Implementation and Evaluation of Mixed-Criticality Scheduling Approaches for Periodic Tasks

Authors: Huang-Ming Huang, Christopher Gill, and Chenyang Lu

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Abstract—Traditional fixed-priority scheduling analysis for periodic task sets is based on the assumption that all tasks are equally critical to the correct operation of the system. Therefore, every task has to be schedulable under the scheduling policy, and estimates of tasks’ worst case execution times must be conservative in case a task runs longer than is usual. To address the significant under-utilization of a system’s resources under normal operating conditions that can arise from these assumptions, three main approaches have been proposed: priority assignment, period transformation, and zero-slack scheduling. However, to date there has been no quantitative comparison of system schedulability or run-time overhead for the different approaches. In this paper, we present what is to our knowledge the first side-by-side evaluation of those approaches, for periodic mixed-criticality tasks on uniprocessor systems, under a mixed-criticality scheduling model that is common to all three approaches. To make a fair evaluation of zero-slack scheduling, we also address two previously open issues: how to accommodate execution of a task after its deadline, and how to account for previously unidentified forms of interference between mixed-criticality tasks. Our simulations show that while priority assignment and period transformation are most likely to be able to schedule a randomly selected task set, a small fraction of the task sets are schedulable only under the zero-slack approach. Our empirical evaluation demonstrates that user-space implementations of mechanisms to enforce period transformation and zero-slack scheduling can be achieved on Linux without kernel modification, with suitably low overhead for mixed-criticality real-time task sets.

I. INTRODUCTION

Traditional fixed-priority scheduling analysis for periodic task sets assumes that all tasks are equally critical to a system’s correct operation; thus, every task has to be schedulable under the scheduling policy. To meet this assumption, the estimation of worst case execution time for tasks has to be conservative in order to accommodate the special case when a task runs longer than average. Such a conservative approach can lead to under-utilization of a system under normal operating conditions.

Mixed criticality models: To address this issue, Vestal et al. [1, 2] and de Niz et al. [3, 4] have developed alternative mixed-criticality models for systems in which tasks are not equally critical. In the first model [1, 2], each task \( \tau_i \) may have a set of alternative execution times \( C_i(\ell) \), each having a different level of confidence \( \ell \). A task \( \tau_i \) is also assigned a criticality level \( \zeta_i \), which corresponds to the required level of confidence for the task and is used in schedulability analysis.

The second model [3, 4] is a special case of the first one, where each task can specify only two execution times: a normal worst case execution time \( C_i^\nu \) and an overload budget \( C_i^\sigma \). Assuming all confidence and criticality levels are positive integers, with larger values indicating higher confidence and higher criticality, the execution times of a task \( \tau_i \) are

\[
C_i(\ell) = \begin{cases} 
C_i^\nu & \text{if } \zeta_i > \ell, \\
C_i^\sigma & \text{otherwise}.
\end{cases}
\]

Mixed criticality enforcement: To ensure that no lower-criticality task prevents a higher-criticality task from meeting its deadline, a scheduler could use the criticality of a task directly as its scheduling priority. However, this would unnecessarily penalize lower-criticality tasks when the system is not overloaded.

To improve the schedulability of lower-criticality tasks while preserving the mixed-criticality scheduling guarantee, Vestal et al. [1, 2] proposed the use of Audsley’s priority assignment scheme [5] and the period transformation technique proposed by Sha et al. [6]. The priority assignment approach is based on two important observations: (1) the response time of a task is determined if the set of its higher priority tasks (\( \Gamma^h \)) is known, regardless of the relative priority ordering within \( \Gamma^h \); and (2) if a task is schedulable at a given priority level, then it remains schedulable when it is assigned a higher priority. The algorithm operates in increasing priority order, at each step selecting a task and, if it is schedulable, assigning it the current priority and then moving to the next higher priority; otherwise, another task is selected at the current priority level. The algorithm terminates when all tasks are assigned priorities or when no remaining task is schedulable at the current priority.

In the period transformation approach, if a higher-criticality task \( \tau_i \) has a longer period than a lower-criticality task \( \tau_j \), the higher-criticality task is equally sliced into \( n_i \) sections such that \( \tau_i \) has a smaller transformed period and execution time. Task priorities are then assigned in rate monotonic order according to the transformed periods under the assumption (which the run-time environment may need...
to enforce) that \( \tau_i \) can run for no more than \( C_i^0/n_i \) time units within each transformed period.

More recently, de Niz et al. [3, 4] proposed a zero-slack scheduling method for mixed-criticality tasks under the restricted model shown in Equation 1. Because zero-slack scheduling, priority assignment, and period transformation approaches share only that model in common, we will consider it instead of the more general model unless explicitly noted otherwise. Zero-slack scheduling is a bi-modal scheduling policy, where every task has a normal mode and a critical mode. When a task is in its normal mode, it is scheduled based on its priority (assigned by a rate-monotonic or deadline-monotonic policy). When a task \( \tau_i \) is in critical mode, the scheduler will suspend all lower-criticality tasks; in other words, \( \tau_i \) will steal slack from lower-criticality tasks when it is in critical mode.

Limitations of the state of the art: Despite their potential to improve schedulability of mixed-criticality task sets, there has been no practical comparison of the system schedulability or run-time implementation overhead implications for these different approaches. There are also two important issues with the zero-slack scheduling approach which must be addressed in practice to ensure that overloaded lower-criticality tasks cannot impair the schedulability of higher-criticality tasks: (1) since it is difficult to simply halt threads safely atop commonly available operating systems, the implicit assumption that real-time tasks that miss their deadlines are simply dropped rather than being allowed to continue to run must be removed; (2) a particular form of interference that is not accounted for in the previously published analyses of zero-slack scheduling also must be addressed. Finally, after these issues are addressed, efficient mechanisms to support zero-slack scheduling and period transformation atop commonly available operating systems must be implemented, and their expected overheads must be quantified empirically.

Contributions of this work: The primary contribution of this work is a practical implementation and evaluation of different mixed-criticality scheduling approaches, atop a realtime capable version of the commonly available Linux operating system. To our knowledge this is the first system implementation and comparative evaluation of different mixed-critical scheduling approaches for periodic tasks. Specifically, our contributions are four-fold.

- Extensions and improvement of the zero-slack scheduling algorithm and analysis to (i) accommodate execution of tasks beyond their deadlines and (ii) refine the calculation of zero-slack instants to account for forms of interference not considered in previously published analyses.
- Simulations of schedulability under different approaches to mixed-criticality scheduling which show that while the application of Audsley’s priority assignment and period transformation are most likely to be able to schedule randomly selected mixed criticality task sets. Nevertheless, a fraction of the task sets are schedulable only under the zero-slack approach.
- User-space implementation of enforcement mechanisms for zero-slack scheduling and period transformation on Linux.
- Empirical evaluation on a Linux platform which demonstrates the efficiency and efficacy of these mechanisms. Our results show that both zero-slack scheduling and period transformation impose only 0.2% and 0.4% additional overhead respectively, which demonstrates their viability in practice.

