Abstract

The new representations of solutions to the one-dimensional Schrödinger equation
\[-y'' + q(x)y = \omega^2 y\]
and to the perturbed Bessel equation
\[-y'' + \frac{\ell(\ell+1)}{x^2} + q(x)y = \omega^2 y\]
are introduced.

For the first equation a pair of linearly independent solutions has the form

\[c(\omega, x) = \cos \omega x + 2 \sum_{n=0}^{\infty} (-1)^n \beta_{2n}(x) j_{2n}(\omega x)\]

and

\[s(\omega, x) = \sin \omega x + 2 \sum_{n=0}^{\infty} (-1)^n \beta_{2n+1}(x) j_{2n+1}(\omega x),\]

where \(j_n\) are the spherical Bessel functions and the coefficients \(\beta_j\) can be calculated by
the simple recursive procedure.

For the second equation the regular solution \(u_\ell\) satisfying the asymptotics
\(u_\ell(x, \omega) \sim x^{\ell+1}, x \to 0\) has the form

\[u_\ell(\omega, x) = \frac{2^{\ell+1} \Gamma(\ell + 3/2)}{\sqrt{\pi} \omega^\ell} x j_\ell(\omega x) + \sum_{n=0}^{\infty} (-1)^n \beta_n(x) j_{2n}(\omega x),\]

where the coefficients \(\beta_n\) can be obtained by a similar recursive procedure.

The representations are based on the expansion of the integral kernels of the trans-
mutation operators into Fourier-Legendre series and the recent results obtained by the
author jointly with V. V. Kravchenko. It is shown that the partial sums of the series
approximate the solutions uniformly with respect to \(\omega\). Convergence rate estimates and
representations for the derivatives of the solutions are given.

The talk is based on the results obtained with V. V. Kravchenko, Luis J. Navarro
and R. Castillo-Perez.

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