs. [9] and [10] seem to indicate the existence of shockwaves for the same airfoil and under the same testing condition. Furthermore, the results of the experimental studies Ref [11] show the occurrence of multiple shockwaves at a typical model helicopter flight condition. The reason for this anomaly is not completely known; however, it could be that the weak shocks in [9] and [10] do not greatly affect the stall—and thus did not show up in the results of [6].

In support of this view, data from Ref. [12] show that—at relatively low Mach numbers ($M < 0.6$)—the transonic shock wave formation has no effect on either the dynamic stall or the vortex-shedding phenomenon. On the other hand, for higher Mach numbers the static data stall do show clear evidence of shock-induced separation and stall. Additionally, the results of Ref. [13] imply that, as long as the shock-pressure-ratio is less than 1.4, the presence of the shockwave should not result in any boundary layer separation.

Reference [14] has reviewed the recent experimental studies that have been done in order to evaluate the effects of compressibility on the dynamic stall. That reference concludes that compressibility effects can change the physical mechanisms that cause stall to occur. Moreover, compressibility effects to a significant extent restrain the time delay in the onset of both stall and overshoot in the lift.
1.2.3.5. Plunging Oscillation

A common observation is that plunging oscillations give the same stall behavior as do pitching oscillations, considering the fact that an effective fluctuating angle of attack, $\alpha_{\text{eff}} = -\dot{h}/V_\infty$, should be superimposed on the mean angle of attack ($\alpha_0$).

1.3. Dynamic Stall in the Rotating Environment

Helicopters often experience dynamic stall on the retreating side of rotor in forward flight. Figure 1.6 shows the velocity distribution along the blade for a helicopter in forward flight. If the helicopter rotor rotates at a rotational speed $\Omega$ and the free-stream velocity is considered to be $V_\infty$, the tip speed on the advancing side of the rotor will be $\Omega R + V_\infty$. On the Retreating side, the blade moves in the opposite direction of the flight which will result in a tip speed of $\Omega R - V_\infty$ on this side.

![Figure 1.6. Velocity distribution in forward flight](image)

By generalizing, the normal component of the velocity to the blade at any azimuthal angle $\psi$ and any radial position $r$ is $U_T = \Omega r + V_\infty \sin \psi$, so the wind velocity that the blade sees is sinusoidal. This velocity produces a lift per unit length approximately equal to $0.5 \rho U_T^2 a \alpha$. In order to fly straight and not maneuver, this lift must be balanced between the left and right sides.
of the rotor; which means that $\alpha$ (the angle of attack) must be varied azimuthally to compensate for the variation in $U_T$. This requires the blades on the retreating side of the rotor to work at higher angles of attack than on the advancing side, which can make them stall over all or part of the span. Figure 1.7 (from Ref. [1]) shows that, for a helicopter in forward flight at moderate thrust and speed, the angle of attack that results in stall is reached in the third quadrant, and inboard. At high thrust and speed, the region of stall extends outward along the blade.

In hover, because the flow is axisymmetric, stall occurs within an annulus on the rotor disk. As the thrust is increased, the induced velocity also increases with thrust which produces the smallest inflow angle at the tip of the blades. Consequently, the blade angle of attack change becomes greatest at the tip of the blades; and the tip region stalls first—at least for low twist. However, the peak angle of attack moves inboard for large twist angles. Figure 1.8 (from Ref. [1]) shows the angle-of-attack distribution for a hovering rotor.

In autorotation for which the inflow ($V_\infty + \nu$) is upward through the disk, the inflow angle $\phi = \tan^{-1}|V_\infty + \nu|/\Omega r$ is large inboard and decreases toward the tip. As a result, the angle of attack $\alpha = \theta + \phi$ is largest at the root; and the root region stalls first.

![Blade angle-of-attack distribution](image)

**Figure 1.7.** Blade angle-of-attack distribution (in degrees) for a helicopter in forward flight (nonuniform inflow), $C_T/\sigma = 0.08$, $f/A = 0.008$, $\theta = -8^\circ$, $\mu = 0.15, 0.30, 0.45$ [1]
Helicopters may also experience stall during the maneuvers or in aerodynamic turbulence. Pull ups, turns, and many other maneuvers are associated with an increase in the rotor thrust and (consequently) an increase in the blades angles of attack, which may result in occurrence of stall. The nonlinearities associated with this phenomenon give rise to high blade stress, vibrations, and control loads. Hence, stall must be considered as a major factor in the aerodynamic and structural design of the helicopter rotor and control system and must always be included in the analysis of the helicopter performance and aeroelastic behavior.

### 1.3.1. Double Stall Event

As was mentioned in Sec. 1.2.1, an airfoil undergoing dynamic stall often displays a secondary lift peak after the lift has already begun to drop off. This secondary peak is known to be the result of a secondary vortex that first attaches and then sheds from the airfoil—producing additional fluctuation in the airloads. Figure 1.9 is a laser shadow diagram of the flow around a NACA0013 rotor blade and shows the formation of this secondary vortex near the leading edge.

Although several low-order models of dynamic stall have been developed for modeling the dynamic stall in flight simulations, none of these models are capable of capturing the secondary vortex shedding phenomenon. One of the main purposes of this work is to offer a dynamic stall model that can represent both the major stall and the double stall events.
1.3.2. Three-Dimensional Effects on Dynamic Stall

Although the flow around a finite wing is three-dimensional, it is common to consider two-dimensional stall characteristics for an oscillating finite wing. This is because of simplicity of analysis and the lack of insightful, three-dimensional information. The local sweep or yaw angle at a blade element, which is defined for a three-dimensional wing, is one of the parameters that can have a significant effect on the dynamic stall. References [15], [16], [17] and [18] have examined the effects of sweep on stall characteristics of an oscillating wing with NACA0012 airfoil at a constant sweep angle of 0° and 30°, Mach numbers of 0.3 and 0.4, and Reynolds numbers that are nominally full rotor scale. The results show that sweep delays the onset of dynamic stall and also reduces the rate of change of the lift and moment coefficients with angle of attack. Furthermore, for oscillating lift, sweep reduces the magnitude of the hysteresis loops.

Apart from the three-dimensional effects associated with swept flows, the problem of three-dimensional unsteady separating flows is still poorly understood. This problem also has been studied in Refs. [19], [20] and [21] for a cantilevered, semi-span wing that was oscillated in angle of attack through stall and, like the 2-D tests, was designed to simulate the variation in
angle of attack encountered by a helicopter rotor blade. The results of Ref. [21] are for $M = 0.3$, while the results of Refs. [19] and [20] cover a range of Mach number from 0.2 to 0.6, and are for the wing sweep angles of $0^\circ$, $15^\circ$ and $30^\circ$. Figure 1.10 (taken from Ref. [2]) shows the results of the experiments at $k = 0.1$ and $M = 0.2$. The results indicate that, for oscillations below stall, hysteresis loops are similar to ones expected based on 2-D considerations. However, quasi-steady effects reduce the average lift-curve slope towards the tip of the wing. The only section that shows unusual behavior in both steady and unsteady airloads hysteresis loops is the outermost wing station. The reason for that is the influence of the tip vortex of the wing.

These results also show that the dynamic stall characteristics of the 3-D wing are qualitatively similar to the ones on the two-dimensional oscillating airfoil. Because of the influence of the tip vortex, the quasi-steady angle of attack reduces while moving outboard from the innermost station of the wing. Hence, the degree of penetration of the dynamic stall also reduces on the outer board sections of the wing. Furthermore, one can note that, while lift and pitching moment overshoot occur, at the outermost wing station, the results in 3-D are less transient than in 2-D. This implies that the tip vortex alone dominates the flow field near the tip, and the angle of attack never becomes large enough to allow vortex shedding and stall to occur.

Reference [22] also has studied the three-dimensional dynamic stall problem using a model helicopter rotor. Figure 1.11 (from Ref. [22]) shows the airloads and angle of attack at different blade positions for two different cases of a nonstalled and a heavily stalled rotor blade. Here $\psi = \Omega t$ and is taken as nondimensional time. Analysis of the pressure data for the heavily stalled case implies that the resultant airloads and flow field from stall are similar to these quantities for an airfoil in pitching oscillation for $90 < \psi < 270$. On the contrary, for $270 < \psi < 360$, the three-dimensional effects produce noticeably larger lift, which suggests exercising more caution in using two-dimensional data to predict the airloads of the helicopter rotor in the deep stall regime. The three-dimensional effects on dynamic stall are studied further in this research document.
Figure 1.10. Lift and pitching moment data for an oscillating cantilevered wing at $k = 0.1$ and $M = 0.2$ [2]
All of the conclusions presented here are based on limited studies, and much further work still needs to be done to analyze the problem of three-dimensional dynamic stall. Hence, most of the mathematical models used for dynamic stall prediction have been formulized and developed based on oscillating 2-D airfoil data. One of the main purposes of this work is to offer a dynamic stall model that also accounts for the three-dimensional effects on dynamic stall.

![Graphs showing airloads and angle-of-attack values at different blade positions for a model helicopter rotor](image)

**Figure 1.11.** Airloads and angle-of-attack values at different blade positions for a model helicopter rotor [22]

### 1.4. Problem Statement and Approach

The main purpose of this thesis is to develop a tool that can be used in preliminary design and real-time flight simulators to simulate dynamic stall in the 3-D, rotating environment. In this work, we concentrate primarily on the modeling of lift coefficient, but some attention will also be given to pitching moment. There are four important aspects of dynamic stall that we wish to address in order to bring such a model to fruition.

1.) We need to develop a unified model for an oscillating airfoil that also captures the double-stall event. It is of primary importance that this be accomplished before we can address the particular effects of the rotating environment. If this is not modeled, then we will not have the fidelity needed to move on to the other effects.
2.) We must determine the effect of yawed flow on dynamic stall. In the rotating environment, the flow is yawed with a significant radial component. Although it is known that radial flow does not affect linear airloads, it is equally well-known that it does affect static stalled airloads. It follows that yawed flow must also affect dynamic stall, and we must ensure that our extension of the Ahaus model can handle this.

3.) We must determine the effect of unsteady free-stream velocity on dynamic stall and decide how to include it in the model. The Ahaus model [23], on the surface, allows for an unsteady free-stream and gives the exact answer for unstalled loads with unsteady free-stream velocity. It is unknown, however, whether or not the approach of Ahaus will be adequate for stall with unsteady free-stream.

4.) Blade-element theory generally assumes no coupling between adjacent blade stations. However, in the rotating environment, there are two types of coupling that must be considered. First, because of the three-dimensional wake, vortices shed at one blade section can induce flow at other blade sections. Thus, a 3-D induced flow model is needed. Second, when a blade stalls in the rotating environment, it is clear from geometry that the shed vortex will be convected along the blade span and will, consequently, affect other blade stations. This effect must also be included in the Ahaus model, which is described below.

Figure 1.12 illustrates the basic components of the Ahaus-Peters unified model. As one can see, the airfoil motions are transformed into a generalized set of coordinates \((h_n)\) to make the theory applicable to arbitrary airfoil deformations. Then, these generalized coordinates are combined with the free-stream and induced flow velocities to provide the boundary condition for the linear airfoil theory. The linear airloads theory computes the desired generalized linear airloads due to dynamic motions of the generalized displacements. These same boundary conditions are also used as inputs to a representation of the nonlinear static data in order to determine a preliminary static stall correction to the airloads. This static correction drives the Ahaus-Peters dynamic stall model (modified from ONERA) to give the change (due to dynamic stall) in blade circulation, pitching moment, and all other generalized loads. Finally, the total loads are found by superposition of the linear loads and the dynamic stall corrections. These total loads are fed into the induced flow model (Peters-Karunamoorthy 2D finite-state induced flow model) to give the closed loop induced flow due to the shed wake.
Figure 1.12. Schematic of Ahaus-Peters unified airloads model [23]

This model cannot produce the secondary lift peak often seen in dynamic stall data. In order to model the secondary peak, another second-order differential equation is added to the Ahaus-Peters airloads model. This new equation is to be driven by a constant pulse of duration $\Delta \tau$, and is meant to match the physics of the effect of the secondary vortex on lift.

Next, we consider the approach we will take to handle 1.) in the above list, the addition of the secondary lift peak. Figure 1.13 shows the schematic of the modified Ahaus-Peters airloads model. It is assumed that the pitch rate at the stall angle ($\dot{\alpha}_s$) can be an indication of the initial angle of attack at which the pulse should begin. So whenever the pitch rate at the stall angle is greater than the critical angular velocity for pulse to occur, a pulse is generated and the second dynamic stall parameter is added to the loss in blade circulation due to the primary dynamic stall process. Another major component that has been included in the new airloads model is the hysteresis of the static data—which is normally ignored in the dynamic stall model. In other words, as long as the pitch rate at the stall angle is less than the upper critical angular rate for hysteresis, the hysteresis of the static data ($\Delta \alpha$) is added to the boundary conditions ($\alpha$) and then the resultant angle ($\alpha_{\text{eff}}$) is used to determine the static stall correction to the lift ($\Delta C_L$). We will find, in the results to follow, that the effect of hysteresis seems to wash out at higher frequencies. This is the general approach we will formulate in Chapter 3 of this thesis.
Next, we consider the effect of sweep angle on blade dynamic stall. Here, we will follow the approach of the original Ahaus model. In particular, we will assume that the static $C_L$ curve (as modified to include the effect of yaw on static stall) is used to compute $\Delta C_L$ and that this modified lift deficiency is to be run through the same dynamic filter that is used when there is no stall. This will be done in Chapter 4. For item 3.) above—time-varying free-stream velocity—it was originally envisioned that we could just keep the original Ahaus formulation, since it allows for a time-varying flow. However, comparisons with experimental data showed that this was not a good approach. Therefore, based on the physics of the shed stall vortex, a minor modification is made to the Ahaus model in order to accommodate the correct physics. This will be outlined in Chapter 5.

For item 4.) in the above list, the first part (3-D induced flow model) is fairly straightforward. The Peters–Karunamoorthy 2D finite state induced flow model (used by Ahaus in 2-D simulations) is to be replaced by the Peters–He three-dimensional inflow model. The He model is well documented to give good results based on 3-D PIV measurements in NASA wind tunnels [58] so this is a solid approach. The other major effect listed in 4.) above is the effect of radial coupling. Recent flow visualization at Georgia Tech shows that, after the vortex is shed from a section of the blade, it convects in the direction of the free-stream and starts decaying. As a
result, in yawed flow, each blade section would encounter a part of a vortex that has been shed from its neighboring section. This effect is called radial coupling. This effect is addressed in Chapter 6, along with the effect of the 3-D wake. The full formulation of the three-dimensional airloads model is then presented in chapter 6 based on the experimental conditions of [58].

It is also worth mentioning that the models presented in this thesis are not intended to supplant wind-tunnel testing or CFD analysis, but rather are intended as reduced-order models for preliminary design calculations and flight simulation. These models rather make use of experimental and CFD data in order to match the physical parameters of the model to the physics of dynamic stall. Consequently, every major effect on dynamic stall in this thesis is compared against experimental data.
Chapter 2

Literature Review

The general features of dynamic stall and the importance of modeling this phenomenon were explained in the previous chapter. This chapter reviews the different techniques for predicting dynamic stall and also presents in more detail the Ahaus-Peters unified airloads model, which is the framework of the airloads models developed in this work.

2.1. Modeling of Dynamic Stall

The methods used to predict the effects of dynamic stall can be divided to two main categories: computational methods and empirical correlation methods. Computational methods, \textit{i.e.,} potential flow, boundary layer, coupled viscous-\textit{inviscid} interaction and full Reynolds-averaged Navier-Stokes methods) use numerical algorithms to compute the loads over airfoils in unsteady motions, while the empirical methods are based on correlating the force and pitching moment data obtained from various wind tunnel tests to predict the unsteady loads. A review of recent computational methods along with the numerical schemes for their solution can be found in Ref. [24]. In spite of the considerable progress that has been made in computing the unsteady airfoil flows using computational methods, the computed airloads in the stall regime and during the flow reattachment are still not in a quantitatively acceptable correlation with the experimental data. Hence, the prediction of dynamic stall is largely based on empiricism in a realistic rotor design work. This section reviews the different empirical correlation methods that have been used to predict the engineering quantities of an airfoil or wing or rotor blade undergoing the dynamic stall.
2.1.1. Empirical Correlation Methods

The empirical models are developed based on correlation of experimental data obtained from wind-tunnel tests to estimate the unsteady airflow loads on the airfoil. These models are usually sufficient for most rotor design purposes. However, since they are developed based on experimental measurements of a limited number of airfoils at a limited range of Mach numbers, they lose accuracy when applied to different airfoils at different Mach numbers. For high Mach numbers, inaccuracy becomes more significant and the model less reliable. Some of these empirical models that have been commonly used in the helicopter industry are as follows:

2.1.1.1. Boeing-Vertol “gamma” Function Method:

The Boeing-Vertol Method is one of the simplest empirical methods in which the hysteresis effects are produced by considering a delay in the angle of attack. This method was developed by Harris, Tarzanin and Fisher [25] and Gormont [26]. In this method, the onset of dynamic stall is assumed to occur at $\alpha_{DS} = \alpha_{ss} + \Delta \alpha_D$, where $\alpha_{ss}$ is the static stall angle and $\Delta \alpha_D = \gamma \sqrt{\dot{\alpha} c / V_\infty}$. The gamma function is determined empirically from a large number of 2-D oscillating airfoil tests and it depends on the Mach number and the airfoil geometry. Furthermore, it is different for the lift and moment stall. The reference angle of attack, $\alpha_r = \alpha - \gamma \sqrt{\dot{\alpha} c / 2V_\infty \text{sign} (\dot{\alpha})}$, and the equivalent angle of attack that has the unsteady potential flow effects, $\alpha_{eq}$, are used to obtain the lift and moment coefficients from the static airfoil data. An empirically determined center of pressure, $X_{cp}$, is used for calculating the pitching moment.

$$C_L = \left( \frac{\alpha_{eq}}{\alpha_r} \right) C_L(\alpha_r)$$

$$C_D = C_D(\alpha_r)$$

$$C_M = \left( 0.25 - X_{cp} \right)(\alpha_r)$$

(2.1)

The model predicts the overshoot in the lift coefficient over its static value. However, it doesn’t correct the section drag coefficient.
2.1.1.2. UTRC $\alpha, A, B$ Method:

The UTRC $\alpha, A, B$ Method that has been developed by Carta, et al. [27] and Bielawa [28] at United Technologies Research Center (UTRC) is a time domain unsteady aerodynamic model based on the oscillating airfoil tests. In this method, the airloads are determined based on the three independent parameters of the airfoil motion. These parameters are instantaneous angle of attack, $\alpha$, nondimensional angular velocity, $A = \dot{\alpha}c/2V_\infty$, and nondimensional angular acceleration, $B = \ddot{\alpha}c/4V_\infty^2$. Although this method has been somehow successful in predicting the unsteady airloads, the need for generating large data tables for each airfoil and each Mach number makes it impractical for real-time flight simulation.

2.1.1.3. MIT Method:

This method—also known as Johnson’s method—was developed by Wayne Johnson [29], [30] at Massachusetts Institute of Technology. This method is one of the most convenient empirical prediction methods and it works for any airfoil for which the static data are available. Below the static stall angle of attack ($\alpha_{ss}$), the method uses standard airfoil data. When the angle of attack increases from below to above $\alpha_{ss}$, the data are extrapolated from below static stall. As angle of attack increases above $\alpha_{ss}$, the dynamic stall occurs at $\alpha_{DS} = \alpha_{ss} + \gamma\sqrt{\dot{\alpha}c/2V_\infty}$. Here, $\gamma$ is a constant with dimension of angle and it is weakly dependent on the airfoil. When the value of $\gamma$ is not known, it is considered to be equal to 1.0 radian. When vortex shedding occurs, the airloads are assumed to increase linearly over a finite time, until they reach their peak values. The airload peak values are calculated using a formulation that depends on $\dot{\alpha}c/V_\infty$ at the moment of dynamic stall. The airloads keep their peak values until the angle of attack begins to decrease and then they exponentially decay to their static values. The satisfactory prediction of the unsteady lift is achievable by using this method. However, predicted values of pitching moments seem to be less satisfactory.
2.1.1.4. Lockheed Method:

The Lockheed method is an analytical-empirical method developed by Ericson and Reding [31], [32] at Lockheed. In this model, the quasi-steady overshoot of the maximum lift coefficient (and the nose down pitching moment) are assumed to be the result of the boundary layer improvement due to upstream pressure gradient relief induced by pitch rate as well as leading edge plunging. Furthermore, it is also assumed that the upper boundary for this dynamic improvement of the boundary is the static infinite Reynolds number limit. Therefore, the quasi-steady overshoot of both the maximum lift coefficient and the nose down pitching moment are expressed in the form of an increase of the stall angle. In addition to the overshoot of static lift maximum, there is another time lag effect that is assumed to be composed of Karman-Sears circulation lag and the boundary layer convection time lag. This flow phenomenon is considered as transient and is assumed to incorporate the moving separation point effect and the “spilled” vortex effect. The phase-lag time constants and stall-angle delay increments are included in a fictitious effective angle of attack, and this effective angle of attack is used to build the lift and pitching moment from the static stall airfoil characteristics and a linear combination of separate dynamic stall elements. It is worth mentioning that this is the only dynamic stall prediction method that explicitly differentiates between the pitching and plunging motion. However, it should be noted that the Ahaus method does adjust stall with camber. Thus, if one considered pitch rate to be an equivalent camber, the Ahaus model could be used make a distinction between plunge and pitch rates.

2.1.1.5. Time-Delay Method:

The time delay method is a time domain based method and has been developed by Beddoes [33], [34]. In the attached flow region, the airloads behavior is obtained from Duhamel principle using the Wagner indicial response function. In order to produce the increased lag in the unsteady airloads associated with the compressibility effects, some corrections are applied to the Wagner function. The main characteristic of this method is using two nondimensional time delays parameters, which are determined from analyzing the experimental measurement done on different airfoils at a wide range of Mach numbers. These parameters account for the time delay in the onset of the stall, and the time during which the leading edge vortex shedding mechanism
occurs, respectively. The results obtained from this model have been shown to be in a good correlation with the 2-D experimental data. Similar studies also have been done in Ref. [35], at lower Mach numbers.

2.1.1.6. Gangwani’s Method:

This model was developed by Santu T. Gangwani [36], [37] is a time-domain, unsteady aerodynamic model based on oscillating airfoil tests. The formulation of the model sufficiently accounts for the formation and shedding of the vortex from the leading edge during the dynamic stall. Moreover, the effects of the azimuthal variation in aerodynamic sweep angle and Mach number are properly included in the method. Similar to Beddoes’ time delay model, the airloads behavior for the attached flow condition is obtained from the Duhamel integral, using the compressible Wagner function. The formulation of the synthesized unsteady forces and moments is based on the determination of shifted angles of attack with several empirical coefficients being derived from steady and unsteady test data. It has been shown that the synthesized unsteady lift and pitching moment obtained from the model match very well with the two-dimensional experimental data. Furthermore, a good correlation has been shown between the model predicted stall airloads and the experimental measured airloads for a CH-53A helicopter rotor blade during the dynamic stall. The major set-back of the method seems to be in the prediction of the instant at which the reattachment occurs. Another issue with either Beddoes or Gangwani methods is that the Wagner Function assumes a flat, two-dimensional wake. Thus, the method cannot formally be modified to use in the rotating environment (although people often do so, anyway).