II. ZERO-SLACK SCHEDULING AND OUR IMPROVEMENTS

A. Background

The pre-requisite for zero-slack scheduling [3] is to decide the zero-slack instant (ZSI) of each task. Each ZSI may be computed offline and then provided to the scheduler for runtime enforcement. The objective of the ZSI computation is to find the latest possible instant for a task \( \tau_i \) to switch mode in order to reduce its impact on the schedulability of lower-criticality tasks while still maintaining the schedulability of \( \tau_i \). In [3], de Niz et al. detailed such an algorithm for calculating ZSIs for independent task sets on uniprocessor systems.

The algorithm starts with the worst case assumption that \( \tau_i \) is executed only in critical mode. Based on this assumption, it computes the time \( k_i \) needed for a job \( J_{i,1} \) of \( \tau_i \) to execute up to its overload budget \( C_i^0 \) under interference from its higher-criticality tasks. Let the release time of \( J_{i,1} \) be time 0, and the deadline of \( \tau_i \) be \( D_i \). Then \( t = D_i - k_i \) is the instant that \( J_{i,1} \) can switch from the normal mode to the critical mode, so that \( J_{i,1} \) will meet its deadline even when it is overloaded. However, setting the ZSI of \( \tau_i \) to \( D_i - k_i \) so that \( J_{i,1} \) switches mode at that time may be too pessimistic. \( J_{i,1} \) may have executed for a certain amount of time in normal mode, and thus the time budget in critical mode can be overestimated. To reduce this pessimism, the algorithm finds the minimum amount \( (\theta_i) \) of slack available for a task in normal mode and then deducts that slack from the overload budget. With the reduced budget in critical mode, the ZSI \( Z_i \) of \( \tau_i \) then can be moved closer to the deadline. The algorithm repeats this recalculation of \( k_i \) and \( Z_i \) until no more slack is available in normal mode for \( \tau_i \).

How much slack is available for a task \( \tau_i \) to be executed in normal mode is affected by the ZSIs of other tasks which may interfere with \( \tau_i \). That is, there are dependencies for ZSI calculations among tasks. To make the ZSI of a task as late as possible, the algorithm calculates the ZSIs of all tasks with an assumption of maximum interference (i.e., \( \theta_i = 0 \) for all \( \tau_j \)), updates \( \theta_i \) with each computed ZSI, re-calculates all ZSIs with the updated \( \theta_i \), and then continues until the ZSIs of all tasks converge. Since the algorithm relies on the convergence of ZSIs, [3] also provides a proof that the
algorithm will converge as long as the deadline of each task is less than its period.

For convenience of presentation, we will assume that task deadlines equal their corresponding task periods and that task priorities are assigned in accordance with the rate monotonic scheduling policy in all subsequent zero-slack scheduling (ZSRM) examples.

B. Execution After a Missed Deadline

The original zero-slack scheduler [3] is based on the scheduling guarantee that if a task \( \tau_i \) is admitted, it will be able to run up to its overloaded budget \( C_i^{\omega} \) within its deadline as long as no higher-criticality task is overloaded. A task is referred to as schedulable if it satisfies this scheduling guarantee. However, this is based on the assumption that no lower-criticality task misses its deadline or that if it does it is simply dropped rather than allowing it to execute beyond its deadline.

### Table I: A two task ZSRM example

<table>
<thead>
<tr>
<th>Task</th>
<th>( C_i^{\omega} )</th>
<th>( C_i^{\omega} )</th>
<th>Period</th>
<th>Criticality</th>
<th>Priority</th>
<th>ZSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td>2</td>
<td>Low</td>
<td>6</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>High</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 1: Zero-slack rate-monotonic scheduling of the task set in Table I

Figure 1 illustrates that, in the example task set shown in Table I, the overloading of a lower-criticality task could trigger the deadline miss of a higher-criticality task under the original zero-slack scheduling approach in [3]. In this example, all jobs of \( \tau_1 \) are overloaded and run for 5 time units, and \( J_{2,1}, J_{2,2}, \) and \( J_{2,3} \) are also overloaded and run for 3 time units. From Figure 1, we can see that job \( J_{1,1} \) misses its deadline because the lower-criticality task \( \tau_2 \) misses its deadline. Furthermore, \( J_{2,3} \) also misses its deadline. In other words, \( J_{1,2} \) and \( J_{1,3} \) break the scheduling guarantee even though no higher-criticality task is present.

In theory, this problem can be solved by terminating a job when it misses its deadline. In practice, this may be problematic because the target job could be holding resources such as mutexes, which could lead to deadlocks and other problems. Except in special cases where jobs of the same task cannot share resources or where tasks can be made aware of their deadlines’ expirations and can cooperatively release resources and halt execution, that approach is thus impractical on standard platforms.

A better approach is for the scheduler to demote task \( \tau_i \) to the lowest priority when it misses its deadline; at the same time, all lower-criticality tasks have to be suspended and can be restored to their original priorities only if the job that missed its deadline terminates. \( \tau_i \) can miss its deadline only if one or more higher-criticality tasks is overloaded, and thus neither \( \tau_i \) nor its lower-criticality tasks are required to remain schedulable. In the scenario where more than one task misses its deadline, only the ones at the highest criticality level among them will be in the runnable state with the lowest priority level; the others will be suspended.

We refer to this new scheduling rule as the demotion-on-deadline rule. Figure 2 shows the schedule for the task set in Table I using the demotion-on-deadline rule.

C. Unaccounted Interferences

Let \( \pi_i \) be the priority of task \( \tau_i \), with a larger value representing higher priority. The original analysis of ZSI calculation [3] is based on a particular worst case phasing assumption: given a job \( J_{i,1} \) of task \( \tau_i \) which is released at time 0, \( J_{i,1} \) suffers the maximum interference while in normal mode from \( \tau_j \) when the jobs of higher-or-same-priority tasks are released at time 0. In addition, the instant is also aligned to the ZSIs of the jobs from the set of tasks with lower priority and higher criticality than task \( \tau_i \), \( L_i^{hc} = \{ \tau_j | \pi_j < \pi_i \text{ and } \zeta_j > \zeta_i \} \). However, that formulation considers only tasks that can interfere directly with \( \tau_i \), although some tasks which cannot preempt \( \tau_i \) directly may also interfere with \( \tau_i \) through tasks from the set of tasks with greater or equal priority and lower criticality than task \( \tau_i \), \( H_i^{lc} = \{ \tau_j | \pi_j \geq \pi_i \text{ and } \zeta_j < \zeta_i \} \).

