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On singular integral operators with linear-fractional involutions

Abstract

We denote the Cauchy singular integral operator along a contour \( \mathcal{L} \) by \( (S_\mathcal{L}\varphi)(t) = \frac{1}{\pi i} \int_{\mathcal{L}} \frac{\varphi(\tau)}{\tau - t} d\tau \) and the identity operator by \( (I_\mathcal{L}\varphi)(t) = \varphi(t) \).

In the paper [1,2] we constructed a similarity transformation \( F^{-1}AF = D \), between the singular integral operators \( A \) with the rotation operator \( W_T \) through the angle \( 2\pi/m \) on the unit circle \( T \), acting on the space \( L^2_2(T) \), and a certain matrix characteristic singular integral operator without shifts acting on the space \( L^2_2(T) \). For \( m = 2 \), we have \( (W_T\varphi)(t) = \varphi(-t) \),

\[
A = a_0I_T + b_0S_T + a_1W_T + b_1S_TW_T, \quad A \in [L^2_2(T)], D = uI_T + vS_T, \quad D \in [L^2_2(T)].
\]

the right hand and left-hand side we reduced

\[
B_R = aI_R + bQ_R + cS_R + dQ_RS_R, \quad B_R \in [L^2_2(\mathbb{R})], \quad \mathbb{R} = (+\infty, -\infty),
\]

where involution \( (Q_R\varphi)(x) = \sqrt{\delta^2 + \beta x - \delta^2} \varphi[\alpha(x)] \), with \( \alpha(x) = \frac{\delta x + \beta}{\delta x - \delta^2} \), \( \delta^2 + \beta > 0 \),

to a matrix characteristic singular integral operator without shift:

\[
\mathcal{H}BF = D_{R_+}, \quad D_{R_+} = uR_+I_{R_+} + vR_+S_{R_+},
\]

acting on the space \( L^2_2(\mathbb{R}_+, x^{-\frac{1}{2}}) \), \( \mathbb{R}_+ = (0, +\infty) \). We will refer to the formulas as operator equalities. Different applications of operator equalities to singular integral operators and to boundary value problems are considered.