2.1.1.7. Leishman-Beddoes Method:

The Leishman-Beddoes Model was initially developed by Beddoes [38], and by Leishman and Beddoes [39], [40] with several revisions by Leishman [41] and Tylor and Leishman [42]. This model consists of an attached flow model for unsteady (linear) airloads, a separated flow model for nonlinear airloads, and a dynamic stall model for the leading edge vortex induced airloads. The model was developed with the main purpose of calculating lift loads on helicopter profiles. However, it also describes the dynamic variation of moment and drag forces. The forces are
computed as normal force, tangential force and pitching moment. In order to model the attached flow behavior, the compressible indicial response functions for a step increase in the angle of attack are used. The indicial response functions are composed of the circulatory and the non-circulatory components. These step response solutions are superimposed using a more accurate finite difference approximation to the Duhamel’s superposition Integral to yield the circulatory, $C_n^c$, and non-circulatory normal forces, $C_n^l$. The total attached flow normal force is then obtained by adding the circulatory and non-circulatory normal forces.

In the separated flow model, the experimental lift stall characteristics are used with the Kirchhoff-Helmholtz model to determine the effective separation point variation. In a model for trailing-edge separation, as obtained from the Kirchhoff theory, the airfoil normal force is approximated as:

$$C_n = C_{n\alpha} (M) K_N (f) (\alpha - \alpha_{0L})$$

(2.2)

where $C_{n\alpha}$ is the normal force (lift) curve slope at the appropriate Mach number, $K_N = 0.25(1 + \sqrt{f})^2$ is the Kirchhoff expression and $f$ is the trailing edge separation point. Equation (2.2) is inverted and solved for the trailing edge separation point. Then the relation between $f$ and $(\alpha - \alpha_{0L})$ is generalized empirically using the following relation:

$$f = \begin{cases} 
1 - 0.3 e^{(|\alpha - \alpha_{0L} - \alpha_s|)/S_1} & |\alpha - \alpha_{0L}| \leq \alpha_{ss} \\
0.04 - 0.66 e^{(|\alpha - \alpha_{0L} - \alpha_s|)/S_2} & |\alpha - \alpha_{0L}| > \alpha_{ss} 
\end{cases}$$

(2.3)

The constants $S_1, S_2$ and $\alpha_{ss}$ depend on Mach number and are obtained using the experimental lift stall characteristics. It should be noted that $f = 1$ represents fully attached flow, while $f = 0$ represents fully separated flow. Moreover, $f = 0.7$ ($|\alpha - \alpha_{0L}| = \alpha_{ss}$) closely corresponds to the static stall angle of attack.

A delayed angle of attack, $\alpha_d$, is calculated to account for the effects of hysteresis evident in oscillatory pitch data under quasi static condition, the lag in the leading edge pressure response and also the lag in the unsteady boundary layer response. Then the dynamic value of the separation point position, $f_d$, is calculated from $f(\alpha_d)$ with a time lag, and the normal force, $C_n^f$,
is evaluated using \( f_d \). For pitching moment, the reattachment is handled differently than separation. Thus, different \( \alpha_d \) equations are considered for lift and moment.

This approach also includes a model for the vortex lift, assuming that the vortex lift contribution can be viewed as an excess accumulation of the circulation that is held in the vicinity of the airfoil until some critical condition is reached. It is assumed that, the increment in vortex lift is determined by the difference between the linear lift and the nonlinear lift obtained from the Kirchhoff flow equation along with the dynamic value of the separation point position, \( f_d \). The total accumulated vortex lift, \( C_n^v \), is allowed to decay exponentially with time, but may also be updated by a new increment. Finally, the total normal force is obtained by superimposing all of the components.

\[
C_n = C_n^l + C_n^f + C_n^v
\]  

Similar expressions for the drag force or the pitching moment can be found in the literature [40], [41] and [42]. The validation of the model has been verified against the experimental flow measurements [40].

The Leishman-Beddoes model was adopted into the UMARC code while Leishman was a professor at the University of Maryland. Since many industries use UMARC, the Leishman-Beddoes method then found its way into other industrial codes, including the wind turbine code Aerodyne and the Army code RCAS. However, there are certain aspects of the Leishman-Beddoes methodology that are not ideal, and this is what led to other methodologies following ONERA (see following section). Perhaps the most glaring aspect of the Leishman formulation is that it utilizes the two-state Jones indicial approximation to the Wagner Function as its unsteady aerodynamic theory. The two-state model of Jones approximation has its own inaccuracies, as shown by Peters, et al [43] where it is shown that, for the case of unsteady free-stream, the Johnson methodology (used herein) gives a perfect match to the exact solution of Isaacs, while Leishman has inaccuracies.

However, an even more important drawback is the fact that it is well-known [44] that the Wagner function (which is the indicial version of the Theodorsen Function) is for a flat wake behind an infinite wing; and, as such, it is inaccurate for unsteady aerodynamics of a rotating wing [45]. In fact, Peters has shown that—for a rotating wing—it is better to use no unsteady
aerodynamics at all than to use the Wagner function [46]. Therefore, it is problematical from a physics point of view to use the Leishman or Leishman-Beddoes model for rotorcraft. A third drawback of Leishman-Beddoes is that the complete set of equations necessary to implement the model can fill close to 40 pages of text. Thus, the implementation is more cumbersome that that of the ONERA methods, as we will see in the following section.

2.1.1.8. ONERA Method:

The ONERA method has been developed by Tran and Petot [47], Tran and Falchero [48], Petot [49], McAlister, Lambert and Petot [50], with various modifications by Peters [51]. In the ONERA model the unsteady behavior of the stall is modeled with a set of differential equations, with coefficients based on small-amplitude oscillations in a wind tunnel. The latest version of model, which is called the ONERA Edlin model, has been presented by Petot and can be found in [52] and [53]. Following set of differential equations represent the original form of the ONERA model:

\[ C_L = C_{L1} + C_{L2} \]  
\[ \dot{C}_{L1} + \dot{\lambda} C_{L1} = \dot{\lambda} \dot{\theta} + \left( \dot{\lambda} \dot{s} + \dot{\delta} \right) \dot{\theta} + \dot{s} \dot{\theta} \]  
\[ \ddot{C}_{L2} + \dot{\eta} \dot{C}_{L2} + \dot{\omega} \dot{C}_{L2} = -\ddot{\omega} \left[ \Delta C_L + \dot{\theta} \frac{\partial \Delta C_L}{\partial \theta} \right] \]

where \( C_L \) represents the dynamic lift coefficient and has a linear part \( (C_{L1}) \) and a nonlinear part \( (C_{L2}) \). \( \Delta C_L \) is the difference between the linear static lift coefficient and the measured, stalled, static-lift coefficient and \( \dot{\lambda}, \dot{\theta}, \dot{s}, \dot{\delta}, \dot{\omega} \) are the coefficients that would be determined by parameter identification. Below the static stall angle \( (C_{L2} = 0) \) one just needs to solve Eq. (2.6). However, in the stall regime one needs to solve the whole system of equations in order to determine the unsteady airloads. The results obtained revealed that the model is just well suited to the data sets that are generated with regard to its small-amplitude and high-frequency requirement.

Peters and Rudy [51], [54] indicated that the ONERA model can alternatively be written either in terms of nondimensional circulation \( \bar{\Gamma} = \bar{U}C_L \) or nondimensional lift \( \bar{L} = \bar{U}^2 C_L \) (\( \bar{U} \) is
the nondimensional velocity). However, they noticed that, for unsteady free-stream velocity, the formulation of the model is best behaved and agrees mostly with experimental data in terms of circulation. The following set of equations, documented in [51], shows Peters’ representation of ONERA model for a blade embedded reference frame and small angle-of-attack assumption with velocities divided into $x$ and $y$ components:

$$\tilde{k} \Gamma_1 + \hat{\lambda} \Gamma_1 = \hat{\lambda} \tilde{a} \tilde{U}_y + \hat{\delta} \hat{\delta} \hat{\delta}$$  \hspace{1cm} (2.8)

$$\tilde{k}^2 \Gamma_1 + \tilde{k} \hat{\eta} \Gamma_1 + \hat{\omega}^2 \Gamma_2 = -\hat{\omega}^2 \left[ \Gamma_1 \Delta C_L + \hat{\delta} \hat{\delta} \left( \hat{\delta} \tilde{a} \tilde{U}_y + \hat{\delta} \tilde{a} \tilde{U}_y \right) \right]$$  \hspace{1cm} (2.9)

Here $\tilde{k} = b/r$, $\tilde{b} = b/R$ and $\hat{\delta}$ is the rotation rate of the airfoil with respect to the air mass. ONERA identifies these stall coefficient parameters by dynamic perturbation about a number of mean angles of attack. From the tests run by ONERA and NASA [49], [50], the functional form of the stall coefficients was determined to be:

$$\hat{\omega} = \omega_0 + \omega_2 \left( \Delta C_L \right)^2$$

$$\hat{\eta} = \eta_0 + \eta_2 \left( \Delta C_L \right)^2$$  \hspace{1cm} (2.10)

$$\hat{\epsilon} = \epsilon_0 + \epsilon_2 \left( \Delta C_L \right)^2$$

Peters pointed out that some other modifications are needed to be made to his formulation of the ONERA model. He suggested that the crude, first-order approximation for linear circulation expressed by Eq. (2.8) should be replaced by a more general induced flow theory. This not only gives a more accurate representation of linear circulation, but also makes it possible to replace the two-dimensional induced flow model with a three-dimensional model when dealing with three-dimensional dynamic stall. It also reduces the system of two differential equations needed to present the dynamic stall to just one second-order differential equation. Furthermore, Peters suggested that the model should be modified so that it includes the effect of the unsteady free-stream. Modifications for the Peters dynamic stall model (as modified from ONERA) were made by Peters and Johnson [55] and Ahaus and Peters [23]. In this thesis, the Ahaus-Peters
model is used for modeling the dynamic stall. This model is explained in section 2.2.3 of this thesis.

2.2. Development of Ahaus-Peters Unified Airloads Model

As mentioned earlier, the Ahaus-Peters unified airloads model is used as the framework for the models developed in this thesis. The unified airloads model is illustrated schematically in Fig 1.12. This section explains the basic components of this model.

2.2.1. Finite-State Airloads Theory

The unified airloads theory uses Peters-Johnson state-space airloads theory [55] to determine all generalized forces on the airfoil. The Peters-Johnson state-space airloads theory accommodates either a two- or three-dimensional induced flow model and was originally derived so that it could include dynamic stall. This section summarizes the derivation and development of the theory.

2.2.1.1. Aerodynamic Theory

Figure 2.1 shows a thin airfoil of arbitrary shape moving through a still fluid in two dimensions. The coordinate system is located at the center of the airfoil and it can move with a uniform horizontal velocity, \( u_0 \), a uniform vertical velocity, \( v_0 \), and a velocity gradient \( v_1 \). The airfoil can have small motions in the \( y \) direction, \( h(x, t) \), which means that \( h \ll b, \partial h/\partial x \ll 1 \) and \( \partial h/\partial t \ll u_0 \) while the frame motion can be arbitrary large.

The airfoil motion produces an upwash velocity relative to the blade of:

\[
w = u_0 \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} + v_0 + \frac{v_1}{b} x
\]  

(2.11)
On the other hand, at the blade section there is also a downwash velocity, $\lambda$, due to the external vorticity, and $\vec{v}$ due to the vorticity representing the blade surface (bound vorticity). From the strength of the vortex sheets representing the airfoil and shed wake, these induced velocities are:

\[
\vec{v} = \frac{-1}{2\pi} \int_{-b}^{b} \gamma_b(\xi, t) \frac{d\xi}{x - \xi} \quad (2.12)
\]

\[
\lambda = \frac{-1}{2\pi} \int_{-b}^{b} \gamma_w(\xi, t) \frac{d\xi}{x - \xi} + \zeta \quad (2.13)
\]

where $\zeta$ is the velocity due to the external bound vorticity. The boundary condition of no flow through the wing implies that:

\[
\lambda + \vec{v} - w = 0 \quad (2.14)
\]

Plugging Eq. (2.11) into Eq. (2.14), one can get:

\[
\vec{v} = u_0 \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} + v_0 + v_1 \frac{x}{b} - \lambda \quad (2.15)
\]
The differential pressure on the airfoil surface can be obtained from the linearized unsteady Bernoulli equation:

\[
\Delta P = \rho \left( u_0 \frac{\partial \Phi_v}{\partial x} + \frac{\partial \Phi_v}{\partial t} \right) \tag{2.16}
\]

The velocity parallel to the blade surface is \( u = \frac{\partial \Phi_v}{\partial x} \), and the blade vorticity strength is \( \gamma_b = \Delta u \). So:

\[
\frac{\partial \Phi_v}{\partial x} = \Delta u = \gamma_b \tag{2.17}
\]

\[
\frac{\partial \Phi_v}{\partial t} = \frac{\partial}{\partial t} \int_{-b}^{b} \Delta u dx = \frac{\partial}{\partial t} \int_{-b}^{b} \gamma_b dx \tag{2.18}
\]

As a result:

\[
\Delta P = \rho u_0 \gamma_b + \rho \int_{-b}^{b} \frac{\partial \gamma_b}{\partial t} d\xi \tag{2.19}
\]

which is the differential pressure on the airfoil surface. These equations define the airloads theory. However, these equations should be combined with an induced flow theory (wake model) to close the system. Independent of the particular wake model, there is an additional equation that reflects that all vorticity in the flow (from which induced flow comes) must follow the convection equation. This additional equation is derived in Appendix A of Ref. [55]:

\[
\frac{\partial \lambda}{\partial t} + u_0 \frac{\partial \lambda}{\partial x} = \frac{1}{2\pi} \frac{d\Gamma/dt}{b - x} + \bar{u}_0 \frac{\partial \zeta}{\partial x} \tag{2.20}
\]

where \( \bar{u}_0 \) is the velocity with respect to the air mass of any external bound vortex and \( \Gamma \) is the total circulation on the airfoil.

\[
\Gamma = \int_{-b}^{b} \gamma_b dx \tag{2.21}
\]
2.2.1.2. Glauert Expansions

Writing the airloads equations in terms of blade deformations, frame motions, and generalized loads is advantageous. To do so, one needs to expand all quantities in terms of Glauert variables.

\[ x = b \cos \phi \]  
\[ (-b \leq x \leq +b, \ 0 \leq \phi \leq \pi) \]  

From Ref. [56], one can expand the bound vorticity on the airfoil and the differential pressure on the airfoil surface in the following form:

\[ \gamma_b = 2 \left[ \frac{+\gamma_s}{\sin \phi} - \frac{\gamma_0 \cos \phi}{\sin \phi} + \sum_{n=1}^{\infty} \gamma_n \sin(n\phi) \right] \]  
\[ \Delta P = 2 \rho \left[ \frac{+\tau_s}{\sin \phi} - \frac{\tau_0 \cos \phi}{\sin \phi} + \sum_{n=1}^{\infty} \tau_n \sin(n\phi) \right] \]

Substitution of Eq. (2.23) and Eq. (2.24) into Eq. (2.19), yields:

\[ u_0 \gamma_s = \tau_s, \ u_0 \gamma_0 = \tau_0 \]

\[ b \left( \dot{\gamma}_0 - \frac{1}{2} \dot{\gamma}_2 \right) + u_0 \dot{\gamma}_1 = \tau_1 - \frac{\hat{\Gamma}}{\pi} \]  
\[ \frac{b}{2n} \left( \dot{\gamma}_{n-1} - \dot{\gamma}_{n+1} \right) + u_0 \dot{\gamma}_n = \tau_n - \frac{\hat{\Gamma}}{n\pi} \quad n \geq 2 \]

Eq. (2.25) relates the pressure and the vorticity to each other. In order to satisfy the Kutta condition at the trailing edge for rotors with both forward and reverse flow, one can define:

\[ \tau_s = f \tau_0 \]  

where \( f \) is the reverse flow parameter and can be obtained from Eq. (2.27).
\[ f = 1 \quad \text{(no reverse flow)} \]
\[ f = u_0 / |u_0| = \text{sgn}(u_0) \quad \text{(full reverse flow)} \]
\[ f = u_0 \sqrt{u_0^2 + (v_0 + \dot{h}_0 - \lambda_0)^2} = \cos \alpha \quad \text{(weak reverse flow)} \]

Induced flow normal to the airfoil due to the bound vorticity can be written in Glauert series expansion by placing Eq. (2.23) into Eq. (2.12).

\[ \bar{v} = \sum_{n=0}^{\infty} \gamma_n \cos(n\phi) \quad (2.28) \]

Similarly, one can also expand the other parameters in Glauert series:

\[ w = \sum_{n=0}^{\infty} w_n \cos(n\phi) \quad (2.29) \]
\[ \lambda = \sum_{n=0}^{\infty} \lambda_n \cos(n\phi) \quad (2.30) \]
\[ \zeta = \sum_{n=0}^{\infty} \zeta_n \cos(n\phi) \quad (2.31) \]
\[ h = \sum_{n=0}^{\infty} h_n \cos(n\phi) \quad (2.32) \]
\[ \frac{\partial h}{\partial x} = \sum_{n=1,3,5}^{\infty} \frac{nh_n}{b} + \sum_{m=1}^{\infty} \cos(m\phi) \left[ \sum_{n=m+1,m+2}^{\infty} \frac{2nh_n}{b} \right] \quad (2.33) \]

The relations between the pressure and the total motion of flow relative to the airfoil can be obtained by substitution of Eq. (2.30) and Eq. (2.31) into Eq. (2.20) and application of the resultant Equation in conjunction with Eq. (2.25), Eq. (2.14) and Eq. (2.29).
\[ u_0 (w_0 - \lambda_0) = \tau_0 \]

\[ b \left( \dot{w}_0 - \frac{1}{2} \dot{w}_2 \right) + u_0 w_1 = \tau_1 + \ddot{u}_0 \zeta_1 \]

\[ \frac{b}{2n} (\dot{w}_{n-1} - \dot{w}_{n+1}) + u_0 w_n = \tau_n + \ddot{u}_0 \zeta_n \quad n \geq 2 \]

Equation (2.34) presents the general airloads model. This equation can be transformed into a more applicable form by defining the relative velocities, \( w_n \), in terms of the actual airfoil deformations, \( h_n \). This is obtained from Eqs. (2.29), (2.30), (2.32) and (2.33) into Eq. (2.11).

\[ w_0 = v_0 + \dot{h}_0 + u_0 \sum_{n=1,2,3}^{\infty} \frac{nh_n}{b} \]

\[ w_1 = v_1 + \dot{h}_1 + u_0 \sum_{n=2,4,6}^{\infty} \frac{nh_n}{b} \]

\[ w_m = \dot{h}_m + 2u_0 \sum_{n=m+1,m+3}^{\infty} \frac{nh_n}{b} \quad m \geq 2 \]

### 2.2.1.3. Generalized Loads and Drag

The generalized loads would be obtained from the following relation (from virtual work):

\[ L_n = - \int_{-\theta}^{\theta} \Delta P \cos(n\phi) \, dx = - \int_{0}^{\pi} b\Delta P \cos(n\phi) \sin \phi \, d\phi \]

Equation (2.36) gives the following expression for the generalized forces in terms of the generalized airfoil deformations:

\[ \frac{1}{2\pi\rho} \{L_n\} = -b^2 \left[ M \right] \{\ddot{h}_n + \ddot{v}_n\} - bu_0 \left[ C \right] \{\dot{h}_n + v_n - \lambda_0\} - u_0^2 \left[ K \right] \{h_n\} \]

\[ -b \left[ G \right] \{\dot{u}_0 h_n + \ddot{u}_0 \zeta_n - u_0 v_n + u_0 \lambda_0\} \]

\[ n \geq 2 \]
Furthermore, the total bound vorticity can be obtained by substitution of Eq. (2.23) into Eq. (2.21) with application of Eqs. (2.34) and (2.35) respectively:

\[
\frac{1}{2\pi} \Gamma = b \{1\}^T [C - G] \{\dot{h}_n + v_n - \lambda_1\} + u_0 \{1\}^T [K] \{h_n\}
\]  

(2.38)

Although the local lift is always perpendicular to the local airfoil surface, there is also a leading edge suction load along the airfoil. These loads combine and can create a component of lift in the direction of the free-stream, which can be either induced drag or a propulsive force. The total drag is given by:

\[
D = \int_0^\pi b (\Delta P)(\partial h/\partial x) \sin \phi d\phi - 2\pi \rho bf (w_0 - \lambda_0)^2
\]  

(2.39)

By substituting Eq. (2.24) into Eq. (2.39) and, Once again, using Eqs. (2.34) and (2.35) respectively, one can get the flowing expression for the induced drag:

\[
\frac{1}{2\pi \rho} D = -b \{\dot{h}_n + v_n - \lambda_0\}^T [S] \{\dot{h}_n + v_n - \lambda_0\} + b \{\dot{h}_n + v_n\}^T [G] \{h_n\} - u_0 \{\dot{h}_n + v_n - \lambda_0\}^T [K - H] \{h_n\} + \{\dot{u}_0 h_n + u_0 \zeta_n - u_0 v_n + u_0 \lambda_0\}^T [H] \{h_n\}
\]  

(2.40)

Appendix A defines the different matrices and vectors in Eqs. (2.37) - (2.40).

2.2.2. Dynamic Inflow Model

In order to calculate the generalized loads and circulation, Eqs. (2.37) - (2.39), the state-space airloads theory, Eq. (2.40), must be combined with an induced flow theory. To do so, one needs to have knowledge of \(\lambda_0\) and \(\lambda_1\). The original Ahaus-Peters unified airloads model [23] uses Peters-Karunamoorthy two-dimensional, finite-state induced flow theory [57], since it was developed for a two-dimensional flow over an airfoil. In this thesis, Peters-Karunamoorthy two-dimensional induced flow model is used as well, to study the two-dimensional flow. For a three-dimensional flow over a finite wing, however, Peters-He three-dimensional finite state wake
model [58] is the model of interest. Section 2.2.2.1 summarizes the Peters-Karunamoorthy two-dimensional induced flow model for the sake of reviewing the original Ahaus-Peters unified airloads model. The Peters-He three-dimensional finite state wake model is presented later, in chapter four of this thesis.

2.2.2.1. Peters-Karunamoorthy two-dimensional finite state induced flow Model

The derivation of Peters-Karunamoorthy 2D finite state induced flow Model is presented in this section. In Ref. [57] nondimensional parameters has been used to derive the differential equations that would define the velocity components. Furthermore, it has been assumed that there is no external bound vorticity, so $\zeta = 0$. Following the procedure of Ref. [57] we have used nondimensional parameters, but then converted the final equations to dimensional form.