### Table II: Example task set for worst case phasing condition

<table>
<thead>
<tr>
<th>( C_i^{\omega} )</th>
<th>( C_i^{\omega} )</th>
<th>Period</th>
<th>Criticality</th>
<th>Priority</th>
<th>ZSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>Med</td>
<td>8</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>4</td>
<td>5</td>
<td>15</td>
<td>Low</td>
<td>9</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>High</td>
<td>0</td>
</tr>
</tbody>
</table>

For example, consider the task set in Table II scheduled with ZSRM. Figure 3 shows a schedule where the second
and third jobs of τ3 are overloaded and run for 3 and 5 time units respectively; in addition, the first job of τ2 runs only for 1 time unit (valid because the specification does not require τ2 to run for at least \( C^N_i \) time units). By the original analysis, the normal mode slack vector of τ2 is \( \{4,3\} \), i.e., τ2 can run for at least three time unit starting from time 4. However, as is shown in Figure 3, J1,2 does not have any slack before its ZSI (at time 18). If J1,2 is also overloaded and runs for more than 2 time units, τ1 would miss its deadline because of the interference from lower-criticality task τ3, and thus the scheduling guarantee would be violated.

We therefore introduce a revised ZSI calculation algorithm which addresses the previously unaccounted for interference. We define \( \theta_j(\zeta_m) \) to be the minimum slack that can be used by \( \tau_j \) before \( Z_i \) at criticality level \( \zeta_m \). This value is initialized to 0 for all tasks and can be increased during the ZSI calculation.

Let \( C_j(\zeta_m) \) be the maximum execution time of \( \tau_j \) at criticality level \( \zeta_m \), as described in Equation 1. In addition, let \( I^j_\tau \) be the effective execution interval of \( \tau_j \) that can interfere with \( \tau_i \) at criticality level \( \zeta_m \), which can be expressed by the following equation if the priority assignment is rate-monotonic or deadline-monotonic:

\[
I^j_\tau(\zeta_m) = \begin{cases} 
\max(C_j(\zeta_m) - \theta_j(\zeta_m), 0) & \text{if } \tau_j \in L_i^{hc} \text{ or } L_i^{ec} \\
C_j(\zeta_m) & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (2)

The rationale for this equation is based on the observation that when \( \tau_j \in L_i^{hc} \), the minimum amount of time \( \tau_j \) spends in its normal mode is \( \theta_j \) and \( \tau_j \) only interferes with \( \tau_i \) when \( \tau_j \) is in critical mode; therefore, \( I^j_\tau \) is \( C_j(\zeta_m) - \theta_j(\zeta_m) \) (bounded by 0 if \( C_j(\zeta_m) < \theta_j(\zeta_m) \)).

### D. Worst Case Alignment

To obtain \( \theta_j \), we need to know the maximum possible amount of interference \( \tau_i \) can suffer from other tasks. Let \( H_i^{hc} = \{ \tau_j | \pi_j \geq \pi_i \text{ and } \zeta_j > \zeta_i \} \) be the set of tasks with greater or equal priority and higher criticality than task \( \tau_i \). Let \( H_i^{ec} = \{ \tau_j | \pi_j \geq \pi_i \text{ and } \zeta_j = \zeta_i \} \) be the set of tasks with greater or equal priority than, and the same criticality as, task \( \tau_i \). Let \( \Gamma_i = L_i^{hc} \cup H_i^{hc} \cup H_i^{ec} \cup L_i^{ec} \) be the set of interfering tasks for task \( \tau_i \) in normal mode. The ZSI calculation in [3] assumed that all release times of tasks from \( \Gamma_i - L_i^{hc} \) are aligned to the release time of \( \tau_i \), as are the ZSIs of tasks from \( L_i^{hc} \). However, as we show in Figure 3, this may not be the worst case phasing condition.

The key to the worst case phasing condition for zero-sack scheduling is the alignment between \( \tau_i \) and the other tasks in \( \Gamma_i \). Let \( t_{1,i} \) be the time at which job \( J_{1,i} \) is released; let \( t_{2,i} \) be the last job of \( \tau_j \in \Gamma_i \) which is released no later than \( t_{1,i} \). To maximize the interference with \( \tau_j \), the time when \( J_{2,i} \) is able to interfere with \( \tau_i \) should be no earlier than \( t_{2,i} \) and as close to \( t_{1,i} \) as possible. For the example in Figure 3, \( \tau_j \) suffers the maximum interference from \( \tau_3 \) when the instant \( t_{3,i} \) at which \( J_{3,2} \) starts execution aligns with the release time of \( J_{1,2} \), and then \( J_{3,2} \) terminates. Based on this observation, in Theorem 1 we formally state the conditions for the worst case phasing that maximizes the interference that must be considered for the ZSI calculation.

**Theorem 1.** Given two jobs \( J_{1,i} \) and \( J_{2,i} \) of tasks \( \tau_i \) and \( \tau_j \) respectively, let \( L_i^{hc} = \{ \tau_j | \pi_j < \pi_i \text{ and } \zeta_j < \zeta_i \} \) and \( L_i^{ec} = \{ \tau_j | \pi_j < \pi_i \text{ and } \zeta_j = \zeta_i \} \). Let \( t_{1,i} \) and \( t_{2,i} \) be the times when \( J_{1,i} \) is released and when it finishes its execution, respectively. Let \( t_{3,i,b} \) be an instant between \( t_{1,i} \) and \( t_{2,i} \) before \( J_{2,i} \) starts to execute, and let \( t \) be a time interval starting from \( t_{3,i,b} \). Further, given that no task \( \tau_k \) that satisfies the following two conditions is executed within the interval \( [t_{3,i,b}, t_{3,i}] \): (1) \( \tau_k \in L_i^{hc} \cup L_i^{ec} \) and \( \zeta_k \geq \zeta_i \), and (2) \( \tau_k \in L_i^{hc} \) and \( \tau_k \) is in normal mode, then \( J_{1,i} \) suffers the maximum interference from \( \tau_j \) in the time interval \( t \) if its release time \( t_{r,i} \) is aligned with the time \( t_{3,i} \).

**Proof:** Illustrated in Figure 4, \( J_{2,i} \) cannot be executed before \( t_{3,i} \); therefore, \( J_{1,i} \) will always suffer less interference from the subsequent jobs of \( J_{2,i} \) within the interval \( t \) if \( t_{r,i} \in [t_{3,i,b}, t_{1,i}] \). By definition, any task \( \tau_k \) that satisfies the above two conditions cannot be executed within the interval \( [t_{3,i}, t_{3,i}^f] \). If \( \tau_k \) is executed before \( t_{3,i} \), \( J_{1,i} \) must have been finished; i.e., \( t_{3,i}^f < t_{3,i} \). In this case, \( J_{2,i} \) could not produce maximum interference with \( J_{1,i} \). Therefore, the worst case occurs when task execution within \( [t_{3,i}^b, t_{3,i}] \) can interfere with \( J_{1,i} \), and \( J_{1,i} \) will suffer less interference if \( t_{r,i}^f \in (t_{3,i}^b, t_{3,i}] \). As a consequence, \( J_{1,i} \) suffers the maximum interference from \( \tau_j \) when \( t_{r,i}^f = t_{3,i}^f \).

