A New Necessary Condition for the Hyponormality of Toeplitz Operators on the Bergman Space

Abstract

It is a well known result of C. Cowen that, for a symbol $\varphi \in L^\infty$, $\varphi = f + g$ ($f, g \in H^2$), the Toeplitz operator $T_\varphi$ acting on the Hardy space of the unit circle is hyponormal if and only if $f = c + T_\bar{h} g$, for some $c \in \mathbb{C}, h \in H^\infty, \|h\|_\infty \leq 1$. In this talk we will consider possible versions of this result in the it Bergman space case.

Concretely, we consider Toeplitz operators on the Bergman space of the unit disk, with symbols of the form

$$\varphi \equiv \alpha z^n + \beta z^m + \gamma \bar{z}^p + \delta \bar{z}^q,$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ and $m, n, p, q \in \mathbb{Z}^+$, $m < n$ and $p < q$. By letting $T_\varphi$ act on vectors of the form

$$z^k + cz^\ell + dz^r (k < \ell < r),$$

we study the asymptotic behavior of a suitable matrix of inner products, as $k \to \infty$. As a consequence, we obtain a rather sharp inequality involving the above mentioned data:

$$|\alpha|^2 n^2 + |\beta|^2 m^2 - |\gamma|^2 p^2 - |\delta|^2 q^2 \geq 2 |\bar{\alpha}\beta mn - \bar{\gamma}\delta pq|.$$

This result is intended to be a precursor of basic necessary conditions for joint hyponormality of commuting tuples of Toeplitz operators acting on Bergman spaces in several complex variables.

The talk is based on joint work with Željko Ćučković (University of Toledo, USA).

Talk time: 07/19/2016 3:00PM—07/19/2016 3:20PM
Talk location: Crow 206

Special Session: Multivariable operator theory. Organized by H. Woerdeman.