The downwash velocity due to the shed wake can be written in operator notation through the definition of a functional operator $Q[ ]$:

$$Q[f(\bar{x})] = -\frac{1}{\pi} \frac{1}{1} \int_{1}^{\infty} \bar{\gamma}_w f(\bar{x}) d\bar{x} \quad (2.41)$$

Eq. (2.41) can be written as below in the elliptical coordinates:

$$Q[g(\eta)] = -\frac{1}{\pi} \frac{1}{0} \int_{0}^{\infty} \bar{\gamma}_w g(\eta) \sinh \eta d\eta \quad (2.42)$$

where $\bar{x} = \sin h\eta$ in the wake. From Eqs. (2.12), (2.13), (2.14), (2.21) and (2.41) one can write:

$$\Gamma = \frac{1}{2} Q[1] \quad (2.43)$$

Moreover, using Eqs. (2.13), (2.30) and (2.42) one can obtain:

$$\bar{\lambda}_0 = \frac{1}{2} Q[1/\sinh \eta]$$

$$\bar{\lambda}_n = Q[e^{-n\eta}/\sinh \eta] \quad (2.44)$$
Application of Eqs. (2.42), (2.13) and (2.20) along with Integration by parts yields the following differential equation for this functional in the elliptical coordinates:

$$
\dot{Q}[g(\eta)] = 2\dot{\Gamma}g(0) + Q\left[\frac{dg/d\eta}{\sinh \eta}\right] 
$$

(2.45)

From Eq. (2.45), a dimensional equation can be formed for the velocity coefficients.

$$
b\dot{\lambda}_0 - (b/2)\dot{\lambda}_2 + u_0\dot{\lambda}_1 = \dot{\Gamma}/\pi
$$

$$
(b/2n)(\dot{\lambda}_{n-1} - \dot{\lambda}_{n+1}) + u_0\dot{\lambda}_n = (1/n\pi)\dot{\Gamma}
$$

(2.46)

However, an additional relationship involving \(\lambda_0\) is needed to complete this set. From Eq. (2.44), one can see that if:

$$
1 \approx \frac{1}{2} \sum_{n=1}^{N} b_n X^n, \quad 0 < X \leq 1
$$

(2.47)

where \(X = e^{-\eta}\), then it follows that:

$$
\lambda_0 \approx \frac{1}{2} \sum_{n=1}^{N} b_n \lambda_n
$$

(2.48)

Reference [57] shows that defining \(b_n\) from the augmented least square approach produces the best approximation to classical aerodynamic theories:

$$
b_n = (-1)^{n-1} \frac{(N+n-1)!}{(N-n-1)! (n!)^2}, \quad 1 < n < N - 1
$$

$$
b_N = (-1)^{(N-1)}
$$

(2.49)
Equations (2.23), (2.21) and (2.34), are combined and differentiated to give the right hand side of Eq. (2.46):

\[
\dot{\Gamma} = 2\pi b \left[ f \left( \tilde{w}_0 - \frac{1}{2} \{ b \}^T \{ \dot{\lambda} \} \right) + \frac{\dot{w}_0 - \dot{\lambda}_0}{2} \right] + \Gamma_0
\]  

(2.50)

where \( \dot{\Gamma}_0 \) is the change in total bound circulation due to stall. Substituting Eq. (2.50) into Eq. (2.46) and using Eq. (2.35), yields:

\[
\{ \dot{\lambda} \} = [A]^{-1} \left[ \{ c \} \{ e \}^T \{ \dot{v}_n + \dot{h}_n \} + \frac{u_0}{b} \{ f \}^T \{ \dot{h}_n \} \right.
\]

\[
\left. + \frac{\dot{u}_0}{b} \{ f \}^T \{ h_n \} + \frac{\Gamma_0}{2b\pi} - \frac{u_0}{b} \{ \dot{\lambda} \} \right]
\]  

(2.51)

which is the form of induced flow equations used in this thesis for two-dimensional flow around a thin airfoil. The vectors and matrices in Eq. (2.51) are defined in Appendix A.

2.2.3. Dynamic Stall Model

As mentioned in sec. 2.1.1.8, in this thesis the Ahaus-Peters model is used for modeling the dynamic stall.

\[
 \frac{b^2}{U^2} \dot{\Gamma}_n + \eta \frac{b}{U} \dot{\Gamma}_n + \omega^2 \Gamma_n = -bU \omega \left[ \Delta C_n + e \frac{b}{U} \frac{d\Delta C_n}{dt} \right]
\]  

(2.52)

Equation (2.52) presents this model, where \( \Delta C_n \) is the difference between the thin airfoil values for the \( n \)th generalized load and the experimental data. Furthermore, \( U = \sqrt{u_0^2 + (v_0 + \dot{h}_0 - \lambda_0)^2} \) is the total flow at the mid-chord. The Ahaus-Peters dynamic stall model is in dimensional form. Since this model uses real time, it is possible to include the unsteady free-stream rigorously. The stall model involves an expansion of \( \eta, \omega \), and \( e \) in two-term series involving \( \Delta C_n \) analogous to Eq. (2.10). That implies that, for each generalized
loading, there are six free parameters in the dynamic stall equation that need to be determined by parameter identification. These free parameters are called the stall parameters.

### 2.2.4. Unified Model

The schematic of the unified airloads model is shown in Fig. 1.12. The inputs of the model are the airfoil motions. As mentioned earlier in section 1.4, airfoil motions are transferred into a generalized set of coordinates, \( h_n \). This transformation is done by means of a transformation matrix, \([T]\). Figure 2.2 shows a rigid body airfoil pitching about an axis at \( x = ab \). This is the case that has been studied in this thesis. For this case, the \( h_n \) expansions of the airloads theory can be obtained as:

\[
\begin{align*}
h_0 &= h - b\alpha \\
h_1 &= b\alpha \\
h_n &= 0 \quad n \geq 2
\end{align*}
\]

When camber is present, the appropriate \( h_2 \) can be included. Equation (2.53) can be cast in a matrix form in order to express the total \( h_n \) in terms of the airfoil degree of freedom, \( \alpha \):

\[
\{h_n\} = [T]\begin{bmatrix} h \\ \alpha \end{bmatrix}
\]

(2.54)

where:

\[
[T] = \begin{bmatrix} 1 & -ba \\ 0 & b \\ 0 & 0 \\ \vdots & \vdots \end{bmatrix}
\]

(2.55)

The generalized coordinates are combined with the flow field geometry \((u_0, v_0, v_1)\) and induced flow velocities from the shed wake to provide the boundary condition for the linear
Having the boundary conditions, the linear airloads theory computes the desired generalized loadings; and then each generalized loading, \( L_n \), is corrected with its own type stall correction, \( \Gamma_n \), obtained from Eq. (2.52).

\[
L_n = L_{n(\text{linear})} + \rho U \Gamma_n
\]  

(2.56)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.2.png}
\caption{Solid Airfoil pitching about an axis at \( x = ab \)}
\end{figure}

In order to obtain good correlation with experimental data, static correction factors must be added to the theory to account for the effects of thickness, viscosity, and compressibility. For the purpose of this thesis, we consider the illustrative example of Fig 2.2 once more. From thin airfoil theory, the lift curve slope of this thin airfoil is equal to \( 2\pi \). However, extensive wind tunnel data obtained from analysis of the NACA 0012 airfoil by Ref. [59], reveals that the lift curve slope for the thin airfoil is a function of the Reynolds number as well as the Mach number. At a range of Reynolds number between \( 2 \times 10^6 \) and \( 2 \times 10^7 \), the following approximation has been made for the lift curve slope per radian [59]:
\[
C_{l_w} = \frac{5.8728 + 0.2997 \log \left( \frac{\text{Re}}{10^6} \right)}{\sqrt{1 - M^2}}
\]  

(2.57)

Nevertheless, if static experimental \(C_L\) data are available at the desired Mach and Reynolds numbers, the best practice to determine \(C_{l\alpha}\) is to use Eq. (2.58) and find the values of \(C_{l\alpha}\) and \(\alpha_{0L}\) that give the best fit of the experimental data.

\[
C_{L(\text{exp})} = C_{l_w} \sin (\alpha - \alpha_{0L})
\]

(2.58)

In order to include this correction in the theory, the lift components obtained from the linear airfoil theory must be multiplied by a static lift correction factor, \(f_L = C_{l\alpha}/2\pi\). In conclusion, the total corrected lift for the airfoil of Fig 2.2 can be obtained from Eq. (2.61).

\[
L_x = -f_L D + \rho \left( v_0 + \dot{h}_0 - \lambda_0 \right) \Gamma_L
\]

(2.59)

\[
L_y = -f_L L_v + \rho u_0 \Gamma_L
\]

(2.60)

\[
L = \sqrt{L_x^2 + L_y^2}
\]

(2.61)

where \(D\) and \(L_0\) are the complete unsteady values from Eqs. (2.40) and (2.37), respectively. Note that the reason for the negative sign in front of \(L_0\) term in Eq. (2.60) is that \(L_0\) obtained from Peters-Johnson state-space airloads theory corresponds to the plunging motion and the negative plunging load means a positive lift and vice versa. Similarly, in Eq. (2.59), \(D\) is written as \(-D\) in the negative \(x\) direction because induced drag is propulsive for positive \(\Gamma_L\) and positive \((v_0 + \dot{h}_0 - \lambda_0)\).

As a final step, the corrected total loads (linear loads plus the correction due to the stall and corrected by means of the correction factors) are fed into the induced flow model and this completes the loop.
Chapter 3

Modeling the Secondary Lift Peak

As mentioned in section 1.3.1, an airfoil undergoing dynamic stall sometimes experiences the double-dynamic-stall phenomenon, which appears as a secondary lift peak after the lift has begun to decrease. The Ahaus-Peters unified airloads model, as discussed in section 2.2, cannot capture the double-dynamic-stall phenomenon. In order to make the model capable of modeling the secondary lift peak, a modified dynamic stall model is used within the context of unified airloads model. The block-diagram presentation of the modified Ahaus-Peters airloads model was previously shown in Fig. 1.13. This chapter explains the different components of the modified dynamic stall model. The modified dynamic stall model is developed based on the results obtained from the analysis of a NACA0012 airfoil at $M = 0.1$ and is validated by comparison with the experimental lift data for a VR-12 airfoil at $M = 0.2, 0.3$ and $0.4$.

3.1. Modified Stall Model

The original Ahaus-Peters dynamic stall model—expressed by Eq. (2.52)—is a second-order differential equation that gives the loss in blade circulation due to the primary dynamic stall process. In particular, the loss in the circulation due to dynamic stall is expressed in a nondimensional form ($\Gamma_L$) that can be obtained from Eq. (3.1).

$$\ddot{\Gamma}_L + \eta \dot{\Gamma}_L + \omega^2 \Gamma_L = -\omega^2 \left( \Delta C_L + e \Delta C_L^* \right)$$ (3.1)

This dynamic equation imparts a time delay and overshoot to the loss of circulation due to the lift stall. Figure 3.1 compares the lift coefficient results obtained from the Ahaus-Peters dynamic stall model with the experimental data for a NACA 0012 airfoil that is oscillating in
pitch with a mean angle of 15° and an oscillation amplitude of 10°, where the secondary peaks in
the lift data occur. It is obvious that the model in not capable of capturing the secondary stall event. In fact, none of the current empirical models is capable of modeling the secondary lift stall peak. The purpose of this section is to modify the current Ahaus-Peters model so that it can also capture this secondary peak in the lift data.

A study of double dynamic stall on the NACA 0012 Airfoil data of Ref. [8] at $M = 0.10$
shows that the initial lift peak is followed by a damped oscillation in lift at all reduced
frequencies over the range $0.02 < k < 0.25$. In order to generate the secondary lift peak and its
decay we add another second-order circulation equation that is driven by a simple pulse. This
introduces a second dynamic stall parameter to the model as follows.

$$\ddot{\Gamma}_s + 2\zeta_s \omega_s \dot{\Gamma}_s + \omega_s^2 \Gamma_s = \omega_s^2 f_s(\tau) = F_s(\tau)$$

(3.2)

The forcing function is assumed to be a constant pulse of duration $\Delta \tau$. It is known that the
physical basis of the secondary lift peaks in stall is the attaching, detachment, and reattachment
of vortices to the airfoil. This pulse is meant to represent whatever physical behavior in the
boundary layer causes this series of attachments and detachments. The pulse is written as:

$$f_s(\tau) = h_s [u(\tau_i) - u(\tau_c)]; \quad \tau_c = \tau_i + \Delta \tau$$

(3.3)

where $u(\cdot)$ is the step function. It is hoped that, by parameter identification, the parameters of
the model can be determined based on experimental data.

The parameters in Eq. (3.1) ($\eta$, $\omega$, and $e$) are expanded in a two-term series involving $\Delta C_L$
alogous to Eq. (2.10). That implies that the model has six free parameters. The secondary
stall model involves five parameters: $\zeta_s$, $\omega_s$, $h_s$, $\tau_i$, and $\Delta \tau$. The combination of the two models in
Eqs. (3.1) and (3.2) offers the possibility to model the complete dynamic stall process in a time-
domain, low-order model by proper choice of all of the parameters.
Figure 3.1. Ahaus-Peters stall model vs. the experimental lift data for NACA 0012 airfoil at $M = 0.1$ and $k = 0.02, 0.5, 0.15$
3.1.1. Determination of Parameters

Dynamic stall data for the NACA 0012 airfoil at $M = 0.1$ under the following forcing function can be found in Ref. [8].

$$\theta = \theta_0 + \theta_1 \sin(k \tau); \quad \theta_0 = 15^\circ, \quad \theta_1 = 10^\circ, \quad k = 0.004, 0.02, 0.05, 0.15, 0.25 \quad (3.4)$$

The data at $k = 0.004$ are virtually quasi-static due to the low frequency. The data at $k = 0.25$ are dominated by the primary stall. The data at $k = 0.02$, $k = 0.05$ and $k = 0.15$ seem to have the richest information. Therefore, it was decided to train our model based on these three intermediate frequencies. After the process is over, we can go back and see how the model behaves for the other cases of $k = 0.004$ and 0.25. In general, for this approach, one should use the best data available for the training (i.e., for the choosing of parameters). The data at these conditions are used to find the parameters for the above form of the stall model, Eqs. (3.2) and (3.3), in order to understand if that model is viable. In the following, we describe the procedure for initially finding the parameters for each test case. After that, we will decide how to represent those parameters in a form that is appropriate for real-time flight simulations in terms of a general form independent of reduced frequency.

First, a smooth curve is drawn through the $C_L$ loops to give the baseline (Fig. 3.2). The smooth curve is only an intermediate step, and results of the final model are only weakly dependent on the choice of this curve, but the smooth curve makes the model identification easier. The smooth curve is then subtracted from the experimental data to give the secondary stall peak and its decay. This residual curve is plotted versus reduced time, Fig. 3.3. Based on that process, it was determined that—at all of the reduced frequencies in Ref. 4—the residual is a simple damped oscillation with damping ratio $\zeta_s = 0.2055$. Based on this, we were able to do parameter identification to best match the transient behavior at each reduced frequency. The unknown parameters are the natural frequency $\omega_s$; the height of the pulse, $h_s$; the start of the pulse, $\alpha_p$; and the width of the pulse, $\Delta \tau$. 
Figure 3.2. Smooth curves and the experimental lift data for NACA 0012 airfoil at $M = 0.1$ and $k = 0.02, 0.5, 0.15$
Figure 3.3. Secondary Lift stall $\Gamma_s$ versus reduced time
The identified pulse parameters at these three reduced frequencies are given in Table 3.1, where \( h_s \Delta \tau \) denote the resultant area under the pulse (which defines the end time).

<table>
<thead>
<tr>
<th>Pulse Parameter</th>
<th>Reduced Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>( \zeta_s )</td>
<td>0.2055</td>
</tr>
<tr>
<td>( \omega_s )</td>
<td>0.4897</td>
</tr>
<tr>
<td>( h_s )</td>
<td>0.160</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>17.3°</td>
</tr>
<tr>
<td>( \Delta \tau )</td>
<td>6.56</td>
</tr>
<tr>
<td>( h_s \Delta \tau )</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 3.1. Numerical values of the pulse parameters for NACA0012 airfoil

\((M = 0.1 \text{ and } k = 0.02, 0.05, 0.15)\)

Figure 3.3 shows the generating pulses and the modeled secondary stall circulation as compared with the experimental data for these three reduced frequencies. Note that the first, second, and third peaks are all well-modeled by the present approach. The experimental result at \( k = 0.02 \) seems to rise rather slowly at first, but this is just an artifact of how the smooth curve is sketched. The main point of the figure is that the dynamic behavior of the secondary stall phenomenon is well modeled by this simple, second-order system with a pulse input.
3.1.1.1. Determining Constants for Secondary Lift Stall

The next step is to determine a general function to give these physical stall parameters in terms of instantaneous properties of the flow and determining constants that are universal. These will be developed from the parameters for the generated pulses of the NACA 0012 airfoil at \( M = 0.1 \) and \( k = 0.02, 0.05 \) and \( 0.15 \) in order to formulate general expressions for the height, width and the undamped frequency of the pulse. It is assumed that the angular rate at the point at which the angle reaches static stall \( \dot{\alpha}_{ss} \) might be an indication of the initial angle of attack at which the pulse should begin. A fit of that data yields an equation for the onset of the pulse in terms of 1.) a critical angular rate \( \dot{\alpha}_c \) (the smallest angular rate at which the pulse can begin), 2.) a reference angle of attack \( \alpha_c \) (the smallest angle of attack at which the pulse can begin), and 3.) \( K \) (the coefficient of change of critical angle with angular rate).

\[
\begin{align*}
\dot{\alpha}_{ss} < \dot{\alpha}_c & \quad \text{no pulse occurs} \quad (3.5) \\
\dot{\alpha}_{ss} \geq \dot{\alpha}_c & \quad \alpha_p = \alpha_c + K \left( \alpha_{ss} - \dot{\alpha}_c \right) \left( \dot{\alpha}_{ss} - \alpha_c \right)^{1/2}
\end{align*}
\]

For the sake of convenience, angles are expressed in terms of degrees and angular rates are expressed in terms of degrees per unit of reduced time which is nondimensional. Thus, the units of \( K \) are \((deg)^{1/2}\).

A fit of the data for the NACA 0012 airfoil indicates that both the height of the normalized pulse \( h_s \) and the area under the pulse \( h_s \Delta \tau \) are linear in the onset angle.

\[
\begin{align*}
h_s &= \left[ h_i + h_e \left( \alpha_p - \alpha_c \right) \right] u \left( \alpha_{ss} - \dot{\alpha}_c \right) \left( \alpha_{ss} - \alpha_c \right) \left( \dot{\alpha}_{ss} - \alpha_c \right) \left( \dot{\alpha}_{ss} - \alpha_c \right)^{1/2} \quad (3.6) \\
h_s \Delta \tau &= p_s = \left[ p_i + p_e \left( \alpha_p - \alpha_c \right) \right] u \left( \alpha_{ss} - \dot{\alpha}_c \right) \left( \alpha_{ss} - \alpha_c \right) \left( \dot{\alpha}_{ss} - \alpha_c \right) \left( \dot{\alpha}_{ss} - \alpha_c \right)^{1/2} \quad (3.7)
\end{align*}
\]

where \( u() \) is the step function. From the context of Eqs. (3.6) and (3.7), one can see that the parameters \( h_i, h_e, p_i \) and \( p_e \) are the slope and intercept of the linear curves that do a good job of fitting the experimental data. From these equations arise the formulas both for the length of the pulse and for the undamped natural frequency to be used in Eq.(3.2).
\[ \Delta \tau = \frac{p_s}{h_s} \]  
\[ \omega_s = \frac{\pi}{\Delta \tau \left(1 - \xi_s^2\right)^{1/2}} \]  

The numerical values of the parameters in Eqs. (3.5)-(3.9) for the NACA0012 airfoil at \( M = 0.1 \) can be found in Table 3.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NACA 0012</th>
<th>VR-12</th>
<th>Mach Number</th>
<th>Mach Number</th>
<th>( M = 0.1 )</th>
<th>( M = 0.2 )</th>
<th>( M = 0.3 )</th>
<th>( M = 0.4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_0 )</td>
<td>0.248</td>
<td>0.393</td>
<td>0.333</td>
<td>0.584</td>
<td>( \omega_2 )</td>
<td>0.178</td>
<td>0.304</td>
<td>0.094</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>0.391</td>
<td>0.786</td>
<td>0.489</td>
<td>1.862</td>
<td>( \eta_2 )</td>
<td>0.659</td>
<td>0.980</td>
<td>0.141</td>
</tr>
<tr>
<td>( e_0 )</td>
<td>-1.594</td>
<td>-0.430</td>
<td>-2.625</td>
<td>-2.745</td>
<td>( e_2 )</td>
<td>1.042</td>
<td>-0.481</td>
<td>3.233</td>
</tr>
<tr>
<td>( \alpha_{ss} )</td>
<td>13.2°</td>
<td>14.27°</td>
<td>12.7°</td>
<td>10.51°</td>
<td>( \zeta_s )</td>
<td>0.2055</td>
<td>0.1789</td>
<td>0.2955</td>
</tr>
<tr>
<td>( h_i )</td>
<td>0.042</td>
<td>0.177</td>
<td>0.150</td>
<td>0.100</td>
<td>( h_c )</td>
<td>0.061 deg(^{-1})</td>
<td>0.100 deg(^{-1})</td>
<td>0.116 deg(^{-1})</td>
</tr>
<tr>
<td>( p_l )</td>
<td>0.747</td>
<td>1.053</td>
<td>1.420</td>
<td>0.839</td>
<td>( p_e )</td>
<td>0.157 deg(^{-1})</td>
<td>0.284 deg(^{-1})</td>
<td>0.138 deg(^{-1})</td>
</tr>
<tr>
<td>( \alpha_c )</td>
<td>0.104°</td>
<td>0.415°</td>
<td>0.352°</td>
<td>0.386°</td>
<td>( \alpha_{end} )</td>
<td>25.0°</td>
<td>20.0°</td>
<td>20.0°</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>0.500°</td>
<td>0.454°</td>
<td>0.963°</td>
<td>0.496°</td>
<td>( \alpha_2 )</td>
<td>1.500°</td>
<td>0.860°</td>
<td>2.074°</td>
</tr>
</tbody>
</table>

**Table 3.2.** Numerical values of stall, pulse and hysteresis parameters for NACA 0012 airfoil at \( M = 0.1 \) and for VR-12 airfoil at \( M = 0.2, 0.3 \) and 0.4
3.1.1.2. Determining Constants for Primary Lift Stall

The black solid curves in Fig. 3.4 present the lift coefficient smooth curves at the three reduced frequencies \( k = 0.02, 0.05, \) and 0.15. These curves have the oscillations removed, leaving only the primary stall behavior. (The removed oscillations are the oscillations fit by the second order system with a double pulse mentioned earlier.) The dynamic stall model in Eq. (3.1) now needs to be trained to match these smooth curves (without the oscillations). The problem is considered as an optimization of the six free parameters \((\omega_0, \omega_2, \eta_0, \eta_2, e_0, e_2)\) with side constraints on each of the parameters. The side constraints are applied to ensure that the dynamic stall differential equation is stable for the obtained optimum values of the parameters (Table 3.3).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_0)</td>
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<td>+(\infty)</td>
</tr>
<tr>
<td>(\omega_2)</td>
<td>(-\omega_0/\left(\Delta C_L(\text{max})\right)^2)</td>
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</tr>
<tr>
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<td>0</td>
<td>+(\infty)</td>
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<td>(\eta_2)</td>
<td>(-\eta_0/\left(\Delta C_L(\text{max})\right)^2)</td>
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</tr>
<tr>
<td>(e_0)</td>
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<td>+(\infty)</td>
</tr>
<tr>
<td>(e_2)</td>
<td>(-\infty)</td>
<td>+(\infty)</td>
</tr>
</tbody>
</table>

Table 3.3. Side Constraints applied on the stall parameters \((\Delta C_L(\text{max}) \approx 1.7515)\)

The dynamic stall differential equation is solved for the nondimensional circulation loss in the blade due to the primary dynamic stall of the lift \((\bar{\Gamma}_L)\) numerically using the ode45 solver in MATLAB. The value of \(\bar{\Gamma}_L\) is added to the linear lift coefficient value \((C_L\text{linear})\) as obtained from Peters-Johnson state-space airloads theory and corrected by means of the static lift correction factor \((f_L)\), discussed in section 2.2.4, to give the total lift coefficient value \((C_L)\). The fitness function is considered to be a sum of the error between the experimental data and the results obtained from the mathematical model at each of the three reduced frequencies.
Figure 3.4. NACA 0012 fit of the smooth lift coefficient curves which includes the shift in angle of attack ($M = 0.1$)
In Eqs. (3.10) and (3.11), $E_k$ is the normalized root-mean-square error between the experimental data and the numerical solution at each reduced frequency, $C_L(\text{exp})$ is the experimental lift data, $C_L(\text{num})$ is the numerical solution obtained from the mathematical model, $N$ is the number of points at which the error is evaluated and $F$ is the fitness function. In Eq. (3.11), the error norms are evaluated at 0.1 intervals from 0° to 25°. Genetic Algorithm toolbox in MATLAB is used to find the optimum set of parameters that minimize the fitness function. Table 3.4 shows the optimization options setup in the genetic algorithm toolbox.
### MATLAB Genetic Algorithm Toolbox Setup

<table>
<thead>
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<td>Crossover Fraction</td>
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</tbody>
</table>

**Table 3.4.** Optimization options setup in *MATLAB Genetic Algorithm* toolbox
3.1.1.3. Identification of Constants for Static Hysteresis

The data for \( k = 0.004 \) (which is virtually quasi-static) shows a hysteresis of the lift curve, which is typical of quasi-static lift data into stall. During the training by optimization at higher values of \( k \) (0.02, 0.05, and 0.15) it was found that inclusion of this same type of hysteresis as found in the static data (i.e., hysteresis independent of velocity) greatly improves the fidelity of the model. To capitalize on this, the following representation of velocity-independent hysteresis was developed to improve correlation.

