A Nagy-Foias model for commuting pairs of contractions

Abstract

The starting point for the Nagy-Foias model for a contractive operator $T$ on Hilbert space is Sz.-Nagy’s observation that $T$ has a canonical minimal unitary dilation to a larger Hilbert space. For a pair $T = (T_1, T_2)$ of commuting contractions, Ando’s theorem asserts that there exist commuting unitary dilations of $T$ to larger Hilbert spaces, and one might aspire to extend the Nagy-Foias model to such operator pairs. However, the dilations provided by Ando’s theorem are far from being canonical, and this fact appears to rule out a good model theory for $T$.

It has recently been shown that nevertheless there is such a model theory, though it requires a slight shift in perspective. One focuses on the commuting pair $(S, P)$, where $S = T_1 + T_2$, $P = T_1T_2$. The operator pair $(S, P)$ is a $\Gamma$-contraction, which means that the set

$$\Gamma = \{ (z + w, zw) : |z| \leq 1, |w| \leq 1 \}$$

is a spectral set for $(S, P)$. One constructs a canonical dilation, and thereafter a functional model, not for the individual operators $T_1, T_2$, but for the pair $(S, P)$. The model parallels closely the original Nagy-Foias model.

This line of investigation was begun by Agler and Young, and successfully developed and brought to a conclusion by members of the Indian school of operator theory.

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