Corrigendum to: Operator monotone functions and Löwner functions of several variables

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Corrigendum to:
Operator monotone functions and Löwner functions of several variables

By Jim Agler, John E. McCarthy, and N. J. Young

Abstract

We fix a gap in the proof of Theorem 7.24 in Ann. of Math. 176 (2012), 1783–1826.

There is a gap in the proof of Theorem 7.24 in [1], though the statement of the theorem is correct.

In the proof of necessity, we argue that $A$ is in $\mathcal{G}$ by contradiction. If it were not, invoking the Hahn-Banach separation theorem would yield a real skew-symmetric matrix $K$ and a constant $\delta \geq 0$ such that $\text{tr}(\Gamma K) \geq -\delta$ for all $\Gamma$ in $\mathcal{G}$, and $\text{tr}(\Lambda K) < -\delta$. In the proof we assumed that $\delta = 0$, but this assumption is unjustified.

Instead, we argue as follows. Define $\Delta$ by

$$\Delta^r_{ij} = (x^r_j - x^r_i)K_{ji}, \quad i \neq j,$$

and with the diagonal entries $\Delta^r_{ii}$ chosen so that each $\Delta^r \geq 0$ and so that

$$\mu^r := \sum_{i=1}^{n} f_{r,i} \Delta^r_{ii}$$

is minimal over all choices of $\Delta_{11}^r, \ldots, \Delta_{nn}^r$ such that $\Delta \geq 0$. (A minimal choice exists, since all the $f_{r,i}$ are strictly positive by assumption.) Then $\Delta$ is in $\text{SAM}_n^d$, and

$$[\Delta^s, S^r]_{ij} = (x^s_j - x^s_i)K_{ji}(x^r_j - x^r_i) = [\Delta^r, S^s]_{ij}.$$
As $f$ is locally $M_n$ monotone, we must have then that $D\Delta f(S) \geq 0$ by Lemma 7.3. As

$$-\delta > \text{tr}(AK) = \sum_{1 \leq i, j \leq n} [D\Delta f(S)]_{ij} - \sum_{r=1}^{d} \sum_{i=1}^{n} \Delta_{ii}^r f_{r,i},$$

we get that

\[
\sum_{r=1}^{d} \mu^r - \delta > \sum_{1 \leq i, j \leq n} [D\Delta f(S)]_{ij} \geq 0. \tag{0.2}
\]

By Duffin’s strong duality theorem [2], the minimum $\mu^r$ in (0.1) satisfies

\[
\mu^r = \min_{i \neq j} \Delta_{ij}^r A^r(i, j), \tag{0.3}
\]

where $A^r$ range over the set of real positive matrices such that the diagonal entries of $A^r$ are $f_{r1}, \ldots, f_{rn}$ for each $r$.

For each such $A = (A^1, \ldots, A^d)$, let $\Gamma$ be the corresponding element of $\mathcal{G}$: $\Gamma_{ii} = 0$ and

$$\Gamma_{ij} = \sum_{r=1}^{d} (x_j^r - x_i^r)A^r(i, j) \quad \text{for } i \neq j.$$

We have

$$-\delta \leq \text{tr} \Gamma K$$

$$= \sum_{i \neq j} \sum_{r=1}^{d} (x_j^r - x_i^r)A^r(i, j)K_{ji}$$

$$= \sum_{r=1}^{d} \sum_{i \neq j} \Delta_{ij}^r A^r(i, j).$$

Hence, by equation (0.3), $-\delta \leq \sum_{r=1}^{d} (-\mu^r)$, so $\sum_{r=1}^{d} \mu^r \leq \delta$. This contradicts (0.2), so it follows that $\Lambda \in \mathcal{G}$, and necessity is proved.

References


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