Beginning when the angle of attack reaches the stall angle, the effective angle of attack at which \( \Delta C_L \) is computed (from the static data with no hysteresis) is shifted according to the following equation.

\[
\alpha_{\text{eff}} = \alpha + \Delta \alpha \left( \frac{\alpha - \alpha_{ss}}{\alpha_{\text{end}} - \alpha_{ss}} \right)^2; \quad \alpha_{ss} \leq \alpha < \alpha_{\text{end}}, \quad \frac{d\alpha}{d\tau} \geq 0
\]

\[
\alpha_{\text{eff}} = \alpha + \Delta \alpha \left( \frac{\alpha - \alpha_{ss}}{\alpha_{\text{end}} - \alpha_{ss}} \right); \quad \alpha_{ss} \leq \alpha < \alpha_{\text{end}}, \quad \frac{d\alpha}{d\tau} < 0 \quad (3.12)
\]

\[
\alpha_{\text{eff}} = \alpha + \Delta \alpha; \quad \alpha \geq \alpha_{\text{end}}
\]

where \( \alpha_{\text{end}} \) is the angle of attack at which the hysteresis ends. For the NACA 0012 airfoil at \( M = 0.1, \alpha_{\text{end}} = 25^\circ \). At lower frequencies, the optimum maximum shift (as determined by parameter identification) is \( \Delta \alpha_0 \). However, the identified optimum static hysteresis was found to decrease at higher reduced frequencies. The following schedule gives the maximum shift as a function of the time rate of change of angle of attack at stall onset.

\[
\Delta \alpha = \Delta \alpha_0 = C \left( \alpha_2 - \alpha_1 \right); \quad 0 \leq \alpha_{ss} < \alpha_1
\]

\[
\Delta \alpha = C \left( \alpha_2 - \alpha_{ss} \right); \quad \alpha_1 \leq \alpha_{ss} < \alpha_2 \quad (3.13)
\]

\[
\Delta \alpha = 0'; \quad \alpha_2 \leq \alpha_{ss}
\]
where $C$, $\dot{\alpha}_1$ and $\dot{\alpha}_2$ are the coefficients of change, lower critical angular rate and the upper critical angular rate for hysteresis respectively, as shown in Table 3.2.

In Ref. [23], for a NACA 0012 airfoil at $M = 0.3$, a fixed shift of $0.9^\circ$ was used to compute $\Delta C_L$ for angles of attack going up to $15^\circ$. That shift was similarly found to improve correlation, especially on the return stroke of the airfoil. Equations (3.12) and (3.13) give a variable shift that is smaller for increasing alpha than it is for decreasing alpha. For a NACA 0012 airfoil at $M = 0.1$, the shift at $\alpha = 15^\circ$ for decreasing alpha (at lower frequencies) is only $0.3^\circ$, which is smaller than the $0.9^\circ$ used by Ahaus, Ref. [23]. The shift at $25^\circ$ (for lower frequencies) is $1.8^\circ$, which is double that used by Ahaus. The important point is that use of the different paths for positive and negative pitch rates creates a hysteresis loop that better matches the data.

### 3.1.1.4. Final Fit of Smooth Curves

Training of the model by the genetic algorithm gives the following stall parameters for a NACA0012 airfoil at $M = 0.1$.

$$\omega = 0.248 + 0.178(\Delta C_L)^2$$

$$\eta = 0.391 + 0.659(\Delta C_L)^2$$ (3.14)

$$e = -1.594 + 1.042(\Delta C_L)^2$$

Figure 3.4 shows the good fit of the smooth curves with this stall model, which includes the shift in angle of attack due to static hysteresis.

From the analysis mentioned in section 3.1.1.1, one could program a time-domain code that would model the second stall peak and its decay. Whenever the angle of attack reaches the stall angle of attack ($\alpha_{ss}$), the code would check to make sure $\dot{\alpha}_s > \dot{\alpha}_c$. If so, then if the angle of attack continued to increase beyond the critical angle ($\alpha_c$), the program would create a pulse $f_s(\tau)$—beginning at $\alpha_p$—and continuing for $\Delta \tau$ radians of reduced time. That pulse would drive Eq. (3.2) to give the effect of the second stall peak on lift. When added to the circulation loss due to the original stall equation, this gives a unified, reduced-order model for stall with a secondary lift peak.
Figure 3.5 shows results with the total stall model (i.e., with hysteresis and the double-pulse model) as compared to the experimental data of the training set, $k = 0.02, 0.05,$ and $0.15$. The results are excellent with error norms of each case of $E = 0.0578, 0.0617$ and $0.0829$, respectively. In comparison, the error norms with the model that does not have the secondary-stall peak model (Fig. 3.1) are, $E = 0.0793$, $E = 0.1282$, and $E = 0.1406$. Since no other method includes the secondary peak correction, one can expect a similar improvement in error norm over all existing methods. Thus, a major improvement has been achieved. It is also interesting to compare the method on data at reduced frequencies outside the training set. Figure 3.6 shows results at $k = 0.004$ and $k = 0.25$, frequencies above and below the training range. Once again, the error norms are excellent and are $E = 0.0825$ and $E = 0.097$, respectively, similar to the norms in the training range. This implies that the method is very robust.

On first glance, one might think that—since we have now doubled the number of states in the model by addition of the secondary circulation—the new model will be twice as costly to run in real time as was the old. However, it should be noted that the secondary stall model, Eq. (3.2), can be solved in closed form off-line, before any simulation begins. This is because the forcing function is always a finite pulse, which is the sum of two, staggered step functions. Since step response is well known in closed form for a second-order system, it follows that the necessity for carrying more states is eliminated. The secondary stall effects can be added to the response in closed form whenever the critical angular rate is reached at the static stall angle.
Figure 3.5. NACA 0012 lift coefficient data at the three reduced frequencies $k = 0.02, 0.05$ and $0.15$ ($M = 0.1$)
Figure 3.6. NACA 0012 lift coefficient data at $k = 0.004$ and $k = 0.25$ ($M = 0.1$), not used in the training
3.1.2. Physical Parameters and Determining Constants

It is important to distinguish here between the physical parameters of the stall model (which can change with angle of attack and rate of angle attack) and the determining constants of the model. For example, in the basic stall model of Eq. (3.1) there are three physical parameters for lift: $\eta$, $\omega$, and $e$. These parameters, however, depend upon angle of attack, so they cannot be fundamental determining constants of the model. Thus, they are expanded in a two-term series involving $\Delta C_L$ analogous to Eq. (2.10) which represents these three parameters in terms of six determining constants: $\eta_0$, $\omega_0$, $e_0$, $\eta_2$, $\omega_2$, and $e_2$. (Note that $\alpha_{ss}$ is not a constant of the model but is the static stall angle from the static data).

There are another three similar physical parameters each for pitch moment and drag (making nine total parameters). However, earlier work of Ahaus and Peters (Ref. [23]) has demonstrated (as did the work of ONERA before us), that the moment and drag parameters can often be set equal to the lift parameters with reasonable results. Thus, in that case, the nine physical parameters of the primary stall model are represented by only six determining constants. For any airfoil at any Mach number, these six constants are fixed and are used for all angles, all reduced frequencies, and all angle-of-attack time histories. They are given in Table 3.2 for the airfoils and Mach numbers used here.

In terms of the secondary stall model, a similar situation occurs. Table 3.1 gives six physical parameters that determine the secondary stall model of Eqs. (3.2) and (3.3). These six are identified in Table 3.1 at three reduced frequencies to have an idea of their typical values. However, these six are not the determining constants of the model because the determining constants cannot depend on angle of attack or reduced frequency. Rather, Eqs. (3.5-3.9) show how these six physical parameters can be expressed in terms of eight determining constants. The eight determining constants, as identified for the NACA 0012 airfoil, are given in Table 3.2.

These are the constants of the Pulse Parameters and are: $\zeta$, $h_i$, $h_e$, $p_i$, $K$, $p_e$, $\alpha_c$ and $\alpha_{c\ast}$. The constants do not depend on reduced frequency and are valid for all cases. They were identified based on minimizing the total error between data and model of all three reduced frequencies simultaneously, $k = 0.02, 0.05$, and $0.15$.

The formulas in Eqs. (3.5-3.9) show precisely how the six secondary stall parameters are found in terms of the eight constants. These, then, are valid at all reduced frequencies, all angles
of attack, and all angle-of-attack time histories. It follows that any airfoil at any Mach number will only have a single set of stall parameters, as indicated in Table 3.2. The same is true for the five static hysteresis parameters. There is only one set of these for any airfoil at any Mach number, and these remain unchanged for any angle-of-attack time history at any reduced frequencies.

The determining constants for the stall parameters and static hysteresis used here have been shown in Ref. [23] to be equally valid for pitching moment and drag, also. However, in this thesis we have only looked at the secondary stall model with respect to lift correlations. In continuing work, we will look at the effect of the secondary pulse on pitching moment and drag as well. We are hoping that, as with the primary stall model and the hysteresis model, we will need few (if any) additional parameters to model moment and drag. Thus, we anticipate that about 19 to 25 total independent determining constants will be required for lift, pitching moment, and drag.

3.2. Validation of The modified Stall Model on VR-12 Airfoil

In order to validate the method further, the offered model was used to generate the secondary lift stall peaks of the VR-12 airfoil at three different Mach numbers of \( M = 0.2, 0.3 \) and 0.4. The experimental data for this airfoil are given in Ref. [60]. The obtained values of pulse, hysteresis and stall parameters are shown in Table 3.2.

Having the values of the pulse, hysteresis and stall parameters at \( M = 0.2, 0.3 \) and 0.4, one can find any of these parameters at any desired Mach number in the range of 0.1 and 0.3 by expanding the corresponding parameter at these three Mach numbers and finding the expansion factors \( a_1, a_2 \) and \( a_3 \).

\[
\text{Pulse Parameter} = a_1 + a_2 M + a_3 M^2
\]  

(3.15)

where \( a_1 = 18.75^\circ, a_2 = -18.75^\circ \) and \( a_3 = 0^\circ \). For example, the value of critical angle at any desired Mach number can be obtained from Eq. (3.16).

\[
\alpha_c = 18.75^\circ \left(1 - M\right)
\]  

(3.16)
Figures 3.7-3.9 show the resultant lift coefficient data from the model for Boeing VR-12 airfoil at two Reduced frequencies of $k = 0.05$ and 0.1 and $M = 0.2, 0.3$ and 0.4, as compared to the experimental data. The resultant fit of the trained model to the airfoil behavior is remarkable. The same framework that gave good correlation for the NACA 0012 now gives good correlation for the VR-12. Almost every detail of the second and third peaks is captured by the model. Furthermore, the training was not difficult; and the values of the various parameters for the VR-12 (as seen in Table 3.2,) are directly in line with the values for the NACA 0012.
Figure 3.7. VR-12 airfoil lift coefficient data at the two reduced frequencies $k = 0.05$ and $0.1 (M = 0.2)$
Figure 3.8. VR-12 airfoil lift coefficient data at the two reduced frequencies $k = 0.05$ and $0.1$ ($M = 0.3$)
Figure 3.9. VR-12 airfoil lift coefficient data at the two reduced frequencies $k = 0.05$ and $0.1$ ($M = 0.4$)
It should be noted that the correlation on the downstroke of each cycle is not as good as that for the NACA 0012. This lack of correlation was also noted by Ahaus, Ref. [23]. It is not a result of the new second-stall model, but is a feature of the original Ahaus-Peters model. One very simple change to the Ahaus-Peters model that might be able to correct this would be to allow the stall parameters for the primary model to be different for the upstroke than for the downstroke. This could easily be accomplished by a $\text{sgn} (\dot{\alpha})$ in the parameters with no changes to the secondary-stall model. Figure 3.10 shows the result obtained from using a different stall parameters for the downstroke of the lift curve of VR-12 airfoil at $M = 0.4$. As one can see, a good agreement with the experimental data is obtained on both upstroke and downstroke of the lift cycle. Equation (3.17) shows the stall parameters for the downstroke.

$$\omega = 1.084 + 2.975 (\Delta C_L)^2$$
$$\eta = 2.925 + (\Delta C_L)^2$$
$$e = 0.692 - 2.806 (\Delta C_L)^2$$

Equation (3.17) shows the stall parameters for the downstroke.

The model presented in this chapter, is the only empirically based method presently available that can model the effect of the secondary stall peaks. For cases in which the secondary peaks are not important, the secondary equations can be eliminated and the model reduced back to the Ahaus model. Similarly, for airfoils in which the static hysteresis is not important, that part can be eliminated as well. The biggest drawback of the present methodology for identifying the determining constants of the secondary stall model is the requirement for drawing the smooth curves through the data, which implies a certain amount of art. On the other hand, now that the form of the equations for stall is known, it may be possible to identify the determining constants directly through parameter identification as has been done for the primary stall constants.
Figure 3.10. VR-12 airfoil lift coefficient data at the two reduced frequencies $k = 0.05$ and $0.1$, using two different set of stall parameters for the upstroke and downstroke of the lift curve ($M = 0.4$)
Chapter 4

Effect of Yawed Flow on Dynamic Stall

Two-dimensional stall characteristics are used in most dynamic stall models—despite the fact that the flow around the rotor is three-dimensional. The rationale for this is that limited, three-dimensional studies indicate that 3-D stall is qualitatively similar to stall of a two-dimensional airfoil. The purpose of the next three chapters is to present a dynamic stall model that includes some of the predominant three-dimensional effects so that the model can be more than just qualitatively accurate. To this end, the Ahaus-Peters modified dynamic stall model is used as a framework and is modified to account for the three-dimensional effects. This chapter presents the effects of yawed rotational motion on the stalled airloads.

4.1. Modeling Yawed Flow

Figure 4.1 shows a helicopter during forward flight. The blades rotate with angular velocity of $\Omega$ and the free-stream velocity is considered to be $V_\infty$. As a result, each blade station sees a normal component of velocity, $U_T$, and a radial component, $U_R$. The normal component is composed of two contributions. One is due to the blade rotation $\Omega r$, and the other is due to the forward speed of the helicopter, $V_\infty \sin \psi$. Radial velocity results from the forward speed of the helicopter.

\[
U_T = \Omega r + V_\infty \sin \psi \quad (4.1)
\]
\[
U_R = V_\infty \cos \psi \quad (4.2)
\]

The yaw (sweep) angle, due to radial velocity is defined in terms of $U_T$ and $U_R$ as:
\[ \Lambda(\bar{r}, \psi) = \tan^{-1}\left( \frac{U_r}{U_T} \right) = \tan^{-1}\left( \frac{\frac{V_{\infty} \cos \psi}{\Omega R}}{\Omega R \left( \frac{r}{R} + \frac{V_{\infty}}{\Omega R} \sin \psi \right)} \right) = \tan^{-1}\left( \frac{\mu \cos \psi}{\bar{r} + \mu \sin \psi} \right) \]  

(4.3)

where \( \mu = \frac{V_{\infty}}{\Omega R} \), and it is called the advance ratio.

\[ \mu = \frac{V_{\infty}}{\Omega R} \]

Figure 4.1. Velocity components relative to a blade section of a helicopter in forward flight

Figure 4.2 (adopted from Ref. [2]) shows the Iso-sweep angles over the rotor disk in forward flight for two different advance ratios of \( \mu = 0.05 \) and \( \mu = 0.30 \). As one can see, the helicopter rotor blade is unswept in only two positions of 90° and 270° and the blade section undergoes wide variation in sweep angle while moving through the azimuth plane. Furthermore, as the advance ratio (i.e. the forward flight speed) increases, the value of sweep angle becomes large enough to create significant effects on the aerodynamic forces and consequently the dynamic stall characteristics. In conventional rotor analysis, it is assumed that the radial component of the velocity does not affect the sectional loads; and the airloads are calculated considering the velocities in a plane containing the airfoil chord. However, research results—such as that of Ref. [61]—show that, although this assumption is quite acceptable in the low angle-of-attack region, in the high angle-of-attack region a much higher lift coefficient is obtained for larger sweep
angles. Therefore, conventional, unswept theories may not be sufficient for the rotor blade aerodynamic analysis in a highly complex three dimensional flow field.

![Iso-sweep angles over the rotor disk in forward flight for $\mu = 0.05$ and $\mu = 0.30$](image)

**Figure 4.2.** Iso-sweep angles over the rotor disk in forward flight for $\mu = 0.05$ and $\mu = 0.30$ [2]

The effect of sweep due to rotational motion has been studied both experimentally and computationally by several investigators. The earlier experimental studies of the radial flow effects were conducted by Dannenberg (Ref. [62]) and Purser and Spearman (Ref. [61]) using a two-dimensional wing. Harris in Ref. [63] also studied the problem by reducing three-dimensional data of a rectangular wing with the aspect ratio of 6.0 and discovered that a unit span of a yawed infinite wing operates at higher section lift coefficients than does the same unit span unyawed wing. Reviewing Refs. [62] and [61] data for two-dimensional wing, Harris, *et al* (Ref. [64]) suggested Eq. (4.4) to capture the increasing trend of the section maximum lift coefficient versus the yaw angle.

$$
C_{L_{(\text{max})}}(\Lambda) = \frac{C_{L_{(\text{max})}}(\Lambda = 0)}{\cos \Lambda} \tag{4.4}
$$

Gormont (Ref. [65]) used the Harris, *et al* approximation (Eq. (4.4)) and offered Eq. (4.5) to adjust the two dimensional static $C_{L} - \alpha$ data to the required yaw angle.
In the nonstalled regime, this formulation results in higher lift curve slopes for angles of attack. However, since the yawed flow data do not show any increase in the lift curve slope in the linear range, he limited the formulation to:

\[ C_l(\Lambda) \leq \left( \frac{dC_l}{d\alpha} \right)_{\text{Linear}} \cos \Lambda \]  

\[ (4.6) \]

Figure 4.3 compares the predicted yawed lift from Eqs. (4.5) and (4.6) with the experimental measurements of Ref. [61] and also Refs. [66] and [67] for NACA 0012 airfoil. As one can see, the predicted yawed lift correlates well with these experimental data.

Figure 4.3. Comparison of Gormont’s predicted yawed lift with the experimental measured data [65]
To find out whether this trend of increased sectional lift coefficient versus the yaw angle would be conveyed into the dynamic stall regime, a comprehensive experimental study of the problem was performed by UTRC (Refs. [15-18]) using an oscillating NACA 0012 airfoil at sweep angles of 0° and 30°, pitching amplitude of 8° and 10°, mean angle of attack of 0° to 15°, normal Mach number of 0.3 and 0.4, and reduced frequency from 0.038 to 0.125. The results of these studies showed that, in the attached flow region, unsteady effects related to sweep are negligible. However, in the stall region, flow sweep should be considered. The studies also revealed that, in general, sweep tends to delay the onset of dynamic stall. Furthermore, the lift hysteresis loops obtained for swept wings are narrower than those for unswept wings. For both swept and unswept wings, the peak of the lift-coefficient increased in magnitude and shifted to higher angles as the frequency increased. Moreover, increasing the pitching amplitude at constant frequency delayed the onset of dynamic stall. It was also observed for unswept wings that an increase in the Mach number, within the range of Mach numbers tested, triggers dynamic stall at a smaller angle; while for swept wings this effect is not noticeable. The results of these comprehensive studies validate the use of simple models that make corrections to the angle of attack based on the steady, unyawed wind tunnel measurements, such as the ones suggested by Harris, et al (Ref. [64]) and Gormont (Ref. [65]), in order to capture the effects of yawed flow.

Barwey and Peters (Ref. [68]) revisited the empirical fit suggested by Harris, et al (Eq. (4.4)) and made further changes to it. They re-plotted data of Ref. [61] versus the wind-axis yaw angle which they called $\Lambda_T$ in order to distinguish it from Purser and Spearman’s yaw angle and reported that Eq. (4.4) does a reasonable job at low yaw angles, $\Lambda_T \leq 30^\circ$, but it deviates from the data after that. They pointed out that Eq. (4.4) gives an infinite lift coefficient at $\Lambda_T = 90^\circ$ which is unacceptable from a numerical aspect and unsupported by the data. They found the root of the problem in the fact that Eq. (4.4) only uses the maximum sectional lift coefficient to fit the data and it neglects the linear lift curve slope which defines how and where the maximum lift coefficient reaches. To that end, they chose two independent parameters of lift curve slope, $a$, and the unyawed $C_{L_{(\text{max})}}$ to generate the yawed lift characteristics.

Barwey and Peters modeled the increasing trend of the maximum lift coefficient versus the yaw angle in the series form of Eq. (4.7). Since they chose to use the yaw angle in the wind-axis system, $\Lambda_T$, rather than the one in the blade-axis system, $\Lambda$, all of their formulas are written in
Flow geometry in both wing-axis (blade-axis) and wind-axis systems is reviewed in Appendix B of this thesis.

\[ C_{L\text{(max)}} \big|_{\alpha_r} = b_0 + \sum_{n=1}^{N} b_n \sin^{2n} (\Lambda_r) \]  

(4.7)

In Eq. (4.7) \( b_n \) are coefficients determined by data analysis. It is clear that \( b_0 \) must be equal to the unyawed \( C_{L\text{(max)}} \). Furthermore, it is unnecessary to use more than two of the unknown coefficients \( (b_n) \), so only two independent parameters, \( b_1 \) and \( b_2 \), fit the data. Barwey and Peters showed that Eq. (4.7) does an excellent job of capturing the experimental data of Ref. [61] for the whole range of yaw angles. However, in order to use Eq. (4.7), one needs to determine the unknown coefficients \( (b_n) \) which requires access to the two dimensional measurements of the steady \( C_L - \alpha \) data at various yaw angles, while the simpler model offered by Harris, et al and Gormont does not have this requirement.

