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A band formula for a Toeplitz commutant lifting problem

Abstract

The band method plays a fundamental role in solving a Toeplitz and Nehari interpolation problem; see [2]. The solution to the Nehari problem involves the inverses of $I - HH^*$ and $I - H^*H$ where $H$ is the corresponding Hankel matrix. Here we will derive a similar result for a certain commutant lifting problem.

Let $\Theta$ be an inner function in $H^\infty(E,Y)$ and $H(\Theta)$ the subspace of $\ell_2^2(Y)$ defined by

$$H(\Theta) = \ell_2^2(Y) \ominus T_\Theta \ell_2^2(E)$$

where $T_\Theta$ is the Toeplitz operator determined by $\Theta$. Clearly, $H(\Theta)$ is an invariant subspace for the backward shift $S_Y$. Consider the data set $\{A, T', S_Y\}$ where $A$ is a strict contraction mapping $\ell_2^2(U)$ into $H(\Theta)$, the operator $T'$ on $H(\Theta)$ is the compression of $S_Y$ to $H(\Theta)$, that is,

$$T' = \Pi_{H(\Theta)} S_Y | H(\Theta)$$

Here $\Pi_{H(\Theta)}$ is the orthogonal projection from $\ell_2^2(Y)$ onto $H(\Theta)$. Moreover, $A$ intertwines $S_U$ with $T'$, that is, $T'A = AS_U$. Given this data set the commutant lifting problem is to find all contractive Toeplitz operators $T_\Psi$ such that

$$\Pi_{H(\Theta)} T_\Psi = A. \quad (1)$$

This lifting problem includes the Nevanlinna-Pick and Leech interpolation problems. Using two different methods we will show that the set of all solutions are given by

$$\Psi = (\Upsilon_{12} + \Upsilon_{11}g)(\Upsilon_{22} + \Upsilon_{21}g)^{-1}.$$

Here $g$ is a contractive analytic function acting between the appropriate spaces. Analogously to the band formulas in the Nehari interpolation problem, $\Upsilon_{jk}$ are determined by the inverses of $I - AA^*$ and $I - A^*A$. The proofs rely on different techniques. Finally, this is joint work with S. ter Horst and M.A. Kaashoek.

References


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Talk location: Cupples I Room 113

Special Session: State space methods in operator and function theory. Organized by J. Ball and S. ter Horst.