**E. Refining the ZSI Calculation**

Since rate-monotonic scheduling is a special case of deadline monotonic scheduling where deadlines of tasks are equal to their periods, we analyze the worst case phasing of zero-sack deadline monotonic scheduling instead of zero-sack rate-monotonic scheduling (while in [3] the shift from ZSRM to ZSDM involves only a single variable, the shift...
the total amount of time demand generated by jobs of \( \tau \) prevent demotion-on-deadline rule discussed in Section II-B would finishes execution at criticality level after a job \( \Gamma \)

\[ \text{time demand generated by a task set of } \tau \]

However, that equation fails to capture the worst time demand for when \( \mathcal{J} \in H_i^{|c|} \) (e.g., let \( i = 1 \) and \( j = 3 \) for the example in Figure 3). Similarly, that equation also fails to capture the time demand for \( \mathcal{J}_i(\zeta_m, \mathcal{J}_j, t) \) when \( \mathcal{J}_j \in H_i^{|c|} = \{ \mathcal{J}_j \in H_i^{|c|} | H_i^{|c|} \cap H_i^{|c|} \neq 0 \} \). To correct these problems, it is necessary to derive new demand functions based on Theorem 1. For brevity, we provide only the formulas obtained, and defer a more detailed explanation to the Appendix.

\[
\delta_i^c(\zeta_m, \mathcal{J}_j, t) = \left( 1 + \max \left( \left[ \frac{t - \phi_i^c(\zeta_m)}{T_j} \right], 0 \right) \right) I_j^i(\zeta_m),
\]

for \( \mathcal{J}_j \in H_i^{|c|} \), and

\[
\delta_i^c(\zeta_m, \mathcal{J}_j, t) = \left( 1 + \max \left( \left[ \frac{t - \phi_i^c(\zeta_m)}{T_j} \right], 0 \right) \right) I_j^i(\zeta_m),
\]

for \( \mathcal{J}_j \in H_i^{|c|} \),

where \( \phi_i^c(\zeta_m) \) and \( \phi_i^c(\zeta_m) \) are the worst case phasings for \( \mathcal{J}_i \) in normal and critical mode, respectively.

Given that the interference function returns the maximum amount of interference a task \( \mathcal{J}_i \) can suffer, we can use it to compute the minimum amount of slack available for a job \( \mathcal{J}_j \) of \( \mathcal{J}_i \) in a time interval. In addition, we are interested in the slack which starts no later than \( t \); therefore, we define the available slack function with respect to \( \mathcal{J}_i \) as

\[
S_i^a(t) = \text{max} \{ t' - \Delta_i^a(\zeta_i, \Gamma_i^a, t') | (\forall t' < t) \cup (\forall t' \geq t \text{ where } \Delta_i^a(\zeta_i, \Gamma_i^a, t') = \Delta_i^a(\zeta_i, \Gamma_i^a, t')) \}.
\]

With these interferences and available slack functions, we can compute the ZSI of a task using Algorithm 1.

**Algorithm 1 GetSlackZeroInstant(\( \mathcal{J}_i \))**

1: \( x \leftarrow 0 \)
2: repeat
3: \( x' \leftarrow x \)
4: \( C_i^k \leftarrow \text{max}(C_i^k - x, 0) \)
5: \( k \leftarrow \mathcal{K}(C_i^k, D_i, \Gamma_i^c, \Delta_i^c(\zeta_i)) \)
6: \( Z_i \leftarrow \text{max}(D_i - k, 0) \)
7: \( x \leftarrow S_i^a(\Gamma_i^c, Z_i) \)
8: until \( x = x' \) or \( Z_i = D_i \)
9: return \( Z_i \)

An important property in Algorithm 1 is that the calculation of \( Z_i \) is dependent on \( Z_j \) of another task \( \mathcal{J}_j \) only when \( \zeta_j > \zeta_i \). Based on this, we can calculate the ZSIs of a task set in decreasing criticality order. In [3], ZSIs are computed in an unspecified order, and then the algorithm keeps looping until all ZSIs of a task set are stabilized. Our algorithm improves on [3] by computing ZSIs in a deterministic order to avoid unnecessary computation, and it is guaranteed to terminate.
III. Schedulability Evaluation

Although our revised ZSI calculation algorithm corrects the unaccounted for interference issue from [3], it does so by making some task sets unschedulable. This observation motivates a broader quantitative comparison of how the different mixed-criticality scheduling approaches affect the schedulability of tasks. Under the common mixed-criticality model we consider in the evaluation presented in this section, a task is schedulable if and only if it has enough slack to finish its overload execution budget by its deadline, and no higher criticality task is overloaded. A task set is schedulable if and only if all its constituent tasks are schedulable.

We randomly generated 1000 task sets and tested the schedulability of each set under criticality monotonic (CM) scheduling, Audsley’s method (AU), rate monotonic scheduling with (PT) and without (RM) period transformation, and two variations of zero-slack scheduling with our improved ZSI calculation algorithm. One variation incorporates the less constrained worst phasing condition specified by Algorithm 1 (ZN), and the other assumes all tasks start simultaneously so that the phasing assumption can be less pessimistic (ZS). Notice that the result of ZS is not derived from the algorithm in [3]; instead, the result came from simulating the dispatching of a given task set with the zero slack scheduler. We chose the comparison with ZS because we wanted to know how pessimistic the theoretical analysis based on Theorem 1 could be.

The generated task sets consisted of \( n \) (between 3 and 10) independent tasks with total CPU utilization \( U \) of 0.8 or 0.9. In our evaluation, we used both harmonic and non-harmonic task sets. For harmonic task sets, the period of a task was \( 2^k \times 1000 \), where \( k \) was a normally distributed random integer between 0 and 5. For non-harmonic task sets, the period of a task was \( k \times 2000 \), where \( k \) was a normally distributed random integer number between 1 and 16. Each task \( \tau_i \) was associated with a utilization factor \( u_i \), a normally distributed random integer number between 1 and 1000. The worst case execution time under normal mode \( C_i^u \) of \( \tau_i \) was then obtained from the equation \( C_i^u = T_i \times u_i / \sum_{j \in T_i} u_j \). The overload budget \( C_i^o \) of \( \tau_i \) was always \( 1.2 C_i^u \). The criticality level of each task was also randomly assigned.