Barwey and Peters further examined radial flow effects on lift hysteresis in dynamic stall by including these effects in the ONERA dynamic stall model (presented in section 2.1.1.8), where the ONERA crude first-order approximation of the linear lift is replaced by the finite-state dynamic induced flow model. They used Peters-Karunamoorthy two-dimensional, finite-state induced flow theory (discussed in section 2.2.2.1), since they were studying the two-dimensional flow over an airfoil. This was achieved by calculation of both \( \Delta C_L \) and the static stall angle, \( \alpha_{\text{stall}} \), based on the lift-curve slope in the linear range, \( \alpha \), and also the maximum lift, \( C_{L\text{(max)}} \), at a particular yaw angle (Eq. (4.7)) and then use of that \( \Delta C_L \) in the ONERA dynamic stall equation. It was assumed that dynamic stall coefficients, \( \eta_0, \eta_2, \omega_0, \omega_2, e_0, e_2 \), in Eq. (2.10) are independent of the yaw angle. Barwey and Peters compared the results obtained from their model with the lift hysteresis data of UTRC for NACA 0012 airfoil section at yaw angles of 0° and 30°, reduced frequency of 0.037, 0.075, 0.093, Mach number of 0.4, and mean and oscillatory pith angle of 12° and 10°, respectively, Ref. [15]. They reported a reasonable correlation between the lift hysteresis data obtained from their model and the experimental measurements. Leishman in Ref. [41] also developed a method based on the empirical fit of the static data in order to capture the effects of yawed flow. He included this model in the Leishman-Beddoes dynamic stall model (discussed in section 2.1.1.7) and reported a good
correlation between the dynamic stall loops of lift, drag and pitching moment obtained from his model and the UTRC experimental data.

4.2. Other Three-Dimensional Studies

Dumitrescu and Cardoș in Ref. [69] studied the influence of three-dimensional, rotational flowfield over the inboard boundary layer that develops on the blades in deep stall and developed a three-dimensional flow-field model that explains the stall process over a rotor blade. Their model is based on the consideration that—at high tip speed operating conditions—the flow at the root of the blade behaves like the rotating flow over a stationary disk with strong, three-dimensional rotational effects. They conclude that stall over the inboard blade sections is delayed due to the rotation. They describe this stall-delay process as: 1.) the generation of additional circulation around the blade sections by the wake at the blade root area, and 2.) the subsequent occurrence of a spanwise circulation decay due to dissipation within the twisting boundary layer on the upper side surface of inboard blade.

The effect of rotation on dynamic stall has also been studied numerically by several investigators. Potsdam, et al in Ref. [70] and Sankaran, et al in Ref. [71] used coupled CFD/CSD technique to estimate the airloads over the UH-60A Blackhawk helicopter rotor across a range of flight conditions. Although they report a quite reasonable correlation between the airload predictions and the data from the UH-60A Airload Program—as well as the state-of-the-art rotorcraft simulation codes—the results achieved from dynamic stall in forward flight are too complex to analyze (Gardner and Richter, Ref. [72]). Gardner and Richter in Ref. [72] used CFD to investigate the dynamic stall behavior of an untwisted single rotor blade of constant OA209 cross-section in the presence of rotation. The computations were performed at Mach number of 0.31 and Reynolds number of $1.15 \times 10^6$ with pitching motion of $\alpha = 13^\circ \pm 7^\circ$ around the quarter chord axis. Furthermore, the reference plane was set to be at the mid-point between the root and tip of the blade. The results of their studies showed that the primary effect of rotation is to reduce the slope of the lift curve at the reference section and to delay the separation. They report a delayed stall at the reference section along with an increase in the lift peak magnitude and a 25% decrease in the moment peak magnitude. Moreover, the lift and moment peak points were more separated in time in the presence of rotation than without
rotation. They also noticed a major change in the forces after stall and reattachment as well as an earlier reattachment of the flow compared to the case without rotation.

In this section we will model the experimental lift data of UTRC for NACA 0012 airfoil section at sweep angles of 0° and 30°, Ref. [15]. To this end, the method of Barwey and Peters, Eq. (4.7), is used in order to predict the effect of yaw on maximum lift coefficient; and then these effects are included in the dynamic stall model of chapter 3, in order to apply the effects of yaw on the stalled lift. Then, the results obtained are compared with the lift hysteresis data of Barwey and Peters (Ref. [68]) and Leishman (Ref. [41]).

4.3. Dynamic Stall Model

The UTRC experimental data show a secondary lift peak. Hence, the most comprehensive version of the Peters et al dynamic stall model, presented in chapter 3, is chosen—capable of capturing the secondary lift stall peak—to model the effect of yaw on the secondary peak. This model combines the Ahaus-Peters dynamic stall model, Ref. [23], with a secondary stall equation.

Numerical values of the stall parameters, used in primary stall model (Eq. (3.1)), are given in Table 4.1. These are equal to the stall parameters that are used by Barwey and Peters in Ref. [68]. Following the approach of chapter 3, a fit of the unswept data of Ref. [15] yields an equation for the onset of the pulse, $\alpha_p$, the height of the normalized pulse, $h_s$, and the area under the pulse, $p_s$.

\[
\alpha_{ss} < \alpha_c \quad \text{no pulse occurs} \quad (4.8)
\]

\[
\alpha_{ss} \geq \alpha_c \quad \alpha_p = \alpha_c + K \left( \alpha_{ss} - \alpha_c \right)^2
\]

\[
h_s = \begin{cases} 
  h_t + h_m \left( \alpha_p - \alpha_c \right) + h_e \left( \alpha_p - \alpha_c \right)^2 & \alpha_c < \alpha_p < \alpha_d \\
  h_{\text{max}} & \alpha_p \geq \alpha_d
\end{cases} \quad (4.9)
\]

\[
p_s = h_s \Delta \tau = \begin{cases} 
  p_t + p_m \left( \alpha_p - \alpha_c \right) + p_e \left( \alpha_p - \alpha_c \right)^2 & \alpha_c < \alpha_p < \alpha_f \\
  p_{\text{max}} & \alpha_p \geq \alpha_f
\end{cases} \quad (4.10)
\]
From Eqs.(4.9) and (4.10), one can calculate the length of the pulse as well as its undamped natural frequency using Eqs. (3.8) and (3.9) respectively.

Furthermore, the same type of hysteresis as mentioned in chapter 3 was used to improve the fidelity of the model.

\[
\alpha_{\text{eff}} = \alpha + \Delta \alpha_0 \left( \frac{\alpha - \alpha_{ss}}{\alpha_{\text{end}} - \alpha_{ss}} \right)^2 ; \quad \alpha_{ss} \leq \alpha < \alpha_{\text{end}} , \quad \frac{d\alpha}{d\tau} \geq 0
\]

\[
\alpha_{\text{eff}} = \alpha + \Delta \alpha_0 \left( \frac{\alpha - \alpha_{ss}}{\alpha_{\text{end}} - \alpha_{ss}} \right) ; \quad \alpha_{ss} \leq \alpha < \alpha_{\text{end}} , \quad \frac{d\alpha}{d\tau} < 0 \tag{4.11}
\]

\[
\alpha_{\text{eff}} = \alpha + \Delta \alpha_0 ; \quad \alpha \geq \alpha_{\text{end}}
\]

Numerical values of the parameters in Eqs. (4.8-4.11) are reported in Table 4.1.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_0 )</td>
<td>0.20</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>0.03</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \eta_2 )</td>
<td>0.23</td>
</tr>
<tr>
<td>( e_0 )</td>
<td>3.0</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>4.0</td>
</tr>
<tr>
<td>( \alpha_{ss} )</td>
<td>8.7°</td>
</tr>
<tr>
<td>( \zeta_s )</td>
<td>0.2</td>
</tr>
<tr>
<td>( h_i )</td>
<td>0.0048</td>
</tr>
<tr>
<td>( h_m )</td>
<td>0.0301</td>
</tr>
<tr>
<td>( h_e )</td>
<td>-0.0037</td>
</tr>
<tr>
<td>( h_{max} )</td>
<td>0.0654</td>
</tr>
<tr>
<td>( p_i )</td>
<td>0.0662</td>
</tr>
<tr>
<td>( p_m )</td>
<td>0.1471</td>
</tr>
<tr>
<td>( p_r )</td>
<td>-0.0219</td>
</tr>
<tr>
<td>( p_{max} )</td>
<td>0.3096</td>
</tr>
<tr>
<td>( K )</td>
<td>8.4261 deg(^{-1})</td>
</tr>
<tr>
<td>( \alpha_c )</td>
<td>10.034°</td>
</tr>
<tr>
<td>( \alpha_d )</td>
<td>14.059°</td>
</tr>
<tr>
<td>( \alpha_f )</td>
<td>13.751°</td>
</tr>
<tr>
<td>( \alpha_* )</td>
<td>0.251°</td>
</tr>
<tr>
<td>( \Delta \alpha_0 )</td>
<td>3.11°</td>
</tr>
</tbody>
</table>

**Table 4.1.** Numerical values of stall, pulse and hysteresis parameters
4.4. Inclusion of the Yawed Flow in Ahaus Model

Inclusion of yawed flow in the dynamic stall model is achieved by calculation of the $\Delta C_L$ based on: 1.) the lift-curve slope in the linear range, $a$, and 2.) the maximum lift, $C_{L(max)}$, at a particular yaw angle, Eq. (4.7). The static lift data of Ref. [15] has the slope of $a = 6.78$ rad$^{-1}$ and stalls at $\alpha_{ss} = 8.7^\circ$. Ref. [68] recommends the following choice of $b_n$ coefficients in Eq. (4.7) in order to predict the $C_{L(max)}$ data of Ref. [15].

$$
\begin{align*}
    b_0 &= C_{L(max)} \big|_{\alpha_y=0} \\
    b_1 &= \frac{1}{2} \left( a - 5C_{L(max)} \big|_{\alpha_y=0} \right) \\
    b_2 &= \frac{1}{2} \left( \frac{a}{2} + 4C_{L(max)} \big|_{\alpha_y=0} \right) \\
    b_3 &= -\frac{1}{2} \left( \frac{a}{2} + C_{L(max)} \big|_{\alpha_y=0} \right) \\
\end{align*}
$$

(4.12)

The linear part of the static $C_L$ data does not change when the yaw angle is varied, as indicated by Gormont in Ref. [65]. However, the stalled part of the static $C_L$ data shifts by the difference between $C_{L(max)}$ at the desired yaw angle, obtained from Eq. (4.7)) and the $C_{L(max)}$ of the unyawed data, see Fig. (4.3). $\Delta C_L$ is calculated based on the static $C_L$ curve at the yaw angle and then that $\Delta C_L$ is used in the primary stall model (Eq. (3.1)). Figure 4.4 shows the unyawed static $C_L$ data from Ref. [15] as compared to the static $C_L$ data at yaw angle of $30^\circ$ obtained from the method explained above.

It is worth mentioning that in Eqs.(4.8-4.11), $\alpha_{\#}$ is calculated based on the yawed static stall angle and the angles $\alpha_c, \alpha_d$ and $\alpha_f$ are shifted by the shift of the static stall angle.
Figure 4.4. Static lift coefficient curve for the NACA0012 Airfoil at $M = 0.4$ and $\Lambda = 0^\circ$ and $30^\circ$

Figure 4.5 compares the UTRC experimental data of Ref. [15] with three different theoretical methods. Data are at zero yaw angle and are at reduced frequencies of 0.037, 0.075, and 0.093. The top row of figures compares the data against results obtained from our approach, which uses the dynamic stall model of chapter 3. The second row of figures compares data with the results of Leishman’s model, which uses the dynamic stall model of Ref. [40]. The third row of figures compares data with the results of Barwey and Peters in Ref. [68], which uses the dynamic stall model of Ref. [23]. These results did not include the new equation to model the secondary peak that is reflected in the top row of figures. For both the top row and bottom row of figures, a single set of dynamic stall parameters is used for the entire correlation at all reduced frequencies. As one can see from the comparisons, our new method gives the best correlation with experimental data at all values of $k$, which demonstrates the robustness of this model.
Figure 4.5. Comparison of the unyawed lift data of Ref. [15] with the results obtained from our model, Leishman’s model and Barwey’s model.
Figure 4.6 shows this same comparison but at the yaw angle of 30°. Our results, shown in
the top row of figures, are for the same stall parameters used on Figure 4.5. In addition, the
static stall data used in our model are derived from the unyawed static stall data via Eqs. (4.7)
and (4.12). These data show that one effect of yaw angle is to diminish the effect of the
secondary lift peak. Our model captures this perfectly and automatically from the original
formulation. In other words, once the Barwey correction to static stall are made based on Eqs.
(4.7) and (4.12), our method predicts this reduction in the secondary peaks. Because the
secondary peaks are reduced, the Leishman model and the Barwey model do better than they did
for zero yaw angle. However, our new model still clearly gives the best correlation with the
experimental data, which validates the present approach of combing the stall model of chapter 3
with Eq. (4.7).

Based on the analysis described in this section, dynamic stall data from unyawed
measurements can be utilized to give good results for dynamic stall in yawed flow without the
need for testing the airfoil for static or dynamic stall in yawed flow. Error norms for our method
as compared to error norms for the Leishmann-Beddoes method are given in Table 4.2 for the
three reduced frequencies at both the 0° and 30° yaw angles (Figs. 4.5 and 4.6). Note that, when
strong secondary lift is present at the 0° yaw angle, our method is 2 to 3 times as accurate as
Leishmann-Beddoes. At 30° yaw, for which the secondary peak effects are reduced, Leishmann is
improved; but our method is still about 60% better, as should be expected. Since Leishmann-
Beddoes is the primary method (other than Ahaus) presently used, one can infer a significant
improvement of our method over all existing methods.

<table>
<thead>
<tr>
<th>Stall Model</th>
<th>$\Lambda = 0^\circ$</th>
<th>$\Lambda = 30^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 0.037$</td>
<td>$k = 0.075$</td>
</tr>
<tr>
<td>Peters et al</td>
<td>0.0522</td>
<td>0.0420</td>
</tr>
<tr>
<td>Leishmann-Beddoes</td>
<td>0.1424</td>
<td>0.1383</td>
</tr>
</tbody>
</table>

Table 4.2. Error norms for Peters et al stall model as compared to error norms
for the Leishmann-Beddoes stall model at $\Lambda = 0^\circ$, $30^\circ$
Figure 4.6. Comparison of the yawed lift data of Ref. [15] with the results obtained from our model, Leishman’s model and Barwey’s model.
Chapter 5

Time-Varying Free-Stream Velocity

Another major effect on dynamic stall is the effect of time-varying free-stream on the aerodynamic loads. Each blade segment of a helicopter rotor blade in forward flight experiences a time-varying flow velocity. The results of several studies show that the effect of unsteady free-stream on the airloads can be significant, especially when the flow separation occurs, which means that—in order to accurately predict the airloads distribution along the blade span—the unsteady free-stream effects must be properly modeled.

5.1. Previous Work

Analytical studies of the effect of time-varying free-stream on the lift of thin airfoils have been performed by several investigators such as Theodorsen (Ref. [73]), Sears (Ref. [74]), Isaacs (Refs. [75,76]), Greenberg (Ref. [77]) and Kottapalli (Refs. [78-80]). Van der Wall and Leishman in Ref. [81] reviewed the theories of these investigators and also presented their own Arbitrary Motion Theory (AMT) for an airfoil undergoing a combination of harmonic pitching, plunging and fore-aft motion, based on indicial response. In order to compare these theories, they assumed simple-harmonic motion with reduced frequency of 0.2. No plunge motion was considered. Three different motions were considered: steady $\alpha$, in-phase $\alpha$, and out-of-phase $\alpha$ were examined. Moreover, it was assumed that the free-stream velocity has sinusoidal variation at reduced frequency of 0.2 and the amplitude of velocity oscillations (with respect to 1.0) are 0, 0.40 and 0.8.
\[
\bar{U}_T = 1 + \frac{\mu}{\bar{F}} \sin(\omega t) = 1 + \lambda_V \sin(\omega t) = 1 + \lambda_V \sin(k\tau)
\]

\[
k = \frac{\omega b}{U_{T_0}} = 0.2
\]

\[
\lambda_V = \frac{V_\infty}{U_{T_0}} \frac{\mu}{\bar{F}} = 0, 0.4, 0.8
\]

\[
\alpha = 1 \text{ (steady), } \alpha = \sin(k\tau) \text{ (in-phase), } \alpha = \cos(k\tau) \text{ (out-of-phase)}
\]

where \(\bar{U}_T\) is the section velocity, \(U_T\), normalized by its mean value, \(U_{T_0}\). Comparison of the lift obtained from the aforementioned theories with the results of the Euler code (Ref. [82]) proved the accuracy of Isaacs and AMT methods along with the shortcomings of the others.

Peters et al in Ref. [83] used the Finite-State Airloads model to examine the effect of free-stream oscillations on the lift of a 2-D airfoil. The Finite State Airloads model needs to be combined with an induced flow model to determine the airloads (section 2.2.2). In order to study two-dimensional flow, the Peters-Karunamoorthy two-dimensional induced flow model (section 2.2.2.1) was used. They considered the most difficult case that had been studied in Ref. [81] (Eq. (5.1)), in which the amplitude of velocity oscillations is assumed to be 80% of free stream. They demonstrated that the results obtained from their model are identical to those of Isaacs or Greenberg and better than the results obtained from the Arbitrary Motion Theory.

In parallel to these analytical researches, several experiments were also conducted to study the problem of unsteady free-stream effects. Pierce, et al (Ref. [84]) studied the effects of the harmonic velocity perturbations on the dynamic stall in a low speed wind tunnel equipped with gust generator to simulate the aerodynamic environment of helicopter. The experiments were performed at Reynolds number of \(2.02 \times 10^5\) with a NACA0012 airfoil model oscillating in pitch about various mean angles of attack near the static stall condition (angles of attack of \(6^\circ, 10^\circ, 14^\circ \pm 4^\circ, 18^\circ \pm 4^\circ\)). The authors report that the varying free-stream has significant effect on the unsteady pitching moment in the vicinity of the static stall and the effect could not be predicted by the analytical methods. However they reported no important effect on the blade stability by the varying free-stream velocity.
Saxena, et al (Ref. [85]) performed experimental studies on the unsteady effects due to a sinusoidally oscillating free-stream over a stationary NACA 0012 airfoil at fixed angles of attack close to the static stall angle. The experiments were carried out at reduced frequencies of 0.18 and 0.9, oscillation amplitude ratio of 0.18, and Reynolds number of $2.5 \times 10^5$. Results of their studies show that, below the static stall angle, the changes in the boundary layer and the local separation are quite negligible; so that the flow can be treated analytically in this region. However, above the static stall angle, the changes are so significant that the fluctuations of the free-stream should be combined with those of the airfoil in the wind tunnel studies of the dynamic stall.

Maresca, et al in Ref. [86] approached the problem by studying the unsteady aerodynamics of an airfoil oscillating horizontally in a uniform flow. They studied the fore-aft motion, plunging motion and the combined fore-aft and plunging motion of the airfoil with respect to the undisturbed flow. These motions simulated the variation of velocity, variation of incidence, and out-of-phase variation of velocity and incidence, respectively. The experiments were performed on a rectangular wing with a NACA 0012 profile and over the Reynolds number range of $5 \times 10^4 \leq Re \leq 4 \times 10^5$. The results of the experiments showed that, for the fore-aft motion at low Mach numbers, the unsteady effects are negligible when the incidence is below the static stall angle. However, when the angle exceeds the static stall angle, unsteady effects become strong, depending on the frequency and amplitude of the velocity oscillations. Similarly, for the plunging motion strong unsteady effects were observed at high incidences and at incidence oscillations large enough to trigger the dynamic stall. Moreover, for the combined fore-to-aft and plunging motion with high-amplitude oscillations, all of the unsteady features observed under horizontal oscillations were reported.

Extension of the work of Maresca, et al, was performed by Favier, et al in Refs. [87] and [88]. With new mechanical equipment, they could drive the airfoil in pitch, fore-to-aft motion, and combined pitch and fore-to-aft motion—which made them capable of studying the airfoil aerodynamic behavior generated by combined velocity and angle-of-attack fluctuations. The $V$ and $\alpha$ cams generated the velocity and angle-of-attack cosinusoidal oscillations with the $\phi$ phase shift between two motions. The results of their studies showed that the aerodynamic response of the system is strongly dependent upon: 1.) phase shift between the periodic laws of incidence and velocity, 2.) mean incidence, 3.) amplitude of velocity variations, and 4.) reduced frequency.
Hird, et al (Ref. [89]) also studied coupled free-stream Mach/pitch oscillations of an SSC-A09 airfoil experimentally in order to evaluate the effect of sinusoidal velocity variation on the aerodynamics loads of a rotorcraft in forward flight. The experiments were conducted at reduced frequencies of 0.025 and 0.05, Mach number of 0.4±0.05, and mean Reynolds number from 2 to 3.5 million for various phase shifts between two motions. The authors concluded that in-phase oscillations of pitch and free-stream velocity increase the lift curve slope and the stall angle compared to the case with pitching oscillations in the uniform flow, while out-of-phase oscillations have the opposite effects.

Naigel, et al (Ref. [99]) used the same set-up in the same wind tunnel as Ref. [89], and further evaluated stall with unsteady free stream. In addition of taking experimental data, they used two different methods to simulate the unsteady free-stream stalled airloads based on the steady ones: 1) interpolating the steady free-stream $C_L$ and $C_M$ at points where instantaneous values of $M, Re$ and $k$ are phased matched to the unsteady flow condition and 2) using the steady free-stream stalled airloads based on matching the non-dimensional parameters ($M, Re$ and $k$) at the point of stall. They report good correlation with the experimental unsteady free-stream data using either method.

Shi and Ming in Refs. [90,91] investigated the aerodynamic behavior of the isolated delta wings and also pitching delta wings coupled with unsteady free-stream in an unsteady wind tunnel. They reported larger lift and pitching moment hysteresis loops for pitching wings coupled with unsteady free-stream. Studying the pressure distribution and flow visualization measurements, they found out that the changes in the leading edge vortex structure are the major reason for this variation.

The effect of time-varying free-stream on dynamic stall has also been studied numerically. Kerho, et al in Ref. [92] used CFD to predict the aerodynamic performance of adaptive leading edge. The numerical analysis was performed on a Sikorsky SSC-A09 section as the baseline airfoil and at Mach number and angle-of-attack range of 0.34-0.76 and 10°±10° respectively. They used the pitch history for the UH-60A Blackhawk helicopter, predicted by the comprehensive rotorcraft analysis code CAMRAD II (Ref. [93]), and coupled it with a variable Mach number to simulate the rotational behavior of the rotorcraft in forward flight. They compared results obtained from the variable Mach number case with those obtained from a constant Mach number case and found that the effect of variable Mach number is to produce a
higher $C_L$ on the upstroke of the $C_L$ versus $\alpha$ curve and also increase the value of $C_{l_{\text{max}}}$. The authors conclude that the reason for the higher $C_L$ values on the upstroke of the $C_L$-versus-$\alpha$ curve is the compressibility effects due to the higher absolute Mach number across much of the upstroke. They further conclude that the increase in the value of $C_{l_{\text{max}}}$ is due to the lower absolute Mach number at the peak of the pitch cycle compared to the constant Mach number case, which delays the onset of shock induced separation and also results in larger effective reduced frequency.

