The core variety of a multisequence in the truncated moment problem

Abstract

Let $K$ denote a nonempty closed subset of $\mathbb{R}^n$, let $m = 2d$, and let $\beta \equiv \beta^{(m)} = \{\beta_i \in \mathbb{Z}_+^n, |i| \leq m, \beta_0 > 0\}$, denote a real $n$-dimensional multisequence of finite degree $m$. The Truncated $K$-Moment Problem concerns the existence of a positive Borel measure $\mu$, supported in $K$, such that

$$\beta_i = \int_{\mathbb{R}^n} x^i d\mu \quad (i \in \mathbb{Z}_+^n, \ |i| \leq m).$$

The core variety of $\beta$, $V \equiv V(\beta)$, is an algebraic variety in $\mathbb{R}^n$ that contains the support of any such $K$-representing measure. In previous work we showed, conversely, that if $V$ is a nonempty compact set, or $V$ is nonempty and is a determining set for polynomials of degree at most $m$ (in particular, if $V = \mathbb{R}^n$), then $\beta$ admits a $V$-representing measure. We describe some additional cases where a nonempty core variety implies the existence of a representing measure.

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