The schedulability test we used for PT and other fixed priority schemes is a modified Joseph-Pandya worst case response time algorithm [7] in which the interference from a higher criticality task \( \tau_j \) can never exceed \( C_j^o \). Let \( R_i \) denote the worst case response time of task \( \tau_i \). Then the PT response time of \( \tau_i \) is the fixed point of

\[
R_i = \sum_{\tau_j: \tau_j \geq \tau_i} \left( \left\lfloor \frac{R_i}{T_j} \right\rfloor C_j^o + \min \left( \left\lfloor \frac{R_i \mod T_j}{T_j/n_j} \right\rfloor C_j^o, C_j^o \right) \right)
\]

and the RM, DM, and AU response times are the fixed point of

\[
R_i = \sum_{\tau_j: \tau_j \geq \tau_i} \left( \left\lfloor \frac{R_i}{T_j} \right\rfloor C_j^o \right)
\]

\[
R_i = \sum_{\tau_j: \tau_j \geq \tau_i} \left( \left\lfloor \frac{R_i}{T_j} \right\rfloor C_j^o \right) + \min \left( \left\lfloor \frac{R_i \mod T_j}{T_j/n_j} \right\rfloor C_j^o, C_j^o \right).
\]

\[\text{(5)}\]

Figures 5 and 6 show the number of schedulable harmonic and non-harmonic task sets with the different methods when CPU utilization is 0.8 or 0.9 for 1000 randomly generated task sets. From the simulation results, we can see that criticality monotonic priority assignment is least likely to make a mixed criticality task set schedulable. Although ZN is better than CM, ZN is even worse than RM, except for the non-harmonic task set with 0.9 CPU utilization. AU and PT perform consistently better than other methods in all scenarios. The schedulability of ZS is slightly better than ZN because ZS has a more strict assumption that all tasks start simultaneously. However, both zero slack scheduling variations are sensitive to the number of tasks in the system, whereas the AU and PT approaches are not. The calculation of zero slack instants requires the use of worst case phasing assumptions for all tasks in \( H_i^{lc} \) and \( H_i^{hc} \) as described in Section II-C, as the number of tasks of a system increases, the more interference a task \( \tau_i \) can suffer from \( H_i^{lc} \) or \( H_i^{hc} \).

Table III shows the number of task sets that are schedulable by AU, PT, and/or ZS(ZN) among the 1000 randomly generated task sets. We do not list the result for RM because schedulable tasks under RM are subsets of those under AU.
Table III shows that even though AU and PT perform better than ZS overall, 21 task sets were schedulable only under ZS or ZN; 14 sets out of the 21 sets are schedulable under both ZS and ZN.

**IV. USER SPACE IMPLEMENTATION AND EVALUATION**

Among the scheduling approaches we have considered for mixed-criticality real-time systems, Audsley’s method and rate monotonic scheduling can be easily implemented atop thread priority mechanisms commonly provided by modern operating systems. To support period transformation, additional bandwidth-preserving server mechanisms are needed to ensure a task will not execute beyond its budget within a transformed period. Enforcement of zero-slack scheduling requires the use of additional timers to trigger mode changes and deadline miss handling. The additional overheads of period transformation and zero-slack scheduling mechanisms, compared to the fixed-priority scheduling approaches are thus of practical interest. In this section, we present a user space implementation of deferrable server and zero-slack scheduling mechanisms on Linux, and evaluate the run-time overhead of the different scheduling policies for mixed-criticality systems.

**A. Zero-slack Scheduling Implementation**

A task is implemented as a thread with a priority assigned in accordance with the application and platform. For Linux, valid priority levels range from 0 to 99, where 99 is the highest priority. Our zero-slack scheduling mechanisms reserve priority levels 1 and 99 for criticality enforcement purposes. Task suspension is emulated by lowering the priority of a task to 0. As a result, application tasks can use only priority levels 2 through 98. To simplify discussion, we assume that the criticality levels assigned to a task set are contiguous positive integers.

Each task is associated with three periodic timers, for the job release, ZSI, and deadline. Expiration of the job release timer is received and handled in the task’s associated thread. An additional enforcement thread with priority 99 is created to wait for all other timer expiration events as well as for job termination events, and to make scheduling decisions based on the events it receives. To handle task suspension and resumption correctly, the enforcement thread maintains a binary heap of criticality levels. The top element of the heap has the highest criticality among the tasks that have been suspended. For convenience, we use $\zeta^{s0}$ and $\zeta^{s1}$ to denote the criticality of the top two elements in the binary heap.

A suspension event with a criticality level is used to trigger the enforcement thread to suspend a subset of tasks. When the enforcement thread receives an suspension event with criticality $x$, it suspends the tasks whose criticality is less than or equal to $x$ and higher than $\zeta^{s0}$. In addition, criticality $x$ is inserted into the binary heap. Notice that $x$ could be smaller than $\zeta^{s0}$ and hence no tasks would be suspended. However, the new value of $x$ should still be inserted into the binary heap so that the enforcement thread can keep track of which tasks are in critical mode.

When the ZSI timer of a job $J_{i,k}$ expires and the job has not finished its execution, a suspension event with criticality $\zeta_i - 1$ is sent to the enforcement thread. Deadline timer expiration of a task $\tau_j$ is handled in the same way as ZSI timer expiration, except that if $J_{i,k}$ misses its deadline, a suspension event with criticality $\zeta_i$ is sent instead. When a job $J_{i,k}$ finishes while in critical mode or after missing its deadline, an event is sent to the enforcement thread to wake up the tasks that were suspended. When the event is received, the enforcement thread restores the priority of each task $\tau_j$ where $\zeta^{s1} < \zeta_j \leq \zeta^{s0}$, and then the top element of the binary heap is removed. Priority restoration is done in non-increasing criticality order. When an awakened job $J_{i,k}$ has already missed its deadline, the priority of $\tau_j$ is changed to 1 instead of $\pi_j$. In addition, $\zeta_j$ is inserted into the binary heap so that tasks with criticality levels less than or equal to $\zeta_j$ will remain suspended.

**B. Period Transformation Implementation**

To support period transformation, we also implemented a deferrable server enforcement mechanism in user space. In the deferrable server approach, a task with a transformed period is executed within a server thread. Each server has a period, a budget, and a priority, all of which are assigned according to the transformation mechanism described in Section III. The server budget is replenished at the beginning of each period. The budget decreases while the server is executing a task and is preserved (until the end of the current period) while the server is idle. A server can execute its respective task as long as its budget has not been exhausted.