Martinat, et al (Ref. [94]) also studied the problem by means of CFD. They performed the analysis on a NACA0012 airfoil with a sinusoidal pitching motion about the quarter chord (angle-of-attack range of $12^\circ \pm 6^\circ$) and sinusoidal and horizontal plunging motion. The reduced frequency of both motions was 0.188. Furthermore, the Reynolds number was $10^5$; and two oscillations were in-phase. Results of their studies show that the effect of the longitudinal oscillation is to enlarge the area of the hysteresis cycle. The reason identified was the increased speed during the upstroke part of the motion and the decreased speed during the backstroke part. Gharali and Johnson in Ref. [95] used CFD as well to simulate the dynamic stall of a pitching NACA 0012 airfoil under the unsteady free-stream velocity at Reynolds number of $10^5$. The angle-of-attack range of $10^\circ \pm 15^\circ$ was chosen for the pitching motion, while the reduced frequency of both motions was considered to be 0.1. They performed simulations at three different variations in reduced-velocity amplitude of 0.4, 0.6, 0.8 and also four different phase differences between two oscillations of 0, $\pi/4$, $\pi/2$, $3\pi/4$ and $\pi$. The results of the simulation revealed that either decreasing or increasing the value of phase shift, $\varphi$, from $\pi/2$ increases the leading edge vortex growth time, which can either increase or decrease the aerodynamics loads. In the other words, increasing the value of $\varphi$ from $\pi/2$ decreases the aerodynamic loads while decreasing the value of $\varphi$ from $\pi/2$ increases the aerodynamic loads.

Most recently, Gosselin (Ref. [96]) also studied the influence of sinusoidally varying free-stream on dynamic stall of a helicopter blade in forward flight using CFD. He performed the simulations on a NACA 0012 airfoil at an average Reynolds number of $3.31 \times 10^6$. An angle-of-attack range of $15^\circ \pm 10^\circ$ was considered for the pitching motion. The average reduced frequency for both oscillations was 0.029; and the reduced velocity variation amplitude was 0.625. Furthermore, out-of-phase shift of $180^\circ$ was assumed between the angle-of-attack and free-stream velocity oscillations. Comparing the results of simulations with the experimental results
for the steady free-stream case, the author reported smaller lift on the upstroke of the $C_L$ versus $\alpha$ curve while a larger lift on the backstroke part of the motion, below the static stall angle. However, above the static stall angle, lift was generally smaller on the upstroke but significantly larger on the backstroke. He also reported an increase in the value of the lift peak by 127% and its delay to later in azimuthal angle compared to the steady free-stream case. It was also observed that, below the static stall angle, the boundary layer did not reattach due to the occurrence of the shock-induced separation. Furthermore, above the static stall angle, the trailing-edge vortex rather than the leading-edge vortex was the dominant feature and generated a large region of reverse flow around the airfoil (Gosselin, Ref. [96]).

In this section, a Finite-State Airloads model (Ref. [55]) is combined with the Peters-Karunamoorthy induced flow model (Ref. [57]) in order to include the effects of time-varying free-stream in prediction of two-dimensional airloads. This induced flow theory is appropriate because it gives answers identical to Isaacs’ exact theory for linear airloads and it can also be easily included in the unified airloads model (Ref. [23]). The unsteady stall model is modified for use with unsteady free-stream in only one aspect. In order to account for the independence of the shed vortex to later changes in free-stream velocity, the Ahaus-Peters stall model is augmented by two simple terms to effect that independence. The new, combined model is applied to determine the lift of a NACA0012 airfoil under coupled angle of attack/free-stream velocity oscillations at different phase shifts between two motions. The results are compared with experimental lift hysteresis data as well as with numerical results obtained from the DSTP model of Ref. [88]. Moreover, the model is used to predict the lift and pitching moment of a SSC-A09 airfoil with full scale Reynolds and Mach number under the Mach number oscillations and the results are compared against the experimental data presented in Ref. [99].

5.2. Unsteady Free Stream with Unified Airloads Model

The Peters et al unified airloads model is explained in Section 2.2 of this thesis. The unified model has three main components as shown in Fig. 1.12: 1.) a linear airloads model (section 2.2.1), 2.) a finite-state induced flow model (section 2.2.2) and 3.) the dynamic stall model (section 2.2.3), which is modified from ONERA. The first two models have been validated against Theodorsen theory, Garrick theory, Isaacs theory, and Greenberg theory, Ref. [43]
(which include unsteady free-stream). The third model has been validated against NACA 1102, VR-12, VR-7, and SC 1095 airfoil stall data for steady free stream, Ref. [23].

Equation (5.2) is the more general form of the dynamic stall model of section (2.2.3) (Eq. (2.52)) which gives the dynamic change (due to dynamic stall) in blade circulation, pitching moment, and all other generalized loads.

\[
\frac{b^2}{U^2} \hat{\Gamma}_n + \eta \frac{b}{U} \hat{\Gamma}_n + \omega^2 \Gamma_n = -bU \omega^2 \left[ \Delta C_n + e \frac{b}{U} \frac{d\Delta C_n}{dt} + f \frac{b}{U} \frac{d\Delta C_z}{dt} \right]
\]  

(5.2)

where the parameters in Eq. (5.2) (\(\eta\), \(\omega\), \(e\) and \(f\)) are each expanded in a two-term series involving \(\Delta C_n\) and the possibility of cross-coupling with other loads, \(\Delta C_z\).

\[
\omega = \omega_0 + \omega_2 \left( \Delta C_L \right)^2
\]

\[
\eta = \eta_0 + \eta_2 \left( \Delta C_L \right)^2
\]

\[
e = e_0 + e_2 \left( \Delta C_L \right)^2
\]

\[
f = f_0 + f_2 \left( \Delta C_L \right)^2
\]  

(5.3)

In Eq. (5.2), the subscripts “n” and “z” can take on the subscript of any desired loading variable (such as \(L\) for lift, \(M\) for pitching moment, or \(D\) from drag). The forcing function for each term, \(\Delta C_n\), is the difference between the linear static value for \(C_n\) and the measured static value of that coefficient. For \(\Gamma_L\), only \(\Delta C_L\) is needed; but, for \(\Gamma_M\), both \(\Delta C_M\) and \(\Delta C_L\) may drive the time-derivative term, as indicated by \(\Delta C_n\) and \(\Delta C_z\). The parameter \(e\) multiplies the primary \(\Delta C_n\), and the parameter \(f\) multiplies any cross-coupling \(\Delta C_z\) (e.g., \(\Delta C_L\) in the \(\Gamma_M\) equation). The parameters \((\omega_0, \omega_2, \eta_0, \eta_2, e_0, e_2, f_0, f_2)\) are determined from parameter identification of a training data set. One unified set of these parameters can be used for all \(C_n\) for both the upstroke and downstroke; or, to obtain extra fidelity, a different set of parameters can be used for each coefficient of interest with different values for upstroke and downstroke. For our correlations of \(C_L\) with Ref. [88], we will use the same parameters for both upstroke and downstroke. The total loads are found by superposition of the linear loads and the dynamic stall corrections, Eq. (2.56).

Lift data for an NACA 0012 airfoil can be found in Ref. [88] for the case of simultaneous angle of attack and free-stream oscillations of the following form:
\[ U_T = U_{T_0} \left( 1 + \lambda_v \cos k \tau \right) \]

\[ \alpha = \alpha_0 + \Delta \alpha \cos (k \tau + \varphi) \] (5.4)

The measurements of Ref. [88] were taken at three mean-incidence angles \( \alpha_0 = 9^\circ, 12^\circ \) and \( 15^\circ \), with angle-of-attack variation \( \Delta \alpha \) maintained at a constant value of \( 6^\circ \). For each value of \( \alpha_0 \) the mean value of \( U_T \) was set at two different values—\( U_{T_0} = 6 \) m/s and \( 9 \) m/s—which results in a velocity variation and average reduced frequency of \( (\lambda_v = 0.356, k = 0.314) \) and \( (\lambda_v = 0.153, k = 0.135) \), respectively. The phase shift between the two oscillations, \( \varphi \), was varied from \( 0^\circ \) to \( 360^\circ \) by steps of \( 45^\circ \).

In order to implement the present model to correlate this data, the six stall parameters of Eq. (5.3) \( (\omega_0, \omega_2, \eta_0, \eta_2, e_0, e_2) \) are obtained from a Genetic Algorithm based on the case \( \alpha_0 = 12^\circ \) with steady free stream \( (\lambda_v = 0) \). These six parameters are then used for correlation of all of the cases (mean incidences of \( \alpha_0 = 9^\circ, 12^\circ, 15^\circ \), reduced frequencies of \( k = 0.314, 0.135 \) and phase shift parameters of \( \varphi = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ \)) with unsteady free-stream.

In addition to correlation of the unsteady data based on the six stall parameters chosen based solely on steady free-stream data at \( 12^\circ \), we also investigate correlation for the case in which the six stall parameters are specifically optimized for each mean value of angle. In other words, at each mean angle of attack, a single optimization is done over the entire data set \( (k = 0.314, 0.135; \text{and } \varphi = 0^\circ, 90^\circ, 180^\circ, 270^\circ) \) to optimize the parameters to give the best fit of correlation for the entire data set. Table 5.1 shows the numerical values of the stall parameters based on the steady data and based on each mean angle of attack. Although there is a wide range in the values for each stall parameter in the table—as a function of \( \alpha_0 \)—the stall model is rather robust, so that the qualitative nature of the response is not overly sensitive to variations in parameters—only the quantitative details are sensitive to these variations.

### 5.3. Augmented Equations

During correlation, it became apparent that one simple modification was needed to the model to match unsteady free-stream data with stall. In particular, Ahau-Peters dynamic stall model is based on the ONERA concept of calculating linear airloads and then adding the stall deficit to that after it is passed through a dynamic filter (with the filter having six parameters). This gives
a delay and overshoot of the stall deficit. The physical basis is that, when the blade stalls, a shed vortex from the airfoil remains for a time above the blade—creating the lower pressure that delays loss of lift and change in other loads.

Table 5.1. Numerical values of the dynamic Stall Parameters for NACA 0012 airfoil

<table>
<thead>
<tr>
<th>Stall Parameter</th>
<th>Based on steady free stream data $\alpha = 12^\circ \pm 6^\circ$</th>
<th>Based on entire data set $\alpha = 9^\circ \pm 6^\circ$</th>
<th>Based on entire data set $\alpha = 12^\circ \pm 6^\circ$</th>
<th>Based on entire data set $\alpha = 15^\circ \pm 6^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>0.428</td>
<td>0.313</td>
<td>0.393</td>
<td>0.122</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0.464</td>
<td>1.613</td>
<td>0.777</td>
<td>0.998</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>0.943</td>
<td>1.455</td>
<td>1.018</td>
<td>0.181</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>0.044</td>
<td>4.990</td>
<td>2.574</td>
<td>2.172</td>
</tr>
<tr>
<td>$e_0$</td>
<td>-0.183</td>
<td>-0.679</td>
<td>0.129</td>
<td>1.065</td>
</tr>
<tr>
<td>$e_2$</td>
<td>-3.884</td>
<td>3.821</td>
<td>-2.167</td>
<td>-3.521</td>
</tr>
</tbody>
</table>

The problem of this approach in an unsteady free-stream is that, after the vortex is shed, the linear model continues to compute $\Gamma_n$ values by multiplication of the linear airload coefficients by the time-varying free-stream. However, the physical shed vortex—that is maintaining the lift and other loads as it passes over the airfoil—is not changing strength as free-stream changes. It is fixed at its shed value. Therefore, two additional terms must be added to the formulation in order to account for the constant value of the shed vortex from which the filtered deficits are being subtracted.

The augmented equations for the $y$ component of $L$ and pitching moment (per unit length) are:

\[
L_{\text{total}} = L_{\text{linear}} + F_c (U_c - U_T) \rho acU_p + \rho U_T \Gamma_L + \rho F_c (U_c - U_T) \Gamma_L
\]

\[
M_{\text{total}} = M_{\text{linear}} - F_c (U_c - U_T) \rho ab^2 U_p e + \rho U_T \Gamma_M + \rho F_c (U_c - U_T) \Gamma_M
\]
where:

\[ F_c = \frac{k\Delta C_L}{C_{\text{Static}} + \Delta C_L} \]  

(5.7)

And

\[ U_c = U_T \] \hspace{0.5cm} \alpha < \alpha_{ss}, \ \alpha \geq 0

\[ U_c = U_T + (U_{ss} - U_T) \frac{\alpha}{\alpha_{ss}} \] \hspace{0.5cm} \alpha < \alpha_{ss}, \ \alpha < 0 \]  

(5.8)

\[ U_c = U_{ss} \] \hspace{0.5cm} \alpha \geq \alpha_{ss}

The first term on the right-hand side of Eq. (5.5) is the linear lift that is obtained from the linear airloads model (Ref. [55]). The second term is the new correction to the linear circulation to bring the velocity used for lift (past the stall angle) back to a value based on the vorticity strength at stall. (The parameter \( U_T \) is the time-varying free-stream velocity that multiplies the circulation correction \( \Gamma_L \) to form the lift correction.) The third term, \( \rho U_T \Gamma_L \), is the normal stall correction to the bound circulation as found from the present Ahaus-Peters stall model (the circulation deficit after passing through the filter, Eq. (2.56)). The final term, \( \rho F_c (U_c - U_T) \Gamma_L \), corrects that deficit to be based on free-stream at stall—making it compatible with the linear value that it is supposed to correct. Similar definitions apply to the pitching moment in Eq. (5.6) where \( \epsilon \) is the lift offset from the quarter-chord.

The philosophy of the corrections is illustrated in Fig. 5.1 and is as follows. The velocity \( U_c \) is set equal to \( U_T \) until the stall angle is reached. (Thus, there is no correction made up until then.) At that point, \( U_{ss} \) is fixed at the value of \( U_T \) at the stall onset. Since \( a \) is the corrected lift-curve slope, \( c \) is the chord, and \( U_p \) is the flow velocity normal to the blade, the second term in Eq. (5.5) corrects the value of the shed circulation to the value at the point of stall. The factor \( F_c \) is added because not all of the circulation is lost at stall, so that the shed circulation is only a percentage of the total (\( F_c < 1 \)). In the fourth term, the same correction is added to stall deficit so that the airloads deficit due to stall is based on the free-stream velocity at stall. Best correlation
can be found by adding $\kappa$ in Eq. (5.7), which defines the correction as a simple percentage of total lift that is lost statically. After the airfoil angle has returned to the stall angle, $U_{ss}$ is linearly phased back to the local $U_T$ as the minimum, unstalled angle is reached. The scaling is based on the angular rate at the stall angle, $\dot{\alpha}_{ss}$, with the value of $U_c$ transitioning back as the angular rate goes to zero, Eq. (5.8). Note that this approach adds the largest correction when the free-stream $U_T$ is the smallest.

Figure 5.1. Philosophy of the corrections to the unified airloads model

5.4. Results for Lift

Figure 5.2 shows correlation with experimental data for the pitch-angle variation $12^\circ \pm 6^\circ$ at two different reduced frequencies, $k = 0.135, 0.314$. The first plot in each reduced-frequency set is for no free-stream variation ($\lambda_W = 0$), and the other plots are for a prescribed free-stream variation having a phase angle (with respect to pitch oscillation) varying from $\varphi = 0^\circ$ to $360^\circ$ in $45^\circ$ increments. Each figure shows the measured data as the solid line, the ONERA method (GBCN) as the dashed line, and the present method as the dash-dot line. A single set of stall parameters is used, based on the combined zero free-stream data at the two reduced frequencies.
Figure 5.2. Comparison of the lift data from the Peters et al augmented lift model and the ONERA-GBCN model, \( \alpha = 12^\circ + 6^\circ \cos(\kappa \tau + \varphi) \) and \( \bar{U}_T = 1 + \lambda \cos k \tau \). (stall parameters of the Peters et al augmented lift model are determined based on the \( \alpha = 12^\circ \pm 6^\circ \) case stall data with steady free stream)
Although a single set of stall parameters is used based on the steady free-stream—and although no additional parameters are optimized based on phase angle, the present method is quite accurate for all frequencies and phase angles. The overall error norm for both reduced frequencies at $\varphi = 0^\circ, 90^\circ, 180^\circ$ and $270^\circ$ is 0.1828 for the ONERA-GBCN method but is only 0.1050 for the present method. This shows the additional accuracy from this new approach. The present approach is much improved over the ONERA method of Ref. [88] for three reasons. First, the method here has a better induced-flow model and a better airloads theory than is represented by the simple, first-order ONERA approximation. Second, the airloads model used here is valid with unsteady free-stream, whereas the ONERA model is not. Third, the new correction terms added to the lift in Eq. (5.5) adjusts for the fixed strength of the shed vortex. Note that no hysteresis or secondary-stall corrections are necessary at this very low Mach numbers, $M < 0.05$.

Interestingly, Ref. [88] notices that the error between their method and the data is largest when the instantaneous free-stream is smallest. Based on that, they develop an alternate method called Empirical Damped Superposition (DSTP) in which they compute dynamic stall loads using only the steady, average value of free-stream. Then they simply divide those loads (which are based on the average free-stream) by a correction factor $[1 + \lambda_\nu \cos(k\tau)]^\delta$. They choose $\delta = 0.5$ to obtain the best correlation. This gives a qualitatively similar correction to our own physics-based method shown in Eq. (5.5), in that it adds the largest correction when free-stream is smallest and thus it gives very similar error norms to our own method. However, it is not a viable method for use in production codes because it fails in the case of no stall since the Isaacs closed-form solution shows clearly that this type of correction cannot be used for linear loads. This is demonstrated in Fig. 5.3, in which the lift coefficient $C_L$ is plotted versus $\Psi^\circ(=k\tau)$ for a sinusoidally varying free-stream $\bar{U}_T = 1 + 0.8 \sin(0.2\tau)$ and steady $\alpha = 1$. As one can see, the Peters et al finite state model yields identical results to the Isaacs closed-form solution, while the DSTP method differs significantly from it. In addition, the DSTP method can never be used in flight simulators because one does not know a priori the average value of flow velocity, and one cannot assume that there is a sinusoidal variation. The present approach requires no such assumptions and, as we will see, gives good results.

Figures 5.4-5.6 show the correlation with experimental data for the pitch-angle variations of $9^\circ\pm6^\circ$, $12^\circ\pm6^\circ$ and $15^\circ\pm6^\circ$ respectively. The stall parameters used to generate these results are identical for all $18 \times 3 = 54$ plots and are obtained by optimization based solely on the $12^\circ\pm6^\circ$ stall data with steady free stream ($\lambda_\nu = 0$). In Figs. 5.4-5.6, the experimental data are shown by a solid
line, the DSTP result is shown by a dotted line and the Peters \textit{et al} augmented lift method result is shown by a dash-dot line.

In cases for which the DSTP results are not provided in Ref. [88], the DSTP results have been generated herein by division of the lift with steady free-stream obtained from the Peters \textit{et al} unified airloads model (without the lift correction discussed in this paper) by the DSTP correction factor 

\[ [1 + \lambda \phi \cos(k \tau) ]^{0.5} \]

In other words, the total lift obtained from the Peters \textit{et al} model, is corrected by the DSTP formulation rather than the formulation presented in this paper. The result is shown by dashed line and is labeled as DSTP (Applied to Peters \textit{et al} model). One can see that, for the majority of the data sets, Peters \textit{et al} augmented lift model does a better job of correlating the experimental data than does the DSTP model.

Figures 5.7-5.9 present the lift obtained from the Peters \textit{et al} model when it uses dynamic stall parameters obtained from the optimization of the entire data set for a given mean angle of attack. These results are compared both with the DSTP and the experimental lift data. As should be expected, an even better correlation with the experimental data is achieved when more data are used for optimizing. However, this correlation improvement is not significant enough to warrant the computational expense. Therefore, a user can choose stall parameters based on an exemplar case with steady free-stream velocity and obtain good results at other mean angles, other reduced frequencies, and other free-stream variations.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_3}
\caption{Lift data from Isaacs closed form solution, Peters \textit{et al} finite state model and the DSTP model \((\alpha = 1 \text{ and } \bar{U}_T = 1 + 0.8 \sin(0.2\tau))\)}
\end{figure}
Figure 5.4. Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, $\alpha = 9^\circ + 6^\circ \cos (k \tau + \phi)$ and $\bar{U}_x = 1 + \lambda_V \cos k \tau$. (stall parameters of the Peters et al augmented lift model are determined based on the $\alpha = 12^\circ \pm 6^\circ$ case stall data with steady free stream)
Figure 5.5. Comparison of the lift data from the Peters *et al* augmented lift model and the DSTP model, $\alpha = 12^\circ + 6^\circ \cos(k\tau + \varphi)$ and $\bar{U}_T = 1 + \lambda_\nu \cos k\tau$. (Stall parameters of the Peters *et al* augmented lift model are determined based on the $\alpha = 12^\circ \pm 6^\circ$ case stall data with steady free stream)
Figure 5.6. Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, $\alpha = 15^\circ + 6^\circ \cos(kt + \varphi)$ and $U_T = 1 + \lambda \cos k \tau$. (stall parameters of the Peters et al augmented lift model are determined based on the $\alpha = 12^\circ \pm 6^\circ$ case stall data with steady free stream)
Figure 5.7. Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, \( \alpha = 9^\circ \pm 6^\circ \cos (kr + \varphi) \) and \( \bar{U}_T = 1 + \lambda_y \cos kr \). (stall parameters of the Peters et al augmented lift model are determined by optimizing the entire data set of \( \alpha = 9^\circ \pm 6^\circ \) case)
Figure 5.8. Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, $\alpha = 12^\circ \pm 6^\circ \cos(k\tau + \phi)$ and $\bar{U}_T = 1 + \lambda_V \cos k\tau$. (Stall parameters of the Peters et al augmented lift model are determined by optimizing the entire data set of $\alpha = 12^\circ \pm 6^\circ$ case)
Figure 5.9. Comparison of the lift data from the Peters et al augmented lift model and the DSTP model, $\alpha = 15^\circ + 6^\circ \cos(kt + \varphi)$ and $U_T = 1 + \lambda r \cos kt$. (stall parameters of the Peters et al augmented lift model are determined by optimizing the entire data set of $\alpha = 15^\circ \pm 6^\circ$ case)
5.5. Simulation of Pitching Moment

Experimental pitching moment and lift data for an SSC-A09 airfoil in the presence of unsteady free-stream velocity are presented in Ref. [99]. Furthermore, Ref. [99] discovers that, for either lift or pitching-moment data—with unsteady free-stream velocity—the unsteady results can be simulated (based on the results for steady free-stream velocity) by a simple procedure.

Interestingly, this suggested procedure is virtually identical to the enhancement formula of our model, as suggested herein. Namely, following the onset of stall, the free-stream velocity (i.e., reduced frequency and Mach number) is frozen at its value at stall. Reference [99] shows that this procedure does equally well at predicting either lift or pitching moment in an unsteady free-stream. Thus, the procedure outlined both in Ref. [100] and herein (based on the physics of the reduced-order model) has been shown to give good results for both $C_L$ and $C_M$ with unsteady free-stream velocity.

In this section, we will further validate this conclusion by correlation of the experimental data of Ref. [99] with our own enhanced model. The data are for an SSC-A09 airfoil with an average reduced frequency, $k = 0.05$, a free-stream Mach number, $M = 0.40 + 0.07 \cos(\omega t)$, and an average Reynolds number of $3.0 \times 10^6$. Although the abstract of Ref. [99] indicates a pitch-angle variation, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$, the figures in that paper indicate a range of pitch angle, $\alpha = 9.70^\circ - 12.55^\circ \cos(\omega t)$. On the other hand, the linear, unstalled region of these loops seems to indicate an actual variation of $\alpha = 8.87^\circ - 13.38^\circ \cos(\omega t)$, which is closer to the value given in the abstract than indicated by the range on the figures. For the simulations herein, we use the third range for angle of attack but plot results versus the data of Ref. [99] on the same pitch range given there.

