Monotonicity in several non-commuting variables

Abstract

Let \( f : (a, b) \rightarrow \mathbb{R} \). The function \( f \) is said to be matrix monotone if \( A \leq B \) implies \( f(A) \leq f(B) \) for all pairs of like-sized self-adjoint matrices with spectrum in \((a, b)\). Classically, Charles Loewner showed that a bounded Borel function is matrix monotone if and only if it is analytic and extends to be a self-map of the upper half plane. The theory of matrix monotonicity has profound consequences for any general theory of matrix inequalities. For example, it might seem surprising that \( X \leq Y \) does not imply that \( X^2 \leq Y^2 \), which is a consequence of Loewner’s theorem. We will discuss commutative and noncommutative generalizations to several variables of Loewner’s theorem.

Talk time: 07/22/2016 2:30PM—07/22/2016 2:50PM
Talk location: Crow 206