Similar to our zero-slack scheduling implementation, a manager thread at highest priority is allocated to manipulate the consumption and replenishment of servers’ budgets. This thread sits in an `epoll_wait` system call and waits for the budget replenishment and exhaustion events which are generated by the POSIX real-time timer APIs. For the budget replenishment events, we use timers with the `CLOCK_MONOTONIC` clock id (wall clock timer) to generate asynchronous timeout signals. To
To measure the overhead imposed by these scheduling mechanisms, we conducted experiments on a testbed consisting of a 6-core Intel core i7 980 3.3GHz CPU with hyper-threading enabled, running Ubuntu Linux 10.04 with the 2.6.33-29-realtime kernel which incorporates the Linux RT-Preempt patch [8]. To avoid task migration among cores, CPU affinity was assigned so that our test program was executed in one particular core. All hardware IRQs except those associated with timers were assigned to cores other than the one for application execution. Each task was implemented with a for loop with a fixed number of iterations, where every 31 iterations yielded a 1 μs workload. In each iteration, the CPU timestamp counter was read and then compared with the counter read from the previous iteration. If the difference was greater than a specified number of ticks (700), we considered the thread to have been preempted and the new timestamp counter was stored. After a specified amount of time, all stored timestamp counters were written to a file, and then the test program terminated.

**ZSRM Measurements:** Our first experiment used the task set in Table IV, where \( \tau_h \) and \( \tau_l \) ran workloads of 4 and 2 ms within their periods, respectively. We found that for approximately every 1000 μs, there was a 3 μs interval that was not used by the task set or by the enforcement thread, which we attributed to the fact that the Linux scheduler was invoked at every quantum, which was about 1 ms on our test machine.

As was mentioned earlier, the enforcement thread can be invoked by job termination as well as by ZSI timers and deadline timers. In this experiment, \( \tau_h \) did not have a deadline timer because there was no higher-criticality task with which it could interfere if it missed its deadline. Similarly, \( \tau_l \) did not have a ZSI timer because there was no lower-criticality task that it needed to suspend when it entered critical mode.

When a timer expires or a job terminates, there is an overhead to switch from the current task thread to the enforcement thread. Depending on the scheduling context, the enforcement thread may demote or promote the priorities of some tasks and then return to the task with the highest priority. Thus the overhead of every enforcement thread invocation is the sum of the overheads for preemption invocation, thread priority adjustment, and preemption return.

**Comparison between Scheduling Policies:** To evaluate the cost of criticality enforcement in our ZSRM and deferrable server implementations, we compared the response times seen for rate monotonic scheduling (RM), ZSRM and periodic transformation (PT), according to the busy intervals from each invocation of \( \tau_l \) until the CPU again became idle. As shown in Figure 7, the average busy intervals for RM, ZSRM, and PT were 7742, 7757, and 7773 μs respectively. That is to say, in this example, ZSRM scheduling only added 0.2% overhead compared to rate-monotonic scheduling, while period transformation added about 0.4% overhead. The difference between ZSRM and PT was mostly because the schedule under PT involved more context switches than the schedule under ZSRM.

<table>
<thead>
<tr>
<th>Task</th>
<th>( C^h )</th>
<th>( C^l )</th>
<th>( h )</th>
<th>( l )</th>
<th>( p )</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_h )</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>2</td>
<td>Low</td>
<td>6</td>
</tr>
<tr>
<td>( \tau_l )</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>High</td>
<td>0</td>
</tr>
</tbody>
</table>

2 ms within their periods, respectively. We found that for approximately every 1000 μs, there was a 3 μs interval that was not used by the task set or by the enforcement thread, which we attributed to the fact that the Linux scheduler was invoked at every quantum, which was about 1 ms on our test machine.

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</tr>
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<td>5</td>
<td>1</td>
<td>High</td>
<td>0</td>
</tr>
</tbody>
</table>
Zero-slack Scheduling Micro Benchmarks: To better understand the overheads of each individual segment of enforcement thread invocation and execution, we developed another test case to measure them when there are different numbers of tasks in the system. In this test case, the ZSIs of tasks were assigned artificially so that overheads could be easily identified and measured rather than using Algorithm 1. The rationale for this artificial assignment is that those overheads are related to the number of threads and the operation performed during mode switching, rather than to the exact instants when they take place. All tasks were set to have 2 ms execution time and had the same period, 50 ms. The lowest priority task $\tau_0$ was assigned to the highest criticality level with $Z_0 = 1$ ms. The rest of the tasks were assigned in such a way that the priority of a task was equal to its criticality level. By varying the number of tasks in the system, we obtained the overhead of preemption, preemption return, and priority adjustment, as shown in Figures 8 and 9.

As is shown in Figure 8, the overheads of preemption and preemption return are not linked to the number of tasks in the system and are about 4 μs and 1 μs respectively.

From Figure 9, we can clearly see that the cost of priority adjustment is linear with regard to the number of the threads to be promoted/demoted. However, in our experiment, the overheads for the first invocations of each such adjustment were always far higher than the rest. As a result, we present the cost of first invocations separately from the others, in the curve labeled “First”. The curve of labeled “2nd Largest” and the curve labeled “Average” show the maximum and mean (respectively) of the rest of the invocations. We observe that the second largest overhead is consistently 2 to 3 μs longer than the average, which occurs when the periodic invocation of Linux scheduling occurs during the priority adjustment.

Based on these results, we can easily estimate the overhead of timer expiration or job termination. For example, the cost of a ZSI timer expiration with 8 tasks to suspend is about $4 + 5 + 1 = 10$ μs.

Deferrable Server Micro Benchmarks: Similar to our zero-slack implementation, the overhead incurred by our deferrable server implementation can also be divided into three parts: thread preemption, preemption return, and manager thread handling. In our experiments, the preemption overhead and preemption return overhead for the deferrable server implementation were very close to what is shown in Figure 8; therefore, we omit those details for brevity. As was mentioned previously, the manager thread is responsible for budget replenishment and exhaustion and for adjusting server thread priorities, as well as for canceling the budget exhaustion timer when a job finishes. Regardless of the different functionalities involved, the average and maximum response times of each manager thread invocation were about 3587 and 22448 cycles, respectively, or about 1 and 6.6 μs, respectively.

V. RELATED WORK

In recent years, multiple papers have been published related to mixed-criticality scheduling. Vestal [1] first proposed a formal model for representing real-time mixed-criticality tasks to support analysis of the safety of software systems based on the RTCA DO-178B software standard. In [1], he used fixed-priority scheduling and provided a preliminary evaluation using three real world mixed-criticality workloads which showed that priority assignment [5] and period transformation [6] improved the utilization of the system, in comparison to deadline monotonic analysis. Baruah and Vestal [2] then studied fundamental scheduling-theoretic issues with fixed task-priority, fixed job-priority, and earliest deadline first (EDF) scheduling policies, under Vestal’s model. Later, Baruah et al. also proposed the use of priority assignment on a per-job basis using Audsley’s approach offline assuming the complete ordering of jobs was known a priori [9] or on-line for sporadic workloads [10].