Interestingly, the slight shift in angles mentioned above can be included in the theory either by a shift in the zero-line for angle of attack measurement or by a shift in induced flow $\lambda_0$ which could be attributed to wall effects. We simply make the shift without a decision on from whence the shift arises. However, it should be noted that the shift seems to go away for the pitching moment data after the major vortex is shed. To account for this, we have included the following formula for the induced flow shift, which washes it out after shedding.
\[ \lambda_0 = -0.026 + 0.2 \frac{(U_c - U_T)}{U_T} \] (5.9)

The static characteristics for the airfoil are determined from data published in Ref. [89], which were taken for the same set-up in the same wind tunnel as Ref. [99]. The only difference is that the earlier results were for Mach number \( M = 0.2 \) rather than the \( M = 0.40 \pm 0.07 \) used for the dynamic data in Ref. [99]. Based on these data, the static correction factor for lift used in the linear lift model is 0.81, which includes a Mach-number correction from 0.2 to 0.4. A similar static correction is made for pitching moment, with the recognition that \( C_M \) involves the square of the stretched length. These Mach number corrections are also applied to the static data. The linear lift model also requires an empirical correction for the effective center of rotation of the airfoil. From the static data, the equivalent center of rotation in the linear airloads model must be taken as 0.23 chords from the leading edge (rather than 0.25). The dynamic stall parameters have been identified based on the data set of Ref. [99] with steady free-stream. They are given in Table 5.2. Then, these same parameters are used for the unsteady free-stream correlations.

<table>
<thead>
<tr>
<th>Stall Parameter</th>
<th>( C_L ) Upstroke</th>
<th>( C_L ) Downstroke</th>
<th>( C_M ) Upstroke</th>
<th>( C_M ) Downstroke</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_0 )</td>
<td>0.938</td>
<td>0.151</td>
<td>0.334</td>
<td>0.179</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>-0.115</td>
<td>2.417</td>
<td>0.165</td>
<td>-0.206</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>4.062</td>
<td>0.138</td>
<td>1.174</td>
<td>0.585</td>
</tr>
<tr>
<td>( \eta_2 )</td>
<td>-3.331</td>
<td>3.621</td>
<td>-0.724</td>
<td>-0.392</td>
</tr>
<tr>
<td>( e_0 )</td>
<td>1.251</td>
<td>-0.513</td>
<td>4.755</td>
<td>4.364</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>-4.234</td>
<td>-3.051</td>
<td>-9.997</td>
<td>-9.976</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9.993</td>
</tr>
</tbody>
</table>

Table 5.2. Numerical values of the dynamic Stall Parameters for SSC-A09
With this as the procedure, we can look at results. Since we need high fidelity results in the steady free-stream case in order to assess the effect of unsteady free-stream velocity, different parameters are used for either $C_L$ or $C_M$ on either the upstroke or the downstroke. Comparisons of theory with experiment for $C_L$ and $C_M$ for the case of steady free-stream velocity are given in Figures 5.10 and 5.12. The correlation is virtually perfect with error norms of 0.0241 and 0.0515 for lift and pitching moments, respectively. In contrast, for cases without a second peak (as is the case here), error norms reported Ahaus are typically 0.12–0.14 for $C_L$ and 0.20–0.25 for $C_M$, Ref. [23]. The very small error norm here attests to both the fidelity of the model and the success of the parameter training. Since this correlation is very good, then based on the conclusions of Ref. [99], one would expect the correlation with the new enhancement (for unsteady free-stream) to be equally good. Figures 5.11 and 5.13 give $C_L$ and $C_M$ for the case of unsteady free-stream, with the same stall parameters that were found from the original training. For the earlier correlation of data from Ref. [88], the value of $\kappa$ in Eq. (5.7) is $\kappa = 1.0$. For the data of Ref. [99], the authors used an equivalent value of $\kappa = 1.8$. Therefore, this is the value used here for $C_L$. For $C_M$, we go back to $\kappa = 1.0$. The static stall angle used is $\alpha_{ss} = 10^\circ$.

As expected, Figs. 5.11 and 5.13 show that the model offered herein does an excellent job of predicting both lift and pitching moments in the presence of unsteady free-stream velocities based on the existing Ahaus model with the suggested enhancement. The error norms are 0.0433 for $C_L$ and 0.0673 for $C_M$. In fact, the agreement is generally better than that published in Ref. [99]. This means that data taken for dynamic stall in steady free-stream can be used to train our model; and then that model can be successfully applied to both lift and moment in the presence of unsteady free-stream velocities.
Figure 5.10. Comparison of the lift data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$ and $M = 0.40$

Figure 5.11. Comparison of the lift data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$ and $M = 0.40 + 0.07 \cos(\omega t)$
Figure 5.12. Comparison of the pitching moment data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$ and $M = 0.40$

Figure 5.13. Comparison of the pitching moment data from the Peters et al augmented model and the experimental data of Ref.[99] for SSC-A09 Airfoil, $\alpha = 9^\circ - 13^\circ \cos(\omega t)$ and $M = 0.40 + 0.07 \cos(\omega t)$
Chapter 6

Three-Dimensional Modeling of Dynamic Stall

This section presents the two major three-dimensional coupling effects on dynamic stall and then incorporates all effects treated herein into a three-dimensional model. The first 3-D effect treated is the three-dimensional, induced-flow model, which couples blade sections through the shed wake of each blade segment. The second 3-D effect treated is radial coupling from the fact that the shed stall vorticies are convected radially along the blade span. Then, the entire model (as synthesized from all effects considered) is combined and applied to the rotor conditions used for correlation of the Langley induced flow measurements, Ref. [58].

6.1 Three-Dimensional Finite State Induced Flow Model

The original Ahaus-Peters unified airloads model uses a two-dimensional induced flow model to predict airloads on a morphing airfoil. It was mentioned earlier in section 2.2.2 that the Peters-Karunamoorthy, two-dimensional induced flow model gives the inflow equation for a flat wake in two-dimensions. However, for a helicopter in hover or forward flight, a three-dimensional, skewed, cylindrical wake must be considered. This can be achieved by replacement of the Peters-Karunamoorthy two-dimensional induced flow model with the Peters-He three-dimensional finite-state wake model in the Ahaus-Peters unified airloads model. In this section, we will present a summary of Peters-He three-dimensional finite state inflow model. Full formulation of theory can be found in Ref. [97].

The basis of the Peters-He three-dimensional finite-state induced flow model is an acceleration potential with a skewed, cylindrical wake. The advantage of this method over others is that instead of discretizing the wake and coupling it to a given blade, the induced flow is expanded at the disk in terms of the modal functions. Furthermore, the expression for the lift
on the disk is also left general and can be defined using any lifting theory. The theory relates the expansion coefficients of the inflow to the expansion coefficients of the arbitrary lift via a set of first-order ordinary differential equations. It is worth mentioning that this model implicitly includes the Theodorsen and Loewy effects (from shed wake) and also the Prandtl-Goldstein tip losses (due to the trailing wake) as proved in Ref. [45].

In the theory, the pressure on the rotor disk is expressed by a Fourier expansion. By addition of more harmonics to the expansion, the pressure converges to the lift concentration on the blade and to zero off the blade. This pressure expansion is expressed by Eq. (6.1) in ellipsoidal coordinates:

\[
\Phi_p (\nu, \eta, \bar{\nu}, \bar{\eta}) = \rho \sum_{m=0}^{\infty} \sum_{n=m+1, m+3, \ldots} P_n^m (\nu) Q_n^m (i\eta) \left[ C_n^m (\bar{\iota}) \cos (m\bar{\eta}) + D_n^m (\bar{\iota}) \sin (m\bar{\eta}) \right] \tag{6.1}
\]

where \(m\) and \(n\) are harmonic and the polynomial numbers respectively, \(\nu, \eta, \bar{\nu}\) are the ellipsoidal coordinates, \(\bar{\iota}\) is the nondimensional time (\(\bar{\iota} = \Omega t\)), \(P_n^m\) and \(Q_n^m\) are the associated Legendre function of first and second kinds respectively and \(C_n^m\) and \(D_n^m\) are arbitrary time-dependent coefficients. Appendix C defines the ellipsoidal coordinates system.

The lift at the disk, where \(\eta = 0, \nu = \sqrt{1 - \bar{r}^2}\) and \(\bar{\psi} = \psi\), can be obtained from the difference in pressures above and below the disk.

\[
\Delta P (\bar{r}, \psi, t) = \rho \sum_{m=0}^{\infty} \sum_{n=m+1, m+3, \ldots} \bar{P}_n^m (\nu) \left[ \tau_n^{mc} (\bar{\iota}) \cos (m\psi) + \tau_n^{ms} (\bar{\iota}) \sin (m\psi) \right] \tag{6.2}
\]

where

\[
\bar{P}_n^m (\nu) = (-1)^m \frac{P_n^m (\nu)}{\rho_n^m} \tag{6.3}
\]

\[
\left( \rho_n^m \right)^2 = \frac{1}{2n+1} \frac{(n+m)!}{(n-m)!} \tag{6.4}
\]

\[
\tau_n^{mc} = (-1)^{m+1} 2Q_n^m (i0) \rho_n^m C_n^m \tag{6.5}
\]

\[
\tau_n^{ms} = (-1)^{m+1} 2Q_n^m (i0) \rho_n^m D_n^m \tag{6.6}
\]
The $\lambda_0$ equation, which defines the first term in the respective Glauert series for induced flow due to shed vorticity (see section 2.2.1), is also obtained in terms of azimuthal harmonics and radial distribution functions.

$$\lambda_0(\bar{r}, \psi_q, \bar{t}) = \sum_{r=0}^{\infty} \sum_{j=r+1, r+3, \ldots} \phi_j^r(\bar{r}) J_0(\bar{b}/\bar{r}) \left[ \alpha_j^r(\bar{t}) \cos(r\psi_q) + \beta_j^r(\bar{t}) \sin(r\psi_q) \right]$$  \hspace{1cm} (6.7)

where

$$\phi_j^r(\bar{r}) = \frac{1}{v} \bar{P}_j^r(v), \quad v = \sqrt{1 - \bar{r}^2}$$  \hspace{1cm} (6.8)

and $J_0$ is the Bessel function. Furthermore, $\bar{b}$ is the blade section semi-chord normalized by the blade radius and $\psi_q$ is the azimuthal location of the $q^{th}$ blade. The radial expansion functions $\phi_j^r(\bar{r})$ are simple polynomials in $\bar{r}$ as shown in Eq. (6.9).

$$\phi_j^r(\bar{r}) = \sqrt{(2j+1)H_j^r} \sum_{q=r, r+2, \ldots}^{j-1} \bar{r}^q \frac{(-1)^{(q-r)/2}(j+q)!!}{(q-r)!!(q+r)!!(j-q-1)!!}$$  \hspace{1cm} (6.9)

where

$$H_j^r = \frac{(j+r-1)!!(j-r-1)!!}{(j+r)!!(j-r)!!}$$  \hspace{1cm} (6.10)

The relation between the induced flow expansion coefficients $(\alpha_j^r, \beta_j^r)$ and the pressure coefficients $(\tau_n^{mc}, \tau_n^{ms})$ is established from the momentum equation for the potential flow.

$$R[M]\{\alpha_j^r\} + [\bar{L}]^{-1}[V]\{\alpha_j^r\} = \frac{1}{2}\{\tau_n^{mc}\}$$  \hspace{1cm} (6.11)

$$R[M]\{\beta_j^r\} + [\bar{L}]^{-1}[V]\{\beta_j^r\} = \frac{1}{2}\{\tau_n^{ms}\}$$  \hspace{1cm} (6.12)
In Eqs. (6.11) and (6.12), the $[M]$-matrix is called the apparent mass matrix and is defined by Eq. (6.13).

\[
[M] = \begin{bmatrix} \cdots & K_{mn}^m & \cdots \\
\end{bmatrix} \tag{6.13}
\]

where $K_{mn}^m \equiv \frac{\pi}{2} H_{n}^m$. Furthermore, $[L]$ is the quasi-steady inflow operator and it has two different forms of $[L^c]$ and $[L^s]$ which are expressed as follows.

\[
[L^c] = \begin{bmatrix} \cdots & \overline{A}_{jn}^{c\prime} & \cdots \end{bmatrix} \cdots \begin{bmatrix} \cdots & \hat{L}_{jn}^m & \cdots \end{bmatrix} \tag{6.14}
\]

\[
[L^s] = \begin{bmatrix} \cdots & \overline{A}_{jn}^{s\prime} & \cdots \end{bmatrix} \cdots \begin{bmatrix} \cdots & \hat{L}_{jn}^m & \cdots \end{bmatrix} \tag{6.15}
\]

where

\[
\overline{A}_{jn}^{c\prime} = \frac{(-1)^{n+j-2r}}{2} \frac{2\sqrt{2n+1}\sqrt{2j+1}}{\sqrt{H_n^r H_j^r}} \frac{(n+j)(n+j+2)}{(n-j)^2 - 1} 
\tag{6.16}
\]

and

\[
\hat{L}_{jn}^{0m} = X^{m} \Gamma_{jn}^{0m} 
\tag{6.17}
\]

\[
\hat{L}_{jn}^{lm} = [X_{m}^{[m+r]} + (-1)^{r} X_{m}^{[m+r]}][\Gamma_{jn}^{lm}] 
\tag{6.18}
\]

\[
\hat{L}_{jn}^{lm} = [X_{m}^{[m+r]} - (-1)^{r} X_{m}^{[m+r]}][\Gamma_{jn}^{lm}] 
\tag{6.19}
\]

In Eqs. (6.17) to (6.19), $l = \min(r, m)$ and
$$X = \tan \left[ \frac{\chi_{\text{eff}}}{2} \right] = \frac{V_{\infty} \sin \chi}{V_{T} + \left| V_{\infty} \cos \chi + \bar{U}_{z} \right|} \tag{6.20}$$

where $\chi$ is the skew angle, $V_{T}$ is the total velocity for momentum theory and $\bar{U}_{z}$ is the average induced flow on the disk. $V_{T}$ and $\bar{U}_{z}$ are defined by Eqs. (6.21) and (6.22) respectively.

$$V_{T} = \sqrt{V_{\infty}^{2} \sin^{2} \chi + (V_{\infty} \cos \chi + \bar{U}_{z})^{2}} \tag{6.21}$$

$$\bar{U}_{z} = \sqrt{3} \alpha^{0}_{z} \tag{6.22}$$

Additionally, $\Gamma_{jn}^{rm}$ functions are defined by Eqs. (6.23) to (6.25).

for $r + m$ even,

$$\Gamma_{jn}^{rm} = \frac{(-1)^{n+j-2r}}{2} \frac{2\sqrt{(2n+1)(2j+1)}}{\sqrt{H^{n}_{j}H^{m}_{j}} \left( j+n \right) \left( j+n+2 \right) \left( (j-n)^{2} - 1 \right)} \tag{6.23}$$

for $r + m$ odd, $j = n \pm 1$,

$$\Gamma_{jn}^{rm} = \frac{\pi}{2\sqrt{H^{n}_{j}H^{m}_{j}}} \frac{\text{sgn}(r-m)}{\sqrt{(2n+1)(2j+1)}} \tag{6.24}$$

for $r + m$ odd, $j \neq n \pm 1$,

$$\Gamma_{jn}^{rm} = 0 \tag{6.25}$$

The theory presented here is a more general form of the Peters-He three-dimensional finite state inflow model, which uses the diagonal matrix of $[V]$ rather than the scalar $V_{\infty}$ in Eqs. (6.11) and (6.12). This eliminates the singularity at hover condition, where $V_{\infty}$ goes to zero. The first diagonal element of the $[V]$-matrix is $V_{T}$, which is given in Eq. (6.21). Each other element is the mass inflow parameter as defined by Eq. (6.27).
\[
[V] = \begin{bmatrix}
V_r \\
V \\
V \\
\vdots
\end{bmatrix}
\] (6.26)

\[
V = \frac{V_\infty^2 \sin^2 \chi + (V_\infty \cos \chi + \bar{\omega}) (V_\infty \cos \chi + 2\bar{\omega})}{V_r}
\] (6.27)

The equations that relate the pressure harmonics \(\tau_{mc}^n, \tau_{ms}^n\) to the blade lift are the final pieces that complete the theory.

\[
\tau_{mc}^n = \frac{1}{2\pi} \sum_{q=1}^{Q} \left[ \int_0^{\rho R} J_0 \left( \frac{m \bar{b} / \bar{r}}{\bar{r}} \right) \phi_n^q \left( \bar{r} \right) d\bar{r} \right] 
\] (6.28)

\[
\tau_{ms}^n = \frac{1}{2\pi} \sum_{q=1}^{Q} \left[ \int_0^{\rho R} J_0 \left( \frac{m \bar{b} / \bar{r}}{\bar{r}} \right) \phi_n^m \left( \bar{r} \right) d\bar{r} \right] \cos(m\psi_q) 
\] (6.29)

\[
\tau_{ms}^n = \frac{1}{2\pi} \sum_{q=1}^{Q} \left[ \int_0^{\rho R} J_0 \left( \frac{m \bar{b} / \bar{r}}{\bar{r}} \right) \phi_n^m \left( \bar{r} \right) d\bar{r} \right] \sin(m\psi_q) 
\] (6.30)

In Eqs. (6.28) to (6.30), \(Q\) represents the number of the blades and \(L_q\) is the blade sectional circulatory lift. Equations (6.1) to (6.30) form a complete unsteady three-dimensional wake model in terms of finite number of states \(\alpha_j^r\) and \(\beta_j^r\).

In order to compute the induced flow from the Peters-He dynamic inflow model, the maximum harmonic number and the maximum number of radial shape functions (for each harmonic) must be truncated at finite values. Table 6.1 shows the number of the radial shape functions for each harmonic in order to obtain radial terms up to a given power of \(\bar{r}\). This gives a consistent and well-converged model. For example, if the highest power of \(\bar{r}\) is 3, then from Table 6.1, there are 2 radial terms for the \(m = 0\) harmonic (2 inflow states for \(m = 0\)) and 2 radial terms for each of the \(m = 1\) sine and cosine harmonics (4 inflow states for \(m = 1\)). Furthermore, there is 1 radial term for each of the \(m = 2\) sine and cosine and 1 radial term for each of the \(m = 3\) sine and cosine harmonics (2 inflow states for \(m = 2\) and 2 inflow states for...
\( m = 3 \). Thus for the highest power of \( \bar{r} \) equal to 3, the total number of the inflow states is 10. In general, there are \((M + 1)(M + 2)/2\) states for the maximum power and maximum harmonic of \( M \).

<table>
<thead>
<tr>
<th>Highest Power of ( \bar{r} )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total Inflow States</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
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<td>1</td>
<td></td>
<td></td>
<td></td>
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<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 6.1. Number of Shape Functions per Harmonic [58]

6.2 Effect of Radial Coupling

Another major effect that should be addressed in the formulation of a three-dimensional airloads model is the effect of radial coupling. Figure 6.1 shows a recently-obtained flow visualization from Georgia Institute of Technology. One can see that after the vortex is shed from a section of the blade, it convects in the direction of the free-stream and starts decaying. As a result, in yawed flow, each blade section encounter a part of a vortex that has been shed from its neighboring section. This is what is called radial coupling.

Because vorticity is being convected downstream along the blade, we need to write a convection equation for vorticity based on the radial velocity component \( U_R \). Basically, at each time step, the vorticity at one section is increased by the vorticity convected from upstream, and the decreased by the vorticity convected downstream to the next section. The appropriate convection equations is:

\[
\frac{\partial \Gamma_L}{\partial t} + U_R \frac{\partial \Gamma_L}{\partial r} = \hat{\Gamma}_L \text{ (no radial coupling)}
\]  

(6.31)
We can take this same concept and incorporate the change due to convection into the dynamic stall equation. To this end, extra terms are added to the original Ahaus-Peters dynamic stall model (Eq. (2.52)) to account for the convection and decaying of the shed vortices along the blade.

\[
b^2 \frac{\partial^2 \Gamma}{\partial t^2} + \eta b U \frac{\partial \Gamma}{\partial t} + b^2 U_r \frac{\partial^2 \Gamma}{\partial t \partial r} + b^2 \dot{U}_r \frac{\partial \Gamma}{\partial r} + \omega^2 U^2 \Gamma = -b U^2 \omega^2 \left[ U \Delta C_L + eb \dot{C}_L \right] \tag{6.32}
\]

**Figure 6.1.** Effect of radial coupling on a NACA0013 rotor blade

\[\psi = 270^\circ, \mu = 0.4, r/R = 0.5, \text{Rotation Rate: } 20.94 \text{ rad/s}\]

Collective Pitch: 10°, Longitudinal Cyclic Pitch: −5°

(Courtesy of Dr. Narayanan Komarath and Vrishank Raghav)
The finite-difference method can be used in order to determine the rate of change of $\Gamma_L$ with respect to $r$, as shown in Fig. 6.2 and Eq. (6.33).

\[
U_R > 0: \quad \Delta r = r_n - r_{n-1}, \quad \bar{a} = 1, \quad \bar{\bar{a}} = 0
\]
\[
U_R = 0: \quad \Delta r = (r_{n+1} - r_{n-1})/2, \quad \bar{a} = 0, \quad \bar{\bar{a}} = 0
\]
\[
U_R < 0: \quad \Delta r = r_{n+1} - r_n, \quad \bar{a} = 0, \quad \bar{\bar{a}} = 1
\]

**Figure 6.2.** Rotor blade sections

Eq. (6.32) can also be written in matrix form.

\[
\begin{bmatrix}
\vdots & \vdots & \vdots & \vdots \\
\frac{b_n^2}{\eta_n b_n U_n} \Gamma_{L_n} & \eta_n b_n U_n & \frac{b_n^2 U_R}{r_n - r_p} \Gamma_{L_n} & C \Gamma_{L_n} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{b_n^2 U_R sgn(U_R)}{r_n - r_p} & C & \omega_n^2 U_n^2 & \Gamma_{L_n} \\
\vdots & \vdots & \vdots & \vdots \\
-\frac{b_n U_n^2 \omega_n^2}{e_n b_n \Delta C_{L_n}} & U_n \Delta C_{L_n} + e_n b_n \Delta \dot{C}_{L_n} & \vdots & \vdots \\
\end{bmatrix}
\]

(6.34)
where

\[
\begin{align*}
p &= n - 1 & \text{if } & U_R > 0 \\
p &= n + 1 & \text{if } & U_R < 0
\end{align*}
\]

and

\[
C = \begin{bmatrix}
\ddots & \ddots & \ddots & \ddots & \ddots \\
\ddots & 1 & -\tilde{d} & \ddots & \ddots \\
-\tilde{a} & 1 & -\tilde{d} & \ddots & \ddots \\
-\tilde{a} & 1 & -\tilde{d} & \ddots & \ddots \\
\ddots & \ddots & \ddots & \ddots & \ddots
\end{bmatrix}
\]

(6.36)

Furthermore, \(\tilde{a}\) and \(\tilde{d}\) are as defined in Eq. (6.33).

### 6.3 Implementation of Three-Dimensional Dynamic Stall Model

In order to obtain a full three-dimensional airloads model, the following changes are made to the original Ahaus-Peters unified airloads model (Fig. 1.12).

