A criterion for the solvability of a $\mu$-synthesis problem

Abstract

We give a solvability criterion for the following $\mu$-synthesis problem. Let $\mu$ be the structured singular value for the diagonal matrices with entries in $\mathbb{C}$.

**Problem.** Given distinct points $\lambda_1, \ldots, \lambda_n$ in the open unit disc $\mathbb{D}$ and target $2 \times 2$ complex matrices $W_1, \ldots, W_n$ such that $\mu(W_j) \leq 1$ for all $j = 1, \ldots, n$, find a holomorphic $2 \times 2$ matrix function $F$ on $\mathbb{D}$ such that $F(\lambda_j) = W_j$ for each $j$, and $\mu(F(\lambda)) \leq 1$ for all $\lambda \in \mathbb{D}$.

By [1, Theorem 9.2], this problem is equivalent to the following interpolation problem: does there exist a holomorphic function $x$ from the disc to the tetrablock $\mathbb{E}$ such that $x(\lambda_j) = (w_{11}^j, w_{22}^j, \det W_j)$ for each $j$? The tetrablock is the domain in $\mathbb{C}^3$ defined by

$$\mathbb{E} := \{(x_1, x_2, x_3) \in \mathbb{C}^3 : 1 - x_1z - x_2w + x_3zw \neq 0 \text{ for all } z, w \in \mathbb{D}\}.$$ 

In this talk we show such an $x$ exists if and only if, for distinct $z_1, z_2, z_3 \in \mathbb{D}$, there are positive $3n$-square matrices $[N_{l,j,k}]$, of rank 1, and $[M_{l,j,k}]$ such that

$$\begin{bmatrix}
1 - \overline{z}_l x_3i - x_1^i z_k x_{3j} - x^i_{1j} \\
x_2^i z_l - 1 \end{bmatrix} \geq [(1 - \overline{z} z_k) N_{l,j,k}] + [(1 - \overline{\lambda}_j) M_{l,j,k}],$$

where $(x_1^j, x_2^j, x_3^j) = (w_{11}^j, w_{22}^j, \det W_j)$ for each $j$.