Anderson et al. [11] developed an extension of Linux to support mixed criticality scheduling on multi-core platforms,
using a bandwidth reservation server to ensure temporal isolation among tasks with different criticalities. Tasks of the same criticality are executed in one container with a predefined period and budget. Intra-container task scheduling for high criticality tasks uses a cyclic executive approach where scheduling decisions are statically pre-determined offline and specified in a dispatching table, whereas EDF can be used for low criticality containers.

Pellizzoni et al. [12] also used a reservation-based approach to ensure strong isolation guarantees for applications with different criticalities. Rather than emphasizing CPU scheduling policies, this work focused on the methodology and tools for generating software wrappers for hardware scheduling decisions. For example, it seems worthwhile to examine how allowing mechanisms to support zero-slack scheduling and period transformation approaches which showed that although the Audsley’s priority assignment and period transformation approaches were most likely to be able to schedule randomly selected task sets, the zero-slack approach was able to schedule some task sets that the others could not. We also implemented mechanisms to support zero-slack scheduling and period transformation atop Linux without modifying the kernel. Our empirical evaluation of those mechanisms showed that, compared to fixed-priority scheduling, they imposed only 0.2% and 0.4% additional overhead respectively, which demonstrates their viability in practice.

The results of this research suggest that it may be productive to extend our evaluation to consider different features of the more general mixed-criticality model presented in [1, 2]. For example, it seems worthwhile to examine how allowing tasks to specify greater or lesser numbers of execution times impacts schedulability and run-time performance, and to consider whether those results might motivate a generalization of the zero-slack scheduling approach to support multiple scheduling mode changes under the more general mixed criticality model.

VI. CONCLUSIONS

In this paper, we have presented what is to our knowledge the first practical side-by-side evaluation of mixed-criticality real-time task scheduling based on priority assignment, period transformation, and zero-slack scheduling. We also have offered refinements to zero-slack scheduling and the calculation of zero-slack instants. In particular, we have characterized a scenario in which a deadline miss of a lower-criticality task could affect scheduling guarantees for a higher criticality task, and provide a simple priority demotion rule to address that problem. We also propose a new worst case phasing condition for zero-slack scheduling and show its correctness, provide an analysis of how much interference a task can suffer from other tasks, and develop a new algorithm for calculating zero-slack instants based on that analysis.

We conducted simulations to examine how the different mixed-criticality scheduling methods may impact task schedulability, which showed that although the Audsley’s priority assignment and period transformation approaches were most likely to be able to schedule randomly selected task sets, the zero-slack approach was able to schedule some task sets that the others could not. We also implemented mechanisms to support zero-slack scheduling and period transformation atop Linux without modifying the kernel. Our empirical evaluation of those mechanisms showed that, compared to fixed-priority scheduling, they imposed only 0.2% and 0.4% additional overhead respectively, which demonstrates their viability in practice.

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ACKNOWLEDGMENT

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REFERENCES

can then be expressed as $\delta_i^n(\zeta_m, \tau_j, t) = \left(1 + \max \left(\frac{t - \phi_j^{i,n}(\zeta_m)}{T_j}, 0\right)\right) I_j^i(\zeta_m).$  

If $H_{i}^{lc} \cap (H_{i}^{hc} \cup L_{i}^{hc}) \neq \emptyset$, the scenario can be complicated. Let $Pre(\tau_j)$ be the set of tasks whose latest jobs, which are released before time 0, terminate before $J_{i,0}$. Suppose that $Pre(\tau_j)$ is known, and then the phasing $r_j^{i,n}(\zeta_m)$ can then be obtained by

$$K \left( C_j(\zeta_m) + \sum_{\tau_k \in Pre(\tau_j)} C_k(\zeta_m), D_i, H_{i}^{hc} \cap H_{j}^{sc} \cup H_{j}^{hc} \cup H_{j}^{sc}, \Delta_i^{n}(\zeta_m) \right).$$

However, due to the different ordering of priority/criticality levels among tasks, there is no absolute ordering among the latest jobs (before time 0) of tasks in $H_{i}^{lc}$. Thus, the interference function of a task $\tau_i$ is a maximum of the functions from all possible execution orderings of its interference tasks. If the priority/criticality levels for the task set in $H_{i}^{lc}$ are completely reversed, the computation time would grow exponentially as the set in $H_{i}^{lc}$ grows.

Even though the exact bound is too hard to compute, a conservative loose bond is possible. Based on Equation 11, the interference grows as the phasing decreases; therefore, we can use the smallest possible phasing interval for all tasks in $H_{i}^{lc}$ to bound the worst case interference. If the interference from $H_{i}^{lc}$ and $H_{j}^{hc}$ is not considered, only tasks from $H_{j}^{hc}$ are left in the interference set. Thus we can bound $r_j^{i,n}$ as $r_j^{i,n} = K(C_j, D_i, H_{i}^{hc} \cup H_{j}^{hc} \cup H_{j}^{sc}, \Delta_i^{n})$, which is exactly the same as Equation 7.

With the availability of $\Delta_i^{n}$, we can now compute the available slack in normal mode as follows: 

$$\theta_i(\zeta_m) = \max(Z_i - \Delta_i(\zeta_m, \Gamma_i^{n}, Z_i), 0).$$

### B. Critical Mode Time Demand Function

When a task $\tau_i$ is in critical mode, only the tasks $\Gamma_i^{c} = H_{i}^{hc} \cup H_{i}^{sc} \cup L_{i}^{hc}$ can interfere with $\tau_i$. Since the tasks from $H_{i}^{lc}$ cannot interfere with $\tau_i$, one may consider the critical mode time demand function is

$$\delta_i^c(\zeta_m, \tau_j, t) = \left[\frac{t}{T_j}\right] I_j^i(\zeta_m),$$

which was also assumed by de Niz et al. Unfortunately, this is correct only when $\tau_j \notin \{\tau_k \in H_{i}^{hc} | H_{k}^{lc} \cap H_{i}^{lc} \neq \emptyset\}$. To see why, let us consider $\tau_2$ from the task set in Table V.

<table>
<thead>
<tr>
<th>$C_m$</th>
<th>$C_m^{o}$</th>
<th>$T$</th>
<th>$\zeta$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>2</td>
<td>4</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>3</td>
<td>4</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Table V: An example task set where the calculation of $Z_2$ is based on Equation 10.
If we use Equation 10 to calculate the interference of $\tau_1$ to $\tau_2$ in critical mode, there would be a slack of 8 time units available for $\tau_2$ for every interval of 10 time units.