1. The dynamic stall model (Eq. (2.56)) is replaced by Eq. (6.34), which includes the effect of radial coupling.
2. The Barwey-Peters approach is followed in order to include the effects of yawed flow in \(\Delta C_L\) (as explained in chapter 4); and then that resultant \(\Delta C_L\) is used in the dynamic stall equation (Eq. (6.34)).
3. The equations for total generalized airloads in the original Unsteady Airloads Model for Dynamic Stall (Eq. (2.56)) is replaced by the augmented ones (Eq. (5.5), (5.6)), in order to capture the effects of unsteady free-stream velocity on the stalled loads.
4. The Peters-Karunamoorthy two-dimensional finite-state induced flow model is replaced by the Peters-He three-dimensional finite-state wake model (explained in section 6.1), which gives the inflow equation for a three-dimensional, skewed, cylindrical wake and also accounts for the other three-dimensional effects such as the Theodorsen and Loewy shed wake effects and the Prandtl-Goldstein tip loss effects.
6.3.1. Simulation Condition

As much as possible, we will simulate the test conditions of the data correlated in Ref. [58]. We assume a set of four blades with NACA0012 airfoil profile. The blades have 13° negative, linear twist, a uniform chord over the inner 75% radius \((c/R = 0.097)\), and a 3-to-1 blade taper over the outer 25 percent. The set has a 25% root cutout. Figure 6.3 gives the schematic of the single-blade profile.

The helicopter flies with the advance ratio of \(\mu = 0.15\) at 3° nose down shaft angle. The tip speed of the blades is 624 ft/s. The blade collective pitch at \(\bar{r} = 0.75\) is 6.86°, lateral cyclic pitch is 1.96° and longitudinal cyclic pitch is -2.26°. So the blade pitch angle in degrees at any radial position \(\bar{r}\) and any azimuthal angle \(\psi\) is:

\[
\theta = 16.01 - 13\bar{r} + 1.96 \cos \psi - 2.26 \sin \psi
\]

![Figure 6.3. Rotor blade shape](image)

For numerical purposes, each blade is divided to 5 constant-chord segments, as shown in Fig. 6.4. Although the 5\(^{th}\), tapered segment has an average chord of \(2c/3\), the element is given a collocation point at 3/4 of the way to the tip (typical of panel methods). Thus, the chord for the element is taken at that point—which is \(c/2\). Each of the five segments has 2 dynamic stall states (10 states total) and its own linear airloads model that must be run simultaneously with the 15 states of the inflow model in the simulation (4 harmonics). Thus, this is a 25-state model.
From 2-D experimental data for the NACA 0012 at similar Mach numbers (correlated earlier in this thesis), it was found that the linear lift-curve slope was $2\pi(1.09)/57.3 = 0.12/\text{deg}$; and that is what we use here. For the He 3-D inflow model, we take the highest power of $\bar{r}$ to be 5, Table 6.1, which gives 4 harmonics of inflow and 15 inflow states, typical for a four-bladed rotor. Since all four blades will have the same vibratory loads, we use an airloads model (linear and stall) for only one blade, and then assume the other blades have identical loads when at the same azimuthal position. This reduces the complexity of the model. First, a run was done without dynamic stall to make sure that the computed inflow from our model agreed with the results published in [58], which themselves were correlated with experimental data. Once that was done (and it was verified that the code was working properly), then we added the stall model and computed loads.

### 6.3.2. Correlation at Test Pitch Angles

Figure 6.5 gives the results for the second blade segment (mean radius $r/R = 0.4375$). The lift coefficient $C_L$ is plotted versus the blade pitch angle $\theta$ in degrees. Two curves are presented. One is with stall dynamics (solid curve), and the other is for stall neglected (dashed curve). It can be seen that the stalled and unstalled cases are very close. This is because the experiment was set up to have little or no stall in order that clean airloads could be measured. Thus, angle of attack goes only to $13.5^\circ$, which is barely into stall. It may seem strange to see such a large hysteresis loop for a case with little or no stall. One must remember, however, that $\theta$ is only the pitch angle, not the angle of attack. The angle of attack involves both the induced flow and the flow from the shaft tilt, each divided by the local free-stream velocity.
Figure 6.5. Lift coefficient data versus the pitch angle for segment 2

\[ \theta = 16.01 - 5.69 \cos \psi + 1.96 \sin \psi \]

Figure 6.6 shows the identical \( C_L \) data plotted versus angle of attack \( \alpha \) rather than pitch angle. It is clear that there is a linear lift behavior with angle of attack equal to 0.12, which is the value determined from static experimental data at this Mach number. There is very little stall hysteresis evident, and very little linear airloads hysteresis appears (due to the low reduced frequency). This linear behavior is clearly seen when results are plotted directly against the local angle of attack.
Figure 6.6. Lift coefficient data versus the angle of attack for segment 2
\[ \theta = 16.01 - 5.69 \cos \psi - 2.26 \sin \psi \]

Figure 6.7 displays \( C_L \) versus \( \theta \) at the 4\textsuperscript{th} segment for this same flight condition as Figs. 6.5 and 6.6. Being further from the rotor center than segment 2 (per cent radius = 0.6875), this segment has absolutely no dynamic stall with an angle of attack that does not exceed 10.5°. The elliptical shape that was seen for segment 2 has been replaced with a figure-8 pattern for \( C_L \) versus pitch angle. As before, however, this elliptical shape is not a result of dynamic stall. It is a result of the complicated nature of angle-of-attack excursions as the blade traverses the induced-flow field.
Figure 6.7 Lift coefficient data versus the pitch angle for segment 4

\[ \theta = 16.01 - 8.94 \cos \psi - 2.26 \sin \psi \]

Figure 6.8 gives \( C_L \) versus \( \alpha \) for segment 4. As it was with segment 2, although \( C_L \) versus \( \theta \) shows a nonlinear behavior (pronounced figure 8), \( C_L \) versus \( \alpha \) is more linear with only a small amount of linear hysteresis appearing; and the lift-curves slope is again 0.12.
Figure 6.8. Lift coefficient data versus the angle of attack for segment 4
\[ \theta = 16.01 - 8.94 \cos \psi - 2.26 \sin \psi \]

6.3.3. High Pitch Angle Case

Since the blade tests in Ref. [58] were designed with little or no stall, the previous correlations did not give opportunity to exercise our stall model. Therefore, we will now double the collective and cyclic pitch used there in order to take the rotor into a simulated stall condition. This would be the equivalent of an attempted 2-g pull-up in which \( C_T/\sigma \) is meant to go from 0.065 to 0.130. The resulting pitch angle in degrees is then:

\[ \theta = 22.27 - 13\bar{r} + 3.92 \cos \psi - 4.52 \sin \psi \]
Figure 6.9. Lift coefficient data versus the pitch angle for segment 2

\[ \theta = 22.27 - 5.69 \cos \psi + 3.92 \sin \psi \]

Figure 6.9 shows the resulting \( C_L \) (with and without stall) for segment 2 at the higher pitch angle. The curve without stall is very similar in nature to the \( C_L \) for segment 2 at the lower pitch angle, Fig. 6.5. The stalled curve, however, now shows the significant stall that one would expect with a maximum angle of attack of 22º.

The effect of stall is more clearly seen in Fig. 6.10 in which \( C_L \) is plotted directly versus angle of attack. The unstalled curve is very similar to the lower pitch case in Fig. 6.6, while the stalled curve shows the heavy stall present at this inboard segment.
Figure 6.10. Lift coefficient data versus the angle of attack for segment 2
\[ \theta = 22.27 - 5.69 + 3.92 \cos \psi - 4.52 \sin \psi \]

Figure 6.11 gives lift coefficient versus pitch angle on segment 4 for this high pitch case. As it was with segment 2, there is a drop in lift due to stall. Note that the looping behavior for the unstalled case is similar to the looping for lower pitch at segment 4, Fig. 6.7. However, the looping behavior with dynamic stall in Fig. 6.11 is dramatically different with a severe loss of lift due to dynamic stall.
Figure 6.11. Lift coefficient data versus the pitch angle for segment 4
\[ \theta = 22.27 - 8.94 \cos \psi - 4.52 \sin \psi \]

Figure 6.12 shows the same \( C_L \) data but versus local angle of attack. Here, the increase in hysteresis and the loss of lift due to stall is clear. Figure 6.12 also shows that, although we doubled the pitch angles in an attempt to double the loads up to a simulated 2-g pull-up simulation, the effect of dynamic stall has been to lower loads such that roughly a 1.5-g pull-up is obtained. Nonetheless, this still is a higher average lift than could have been obtained if there were only static stall to consider. In that case, barely more than 1.2 g’s could have been obtained.
Figure 6.12. Lift coefficient data versus the angle of attack for segment 4
\[ \theta = 22.27 - 8.94 \cos \psi + 3.92 \sin \psi \]

### 6.4 Validation in Rotating Frame

At the beginning of this research, we were hoping to have three-dimensional, rotating-blade, dynamics stall data to correlate. (either from experiments or CFD). However, the data has not yet been obtained. Thus, at this point, all we can say is that every element of our theory has been validated against experimental data except for the radial coupling terms, which must have rotating-frame data for validation. We can also say that we have shown that all our elements can work together in an efficient code, and that they give reasonable results, as one would expect for a realistic rotor. However, we will wait for experimental orcomputational data for a complete, full-up test.
Chapter 7

Future Work

There are still some key aspects to the theory that need to be finished before it is ready to go into production simulation codes. These items we propose as future work beyond this thesis.

7.1. Pitching Moment

Previous work (Ref. [23]) verified the Ahaus-Peters model for pitching moments for unyawed flow with steady free-stream. However, these correlations were not nearly as good as those for lift. Thus, more work needs to be done on pitching moments with stall. In this present work, we have validated the pitching-moment model for two cases with only one airfoil at only one reduced frequency; and we have shown that we can obtain very good results by the use of different stall parameters on the upstroke and downstroke. The two cases are: 1.) one with steady free-stream velocity, and 2.) one with unsteady free-stream velocity. This validation now needs to be extended to more airfoils and more reduced frequencies. Furthermore, the validation needs to be extended to the case of yawed flow. Third, the validation needs to be done for cases with secondary stall peaks. This third area will be the most challenging since it is unclear whether or not the procedure outlined for lift (use of an impulse to drive a second-order equation) will also work for pitching moments. Thus, pitching moment is still a major area to be studied.

7.2. Drag

Another area that needs to be studied is drag. Ahuas (Ref. [23]) validated the drag model for steady free-stream velocity and no yaw; but, in this work, we have not looked at drag in the presence of unsteady free-stream, yawed flow, or secondary stall peak. Although there is not as
much drag data as there is lift and moment data (and although the drag curves do not appear to
deviate that much from quasi-steady theory), this area still needs to be addressed to close the
theory.

7.3. Higher Mach Numbers

The last area for future work is to validate at higher Mach numbers. Our linear theory has been
validated up to \( M = 0.78 \), but the stall model has only been validated up to \( M = 0.47 \). We
definitely need to look at validation at higher Mach numbers. Leishmann and Beddoes have
found that they need an extra state to account from compressibility at higher Mach numbers.
The same may be true for us, and this needs to be determined.

7.4. Three-Dimensional, Rotating-Blade Validation

The crown jewel in the future work would be the ability to correlate against three-dimensional,
rotating-blade dynamic stall data (either experimental or CFD). Although we had hoped that
such data would become available during this work, it has not happened as of yet. Hopefully,
during the follow-on work, we will be able to obtain some rotating-wing data. When that
happens, we will be able to identify the optimum radial coupling term and see whether or not
yawed flow, unsteady free-stream, secondary peak, and radial stall coupling will be enough to
simulate the data. If not, then added modeling will be necessary.
Chapter 8

Summary and Conclusions

8.1. Summary

The Ahaus-Peters dynamic stall model has been modified and verified for use in three-dimensional simulation of stall on a rotating blade. Each of the major components necessary for simulation of stall has been addressed with the results compared with experimental data:

1.) Modeling of secondary stall peak
2.) Treatment of the effect of yawed flow
3.) Modification for unsteady free-stream velocity
4.) Inclusion of three-dimensional induced flow effects
5.) Radial coupling due to convection of stall vortex
6.) Incorporation of all effects into a viable, 3-D code

Items 1-4 are completely vetted through correlation studies for lift coefficient, and items 1-3 have been correlated with a single test condition for pitching moment coefficient. Since no experimental data are available for rotating-frame stall, experimental validation of items 5-6 remains for future work. Table 8.1 lists all of the experimental studies against which the present model has been compared. Agreement is uniformly excellent. The simulation in item 6 above, although having no stall data to compare with, is nonetheless based on parameters for a wind tunnel test in which induced flow was measured, and that induced flow is well-matched by the theory.
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<th>Unsteady Free-Stream</th>
<th>Wake Model</th>
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Table 8.1. Summary of airload correlations. ( ✓: Yes, ✗: No )
8.2. Conclusions

The modeling of the secondary stall peak requires another second-order circulation be added to the system at each blade section. The equation for the secondary peak is driven by a pulse, which is triggered by the angular rate at the static stall angle. Since the equation is linear, the solution to the pulse is known closed-form, so that no new states need to be added to the method. The modeling of yawed flow on stall requires only limited static data. The Barwey, closed-form shift of the static stall curve can then be used for arbitrary yaw angle to shift the $\Delta C_L$ that drives the dynamic stall equation. Thus, once again, no new states are required. For the effect of unsteady free-stream velocity, it was determined that the original ONERA/Ahaus formulation needed to be modified by two quasi-static terms that reflect the fact the shed stall vortex does not change with free-stream velocity as it is transported. Only one new stall parameter is added, $\kappa$, which indicates the percentage of total lift that is taken to be fixed at the stalled value.

For three-dimensional induced flow, the Peters-He wake model has already been shown in other work to adequately model experimental induced-flow velocities. Since this model produces the average induced flow across the chord, $\lambda_0$, that is necessary for the Ahaus stall model, no new correlation is needed. It is simply coupled with the dynamic stall in linear airloads model. For radial coupling, a new radial coupling formulation has been introduced (with one tuning parameter) in the form of a coupling between adjacent blade elements based on radial flow. Once experimental data are available, this approach can be tuned and tested. The fully-operational 3-D code shows that the present approach can be efficiently programmed.
References


Appendix A

Vectors and Matrices of Unified Airload Model

This appendix defines the vectors and matrices used in the Ahaus-Peters unified airload model. In this Appendix, $M$ corresponds to the number of states in the Glauert expansion which results in $(M + 1) \times (M + 1)$ matrices and $(M + 1) \times 1$ vectors. Besides, $N$ refers to the number of inflow states in Peters-Karunamoorthy 2D finite state induced flow Model and it results in $N \times N$ matrices and $N \times 1$ vectors.

In vector $\{e\}$ the first element, $f$, corresponds to the reversed flow parameter which can be defined from Eq. (2.27). Moreover, elements of vector $\{b\}$ are obtained from Eq. (2.49).

\[
b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}, \quad c = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ \vdots \\ 2 \\ N \end{bmatrix}, \quad d = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad e = \begin{bmatrix} f \\ 1/2 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, \quad f = \begin{bmatrix} 0 \\ f \\ 2 \\ 3f \\ \vdots \\ 0 \end{bmatrix}
\]

\[
h_n = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_M \end{bmatrix}, \quad v_n = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ 0 \end{bmatrix}, \quad \lambda_0 = \begin{bmatrix} \lambda_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \lambda_1 = \begin{bmatrix} \lambda_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad 1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\]

\[
[A] = [D + db^T + cd^T + \frac{1}{2} cb^T]
\]
$[C] = \begin{bmatrix} f & 1 & 0 & 0 & 0 & \cdots \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \cdots \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \cdots \\ 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & \ddots \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$

$[D] = \begin{bmatrix} 0 & -\frac{1}{2} & 0 & 0 & \cdots & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} & 0 & \cdots & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{6} & \cdots & 0 \\ 0 & 0 & \frac{1}{8} & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \frac{1}{2N} & 0 \end{bmatrix}$

$[G] = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \cdots \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \cdots \\ 0 & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & \ddots \\ 0 & 0 & -\frac{1}{4} & 0 & \frac{1}{4} & \ddots \\ 0 & 0 & 0 & -\frac{1}{4} & 0 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$

$[H] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \cdots \\ 0 & 0 & \frac{2}{2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \frac{3}{2} & 0 & \cdots \\ 0 & 0 & 0 & 0 & \frac{4}{2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$

$[K] = \begin{bmatrix} 0 & f & 2 & 3f & 4 & \cdots \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \cdots \\ 0 & 0 & -\frac{2}{2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & -\frac{3}{2} & 0 & \ddots \\ 0 & 0 & 0 & 0 & \ddots & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & -\frac{M}{2} \end{bmatrix}$
\[ M_{00} = \frac{1}{2}, \quad M_{11} = \frac{1}{16}, \quad M_{02} = M_{20} = -\frac{1}{4} \]

\[ M_{nn} = \frac{n}{4(n^2 - 1)}, \quad n \geq 2 \quad \Rightarrow \quad [M] = \begin{bmatrix}
\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & \cdots \\
0 & \frac{1}{16} & 0 & -\frac{1}{16} & 0 & \cdots \\
-\frac{1}{4} & 0 & \frac{1}{6} & 0 & \ddots & \ddots \\
0 & -\frac{1}{16} & 0 & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots & -\frac{1}{8M} & \ddots & \frac{M}{4(M^2 - 1)}
\end{bmatrix} \]

\[ [S] = \begin{bmatrix}
f & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{bmatrix} \]
Appendix B

Wing & Wind Coordinate Systems

This Appendix reviews the flow geometry in both wing and wind axis systems. Figure B.1 shows the flow geometry in the wing (blade-fixed) coordinate systems. As one can see, the $x$-axis is along the wing plane and it is taken positive toward the trailing edge, while the $y$-axis is perpendicular to the wing plane and it is taken positive in the positive lift direction (up). The $z$-axis is along the wing (perpendicular to both $x$ and $y$ axes) and its direction is defined using the left-hand rule. The following relations can be written from Fig. B.1.

\[
U_D = \sqrt{U_R^2 + U_T^2} \tag{B.1}
\]

\[
U = \sqrt{U_T^2 + U_P^2} \tag{B.2}
\]

\[
U_S = \sqrt{U_R^2 + U_P^2} \tag{B.3}
\]

\[
U_F = \sqrt{U_R^2 + U_P^2 + U_T^2} = \frac{\sqrt{(U_D^2 + U^2 + U_S^2)}}{2} \tag{B.4}
\]

Figure B.1. Flow geometry in wing-axis system [68]
In Eqs. (B.1)-(B.4), $U_T$ is air velocity of the blade section tangent to the disk plane, $U_P$ is air velocity of the blade section perpendicular to the disk plane, $U_R$ is the radial velocity, $U_D$ is the total velocity in disk plane, $U$ is the blade section resultant velocity perpendicular to the spanwise direction, $U_S$ is the resultant air velocity perpendicular to $U_T$ and $U_F$ is the total resultant velocity. Moreover, the angles between the flow components and the blade are defined as follows.

$$\alpha = \tan^{-1} \left( \frac{U_P}{U_T} \right) \quad (B.5)$$

$$\Lambda = \tan^{-1} \left( \frac{U_R}{U_T} \right) \quad (B.6)$$

where $\alpha$ is the angle of attack and $\Lambda$ is the yaw angle in the wing-axis system. Each of these angles varies between $-180^\circ$ and $+180^\circ$. Figure B.2 shows the flow components in the wind-axis system, where a new set of angles has been considered.

$$\alpha_D = \sin^{-1} \left( \frac{U_P}{U_F} \right) \quad -90^\circ \leq \alpha_D \leq +90^\circ \quad (U_T \geq 0) \quad (B.7)$$

$$\Lambda_T = \sin^{-1} \left( \frac{U_R}{U_F} \right) \quad -90^\circ \leq \Lambda_T \leq +90^\circ \quad (U_T \geq 0) \quad (B.8)$$

where, $\alpha_D$ represents the wind tunnel angle of attack and $\Lambda_T$ is the wind tunnel yaw angle.

Figure B.2. Flow geometry in wind-axis system [68]
Equations (B.7) and (B.8) can be extended to wind tunnel definitions of angle of attack and yaw through ±180° using the following equations.

\[ \alpha_D = 180° - \sin^{-1} \left( \frac{U_r}{U_T} \right) \quad (U_T < 0) \quad (B.9) \]
\[ \Lambda_T = 180° - \sin^{-1} \left( \frac{U_r}{U_T} \right) \quad (U_T < 0) \quad (B.10) \]

It is also worth mentioning that considering no reversed flow, Eqs. (B.7) and (B.8) can be used in order to do the transformation between the wind-axis angles and the wing-axis ones.

\[ \sin \alpha_D = \sin \alpha \cos \Lambda_T \quad (B.11) \]
\[ \sin \Lambda_T = \sin \Lambda \cos \alpha_D \quad (B.12) \]

From Eqs. (B.7) and (B.8), one can see that for the small angles assumption, there is no difference between the angles defined in wind coordinate system and those defined in wing coordinate systems.
Appendix C

Ellipsoidal Coordinate System

As it is mentioned in section 4.1.3, the inflow and pressure distribution are represented in the ellipsoidal coordinate system. In this appendix we will define this coordinate system.

The ellipsoidal coordinate system is defined as follows

\[ x = -\sqrt{1-n^2} \sqrt{1+\eta^2} \cos \psi \] \hspace{1cm} (C.1)
\[ y = \sqrt{1-n^2} \sqrt{1+\eta^2} \sin \psi \] \hspace{1cm} (C.2)
\[ z = -\nu \eta \] \hspace{1cm} (C.3)

where

\[ -1 \leq \nu \leq +1 \] \hspace{1cm} (C.4)
\[ 0 \leq \eta < \infty \] \hspace{1cm} (C.5)
\[ 0 \leq \psi \leq 2\pi \] \hspace{1cm} (C.6)

The \(xz\) view of the ellipsoidal coordinate system is shown in Fig. C.1. The \(\eta = \) constant surfaces represent ellipsoids while the \(\nu = \) constant surfaces represent hyperboloids. Note that \(\eta = 0\) represents the rotor disk, and the sign of \(\nu\) goes from negative to positive crossing the rotor disk. Moreover, \(\psi\) is the azimuthal angle that is measured positive from the negative \(x\)-axis, counterclockwise looking along the positive \(z\)-axis.
Figure C.1. Ellipsoidal coordinate system (viewed in the $xz$ plane)

The inverse of the transformation can also be obtained using Eqs. (C.7)-(C.9).

$$
\nu = -\frac{1}{\sqrt{2}} \text{sgn}(z) \sqrt{(1 - S) + \sqrt{(1 - S)^2 + 4z^2}}
$$  \hspace{1cm} (C.7)

$$
\eta = -\frac{z}{\nu}
$$  \hspace{1cm} (C.8)

$$
\bar{\psi} = \tan^{-1}\left(\frac{y}{-x}\right)
$$  \hspace{1cm} (C.9)

where

$$
S = x^2 + y^2 + z^2
$$  \hspace{1cm} (C.10)
Vita

Ramin Modarres

Degrees

Ph.D., Mechanical Engineering, May 2016
M.Sc., Aerospace Engineering, December 2015
M.Sc., Mechanical Engineering, August 2013
B.Sc., Mechanical Engineering, December 2008

Professional Societies

American Helicopter Society (AHS)
American Institute of Aeronautics and Astronautics (AIAA)

Honors


Journal Publications


Conference Papers


May 2016