If we assume that the jobs $J_{1,1}$, $J_{2,1}$ and $J_{3,1}$ are all released at time 0 and both $J_{2,1}$ and $J_{3,1}$ run for their respective overload budget, then the jobs of $\tau_1$ run for only 2 time units. Based on zero-slack scheduling, $J_{1,1}$ and $J_{1,2}$ would be scheduled at the time slots (5, 7) and (10, 12). When $J_{2,1}$ enters critical mode at time 6, only 3 time units are left for $\tau_2$ between the time interval [6,12), which is less than what is given by Equation 10 (i.e., 8 time units are available within an interval of 10 time units). The reason for the above phenomenon is that $\tau_2$ suspends the lower criticality task $\tau_3$ in critical mode and thus shorten the inter-job arrival times of $\tau_1$.

Let $H_{i}^{hc-} \equiv \{\tau_j \in H_{i}^{hc} | H_{i}^{hc} \cap H_{i}^{hc} \neq \emptyset\}$ be the task set in $H_{i}^{hc}$ whose inter-job arrival time could be shortened by suspending the tasks in $H_{i}^{hc}$. Similar to the calculation of $\phi_{j}^{2-n}$, to estimate the maximum interference from a task $\tau_j \in H_{i}^{hc-}$ to $\tau_1$ in critical mode, we have to minimize the worst case phasing $\phi_{j}^{1}$ in critical mode. Therefore, we assume $\tau_j$ starts execution at the zero slack instant of $\tau_1$. Assuming $Z_{j}$ is known, we can obtain the maximum response time $r_{j}^{c}(\zeta_{m})$ of $\tau_j$ at the criticality level $\zeta_{m}$ based on whether $\theta_{j}(\zeta_{m}) \geq C_{j}(\zeta_{m})$. If $\theta_{j}(\zeta_{m}) \geq C_{j}(\zeta_{m})$, we need only to count the time for $\tau_j$ to run for $C_{j}(\zeta_{m})$ time units in normal mode; i.e.,

$$r_{j}^{c}(\zeta_{m}) = \mathcal{K}(C_{j}(\zeta_{m}), Z_{j}, \Gamma_{j}, \Delta_{j}^{c}(\zeta_{m}))$$

if $\theta_{j}(\zeta_{m}) \geq C_{j}(\zeta_{m})$. Otherwise, the response time is $Z_{j}$ plus the time for $\tau_j$ to run for $C_{j}(\zeta_{m}) - \theta_{j}(\zeta_{m})$ time units in critical mode; i.e.,

$$r_{j}^{c}(\zeta_{m}) = Z_{j} + \mathcal{K}(C_{j}(\zeta_{m}) - \theta_{j}(\zeta_{m}), D_{j} - Z_{j}, \Gamma_{j}, \Delta_{j}^{c}(\zeta_{m}))$$

if $\theta_{j}(\zeta_{m}) < C_{j}(\zeta_{m})$.

Because we assume $\tau_j$ starts execution at its zero slack instant, we can then obtain the phasing of $\tau_j$ by

$$\phi_{j}^{c}(\zeta_{m}) = C_{j}(\zeta_{m}) + T_{j} - r_{j}^{c}(\zeta_{m}).$$

Figure 12 illustrates the calculation of $\phi_{j}^{c}(2)$ for the example in Table V. Since $\tau_2$ does not interfere with $\tau_1$, we can easily see the worst case response time $r_{j}^{c}$ of $\tau_1$ is 7. Because $Z_2$ is not known at the time, we can only assume $Z_2$ aligns to the time when $J_{1,1}$ starts execution so that we can minimize $\phi_{j}^{1}$. Consequently, $\phi_{j}^{1} = C_{1}(2) + T_{1} - r_{j}^{c}(2) = 5$. With the phasing estimation, we can obtain the maximum critical mode time demand function for $\tau_i$ when $\tau_j \in H_{i}^{hc-}$ as follows:

$$\delta_{i}^{c}(\zeta_{m}, \tau_{j}, t) = \left(1 + \max \left(\left\lfloor \frac{t - \phi_{j}^{c}(\zeta_{m})}{T_{j}} \right\rfloor, 0\right)\right)I_{j}^{c}(\zeta_{m}).$$  \hspace{1cm} (11)

C. Available Slack Function

Given that the interference function returns the maximum amount of interference a task $\tau_i$ can suffer, we can use it to compute the minimum amount of slack available for a job $J_{i,1}$ of $\tau_i$ in a time interval. Intuitively, $t - \Delta_{i}^{n}(\zeta_{i}, \Gamma_{i}^{n}, t)$ should be the time available for $\tau_i$ in interval $t$. However, a job released at time $t'$ cannot reclaim the empty slack available before $t'$. Thus, the amount of empty slack before $t$ should be expressed as

$$\max\{t' - \Delta_{i}^{n}(\zeta_{i}, \Gamma_{i}^{n}, t') \mid \forall t' \leq t\}.$$

Furthermore, we are interested in the empty slack which starts no later than $t$; therefore, we define the available slack function with respect to $\tau_i$ as

$$S_{i}^{n}(t) = \max \{t' - \Delta_{i}^{n}(\zeta_{i}, \Gamma_{i}^{n}, t') \mid (\forall t' < t) \cup (\forall t' \geq t \text{ where } \Delta_{i}^{n}(\zeta_{i}, \Gamma_{i}^{n}, t') = \Delta_{i}^{n}(\zeta_{i}, \Gamma_{i}^{n}, t)\}.$$  \hspace{1cm} (12)

Figure 13 illustrates the relationship between $t - \Delta_{i}^{n}(\zeta_{i}, \Gamma_{i}^{n}, t)$ and $S_{i}^{n}(t)$, using $\tau_4$ from the task set in Table $VI$. 

Figure 11: A schedule for the task set in Table V which shows ZSI calculation based on Equation 10 can cause $\tau_2$ to miss deadline.

Figure 12: The illustration of $\phi_{j}^{c}$ using the example task set in Table V.
In [3], de Niz et al. used a slack vector and a procedure \texttt{SlackUpToInstant} to calculate the amount of slack available. The slack vector is a sequence of slack regions ordered by time, where each slack region contains a starting instant and duration. Conceptually, the slack vector used by de Niz et al. is an alternative way to express the interference function where \( \delta_n^i(\zeta_m, \tau_j, t) = \lceil t/T_j \rceil I_i^j(\zeta_m) \) for all \( \tau_j \in \Gamma_n^i \). In contrast, our approach requires different time demand functions for the case of \( \tau_j \in \Gamma_n^i - H_i^{lc} \) and \( \tau_j \in H_i^{lc} \), as shown in Equations 9 and 11 respectively. In other words, the original algorithm for slack vector calculation does not correctly deal with the interference from tasks in \( H_i^{lc} \).

In addition, \texttt{SlackUpToInstant} returns the amount of slack available up to the specified instant, whereas \( S_l^i \) returns the amount of empty slack which starts no later than the instant. The tweak allows us to discover more units of slack.